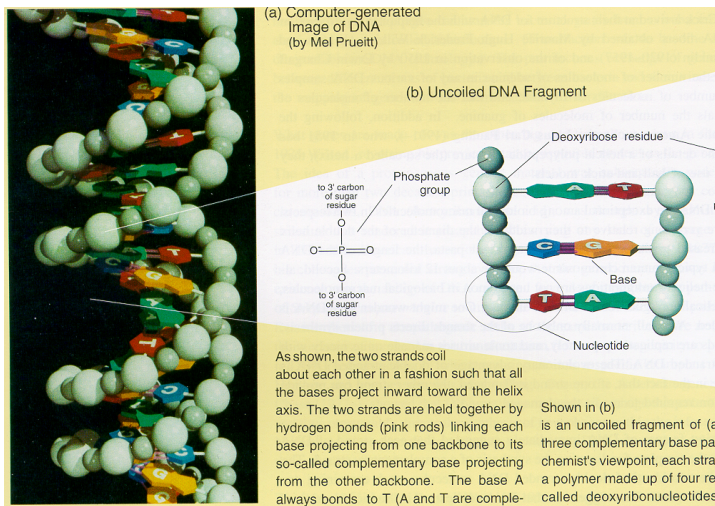


**CSE 527**  
**Autumn 2007**  
 Lectures 8-9 (& part of 10)  
 Motifs: Representation & Discovery

# DNA Binding Proteins

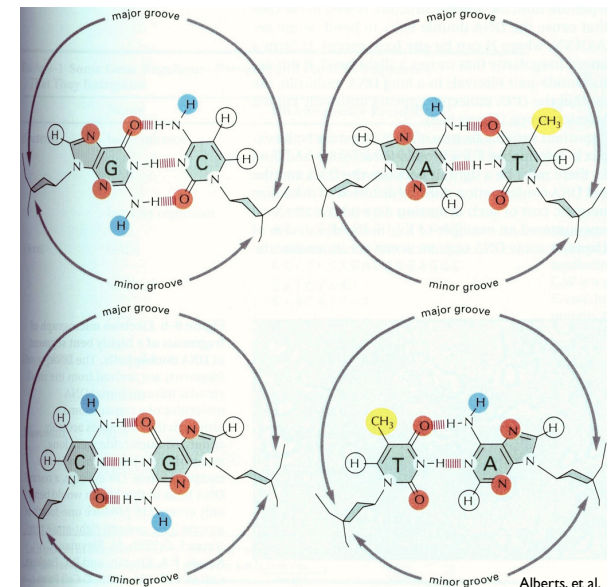
A variety of DNA binding proteins  
 (“transcription factors”; a significant fraction,  
 perhaps 5-10%, of all human proteins)  
 modulate transcription of protein coding  
 genes

## The Double Helix

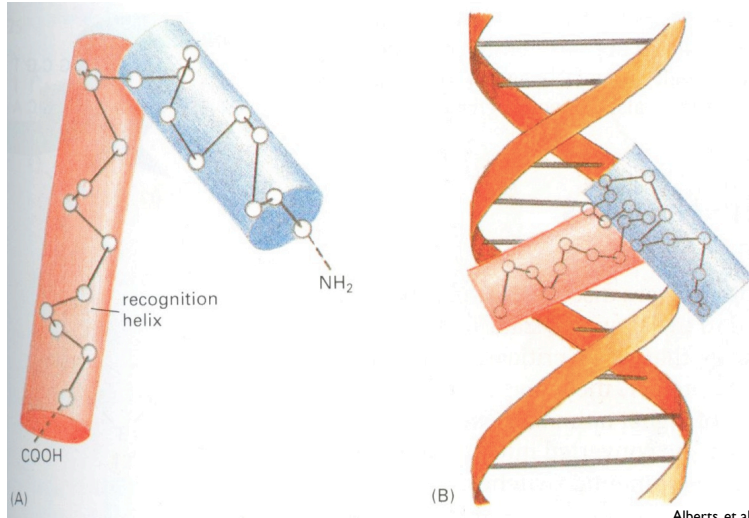


## In the groove

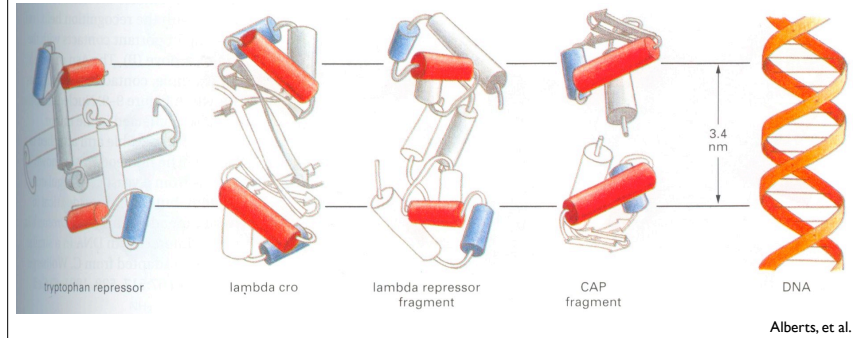
Different patterns of potential H bonds at edges of different base pairs, accessible esp. in major groove



## Helix-Turn-Helix DNA Binding Motif



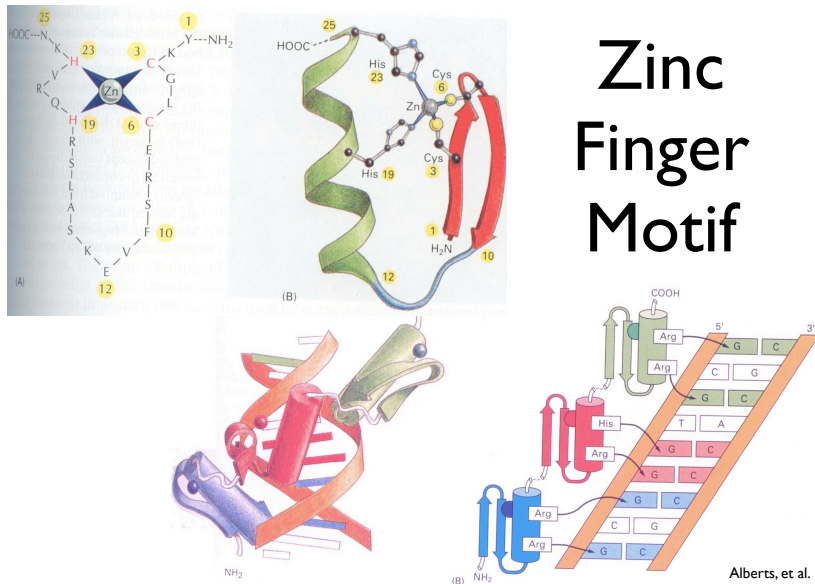
## H-T-H Dimers



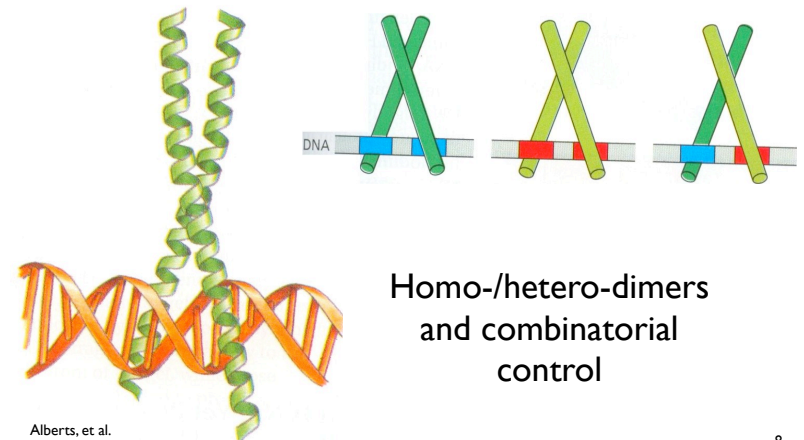
Bind 2 DNA patches, ~ 1 turn apart  
Increases both specificity and affinity

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## Zinc Finger Motif

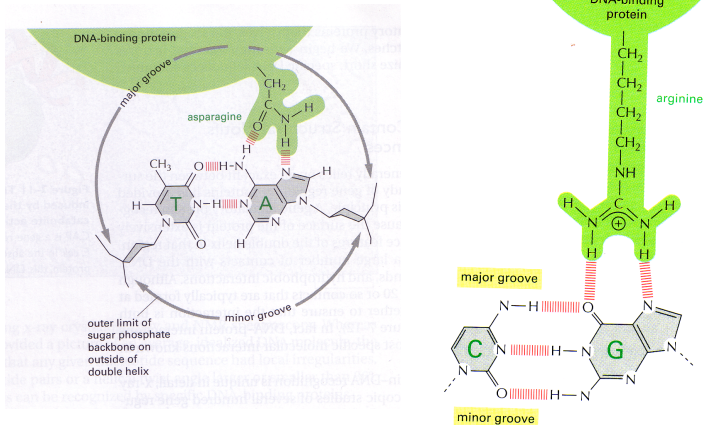


## Leucine Zipper Motif



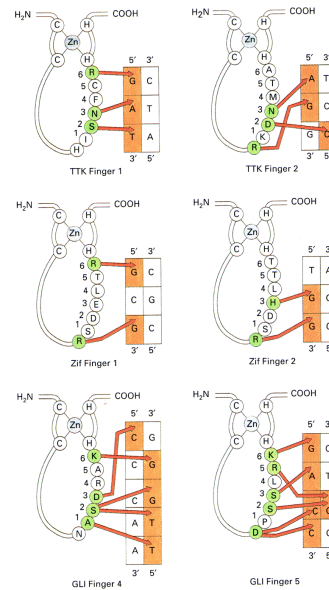
8

# Some Protein/DNA interactions well-understood

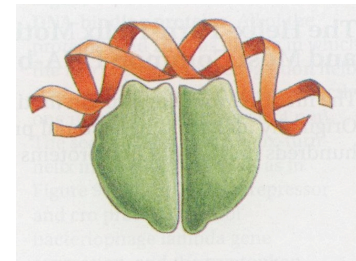


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But the overall DNA binding “code” still defies prediction

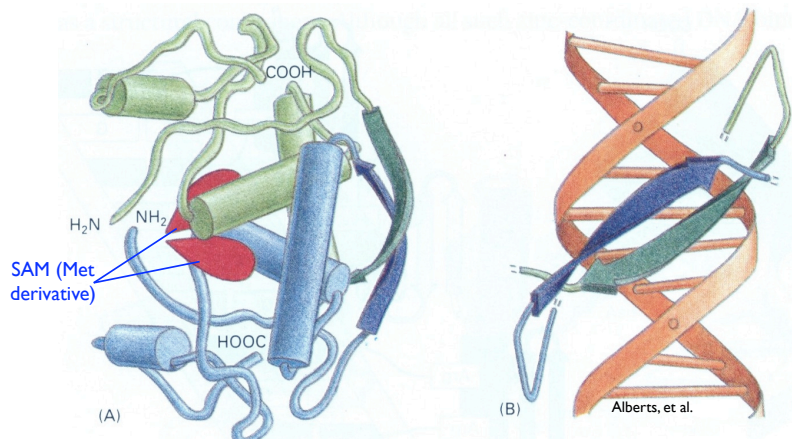


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# Bacterial Met Repressor

a beta-sheet DNA binding domain  
Negative feedback loop:  
high Met level  $\Rightarrow$  repress Met synthesis genes



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# DNA binding site summary

- complex “code”
- short patches (6-8 bp)
- often near each other (1 turn = 10 bp)
- often reverse-complements
- not perfect matches

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# Sequence Motifs

Last few slides described *structural* motifs in proteins

Equally interesting are the DNA *sequence* motifs to which these proteins bind - e.g., one leucine zipper dimer might bind (with varying affinities) to dozens or hundreds of similar sequences

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# *E. coli* Promoters

“**TATA Box**” ~ 10bp upstream of transcription start

How to define it?

Consensus is TATAAT

BUT all differ from it

Allow *k* mismatches?

Equally weighted?

Wildcards like R, Y? ({A,G}, {C,T}, resp.)

TACGAT

TAAAAT

TATACT

GATAAT

TATGAT

TATGTT

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# *E. coli* Promoters

- “**TATA Box**” - consensus TATAAT ~ 10bp upstream of transcription start
- *Not* exact: of 168 studied (mid 80's)
  - nearly all had 2/3 of TAxyzT
  - 80-90% had all 3
  - 50% agreed in each of x,y,z
  - **no** perfect match
- Other common features at -35, etc.

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# TATA Box Frequencies

pos base	1	2	3	4	5	6
A	2	95	26	59	51	1
C	9	2	14	13	20	3
G	10	1	16	15	13	0
T	79	3	44	13	17	96

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# TATA Scores

pos base	1	2	3	4	5	6
A	-36	19	1	12	10	-46
C	-15	-36	-8	-9	-3	-31
G	-13	-46	-6	-7	-9	-46(?)
T	17	-31	8	-9	-6	19

# Scanning for TATA

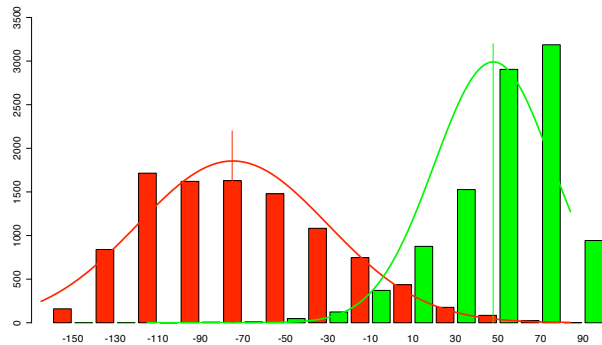
$$\begin{matrix}
 & \begin{matrix} \text{A} & \text{C} & \text{T} & \text{A} & \text{T} & \text{A} & \text{A} & \text{T} & \text{C} & \text{G} \end{matrix} \\
 \begin{matrix} \text{A} \\ \text{C} \\ \text{G} \\ \text{T} \end{matrix} & \begin{matrix} -38 & 19 & 1 & 12 & 10 & -48 \\ -15 & -38 & -8 & -10 & -3 & -32 \\ -13 & -48 & -6 & -7 & -10 & -48 \\ 17 & -32 & 8 & -9 & -6 & 19 \end{matrix} & = -93
 \end{matrix}$$

$$\begin{matrix}
 & \begin{matrix} \text{A} & \text{C} & \text{T} & \text{A} & \text{T} & \text{A} & \text{A} & \text{T} & \text{C} & \text{G} \end{matrix} \\
 \begin{matrix} \text{A} \\ \text{C} \\ \text{G} \\ \text{T} \end{matrix} & \begin{matrix} -38 & 19 & 1 & 12 & 10 & -48 \\ -15 & -38 & -8 & -10 & -3 & -32 \\ -13 & -48 & -6 & -7 & -10 & -48 \\ 17 & -32 & 8 & -9 & -6 & 19 \end{matrix} & = 85
 \end{matrix}$$

$$\begin{matrix}
 & \begin{matrix} \text{A} & \text{C} & \text{T} & \text{A} & \text{T} & \text{A} & \text{A} & \text{T} & \text{C} & \text{G} \end{matrix} \\
 \begin{matrix} \text{A} \\ \text{C} \\ \text{G} \\ \text{T} \end{matrix} & \begin{matrix} -38 & 19 & 1 & 12 & 10 & -48 \\ -15 & -38 & -8 & -10 & -3 & -32 \\ -13 & -48 & -6 & -7 & -10 & -48 \\ 17 & -32 & 8 & -9 & -6 & 19 \end{matrix} & = -95
 \end{matrix}$$

Stormo, Ann. Rev. Biophys. Biophys Chem, 17, 1988, 241-263

# Score Distribution (Simulated)



# Weight Matrices: Statistics

Assume:

$f_{b,i}$  = frequency of base  $b$  in position  $i$  in TATA

$f_b$  = frequency of base  $b$  in all sequences

Log likelihood ratio, given  $S = B_1B_2...B_6$ :

$$\log \left( \frac{P(S| \text{"tata"})}{P(S| \text{"non-tata"})} \right) = \log \frac{\prod_{i=1}^6 f_{B_i,i}}{\prod_{i=1}^6 f_{B_i}} = \sum_{i=1}^6 \log \frac{f_{B_i,i}}{f_{B_i}}$$

Assumes independence

# Neyman-Pearson

Given a sample  $x_1, x_2, \dots, x_n$ , from a distribution  $f(\dots|\Theta)$  with parameter  $\Theta$ , want to test hypothesis  $\Theta = \theta_1$  vs  $\Theta = \theta_2$ .

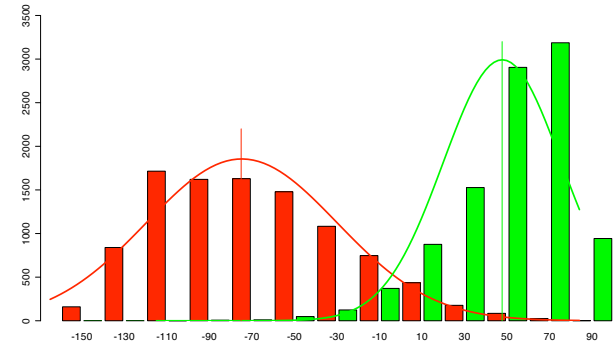
Might as well look at *likelihood ratio*:

$$\frac{f(x_1, x_2, \dots, x_n | \theta_1)}{f(x_1, x_2, \dots, x_n | \theta_2)} > \tau$$

(or *log likelihood ratio*)

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# Score Distribution (Simulated)



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# What's best WMM?

Given 20 sequences  $s_1, s_2, \dots, s_k$  of length 8, assumed to be generated at random according to a WMM defined by  $8 \times (4-1)$  parameters  $\theta$ , what's the best  $\theta$ ?

E.g., what's MLE for  $\theta$  given data  $s_1, s_2, \dots, s_k$ ?

Answer: count frequencies per position.

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# Weight Matrices: Chemistry

Experiments show ~80% correlation of log likelihood weight matrix scores to measured binding energy of RNA polymerase to variations on TATAAT consensus [Stormo & Fields]

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# Another WMM example

8 Sequences:

ATG  
ATG  
ATG  
ATG  
ATG  
GTG  
GTG  
TTG

Freq.	Col 1	Col 2	Col 3
A	.625	0	0
C	0	0	0
G	.250	0	1
T	.125	1	0

LLR	Col 1	Col 2	Col 3
A	1.32	$-\infty$	$-\infty$
C	$-\infty$	$-\infty$	$-\infty$
G	0	$-\infty$	2.00
T	-1.00	2.00	$-\infty$

Log-Likelihood Ratio:

$$\log_2 \frac{f_{x_i,i}}{f_{x_i}}, f_{x_i} = \frac{1}{4}$$

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# Non-uniform Background

- *E. coli* - DNA approximately 25% A, C, G, T
- *M. jannaschi* - 68% A-T, 32% G-C

LLR from previous example, assuming

LLR	Col 1	Col 2	Col 3
A	.74	$-\infty$	$-\infty$
C	$-\infty$	$-\infty$	$-\infty$
G	1.00	$-\infty$	3.00
T	-1.58	1.42	$-\infty$

$$f_A = f_T = 3/8$$

$$f_C = f_G = 1/8$$

e.g., G in col 3 is 8 x more likely via WMM than background, so  $(\log_2)$  score = 3 (bits).

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# Relative Entropy

AKA Kullback-Liebler Distance/Divergence,  
AKA Information Content

Given distributions P, Q

$$H(P||Q) = \sum_{x \in \Omega} P(x) \log \frac{P(x)}{Q(x)} \geq 0$$

Notes:

Let  $P(x) \log \frac{P(x)}{Q(x)} = 0$  if  $P(x) = 0$  [since  $\lim_{y \rightarrow 0} y \log y = 0$ ]

Undefined if  $0 = Q(x) < P(x)$

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# WMM: How “Informative”? Mean score of site vs bkg?

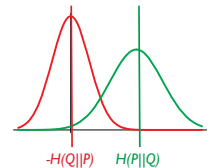
For any fixed length sequence x, let

$P(x)$  = Prob. of x according to WMM

$Q(x)$  = Prob. of x according to background

Relative Entropy:

$$H(P||Q) = \sum_{x \in \Omega} P(x) \log_2 \frac{P(x)}{Q(x)}$$

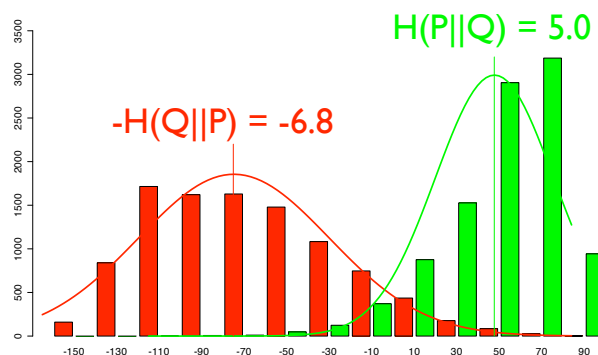


$H(P||Q)$  is **expected log likelihood score** of a sequence randomly chosen from **WMM**;

$-H(Q||P)$  is expected score of **Background**

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# WMM Scores vs Relative Entropy



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For WMM, you can show (based on the assumption of independence between columns), that :

$$H(P||Q) = \sum_i H(P_i||Q_i)$$

where  $P_i$  and  $Q_i$  are the WMM/background distributions for column  $i$ .

# WMM Example, cont.

Freq.	Col 1	Col 2	Col 3
A	.625	0	0
C	0	0	0
G	.250	0	1
T	.125	1	0

Uniform

LLR	Col 1	Col 2	Col 3
A	1.32	$-\infty$	$-\infty$
C	$-\infty$	$-\infty$	$-\infty$
G	0	$-\infty$	2.00
T	-1.00	2.00	$-\infty$
RelEnt	.70	2.00	4.70

Non-uniform

LLR	Col 1	Col 2	Col 3
A	.74	$-\infty$	$-\infty$
C	$-\infty$	$-\infty$	$-\infty$
G	1.00	$-\infty$	3.00
T	-1.58	1.42	$-\infty$
RelEnt	.51	1.42	4.93

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# Pseudocounts

Are the  $-\infty$ 's a problem?

Certain that a given residue *never* occurs in a given position? Then  $-\infty$  just right

Else, it may be a small-sample artifact

Typical fix: add a *pseudocount* to each observed count—small constant (e.g., .5, 1)

Sounds *ad hoc*; there is a Bayesian justification

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# WMM Summary

- Weight Matrix Model (aka Position Specific Scoring Matrix, PSSM, "possum", 0th order Markov models)
- Simple statistical model assuming independence between adjacent positions
- To build: count (+ pseudocount) letter frequency per position, log likelihood ratio to background
- To scan: add LLRs per position, compare to threshold
- Generalizations to higher order models (i.e., letter frequency per position, conditional on neighbor) also possible, with enough training data

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# How-to Questions

Given aligned motif instances, build model?

- Frequency counts (above, maybe with pseudocounts)

Given a model, find (probable) instances

- Scanning, as above

Given unaligned strings thought to contain a motif, find it? (e.g., upstream regions for co-expressed genes from a microarray experiment)

- Hard... rest of lecture.

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# Motif Discovery

Unfortunately, finding a site of max relative entropy in a set of unaligned sequences is NP-hard [Akutsu]

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# Motif Discovery: 4 example approaches

Brute Force

Greedy search

Expectation Maximization

Gibbs sampler

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# Brute Force



Input:

- Seqs  $s_1, s_2, \dots, s_k$  (len  $\sim n$ , say), each w/ one instance of an unknown length  $l$  motif

Algorithm:

- create singleton set with each length  $l$  subsequence of each  $s_1, s_2, \dots, s_k$  ( $\sim nk$  sets)
- for each set, add each possible length  $l$  subsequence not already present ( $\sim n^2 k(k-l)$  sets)
- repeat until all have  $k$  sequences ( $\sim n^k k!$  sets)
- compute relative entropy of each; pick best

problem:  
astronomically sloooow

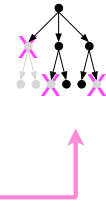
# Greedy Best-First Approach [Hertz & Stormo]

Input:

- Sequence  $s_1, s_2, \dots, s_k$ ; motif length  $l$ ; “breadth”  $d$

Algorithm:

- create singleton set with each length  $l$  subsequence of each  $s_1, s_2, \dots, s_k$
- for each set, add each possible length  $l$  subsequence not already present
- compute relative entropy of each
- discard all but  $d$  best
- repeat until all have  $k$  sequences



usual “greedy” problems

# Expectation Maximization [MEME, Bailey & Elkan, 1995]

Input (as above):

Sequence  $s_1, s_2, \dots, s_k$ ; motif length  $l$ ; background model; again assume one instance per sequence (variants possible)

Algorithm: EM

Visible data: the sequences

Hidden data: where’s the motif

$$Y_{i,j} = \begin{cases} 1 & \text{if motif in sequence } i \text{ begins at position } j \\ 0 & \text{otherwise} \end{cases}$$

Parameters  $\theta$ : The WMM

# MEME Outline

Typical EM algorithm:

- Parameters  $\theta^t$  at  $t^{\text{th}}$  iteration, used to estimate where the motif instances are (the hidden variables)
- Use those estimates to re-estimate the parameters  $\theta$  to maximize likelihood of observed data, giving  $\theta^{t+1}$
- Repeat

Key: given a few good matches to best motif, expect to pick out more

# Expectation Step

(where are the motif instances?)

$$\begin{aligned}
 \hat{Y}_{i,j} &= E(Y_{i,j} | s_i, \theta^t) \xrightarrow{E = 0 \cdot P(0) + 1 \cdot P(1)} \\
 &= P(Y_{i,j} = 1 | s_i, \theta^t) \xrightarrow{\text{Bayes}} \\
 &= P(s_i | Y_{i,j} = 1, \theta^t) \frac{P(Y_{i,j}=1|\theta^t)}{P(s_i|\theta^t)} \\
 &= cP(s_i | Y_{i,j} = 1, \theta^t) \\
 &= c' \prod_{k=1}^l P(s_{i,j+k-1} | \theta^t)
 \end{aligned}$$

where  $c'$  is chosen so that  $\sum_j \hat{Y}_{i,j} = 1$ .

# Maximization Step

(what is the motif?)

Find  $\theta$  maximizing expected value:

$$\begin{aligned}
 Q(\theta | \theta^t) &= E_{Y \sim \theta^t} [\log P(s, Y | \theta)] \\
 &= E_{Y \sim \theta^t} [\log \prod_{i=1}^k P(s_i, Y_i | \theta)] \\
 &= E_{Y \sim \theta^t} [\sum_{i=1}^k \log P(s_i, Y_i | \theta)] \\
 &= E_{Y \sim \theta^t} [\sum_{i=1}^k \sum_{j=1}^{|s_i|-l+1} Y_{i,j} \log P(s_i, Y_{i,j} = 1 | \theta)] \\
 &= E_{Y \sim \theta^t} [\sum_{i=1}^k \sum_{j=1}^{|s_i|-l+1} Y_{i,j} \log (P(s_i | Y_{i,j} = 1, \theta) P(Y_{i,j} = 1 | \theta))] \\
 &= \sum_{i=1}^k \sum_{j=1}^{|s_i|-l+1} E_{Y \sim \theta^t} [Y_{i,j}] \log P(s_i | Y_{i,j} = 1, \theta) + C \\
 &= \sum_{i=1}^k \sum_{j=1}^{|s_i|-l+1} \hat{Y}_{i,j} \log P(s_i | Y_{i,j} = 1, \theta) + C
 \end{aligned}$$

# M-Step (cont.)

$$Q(\theta | \theta^t) = \sum_{i=1}^k \sum_{j=1}^{|s_i|-l+1} \hat{Y}_{i,j} \log P(s_i | Y_{i,j} = 1, \theta) + C$$

Exercise: Show this is maximized by “counting” letter frequencies over all possible motif instances, with counts weighted by  $\hat{Y}_{i,j}$ , again the “obvious” thing.

$s_1$ :	ACGGATT...
	...
$s_k$ :	GC...TCGGAC
$\hat{Y}_{1,1}$	ACGG
$\hat{Y}_{1,2}$	CGGA
$\hat{Y}_{1,3}$	GGAT
$\vdots$	$\vdots$
$\hat{Y}_{k,l-1}$	CGGA
$\hat{Y}_{k,l}$	GGAC

# Initialization

1. Try every motif-length substring, and use as initial  $\theta$  a WMM with, say 80% of weight on that sequence, rest uniform
2. Run a few iterations of each
3. Run best few to convergence  
(Having a supercomputer helps)

# Another Motif Discovery Approach The Gibbs Sampler

Lawrence, et al. "Detecting Subtle Sequence Signals: A Gibbs Sampling Strategy for Multiple Sequence Alignment," *Science* 1993

Sigma-37	223	IIDLTYIQNK	SQKETGDILGISQMHVSR	LQRKAVKKLR	240	A25944
SpoIIIC	94	RFGLDLKKEK	TQREIAKELGISRSYVSR	IEKRALMKMF	111	A28627
NahR	22	VVFNQLLVDR	RVSIITAENLGLTPAVSN	ALKRLRSLQ	39	A32837
Antennapedia	326	FHFNRYLTRR	RRIEIAHALCLTERQIKI	WFOQRMRWK	343	A23450
NtrC (Brady.)	449	LTAALAAATRG	NQIRAADLLGLNRNTLRK	KIRDLDIQVY	466	B26499
DicA	22	IRYRRKNLKH	TQKSLAKALKISHVSVSQ	WERGDSEPTG	39	B24328 (BVECD)
MerD	5	MNAY	TVSRLALDAGVSVHIVRD	YLLRGLLRPV	22	C29010
Fis	73	LDMVMQYTRG	NQTRAAALMMGINRGTLRK	KLKKYGMN	90	A32142 (DNECF)
MAT a1	99	FRKQSLNSK	EKEEVAKKCGITPLQVRV	WFINKRMRSK	116	A90983 (JEBY1)
Lambda cII	25	SALLNKIAML	GTEKTAEAAGVVDKSIQR	WKRDWIPKFS	42	A03579 (QCBP2L)
Crp (CAP)	169	THPDGMQIKI	TRQEIGQIVGCSRETIVGR	ILKMLEQNL	186	A03553 (QRECC)
Lambda Cro	15	ITLKDYAMRF	GQTKTAKDLGVYQSAINK	AIHAGRKIFL	32	A03577 (RCBPL)
P22 Cro	12	YKDVIDHFG	TQRAVAKALGISDAAVSQ	WKEVIPEKDA	29	A25867 (RGBP22)
AraC	196	ISDHLADSNF	DIASVAQHVCLSPRSLSH	LFRQQLGISV	213	A03554 (RGCEA)
Fnr	196	FSPREFRLTM	TRGDIGNYLGLTVETISR	LLGRFQKSGM	213	A03552 (RGECF)
HtpR	252	ARWLDEDNKS	TLQELADRYGVSAAERVRQ	LEKNAMKKLR	269	A00700 (RGEC)
NtrC (K.a.)	444	LTTALRHTQG	HKQEAARLGLWGRNTLTR	KLKELGME	461	A03564 (RGKBCP)
CytR	11	MKAKQETA	TMKDVALKAKVSTATVSR	ALMNPDKVSQ	28	A24963 (RPECCT)
DeoR	23	LQELKRSDKL	HLKDAALGLVSEMTIRR	DLNHSAPVV	40	A24076 (RPECDO)
GalR	3	MA	TIKDVARLAGVSVATVSR	VINNSPKASE	20	A03559 (RPEC)
LacI	5	MKPV	TLYDVAEYAGVSYQTVSR	VVNQASHVSA	22	A03558 (RPECL)
TetR	26	LLNEVEGIEGL	TTRKLAQKLGVEQPTLYW	HVKNKRALLD	43	A03576 (RPECTN)
TrpR	67	IVEELLRGE	SQRELKNELGAGIATITR	GSNSLKAAPV	84	A03568 (RPECW)
NifA	495	LIAALEKAGW	VQAKAARLLGMPTRQVAY	RIQIMDIIMP	512	S02513
SpoIIG	205	RFLVGEEEK	TQKDVADMMGISQSYISR	LEKRIIKRLR	222	S07337
Pin	160	QAGRLIAAGT	PRQKVAIIVDVGVSITLYK	TFPAGDK	177	S07958
PurR	3	MA	TIKDVAKRANVSTTVSH	VINKTRFVAE	20	S08477
EbgR	3	MA	TLKDIAIEAGVSLATVSR	VLNDDPTLNV	20	S09205
LexA	27	DHISQTMGPP	TRAEIAQRLGFRSPNAAE	EHLKALARKG	44	S11945
P22 cI	25	SSILNRIAIR	GQRKVADALGINESQISR	WKGDFIPKMG	42	B25867 (Z1BPC2)
			*****	***		

B	Position in site																	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Arg	94	222	265	137	9	9	137	137	9	9	9	52	222	94	94	9	265	606
Lys	9	133	442	380	9	71	380	194	9	133	9	9	71	9	9	9	71	256
Glu	53	9	96	401	9	9	140	140	9	9	9	53	140	140	9	9	9	53
Asp	67	9	9	473	9	9	224	125	9	67	9	67	67	9	9	9	9	67
Gln	9	600	224	9	9	9	224	9	9	9	9	9	278	63	278	9	9	170
His	240	9	9	9	9	9	125	125	9	9	9	9	125	125	125	9	9	240
Asn	168	9	9	9	9	9	168	89	9	89	9	248	9	168	89	9	89	89
Ser	117	9	117	117	9	9	9	9	9	9	9	819	63	387	63	9	819	9
Gly	151	9	56	9	9	151	9	9	9	1141	9	151	9	56	9	9	56	9
Ala	9	9	112	43	181	901	43	181	215	9	43	9	43	181	112	43	78	9
Thr	915	130	130	9	251	9	9	9	9	9	9	311	130	70	855	9	130	9
Pro	76	9	9	9	9	9	9	9	9	9	9	9	210	210	9	9	9	9
Cys	9	9	9	9	9	9	9	9	295	581	295	9	9	9	9	9	9	9
Val	58	107	9	9	500	9	9	9	156	9	598	9	205	58	9	746	9	58
Leu	9	121	9	9	149	9	93	149	458	9	149	9	37	37	9	177	9	9
Ile	9	166	114	61	323	9	114	166	9	9	427	9	61	9	61	427	9	61
Met	9	104	9	9	9	9	9	198	198	9	104	9	9	198	9	9	9	9
Tyr	9	9	136	9	9	9	9	262	262	9	9	136	136	9	262	9	262	136
Phe	9	9	9	9	9	9	9	9	9	9	108	9	9	9	9	9	9	9
Trp	9	9	9	9	9	9	9	9	9	9	366	9	9	9	9	9	9	366

## Some History

Geman & Geman, *IEEE PAMI* 1984

Hastings, *Biometrika*, 1970

Metropolis, Rosenbluth, Rosenbluth, Teller, & Teller, "Equations of State Calculations by Fast Computing Machines," *J. Chem. Phys.* 1953

Josiah Williard Gibbs, 1839-1903, American physicist, a pioneer of thermodynamics

# How to Average

An old problem:

n random variables:  $x_1, x_2, \dots, x_k$   
 Joint distribution (p.d.f.):  $P(x_1, x_2, \dots, x_k)$   
 Some function:  $f(x_1, x_2, \dots, x_k)$   
Want Expected Value:  $E(f(x_1, x_2, \dots, x_k))$

# How to Average

$$E(f(x_1, x_2, \dots, x_k)) = \int_{x_1} \int_{x_2} \dots \int_{x_k} f(x_1, x_2, \dots, x_k) \cdot P(x_1, x_2, \dots, x_k) dx_1 dx_2 \dots dx_k$$

- Approach 1: direct integration  
(rarely solvable analytically, esp. in high dim)
- Approach 2: numerical integration  
(often difficult, e.g., unstable, esp. in high dim)
- Approach 3: Monte Carlo integration  
sample  $\vec{x}^{(1)}, \vec{x}^{(2)}, \dots, \vec{x}^{(n)} \sim P(\vec{x})$  and average:  
 $E(f(\vec{x})) \approx \frac{1}{n} \sum_{i=1}^n f(\vec{x}^{(i)})$

# Markov Chain Monte Carlo (MCMC)

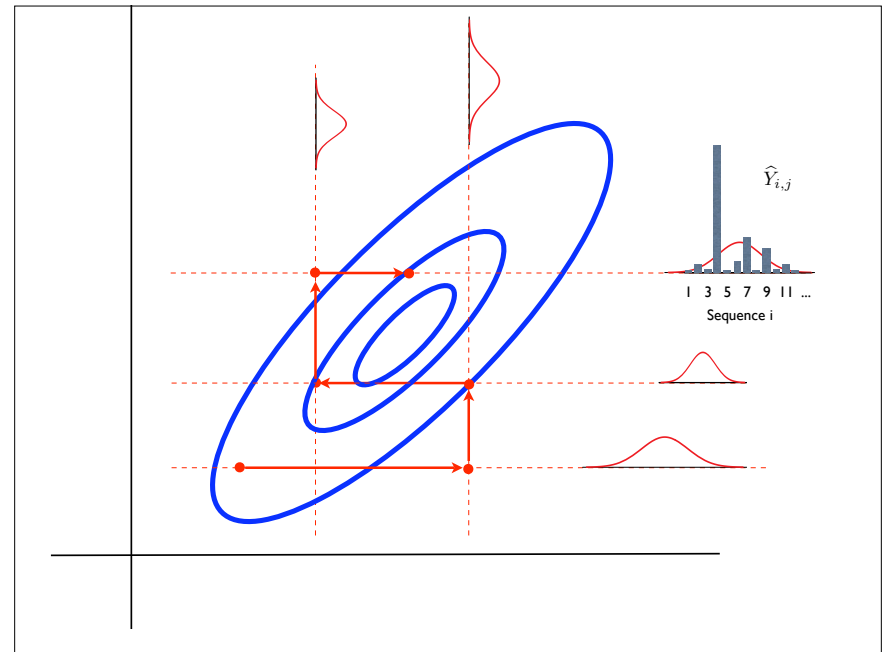
- *Independent* sampling also often hard, but *not required* for expectation
- MCMC  $\vec{X}_{t+1} \sim P(\vec{X}_{t+1} | \vec{X}_t)$  w/ stationary dist =  $P$
- Simplest & most common: Gibbs Sampling  
 $P(x_i | x_1, x_2, \dots, x_{i-1}, x_{i+1}, \dots, x_k)$

Algorithm

for  $t = 1$  to  $\infty$

for  $i = 1$  to  $k$  do :

$$x_{t+1,i} \sim P(x_{t+1,i} | \underbrace{x_{t+1,1}, x_{t+1,2}, \dots, x_{t+1,i-1}}_{t+1}, \underbrace{x_{t,i+1}, \dots, x_{t,k}}_t)$$



- Input: again assume sequences  $s_1, s_2, \dots, s_k$  with one length  $w$  motif per sequence
- Motif model: WMM
- Parameters: Where are the motifs?  
for  $1 \leq i \leq k$ , have  $1 \leq x_i \leq |s_i| - w + 1$
- “Full conditional”: to calc  

$$P(x_i = j \mid x_1, x_2, \dots, x_{i-1}, x_{i+1}, \dots, x_k)$$
 build WMM from motifs in all sequences except  $i$ , then calc prob that motif in  $i^{\text{th}}$  seq occurs at  $j$  by usual “scanning” alg.

## Overall Gibbs Alg

Randomly initialize  $x_i$ 's

for  $t = 1$  to  $\infty$

  for  $i = 1$  to  $k$

    discard motif instance from  $s_i$ ;

    recalc WMM from rest

    for  $j = 1 \dots |s_i| - w + 1$

      calculate prob that  $i^{\text{th}}$  motif is at  $j$ :

Similar to MEME, but it would average over, rather than sample from  $\longrightarrow P(x_i = j \mid x_1, x_2, \dots, x_{i-1}, x_{i+1}, \dots, x_k)$

      pick new  $x_i$  according to that distribution

## Issues

Burnin - how long must we run the chain to reach stationarity?

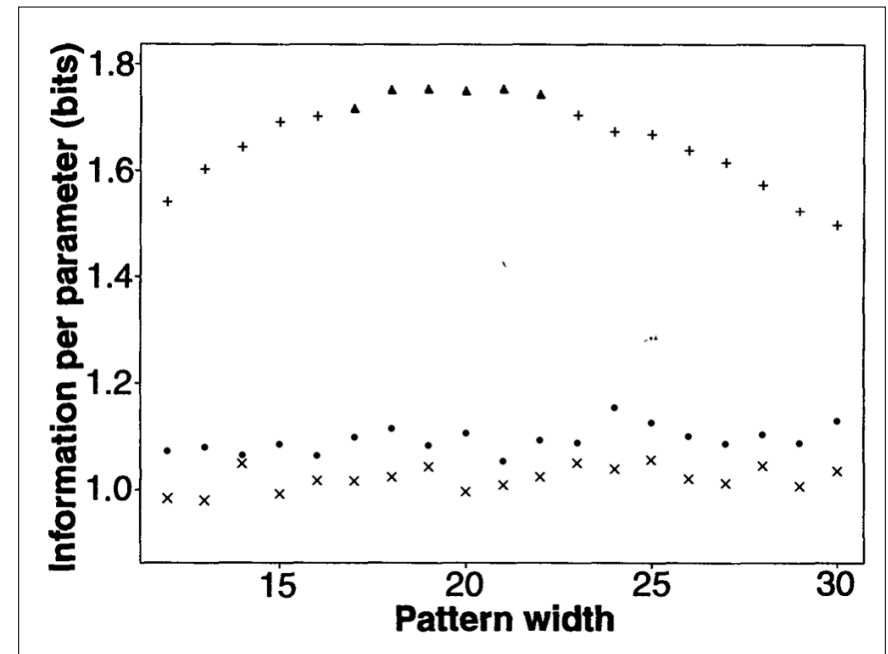
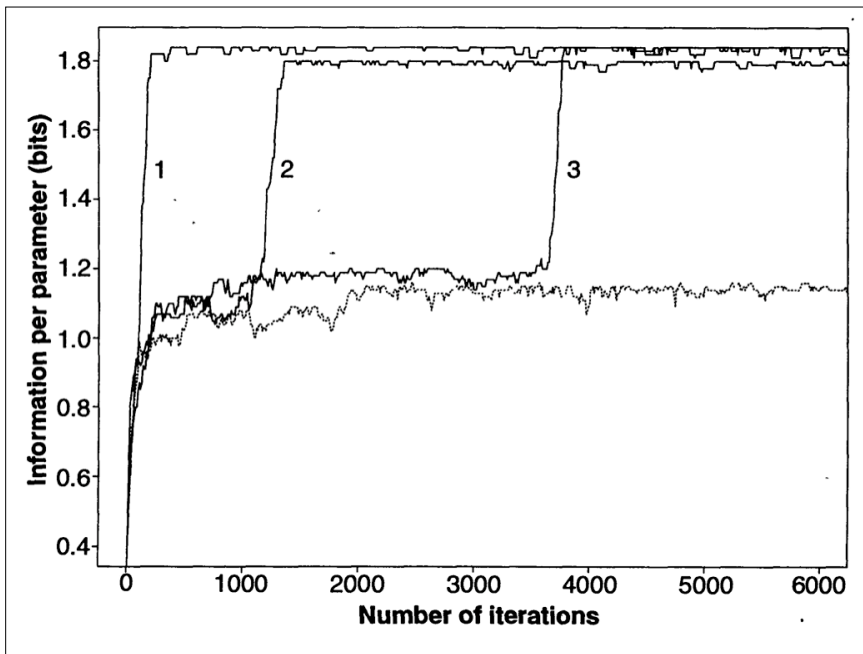
Mixing - how long a post-burnin sample must we take to get a good sample of the stationary distribution? (Recall that individual samples are not independent, and may not “move” freely through the sample space. Also, many isolated modes.)

## Variants & Extensions

“Phase Shift” - may settle on suboptimal solution that overlaps part of motif. Periodically try moving all motif instances a few spaces left or right.

Algorithmic adjustment of pattern width: Periodically add/remove flanking positions to maximize (roughly) average relative entropy per position

Multiple patterns per string



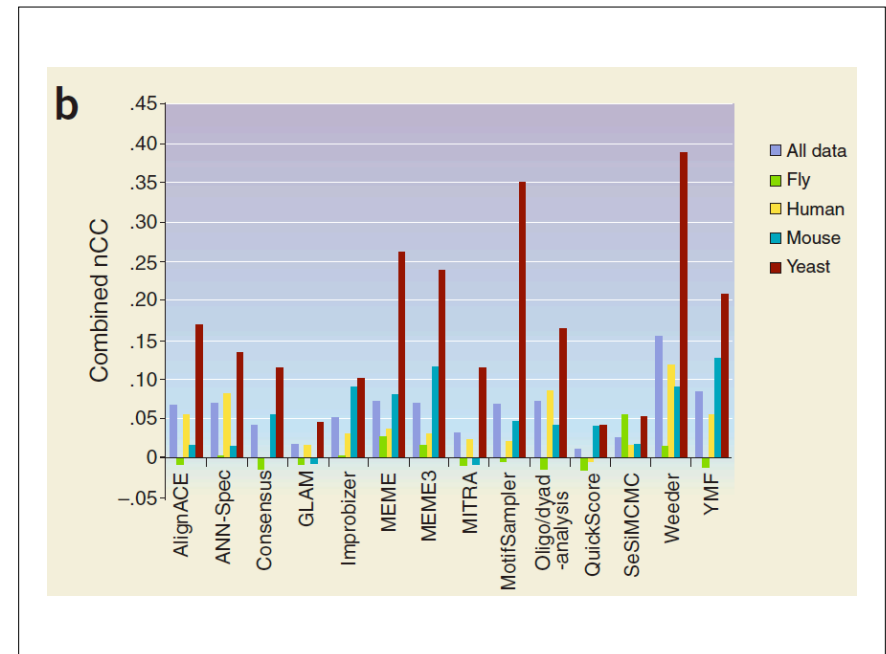
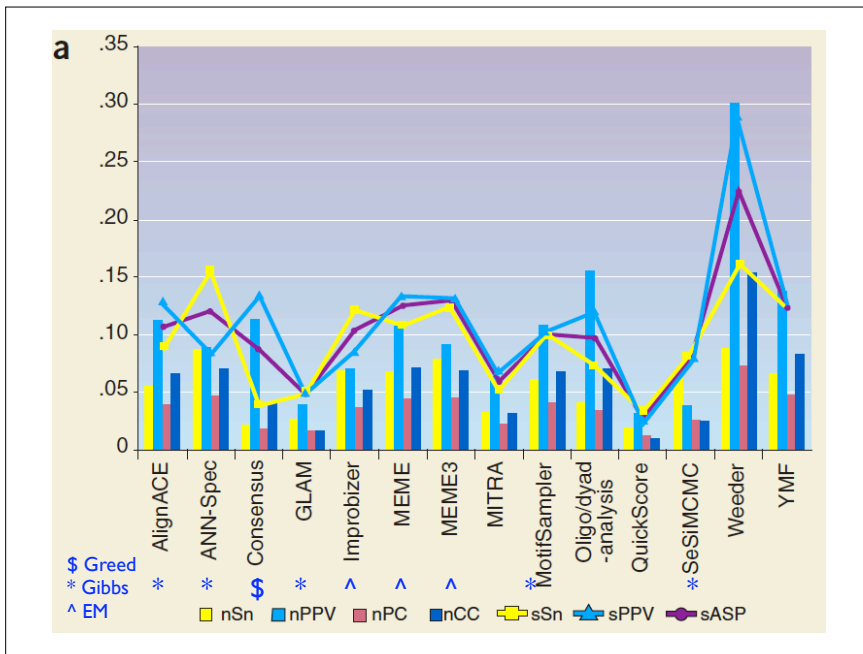
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## Assessing computational tools for the discovery of transcription factor binding sites

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## Methodology

- 13 tools
- Real 'motifs' (Transfac)
- 56 data sets (human, mouse, fly, yeast)
- 'Real', 'generic', 'Markov'
- Expert users, top prediction only



## Lessons

- Evaluation is hard (esp. when “truth” is unknown)
- Accuracy low
  - partly reflects defects in evaluation methodology (e.g.  $\leq 1$  prediction per data set; results better in synth data)
  - partly reflects difficult task, limited knowledge (e.g. yeast > others)
- No clear winner re methods or models

## Motif Discovery Summary

- Important problem: a key to understanding gene regulation
- Hard problem: short, degenerate signals amidst much noise
- Many variants have been tried, for representation, search, and discovery. We looked at only a few:
  - Weight matrix models for representation & search
  - greedy, MEME and Gibbs for discovery
- Still much room for improvement. *Comparative genomics*, i.e. cross-species comparison is very promising