CSE 525: Randomized Algorithms and Probability **Spring 2025**

HW2

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Instructions

- You should think about each problem by yourself for at least an hour before choosing to collaborate with others.
- You are allowed to collaborate with fellow students taking the class in solving the problems. But you **must** write your solution on your own.
- You are not allowed to search for answers or hints on the web. You are encouraged to contact the instructor or the TAs for a possible hint.
- You cannot collaborate on Extra credit problems
- Solutions typeset in LATEX are preferred.
- Feel free to use the Discussion Board or email the instructor or the TAs if you have any questions or would like any clarifications about the problems.
- Please upload your solutions to Gradescope.

In solving these assignments, feel free to use these approximations:

$$1 - x \approx e^{-x}, \qquad \sqrt{1-x} \approx 1 - x/2, \qquad n! \approx (n/e)^n, \qquad \left(\frac{n}{k}\right)^k \leq \binom{n}{k} \leq \left(\frac{en}{k}\right)^k$$

1) Let G be a 3-uniform hypergraph with n vertices and $m \geq n$ edges, i.e., every edge has exactly three vertices. Design a randomize that finds an independent set I in G of expected size at least $\Omega(n^{3/2}/\sqrt{m})$-vertices. Note that $I \subseteq V$ is an independent set if it doesn't contain all 3 vertices of any edge of G.

2) Use the second moment method to show that there is a phase transition for connectivity in a $G(n,p)$ graph. Let $\mathcal{E}$ be the event that $G(n,p)$ is connected. Namely prove the following two facts:

   i) If $p \ll \frac{\log n}{n}$, then $\mathbb{P}[\mathcal{E}] \to 0$ as $n \to \infty$.
   Hint: Let $\mathcal{E}'$ be the event that $G(n,p)$ has an isolated vertex. Prove that $\mathbb{P}[\mathcal{E}'] \to 1$ as $n \to \infty$.

   ii) If $p \gg \frac{\log n}{n}$, then $\mathbb{P}[\mathcal{E}] \to 1$ as $n \to \infty$.