Lecture 1

Introduction to Game Theory

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In this course we will look at problems and issues which fall in the intersection of Computer Science and Economics. Though a lot of research has been done in problems in both Computer Science and Economics, their goals have generally been orthogonal and looking at a problem from these two perspectives at the same time poses new challenges.

1.1 Motivating Examples

To give a flavor of the problems we would be considering in this course let us consider a very standard optimization problem in Computer Science– maximum weighted matching. It is well known that the optimal matching can be computed in polynomial time.

As an example, consider the bipartite graph given in Figure 1.1.

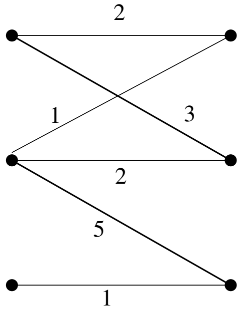

Figure 1.1: An input for the maximum matching problem.

The optimal solution is the set of edges with weight 5 and 3. Now, consider the following real life situation where the vertices on the left side represent customers and the vertices on the right side represent houses being sold by a real estate firm. For any edge (i, j), the weight presents how much customer i is willing to pay for house j. Now if the customer knows that the real estate firm is going to use a maximum weighted matching to decide which house to sell to which customer, a customer may be tempted to lie about how much they "value" a house. For the example in Figure 1.1, the second customer with the weight 5 edge will decrease his value to 3 as she would still get the house at that price.

It is not very obvious how one can reason about such scenarios. In the housing example if the goal of the real estate firm is to make as much revenue as possible (which any sensible firm would do) what is the best way for the firm to decide on which houses to allocate to which customers given that they can lie about how much they are really willing to pay. From the customers' perspective, given the rules by which the firm decides on an allocation, what is her best strategy ? Finally, given the rules that the firm sets up what is going to be the outcome given the customers are going to formulate their own strategies. These are the types of questions game theory helps us answer.

As another example consider the problem of finding the shortest path in the graph. As an optimization problem this is easy to solve. However, note that in the optimization framework, it is implicitly assumed that the weights on edges are the correct weights. However, one can envision a scenario where each edge is owned by separate entity and they have incentive to lie about the true values of the weights– in such a scenario how can one solicit the true values from these entities ?

1.2 Dave's Problems and Challenges

In this section we will outline two problems that Dave talked about in the lecture. These problems are motivated by real world problems and deals with issues which we will consider in this class. The problem statements are not yet very crisply posed and one of the first challenges is to formulate these problems precisely.

1.2.1 ISP Routing problem

In the current internet "architecture", there are a few "backbone" networks– that is, sub-regions of the internet which are controlled by different (and very often competing) entities called Internet Service Providers or ISPs (for eg, Sprint and ATT). To illustrate the problem we will consider the simple model of Figure 1.2.

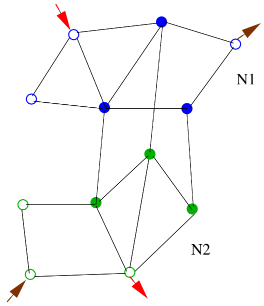

Figure 1.2: A simplified "model" of the internet. The blue nodes belong to ISP $N1$ while the green nodes belong to ISP $N2$. The incoming arrows point to a source of a request and the outgoing arrow comes out from the destination of a request. Packets are routed from one network to the other only along the dotted links.

Figure 1.2 shows the network of two ISPs, $N1$ and $N2$. Now consider a request which originates at the node pointed to by the red arrow whose destination is where the red arrow comes out. Now, $N1$ does not care where in $N2$ it routes the request as long as it gets the request off it's network. Similarly, consider the request denoted by the brown arrows– in this case $N2$ does not care which node in $N1$ it routes the request to as long as it gets the request off it's network. It is not very hard to see that if $N1$ and $N2$ "cooperated", both would have been better off.

Note that in this problem the ISPs are allowed to choose where the traffic leaves there network. The challenges in this problem are threefold–

1. How should one design the network ?
2. How should the existing protocols used by the ISPs change ?,or
3. A combination of the two such that it is efficient for the individual ISPs as well for all the ISPs globally.

The current state of the art in this field are protocols which are designed to minimize the maximum congestion (that is, how much are the links utilized). Typically the ISPs run their network at about 50% of their capacity.

1.2.2 Wireless Network problem

The next problem is even less well formulated than the last one. We will consider wireless networks. For the simplest version of the problem consider a wireless network with a single access point which clients connect to. Figure 1.3 shows a simple wireless network.

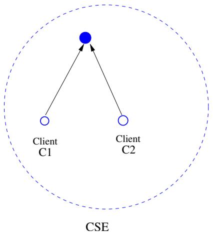

Figure 1.3: A simplified version of a wireless network. The filled in node is the access point.

There are two resources/choices for the person setting up the wireless network–

- *Frequency*: There are a few channel which use different frequencies are thus "orthogonal" in the sense that two signals on two different channels do not interfere.
- *Power*: The more power a client uses the further the reach of the signal and fewer the errors incurred.

The real life situation, however, is more like the one showed in Figure 1.4.

In the scenario depicted by Figure 1.4, there are two networks whose signals interfere with each other. There are a couple of possible solutions–

1. The different networks use different channels. This solution would require cooperation between the two networks– for example getting all wireless users in your apartment building to come together for

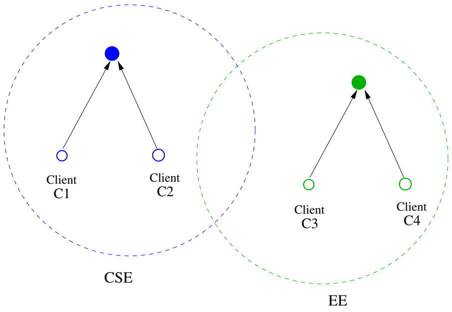

Figure 1.4: A real life situation where two wireless networks (for eg, one belonging to CSE and the other to EE which have departmental buildings next to each other) have interference.

a meeting, let alone coordinate their wireless use seems like a hopeless task. More importantly, there are only a small number of orthogonal channels– this solution would not scale with the number of different wireless networks.

2. Each client uses large power. Using higher power increases the reach. However, this would also lead to a larger interference.

It is not hard to see that if there were just one network then coming up with a layout which would have the least interference is an optimization problem. However, like the previous problems we have considered, this one has an added dimension of having agents which necessarily do not coordinate their activities. The main task here is to design protocols for clients such that coordination happens.

For more details and for a first attempt at using game theory to solve these problems take a look at [1].

1.3 Basic Game Theory Background

Suppose there are two people in the network and they are using TCP as their congestion control protocol. Let C denote the fact that an user is using a correctly implemented protocol and D denote the fact that an user is using a faulty implementation of the protocol (that is, it does not backoff when the TCP protocols says it must). Consider the table in Figure 1.5.

	C	D
C	$-1, -1$	$-9, 0$
D	$0, -9$	$-6, -6$

Figure 1.5: An entry $-x, -y$ implies that the first player incurs a delay of x ms while the second player incurs a delay of y ms.

The "game" in Figure 1.5 is the classical Prisoners Dilemma game. In a nutshell, game theory studies the interaction between rational decision makers. A game is a set of rules and payoffs of different decision makers based on their decisions. Solution of a game is the natural, rational outcome given that rational players are playing the game.

There are various dimensions along which game theory is cut and we would be addressing these concerns in this course.

- **Non-cooperative Game Theory vs. Cooperative Game Theory.** The distinction is in the modeling level of the payoffs. In non-cooperative game theory the payoffs are the level of individuals while in cooperative game theory the modeling is at the level of groups.
- **Strategic (or Normal) vs. Extensive Games.** Strategic games are one round games (they are also known as one shot games) while extensive games are multiple round games. One way to get an extensive game is to take a normal game and repeat it multiple number of times.
- **Complete information vs. Incomplete information Games.** In complete information games, every player knows the whole structure of payoffs while in the incomplete information games, a player need not know all of the payoff structure (for eg, ISPs do not have full information about other ISPs).

1.4 Rest of the Lecture

The rest of the lecture used slides from the Game Theory course by Chris Wallace (see the course website). The slides are self–contained except for the notion of *Pareto Optimal* strategy which we define below (we use notations from the slides).

Definition 1.1 (Pareto Optimal). A strategy profile $s^* \in S$ is *Pareto optimal* if all for every other $s' \in S$ for which there is a player i with $u_i(s') > u_i(s^*)$ there exist another player j such that $u_j(s^*) > u_j(s')$.

References

[1] R. Mahajan, M. Rodrig, D. Wetherall and J. Zahorjan. Experiences Applying Game Theory to System Design. In SIGCOMM Workshop on Practice and Theory of Incentives and Game Theory in Networked Systems (PINS), 2004.