CSE/NEUBEH 528

Lecture 12: Networks that Learn

(Chapter 8)

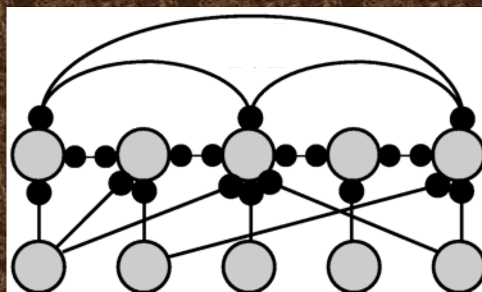


Image from http://clasdean.la.asu.edu/news/images/ubep2001/neuron3.jpg

Lecture figures are from Dayan & Abbott's book

http://people.brandeis.edu/~abbott/book/index.html

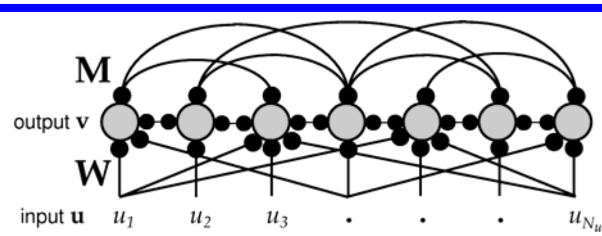
Gameplan for Today

- Plasticity and Learning
  - Unsupervised, Supervised, and Reinforcement learning
- Unsupervised Learning
  - Hebb rule and its variants (Covariance, BCM, Oja rule)
  - Principal Component Analysis (PCA)
  - Temporally Asymmetric Hebbian learning

So far, we have been analyzing networks with *fixed* sets of synaptic weights W and M

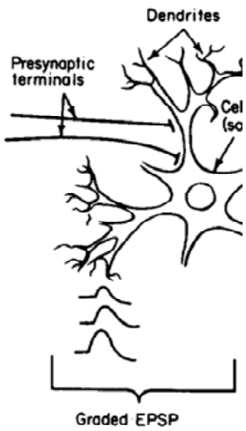
Can these be adapted in response to inputs?

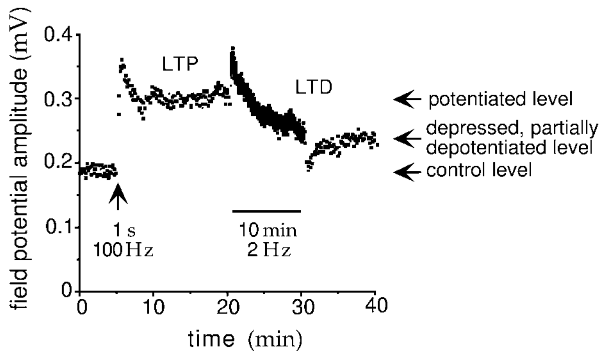
Plasticity and Learning: Adapting the Connections



- Question 1: How do we adapt the synaptic weights W and M to solve useful tasks?
- Question 2: How does the brain do it?

Synaptic Plasticity in the Brain

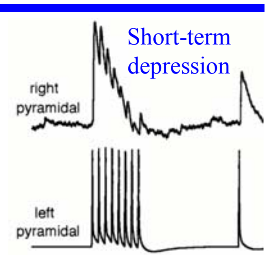




LTP = Long Term Potentiation
LTD = Long Term Depression

Other Forms of Plasticity in the Brain

- Short-Term Synaptic Plasticity
  - Short-term depression/facilitation
  - Dynamics may change on a long-term basis via LTP/LTD
- Changes to intrinsic excitability of cell
  - Density and distribution of various channels (ionic conductances)
  - Not well-studied
- Growth and morphological changes in dendrites
  - Not well-studied
- Addition of new neurons?
  - Hot topic of research these days…



Short-term depression

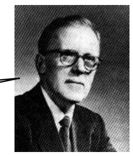
The Theory: Classification of Learning Algorithms

- **Unsupervised Learning**
  - Synapses adapted based solely on inputs
  - Network self-organizes in response to *statistical patterns* in input
  - Similar to Probability Density Estimation in statistics
- **Supervised Learning**
  - Synapses adapted based on inputs and desired outputs
  - External "teacher" provides desired output for each input
  - Goal: Function approximation
- **Reinforcement Learning**
  - Synapses adapted based on inputs and (delayed) reward/punishment
  - Goal: Pick outputs that *maximize total expected future reward*
  - Similar to optimization based on Markov decision processes

Let's start with Unsupervised Learning

Consider a single neuron receiving feedforward inputs from other neurons (e.g. from the retina)

The Grand-Daddy of Unsupervised Learning



- Rule hypothesized by Donald Hebb in 1949
- Hebb's learning rule:

"If neuron A frequently contributes to the firing of neuron B, then the synapse from A to B should be strengthened"

- Related Mantra: *Neurons that fire together wire together*
- Hebb's goal: Produce clusters of neurons ("*cell assemblies*") that fire together in response to a stimulus

Formalizing Hebb's Rule

- Consider a linear neuron: $v = \mathbf{w}^T \mathbf{u} = \mathbf{u}^T \mathbf{w}$
- Basic Hebb Rule: $\tau_w \frac{d\mathbf{w}}{dt} = \mathbf{u}v$ (or $\mathbf{w} \leftarrow \mathbf{w} + \varepsilon \cdot \mathbf{u}v$)
- What is the average effect of this rule?

$$\tau_w \frac{d\mathbf{w}}{dt} = \langle \mathbf{u}v \rangle = Q\mathbf{w}$$

- Q is the input correlation matrix: $Q = \langle \mathbf{u}\mathbf{u}^T \rangle$

Variants of Hebb's Rule

- Pure Hebb only increases synaptic weights (LTP)
  - What about LTD?
- Covariance rules:

$$\tau_w \frac{d\mathbf{w}}{dt} = (\mathbf{u} - \boldsymbol{\theta}_u)v$$

(But: LTD also for no input and some output)

$$\tau_w \frac{d\mathbf{w}}{dt} = \mathbf{u}(v - \theta_v)$$

(But: LTD also for no output and some input)

- BCM rule: $$\tau_w \frac{d\mathbf{w}}{dt} = \mathbf{u}v(v - \theta_v)$$

(Fits biological data better)

Are these learning rules stable?

On Board Analysis, leading up to Oja's rule

What does the Hebb rule do anyway?

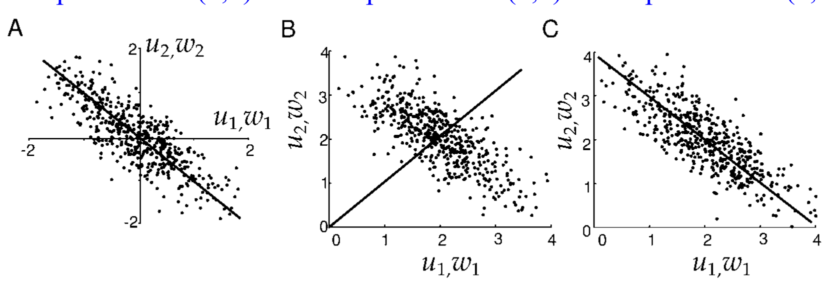
Eigenvector analysis of Hebb rule…

Hebb Rule implements PCA!

Pure Hebb
Input mean = (0,0)

Pure Hebb
Input mean = (2,2)

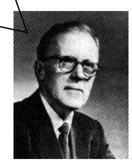
Covariance Rule
Input mean = (2,2)



Hebb rule *rotates* weight vector to align with principal eigenvector of input correlation/covariance matrix (i.e. direction of maximum variance)

Waittaminute…what did Hebb really say?

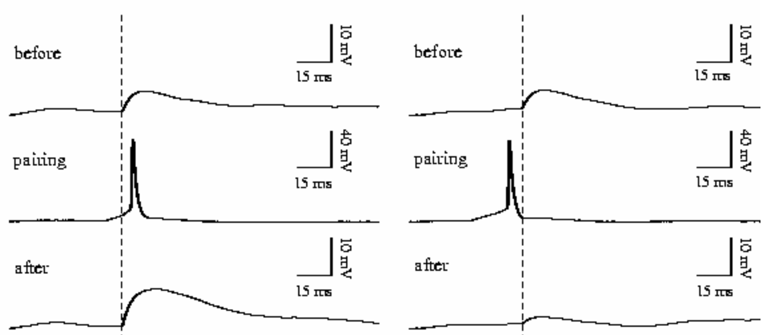
If neuron A frequently **contributes** to the firing of neuron B, then the synapse from A to B should be strengthened



Causality (order of input/output) is important, not just correlation

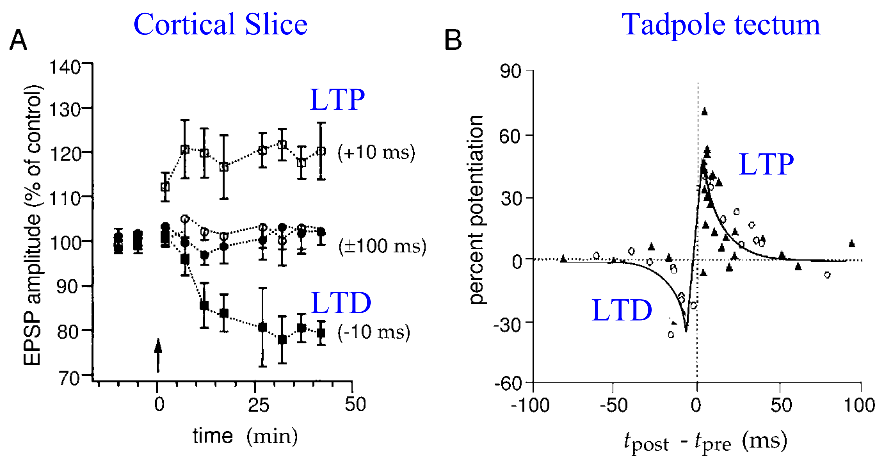
Evidence for Causal Learning Rules: Spike-Timing Dependent Synaptic Plasticity (STDP)

Input before Output Spike   Input after Output Spike



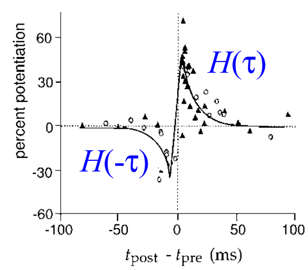
(Note: This is just a simulation I did a while back, not real data!)

STDP in the Vertebrate Brain



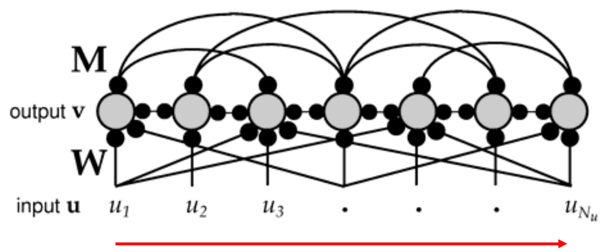
(This is real data!)

Temporally Asymmetric Hebb Rule (STDP)



$$\tau_w \frac{d\mathbf{w}}{dt} = \int_0^\infty \left[\overbrace{H(\tau)\underset{\substack{\uparrow \\ \text{Past inputs}}}{\mathbf{u}(t-\tau)}v(t)}^{\text{LTP}} + \overbrace{H(-\tau)\mathbf{u}(t)\underset{\substack{\uparrow \\ \text{Past outputs}}}{v(t-\tau)}}^{\text{LTD}} \right] d\tau$$

What does STDP do in a Recurrent Network?



Adapt M using STDP, keep W fixed

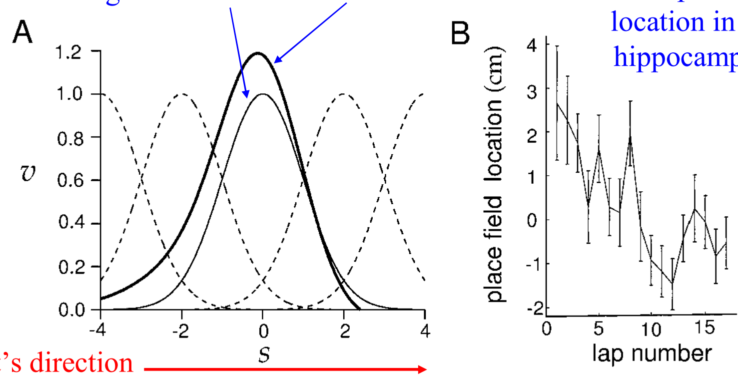
Direction of input sequence

($u_1 \rightarrow u_2 \rightarrow \ldots$)

STDP allows prediction in the navigating rat

Tuning curve before and after

Shift in place field location in rat hippocampus



Rat's direction of motion

Tuning curve shifts, generating anticipatory response

Next Class: Unsupervised Learning

- Things to do:
  - Finish Chapter 8 and Start Chapter 10
  - Watch for the Last Homework (due next Wednesday)
  - Start mini-project