

## HW6

*Submit to gradescope*

## Instructions

- You should think about each problem by yourself for at least an hour before choosing to collaborate with others.
- You are allowed to collaborate with fellow students taking the class in solving the problems. But you **must** write your solution on your own.
- You are not allowed to search for answers or hints on the web. You are encouraged to contact the instructor or the TAs for a possible hint.
- You cannot collaborate on Extra credit problems
- Solutions typeset in LATEX are preferred.
- Feel free to use the Discussion Board or email the instructor or the TAs if you have any questions or would like any clarifications about the problems.
- Please upload your solutions to Gradescope.

---

In solving these assignments, feel free to use these approximations:

$$1 - x \approx e^{-x}, \quad \sqrt{1-x} \approx 1 - x/2, \quad n! \approx (n/e)^n, \quad \left(\frac{n}{k}\right)^k \leq \binom{n}{k} \leq \left(\frac{en}{k}\right)^k$$

1. Show that the cover time of any unweighted  $d$ -regular graph  $G = (V, E)$  is  $O(n^2 \log n)$ .

**Hint:** What is the length of the shortest path between  $u, v \in V$ ?

2. Let  $A \in \mathbb{R}^{n \times n}$  be a random Gaussian matrix where every entry  $A_{i,j}$  is distributed as a  $\mathcal{N}(0, 1)$  and  $A_{i,j} = A_{j,i}$ , and  $A_{i,i} = 0$  for all  $1 \leq i < j \leq n$ . Prove that  $\lambda_{\max}(A) \leq O(\sqrt{n})$  with high probability.