CSE 525: Randomized Algorithms and Probability **Spring 2025**

HW5

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Instructions

- You should think about each problem by yourself for at least an hour before choosing to collaborate with others.
- You are allowed to collaborate with fellow students taking the class in solving the problems. But you **must** write your solution on your own.
- You are not allowed to search for answers or hints on the web. You are encouraged to contact the instructor or the TAs for a possible hint.
- You cannot collaborate on Extra credit problems
- Solutions typeset in LATEX are preferred.
- Feel free to use the Discussion Board or email the instructor or the TAs if you have any questions or would like any clarifications about the problems.
- Please upload your solutions to Gradescope.

In solving these assignments, feel free to use these approximations:

$$1 - x \approx e^{-x}, \qquad \sqrt{1-x} \approx 1 - x/2, \qquad n! \approx (n/e)^n, \qquad \left(\frac{n}{k}\right)^k \leq \binom{n}{k} \leq \left(\frac{en}{k}\right)^k$$

The problems of this HW are hard. To get full points it is enough to solve one of the following problems, although you are encouraged to try both.

1) Let $G \sim G_{n,p}$ with $p = D/n$ and $D > 0$ is some fixed number. Let X be the number of isolated vertices in G (i.e., the number of vertices that have no neighbors). Determine $\mathbb{E}[X]$ and then show that for any $\lambda > 0$,

$$\mathbb{P}\left[|X - \mathbb{E}[X]| \geq 4\lambda\sqrt{Dn}\right] \leq e^{-c\lambda^2},$$

for some universal constant c.

2) Prove the following theorem that we discussed in class: Let $0 < \sigma \leq \tau$. Suppose $y_0, \ldots,$ is a sequence of random variables such that for all $i > 1$, $y_i - y_{i-1} \mid y_1, \ldots, y_{i-1} \sim \mathcal{N}(0, \sigma_i^2)$ where $0 < \sigma_i \leq \tau$. Then:

$$\mathbb{P}\left[|y_\ell| > \lambda \cdot \tau \cdot \sqrt{\ell}\right] \leq 2\exp(-c\lambda^2)$$

for some constant $c > 0$.