On the Formation of Networks and Groups

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Abstract

We provide an introduction to and overview of the volume on *Models of the Strategic Formation of Networks and Groups.*
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1 Introduction

The organization of individual agents into networks and groups has an important role in the determination of the outcome of many social and economic interactions. For instance, networks of personal contacts are important in obtaining information about job opportunities (e.g., Boorman (1975) and Montgomery (1991)). Networks also play important roles in the trade and exchange of goods in non-centralized markets (e.g., Tesfatsion (1997, 1998), Weisbuch, Kirman and Herreiner (1995)), and in providing mutual insurance in developing countries (e.g., Fafchamps and Lund (1997)). The partitioning of societies into groups is also important in many contexts, such as the provision of public goods and the formation of alliances, cartels, and federations (e.g., Tiebout(1956) and Guesnerie and Oddou (1981)).

Our understanding of how and why such networks and groups form and the precise way in which these structures affect outcomes of social and economic interaction is the main focus of this volume. Recently there has been concentrated research focused on the formation and design of groups and networks, and their roles in determining the outcomes in a variety of economic and social settings. In this volume, we have gathered together some of the central papers in this recent literature which have made important progress on this topic. These problems are tractable and interesting, and from these works we see that structure matters and that clear predictions can be made regarding the implications of network and group formation. These works also collectively set a rich agenda for further research.

In this introduction, we provide a brief description of the contributions of each of the papers. We also try to show how these papers fit together, provide some view of the historical progression of the literature, and point to some of the important open questions.

2 A brief Description of Some Related Literatures

There is an enormous literature on networks in a variety of contexts.

The "social networks" literature in sociology (with some roots in anthropology and social psychology) examines social interactions from theoretical and empirical viewpoints. That literature spans applications from family ties through marriage in 15th century Florence to needle sharing among drug addicts, to networks of friendship and advice among managers. An excellent and broad introductory text to the social networks literature is Wasserman and Faust (1994). One particular area of overlap with economics is the portion of that literature on exchange networks. The Bienenstock and Bonacich (1997) paper in this volume (and discussed more below) is a nice source for some perspective on and references to that literature. The analysis of the incentives to form networks and groups and resulting welfare implications, the focus of most of the papers in this volume, is largely complementary to the social networks literature both in its perspective and techniques.

There are also various studies of networks in economics and operations research of transportation and delivery networks.[1] One example would be the routing chosen by airlines which has been studied by Hendricks, Piccione and Tan (1995) and Starr and Stinchcombe (1992). One major distinguishing feature of the literature that we focus on in this volume is that the parties in the network or group are economic or social actors. A second distinguishing feature is that the focus is on the incentives of individual actors to form networks and groups.

Thus, the focus here is on a series of papers and models that have used formal game theoretic reasoning to study the formation of networks and other social structures.[2]

3 Overview of the Papers in the Volume

Cooperation Structures and Networks in Cooperative Games

An important first paper in this literature is by Myerson (1977). Myerson started from cooperative game theory and layered on top of that a

[1]There is also a literature in industrial organization that surrounds network externalities, where, for instance a consumer prefers goods that are compatible with those used by other individuals (see Katz and Shapiro (1994)). There, agents care about who else uses a good, but the larger nuances of a network with links does not play any role. Young (1998) provides some insights into such interactions where network structures provide the fabric for interaction, but are taken to be exogenous.

[2]Also, our focus is primarily on the formation of networks. There is also a continuing literature on incentives in the formation of coalitions that we shall not attempt to survey here, but mention at a few points.

network structure that described the possible communication or cooperation that could take place. The idea was that a coalition of individuals could only function as a group if they were connected through links in the network. Thus, this extends standard cooperative game theory where the modeler knows only the value generated by each potential coalition and uses this to make predictions about how the value of the overall society will be split among its members. One perspective on this is that the members of society bargain over how to split the value, and the values of the different coalitions provide threat points in the bargaining.[3] The enrichment from the communication structures added by Myerson is that it provides more insight into which coalitions can generate value and thus what threats are implicit when society is splitting.

More formally, Myerson starts from the familiar notion of a *transferable utility* game (N, v), where N is a set of players and v is a characteristic function denoting the worth $v(S)$ of each coalition $S \subset N$. He defines a *cooperation structure* as an *non-directed* graph g among the individuals. So, a graph represents a list of which individuals are linked to each other, with the interpretation that if individual i is linked to individual j in the graph, then they can communicate and function together. Thus, a network g partitions the set of individuals into different groups who are connected to each other (there is a path from each individual in the group to every other individual in the group). The value of a coalition S under the network g is simply the sum of the values of the sub-coalitions of S across the partition of S induced by g. For instance, consider a cooperative game where the worth of coalition $\{1,2\}$ is 1, the worth of coalition $\{3,4\}$ is 1, and the worth of coalition $\{1,2,3,4\}$ is 3. If under the graph g the only links are that individual 1 is linked to 2 and individual 3 is linked to 4, then the worth of the coalition $\{1,2,3,4\}$ under the restriction to the graph g is simply $1+1=2$, rather than 3, as without any other links this is the only way in which the group can function. So in Myerson's setting, a network or graph g coupled with a characteristic function v results in a *graph-restricted* game v^g.

In Myerson's setting, an *allocation rule*[4] describes the distribution of payoffs amongst individuals for every pair (v, g). This may represent the natural

[3]From a more normative perspective, these coalitional values may be thought of as providing a method of uncovering how much of the total value that the whole society produces that various individuals and groups are responsible for.

[4]This is somewhat analogous to a solution concept in cooperative game theory.

payoff going to each individual, or may represent some additional intervention and transfers. Myerson characterizes a specific allocation rule which eventually became referred to as the *Myerson value*. In particular, Myerson looks at allocation rules that are fair: the gain or loss to two individuals from the addition of a link should be the same for the two individuals involved in the link; and are balanced in that they are spreading exactly the value of a coalition (from the graph-restricted game) among the members of the coalition. Myerson shows that the unique allocation rule satisfying these properties is the *Shapley value* of the graph-restricted game.[5]

While Myerson's focus was on characterizing the allocation rule based on the Shapley value, his extension of cooperative game theory to allow for a network describing the possibilities for cooperation was an important one as it considerably enriches the cooperative game theory model not only to take into account the worth of various coalitions, but also how that worth depends on a structure describing the possibilities of cooperation.

Network Formation More Generally

While Myerson's model provides an important enrichment of a cooperative game, it falls short of providing a general model where value is network dependent. For example, the worth of a coalition $\{1, 2, 3\}$ is the same whether the underlying network is one that only connects 1 to 2 and 2 to 3, or whether it is a complete network that also connects 1 to 3. While this is of interest in some contexts, it is somewhat limited as a model of networks. For instance, it does not permit there to be any cost to a link or any difference between being directly linked to another individual versus only being indirectly linked.

The key departure of Jackson and Wolinsky (1996) from Myerson's approach was to start with a value function that is defined on networks directly, rather than on coalitions. Thus, Jackson and Wolinsky start with a value function v that maps each network into a worth or value. Different networks connecting the same individuals can result in different values, allowing the value of a group to depend not only on who is connected but also how they are connected. This allows for costs and benefits (both direct and indirect) to accrue from connections. In the Jackson-Wolinsky framework, an allocation rule specifies a distribution of payoffs for each pair network and value

[5]An interesting feature of Myerson's characterization is that he dispenses with *additivity*, which is one of the key axioms in Shapley's original characterization. This becomes implicit in the balance condition given the network structure.

function. One result of Jackson and Wolinsky is to show that the analysis of Myerson extends to this more general setting, and fairness and component balance again lead to an allocation rule based on the Shapley value.

One of the central issues examined by Jackson and Wolinsky is whether *efficient* (value maximizing) networks will form when self-interested individuals can choose to form links and break links. They define a network to be *pairwise stable* if no pair of individuals wants to form a link that is not present and no individual gains by severing a link that is present.

They start by investigating the question of the compatibility of efficiency and stability in the context of two stylized models. One is the *connections model*, where individuals get a benefit $\delta \in [0,1]$ from being linked to another individual and bear a cost c for that link. Individuals also benefit from indirect connections - so a friend of a friend is worth δ^2 and a friend of a friend of a friend is worth δ^3, and so forth. They show that in this connections model efficient networks take one of three forms: an empty network if the cost of links is high, a star-shaped network for middle ranged link costs, and the complete network for low link costs. They demonstrate a conflict between this very weak notion of stability and efficiency - for high and low costs the efficient networks are pairwise stable, but not always for middle costs. This also holds in the second stylized model that they call the *co-author* model, where benefits from links come in the form of synergies between researchers.

Jackson and Wolinsky also examine this conflict between efficiency and stability more generally. They show that there are natural situations (value functions), for which under any allocation rule belonging to a fairly broad class, no efficient network is pairwise stable. This class considers allocation rules which are component balanced (value is allocated to the component of a network which generated it) and are anonymous (do not structure payments based on labels of individuals but instead on their position in the network and role in contributing value in various alternative networks). Thus, even if one is allowed to choose the allocation rule (i.e., transfer wealth across individuals to try to align incentives according to some mild restrictions) it is impossible to guarantee that efficient networks will be pairwise stable. So, the tension between efficiency and stability noted in the connections and co-author models is a much broader problem. Jackson and Wolinsky go on to study various conditions and allocation rules for which efficiency and pairwise stability are compatible.

While Jackson and Wolinsky's work provides a framework for examining the relationship between individual incentives to form networks and overall societal welfare, and suggests that these may be at odds, it leaves open many questions. Under exactly what circumstances (value functions and allocation rules) do individual incentives lead to efficient networks? How does this depend on the specific modeling of the stability of the network as well as the definition of efficiency?

Further Study of the Compatibility of Efficiency and Stability

This conflict between stability and efficiency is explored further in other papers.

Johnson and Gilles (2000) study a variation on the connections model where players are located along a line and the cost of forming a link between individuals i and j depends on the spatial distance between them. This gives a geography to the connections model, and results in some interesting structure to the efficient networks. Stars no longer play a central role and instead chains do. It also has a dramatic impact on the shape of pairwise stable networks, as they have interesting local interaction properties. Johnson and Gilles show that the conflict between efficiency and pairwise stability appears in this geographic version of the connections model, again for an intermediate range of costs to links.

Dutta and Mutuswami (1997) adopt a "mechanism design" approach to reconcile the conflict between efficiency and stability. In their approach, the allocation rule is analogous to a mechanism in the sense that this is an object which can be designed by the planner. The value function is still given exogenously and the network is formed by self-interested players or agents, but properties of the allocation rule such as anonymity are only applied at stable networks. That is, just as a planner may be concerned about the ethical properties of a mechanism only at *equilibrium*, Dutta and Mutuswami assume that one needs to worry only about the ethical properties of an allocation rule on networks which are equilibria of a formation game. That is, the design issue with which Dutta and Mutuswami are concerned is whether it is possible to define allocation rules which are "nice" on the set of equilibrium networks. They construct allocation rules which satisfy some desirable properties on equilibrium graphs. Of course, the construction deliberately uses some ad hoc features "out of equilibrium".

The network formation game that is considered by Dutta and Mutuswami is discussed more fully below, and offers an alternative to the notion of pairwise stability.

The paper by Jackson (2001) examines the tension between efficiency and stability in further detail. He considers three different definitions of efficiency, which consider the degree to which transfers are permitted among individuals. The strong efficiency criterion of Jackson and Wolinsky is only appropriate to the extent that value is freely transferable among individuals. If more limited transfers are possible (for instance, when one considers component balance and anonymity), then a constrained efficiency notion or even Pareto efficiency become appropriate. Thus the notion of efficiency is tailored to whether the allocation rule is arising naturally, or to what extent one is considering some further intervention and reallocation of value. Jackson studies how the tension between efficiency and stability depends on this perspective and the corresponding notion of efficiency used. He shows how this applies in several models including the Kranton and Minehart (1998) model, and a network based bargaining model due to Corominas-Bosch (1999). He also shows that the Myerson allocation rule generally has difficulties guaranteeing even Pareto efficiency, especially for low costs to links. Individuals have incentives to form links to better their bargaining position and thus their resulting Myerson allocation. When taking these incentives together, this can result in over-connection to a point where all individuals suffer.

Stability and Efficiency in Directed Networks Models

Non-directed networks capture many applications, especially those where mutual consent or effort is required to form a link between two individuals. However, there are also some applications where links are directed or unilateral. That is, there are contexts where one individual may form a link with a second individual without the second individual's consent, as would happen in sending a paper to another individual. Other examples include web links and one-sided compatibility of products (such as software). Such settings lead to different incentives in the formation of networks, as mutual consent is not needed to form a link. Hence the analysis of such directed networks differs from that of non-directed networks.

Bala and Goyal (2000a) analyze a communication model which falls in this directed network setting. In their model, value flows costlessly through the network along directed links. This is similar to the connections model, but with δ close to 1 and with directional flow of communication or information. Bala and Goyal focus on the dynamic formation of networks in this directed communications model. The network formation game is played repeatedly, with individuals deciding on link formation in each period. Bala and Goyal use a version of the best response dynamics, where agents choose links in response to what happened in the previous period, and with some randomization when indifferent. In this setting, for low enough costs to links, the process leads naturally to a limiting network which has the efficient structure of a wheel.

Dutta and Jackson (2000) show that while efficiency is obtained in the Bala and Goyal communication model, the tension between efficiency and stability reemerges in the directed network setting if one looks more generally. As one might expect, the nature of the conflict between stability and efficiency in directed networks differs from that in non-directed networks. For instance, the typical (implicit) assumption in the directed networks framework is that an agent can *unilaterally* form a link with any other agent, with the cost if any of link formation being borne by the agent who forms the link. It therefore makes sense to say that a directed network is stable if no agent has an incentive to either break an existing link or create a new one.[6] Using this definition of stability, Dutta and Jackson show that efficiency can be reconciled with stability either by distributing benefits to *outsiders* who do not contribute to the productive value of the network or by violating equity; but that the tension between stability and efficiency persists if one satisfies anonymity and does not distribute value to such outsiders.

Bala and Goyal (2000) also analyze the efficiency-stability conflict for a hybrid model of information flow, where the cost of link formation is borne by the agent setting up the link, but where both agents can access each other's information regardless of who initiated the link. Bala and Goyal give the example of a telephone call, where the person who makes the call bears the cost of the call, but both persons are able to exchange information. In their model, however, each link is *unreliable* in the sense that there is a positive probability that the link will fail to transmit the information. Bala and Goyal find that if the cost of forming links is low or if the network is highly reliable, then there is no conflict between efficiency and stability. Bala and Goyal also analyze the structure of stable networks in this setting.

Modeling the Formation of Networks

[6] In contrast, the implicit assumption in the undirected networks framework is that both i and j have to agree in order for the link ij to form.

Notice that pairwise stability used by Jackson and Wolinsky is a very weak concept of stability - it only considers the addition or deletion of a single link at a time. It is possible that under a pairwise stable network some individual or group would benefit by making a dramatic change to the network. Thus, pairwise stability might be thought of as a necessary condition for a network to be considered stable, as a network which is *not* pairwise stable may not be formed irrespective of the actual process by which agents form links. However, it is not a sufficient condition for stability, In many settings pairwise stability already dramatically narrows the class of networks, and noting a tension between efficiency and pairwise stability implies that such a tension will also exist if one strengthens the demands on stability. Nevertheless, one might wish to look beyond pairwise stability to explicitly model the formation process as a game. This has the disadvantage of having to specify an ad hoc game, but has the advantage of permitting the consideration of richer forms of deviations and threats of deviations. The volume contains several papers devoted to this issue.

This literature owes its origin to Aumann and Myerson (1988), who modeled network formation in terms of the following extensive form game.[7] The extensive form presupposes an exogenous ranking of pairs of players. Let this ranking be $(i_1j_1, \ldots, i_nj_n)$. The game is such that the pair i_kj_k decide on whether or not to form a link knowing the decisions of all pairs coming before them. A decision to form a link is binding and cannot be undone. So, in equilibrium such decisions are made with the knowledge of which links have already formed (or not), and with predictions as to which links will form as a result of the given pair's decision. Aumann and Myerson assume that after all pairs have either formed links or decided not to, then allocations come from the Myerson value of the resulting network g and some graph restricted cooperative game v^g. They are interested in the subgame perfect equilibrium of this network formation game.

To get a feeling for this, consider a symmetric 3-person game where $v(S) = 0$ if $\#S = 1$, $v(S) = 40$ if $\#S = 2$ and $v(N) = 48$. An efficient graph would be one where at least two links form so that the grand coalition can realize the full worth of 48. Suppose the ranking of the pairs is $12, 13, 23$. Then, if 1 and 2 decide to form the link 12 and refrain from forming links with 3, then they each get 20. If all links form, then each player gets 16.

[7] A precursor of the network formation literature can be found in Boorman (1975).

The unique subgame perfect equilibrium in the Aumann-Myerson extensive form is that only the link 12 will form, which is inefficient.

A crucial feature of the game form is that if pair $i_k j_k$ decide not to form a link, but some other pair coming after them does form a link, then $i_k j_k$ are allowed to reconsider their decision.[8] It is this feature which allows player 1 to make a credible threat to 2 of the form "I will not form a link with 3 if you do not. But if you do form a link with 3, then I will also do so." This is what sustains $g = \{12\}$ as the equilibrium link. Notice that after the link 12 has been formed, if 1 refuses to form a link with 3, then 2 has an incentive to form the link with 3 - this gives her a payoff of $29\frac{1}{3}$ *provided* 1 cannot come back and form the complete graph. So, it is the possibility of 1 and 3 coming back into the game which deters 2 from forming the link with 3.

Notice that such threats cannot be levied when the network formation is simultaneous. Myerson (1991) suggested the following simultaneous process of link formation. Players simultaneously announce the set of players with whom they want to form links. A link between i and j forms if both i and j have announced that they want a link with the other. Dutta, van den Nouweland, and Tijs (1998)[9] model link formation in this way in the context of the Myerson model of cooperation structures. Moreover, they assume that once the network is formed, the eventual distribution of payoffs is determined by some allocation rule within a class containing the Myerson value. The entire process (formation of links as well as distribution of payoffs) is a normal form game. Their principal result is that for all superadditive games, a complete graph (connecting the grand coalition) or graphs that are payoff equivalent will be the undominated equilibrium or coalition-proof Nash equilibrium.

The paper by Slikker and van den Nouweland (2000) considers a variant on the above analysis, where they introduce an explicit cost of forming links. This makes the analysis much more complicated, but they are still able to obtain solutions at least for the case of three individuals. With costs to links, they find the surprising result that link formation may not be monotone in link costs: it is possible that as link costs increase more links are formed. This depends in interesting ways on the Myerson value, the way that individual

[8] As Aumann and Myerson remark, this procedure is like bidding in bridge since a player is allowed to make a fresh bid if some player bids after her.

[9] See also Qin(1996).

payoffs vary with the network structure, and also on the modeling of network formation via the Aumann and Myerson extensive form.

Dutta and Mutuswami (1997) (discussed above) use the same normal form game for link formation in the context of the network model of Jackson and Wolinsky. They note the relationship between various solution concepts such as strong equilibrium and coalition proof Nash equilibrium to pairwise stability.[10] They (as well as Dutta, van den Nouweland and Tijs (1998)) also discuss the importance of considering only undominated strategies and/or deviations by at least two individuals in this sort of game, so as to avoid degenerate Nash equilibria where no agent offers to form any links knowing that nobody else will.

Bargaining and Network Formation

One aspect that is present in all of the above mentioned analyses is that the network formation process and the way in which value is allocated to members of a network are separated. Currarini and Morelli (2000) take the interesting view that the allocation of value among individuals may take place simultaneously with the link formation, as players may bargain over their shares of value as they negotiate whether or not to add a link.[11] The game that Currarini and Morelli analyze is one where players are ranked exogenously. Each player sequentially announces the set of players with whom he wants to form a link as well as a payoff demand, as a function of the history of actions chosen by preceding players. Both players involved in a link must agree to form the link. In addition, payoff demands within each component of the resulting graph must be consistent. Currarini and Morelli show that for a large class of value functions, all subgame perfect equilibria are efficient. This differs from what happens under the Aumann and Myerson game. Also, as it applies for a broad class of value functions, it shows that the tension between stability and efficiency found by Jackson and Wolinsky may be overcome if bargaining over value is tied to link formation.

Gerber (2000) looks at somewhat similar issues in the context of coalition formation. With a few exceptions, the literatures on bargaining and on coalition formation, either look at how worth is distributed taking as given that the grand coalition will form, or look at how coalitions form taking as given

[10]See also Jackson and van den Nouweland (2001) for a detailed analysis of a strong equilibrium based stability concept where arbitrary coalitions can modify their links.

[11]See also Slikker and van den Nouweland (2001) and Mutuswami and Winter (2000).

how coalitions distribute worth. Gerber stresses the simultaneous determination of the payoff distribution and the coalition structure, and defines a new solution for general NTU games. This solution views coalitions as various interrelated bargaining games which provide threat points for the bargaining with any given coalition, and ultimately the incentives for individuals to form coalitions. Gerber's solution concept is based on a consistency condition which ties these games together. Gerber shows how this applies in some special cases (including the marriage market which ties in with the networks models) as well as several examples, and illustrates the important differences between her solution and others.

Dynamic Network Formation

The papers discussed above have largely analyzed network formation in static settings (taking an extensive form to be essentially static). The main exception is that of best response dynamics in the directed communications model of Bala and Goyal (2000a).

Watts (2001) also departs from the static modeling tradition.[12] In the context of the connections model of Jackson and Wolinsky, she considers a framework where pairs of agents meet over time, and decide whether or not to form or sever links with each other. Agents are myopic and so base their decision on how the decision on the given link affects their payoffs, given the current network in place. The network formation process is said to reach a *stable state* if no additional links are formed or broken in subsequent periods. A principal result is that a stable state is often inefficient, although this depends on the precise cost and benefit parameters. A particularly interesting result applies to a cost range where a star network is both pairwise stable and efficient, but where there are also some inefficient networks that are stable states. Watts shows that as the number of individuals increases, the probability[13] that a star forms goes to 0. Thus as the population increases the particular ordering which is needed to form a star (the efficient network) becomes less and less likely relative to orderings leading to some other stable states.

[12] The literature on the dynamic formation of networks has grown following Watts' work, and there are a number of recent papers that study various stochastic models of network formation. These include Jackson and Watts (1998, 1999), Goyal and Vega-Redondo (1999), Skyrms and Pemantle (2000), and Droste, Gilles and Johnson (2000).

[13] Links are identified randomly and then agents decide whether to add or delete them.

Networks for the Trade and Exchange of Goods

There has also been study of network models in some other specific contexts. For instance, the two papers by Kranton and Minehart (1998, 2000) focus on networks of buyers and sellers, where goods are to be exchanged between connected individuals, but the terms of trade can depend on the overall set of opportunities that the connected individuals have. The first paper considers buyers with private values who can bid in auctions of sellers to whom they are connected. Buyers gain from being involved in more auctions as they then have a better chance of obtaining an object and at a lower expected cost. Sellers gain from having more buyers participate in their auction as it increases the expected highest valuation and thus willingness to bid, and also increases the competition among buyers. Kranton and Minehart show the striking result that the change in expected utility that any buyer sees from adding a link to some seller is precisely the overall social gain from adding that link. Thus, if only buyers face costs to links, then they have incentives to form a socially efficient network. They also show that if sellers face costs to invest in providing the good for sale, then inefficiency can arise.[14]

In the second paper, Kranton and Minehart (2000) develop a theory of competition in networks which intends to look more generally at how the connection structure among buyers and sellers affects terms of trades. The new concept that they introduce is that of "opportunity paths" which describe the various ways in which individuals can effect trades. The pattern of opportunity paths is central in determining the trades that occur, and Kranton and Minehart provide a series of results indicating how the opportunity paths determine the resulting prices and utilities to the various agents in the network.

Bloch and Ghosal (2000) also analyze how interrelationships among buyers and sellers affect terms of trade. Their analysis is not so network dependent, but focuses more directly on the issue of cartel formation amongst buyers and sellers. In particular, they are concerned with how collusion on one side of the market affects cartel formation on the other side of the market. They build on the bargaining model of Rubinstein and Wolinsky (1990), where a random matching process is a central determinant of the terms of

[14] Jackson (2001) points out that a similar inefficiency result holds in Kranton and Minehart's model if sellers face any costs to links and pairwise stability is required.

trade. They find that there is at most one stable cartel structure, which if it exists consists of equal size structures and cartels each remove one trader from the market. This suggests the emergence of a balance between the two sides of the market.

The paper by Bienenstock and Bonacich (1997) provides discussion of how cooperative game theory concepts can be useful in modeling network exchange. In discussing the way in which notions of transferable utility cooperative game theory can be applied to study exchange of goods in networks, Bienenstock and Bonacich provide a nice overview of the network exchange literature, and some pointed discussion about the alternative behavioral assumptions that can be made, and how utility theory and viewing things as a cooperative game can be a useful lens. An important point that they make is that using game theoretic tools allows for an understanding of how structural outcomes depend on underlying characteristic function and how this relates to the structure itself. That network structure is important in determining power and distribution of resources is a fundamental understanding in most of the work on the subject. Bienenstock and Bonacich (1997) outline why cooperative game theory can be a useful tool in studying this relationship. They discuss the possible use of the kernel in some detail.

Networks in Other Specific Contexts

The remaining papers in the volume, Barbera and Dutta (2000) and Gehrig, Regibeau, and Rocket (2000), are concerned with different issues connected with the organizational structure of firms.

Barbera and Dutta consider a labor-managed firm where there are two types of tasks and two types of workers (skilled and unskilled), with the skilled workers being more productive in the first type of task. Their interest is in the possibility of constructing payment or reward schemes which will induce workers to reveal types correctly, and thereby sort themselves correctly into tasks. They show that using various hierarchical structures in rewards, can provide strong incentives for workers to properly sort themselves into tasks.

Gehrig, Regibeau, and Rocket study the internal organization of firms in a context where the firm has to evaluate cost-reducing R & D projects. They compare hierarchies versus polyarchies. In hierarchies unanimous approval by a number of reviewers is required in order for the project to be approved, while the approval of any one reviewer is sufficient in a polyarchy. They allow for different reviewers to have different hurdles for approval, and then study conditions under which the polyarchy can dominate the hierarchy.

4 Some important Open Questions

We end our introduction to this volume by briefly describing some important issues regarding network and group formation which deserve closer attention, and are suggested through the collection of papers here.

Even a cursory look at the papers in this volume indicates that the conflict between stability and efficiency is of significant interest. Nevertheless, much remains to be known about the conditions under which the conflict will arise. Some of the papers have examined this conflict in the abstract, and others in the context of very pointed and specific models. While we see some themes emerging, we do not yet have an overarching characterization of exactly what leads to an alignment between individual incentives and societal welfare, and what leads these to diverge. Jackson (2001) suggests that at least some of the tension can be categorized as coming from two separate sources: one source is that of a classic externality where individuals do not internalize the full societal effects of their forming or severing a link; and another source is the incentives of individuals to form or sever links in response to how the network structure affects bargaining power and the allocation of value, rather than in response to how it affects overall value. Whether inefficiency can always be traced to one (or both) of these sources and more basically whether this is a useful taxonomy, are open questions.

There are also several issues that need to be addressed in the general area of the formation of networks. It becomes clear from comparisons within and across some of the papers, that the specific way in which one models network stability or formation can matter. This is clearly borne out by comparing the Aumann–Myerson prediction that inefficient networks might form with that of Dutta et al who find efficiency at least in superadditive games. We need to develop a fuller understanding of how the specifics of the process matter, and tie this to different sorts of applications to get some sense of what modeling techniques fit different sorts of problems.

Perhaps the most important (and possibly the hardest) issue regarding modeling the formation of networks is to develop fuller models of networks forming over *time*, and in particular allowing for players who are *farsighted*.

Farsightedness would imply that players' decisions on whether to form a network are not based solely on current payoffs, but also on where they expect the process to go and possibly from emerging steady states or cycles in network formation. We see some of this in the Aumann and Myerson (1988) extensive form, but it is artificially cut by the finiteness of the game. It is conceivable that, at least in some contexts, farsightedness may help in ensuring efficiency of the stable state. For instance, if there are increasing returns to network formation, then myopic considerations may result in the null network being formed since no one (or pair) may want to incur the initial cost. However, the initial costs may well be recouped in the long-run, thereby facilitating the formation of efficient networks. This is only one small aspect of what farsighted models might bring.

More work can also be undertaken in constructing, analyzing, and characterizing "nice" allocation rules, as well as ones that might arise naturally under certain conditions. There are essentially two prominent single-valued solution concepts in cooperative game theory - the Shapley value and the nucleolus. While there is a close connection between characteristic functions and value functions, the special structure of networks may allow for the construction of allocation rules which do not have any obvious correspondence with solution concepts in cooperative game theory.

Also, the papers collected in this volume are all theoretical in nature. Many of them provide very pointed predictions regarding various aspects of network formation, albeit in highly stylized environments. Some of these predictions can be tested both in experiments,[15] as well as being brought directly to the data. The models can also be applied to some areas of particular interest, for example to examine whether decentralized labor markets, which depend a great deal on connections and network structure, function efficiently.

[15]See Charness and Corominas-Bosch (1999) and Corbae and Duffy (2000) for recent examples of testing such predictions.

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