CSE 525: Randomized Algorithms and Probability **Spring 2025**

HW3

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Instructions

- You should think about each problem by yourself for at least an hour before choosing to collaborate with others.
- You are allowed to collaborate with fellow students taking the class in solving the problems. But you **must** write your solution on your own.
- You are not allowed to search for answers or hints on the web. You are encouraged to contact the instructor or the TAs for a possible hint.
- You cannot collaborate on Extra credit problems
- Solutions typeset in LATEX are preferred.
- Feel free to use the Discussion Board or email the instructor or the TAs if you have any questions or would like any clarifications about the problems.
- Please upload your solutions to Gradescope.

In solving these assignments, feel free to use these approximations:

$$1 - x \approx e^{-x}, \qquad \sqrt{1-x} \approx 1 - x/2, \qquad n! \approx (n/e)^n, \qquad \left(\frac{n}{k}\right)^k \le \binom{n}{k} \le \left(\frac{en}{k}\right)^k$$

The problems of this HW are hard. To get full points it is enough to solve one of the following problems, although you are encouraged to try both.

1) Prove that for every integer $d > 1$ there is a finite $c(d)$ such that the edges of any bipartite d-regular bipartite graph in which every cycle has at least $c(d)$ edges can be colored by $d+1$ colors so that
   - For any vertex, there are no two adjacent edges with the same color.
   - There is no two-colored cycle.

2) Let $\mathcal{A}_\infty, \ldots, \mathcal{A}_\setminus$ be a set of bad events defined on independent variables $Z_1, \ldots, Z_m$ with corresponding dependency graph G (ignore conditions of the LLL for this problem). Note that $\mathbb{P}\left[\neg\mathcal{A}_\rangle\right]$ is uniquely defined as a function of the probabilities of Z_i's. Prove or disprove: In the Moser-Tardos algorithm, for a fixed $t > 1$ and $i \in [n]$, the probability that the t-th resampled event is $\mathcal{A}_i$ is at most $\mathbb{P}[\mathcal{A}_i]$. Prove this or find a counterexample.