Homework #1

Due Tuesday, October 20

Recommended Reading: Motwani, Raghavan, chapters 1, 3, 4

Instructions: Choose as many of the following problems as you care to work on, and take each as far as you can. There are some that we don't know how to solve. We don't expect you to tackle all the problems. Keep your answers clear and concise. If you use references, please include citations. If you work with other students on a problem, please submit a single writeup of that problem with all the students names on it.

1. Motwani/Raghavan
   - p. 10, Excercises 1.3 and 1.4
   - p. 26, Problems 1.8, 1.10
   - pp. 64–65, Problems 3.4, 3.6, 3.14
   - pp. 98–99, Problems 4.6, 4.8, 4.13
2. Consider the weighted min-cut problem, where on each edge there is a weight (not necessarily integer) and the goal is to find a cut of minimum total weight. Analyse the randomized algorithm that at each step chooses each edge in proportion to its weight and contracts it.
3. Consider the max-cut problem. Given a graph, partition the vertices into two sets such that a *maximum* number of edges cross the cut. Use the probabilistic method to show that there exists a cut of size at least $E/2$. Does your argument generalize to weighted max-cut?
4. Research Problem 9.2 on page 273 of Motwani/Raghavan.
5. Try designing and, to the best of your ability, analyzing randomized algorithms for finding a satisfying assignment to a boolean formula in 3-CNF. Clearly one can't expect to get a polynomial running time, but algorithms with expected running time $o(2^n)$ may be possible.
6. Try to characterize integer programs for which randomized rounding of the linear programming relaxation will give a "good" solution.
7. Propose problems related to the material that are, to the best of your knowledge, open. Try to attack any of these that appeals to you.