CSE 525: Randomized Algorithms and Probability **Spring 2026**

HW3

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Instructions

- You should think about each problem by yourself for at least an hour before choosing to collaborate with others.
- You are allowed to collaborate with fellow students taking the class in solving the problems. But you **must** write your solution on your own.
- You are not allowed to search for answers or hints on the web. You are encouraged to contact the instructor or the TAs for a possible hint.
- You cannot collaborate on Extra credit problems
- Solutions typeset in LATEX are preferred.
- Feel free to use the Discussion Board or email the instructor or the TAs if you have any questions or would like any clarifications about the problems.
- Please upload your solutions to Gradescope.

In solving these assignments, feel free to use these approximations:

$$1 - x \approx e^{-x}, \qquad \sqrt{1-x} \approx 1 - x/2, \qquad n! \approx (n/e)^n, \qquad \left(\frac{n}{k}\right)^k \leq \binom{n}{k} \leq \left(\frac{en}{k}\right)^k$$

The problems of this HW are hard. To get full points it is enough to solve one of the following problems, although you are encouraged to try both.

1) Let $G = (V, E)$ be an undirected graph and suppose each $v \in V$ is associated with a set $S(v)$ of $8r$ colors, where $r \geq 1$. Suppose further that for each $v \in V$ and $c \in S(v)$ there are at most r neighbors u of v such that c lies in $S(u)$.
   - Prove that there exists a proper coloring of G assigning to each vertex v a color from its class $S(v)$ such that, for any edge $u \sim v$, the colors assigned to u and v are different.
   - Design a randomized polynomial time algorithms. Feel free to assume $|S(v)| \geq c \cdot r$ (for all v) for a constant $c \gg 8$.

2) Let $\mathcal{A}_\infty, \ldots, \mathcal{A}_\backslash$ be a set of bad events defined on independent variables $Z_1, \ldots, Z_m$ with corresponding dependency graph G (ignore conditions of the LLL for this problem). Note that $\mathbb{P}\left[\neg \mathcal{A}_\rangle\right]$ is uniquely defined as a function of the probabilities of Z_i's. Prove or disprove: In the Moser-Tardos algorithm, for a fixed $t > 1$ and $i \in [n]$, the probability that the t-th resampled event is $\mathcal{A}_i$ is at most $\mathbb{P}[\mathcal{A}_i]$. Prove this or find a counterexample.