CSE 525: Randomized Algorithms and Probability **Spring 2026**

HW1

Submit to gradescope

Instructions

- You should think about each problem by yourself for at least an hour before choosing to collaborate with others.
- You are allowed to collaborate with fellow students taking the class in solving the problems. But you **must** write your solution on your own.
- You are not allowed to search for answers or hints on the web. You are encouraged to contact the instructor or the TAs for a possible hint.
- You cannot collaborate on Extra credit problems
- Solutions typeset in LATEX are preferred.
- Feel free to use the Discussion Board or email the instructor or the TAs if you have any questions or would like any clarifications about the problems.
- Please upload your solutions to Gradescope.

In solving these assignments, feel free to use these approximations:

$$1 - x \approx e^{-x}, \qquad \sqrt{1-x} \approx 1 - x/2, \qquad n! \approx (n/e)^n, \qquad \left(\frac{n}{k}\right)^k \leq \binom{n}{k} \leq \left(\frac{en}{k}\right)^k$$

1) Let $\mathcal{F}$ be a family of subsets of $[n] := \{1, 2, \ldots, n\}$, and suppose there are no $A, B \in \mathcal{F}$ satisfying $A \subset B$. Let $\sigma \in S_n$ be a uniformly random permutation of the elements of $[n]$ and consider the random variable

$$X = |\{i : \{\sigma(1), \ldots, \sigma(i)\} \in \mathcal{F}\}|$$

Use expectation of X to show that $|\mathcal{F}| \leq \binom{n}{\lfloor n/2 \rfloor}$.

2) Let $G = (V, E)$ be a bipartite graph with n vertices and a list $S(v)$ of at least $\log_2(n+1)$ colors associated with each vertex $v \in V$. Design a randomized polynomial time algorithm that finds a proper coloring of (vertices) of G assigning to each vertex v a color from its list $S(v)$ with probability at least $1 - 1/n$.