CSE 525: Randomized Algorithms and Probability **Spring 2025**

HW6

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Instructions

- You should think about each problem by yourself for at least an hour before choosing to collaborate with others.
- You are allowed to collaborate with fellow students taking the class in solving the problems. But you **must** write your solution on your own.
- You are not allowed to search for answers or hints on the web. You are encouraged to contact the instructor or the TAs for a possible hint.
- You cannot collaborate on Extra credit problems
- Solutions typeset in LATEX are preferred.
- Feel free to use the Discussion Board or email the instructor or the TAs if you have any questions or would like any clarifications about the problems.
- Please upload your solutions to Gradescope.

In solving these assignments, feel free to use these approximations:

$$1 - x \approx e^{-x}, \qquad \sqrt{1-x} \approx 1 - x/2, \qquad n! \approx (n/e)^n, \qquad \left(\frac{n}{k}\right)^k \leq \binom{n}{k} \leq \left(\frac{en}{k}\right)^k$$

1. Show that the cover time of any unweighted d-regular graph $G = (V, E)$ is $O(n^2 \log n)$.

   Hint: What is the length of the shortest path between $u, v \in V$?

2. Given a sequence $a_0, \ldots, a_{m-1} \in [d]$ and a starting vertex $v_0 \in V$ of a consistently labelled graph $G = (V, E)$, we can traverse G by first going to the unique neighbor of v_0, say v_1, connected with the edge labelled a_0, then going to the unique neighbor of v_1, say v_2, connected with the edge labelled a_1, etc. We say a sequence $a_0, \ldots, a_{m-1}$ **covers** G if for **any** starting vertex $v_0 \in V$, following this sequence we visit **all** vertices of G.

   Given $1 \leq d < n$, prove that there exists a sequence $a_0, \ldots, a_{m-1}$ for some $m = \text{poly}(n, d)$ such that for **any** d-regular consistently labelled n vertex graph $G = (V, E)$ the sequence $a_0, \ldots, a_{m-1}$ covers G.