University of Washington
Department of Computer Science and Engineering
CSE 521, Spring 1998

May 19, 1998

Homework 8, **Due May 27, 1998**

Problem 1. Maximum Flow using a small number of edges:

Prove that the problem of finding a maximum flow in a graph which uses as few edges as possible is NP-complete. Formally, the input is a directed graph G with capacities on the edges, and an integer K, and the question is "does there exist a maximum flow f such that at most K edges have $f(e) > 0$?".

Hint: Give a reduction from Vertex Cover.

Problem 2. Scheduling :

The scheduling problem faced by the University is to assign courses to time slots and rooms, and assign professors, TAs, and students to courses, so that everyone is happy. This is not easy.

Below are a collection of subproblems of University scheduling. Determine which ones are NP-complete. The reductions are straight forward if you start from the right well known NP-complete problem (e.g., graph colouring, three dimensional matching, hamiltonian path, bin packing, travelling salesman).

Some of the problems are solvable in polyomial time. Just give a high level sketch of the algorithm. Edge weighted matching (find the matching which minimizes the sum of the weight of the edges) is solvable in polynomial time.

I have tried to make the subproblems clear. If necessary, use your common sense, and document additional assumptions.

a. Assign courses to timeslots where there is a list of pairs of courses that may not be scheduled at the same time.

b. Assign courses to classrooms to satisfy capacity contraints. Each course has a number of students, and each room has a capacity. A course must be given a room with capacity greater than or equal to the number of students in the course. (For this subproblem, courses are not assigned to time slots, so multiple courses can be assigned to the same room.)

c. Assign courses to timeslots and classrooms to satisfy capacity constraints. At most one course can be assigned to a timeslot/classroom pair.

d. Assign professors to classrooms to minimize the distance that professors must travel from office to class. Assume that each professor teaches only one course. At most K professors may be assigned to any given room, (this is to allow the courses to be scheduled to time slots). Consider the cases where the distance function to minimize is:

i. Sum of the distances that all of the professors travel.

ii. The maximum distance that any professor must travel.

e. Assign professors and TAs to courses. Assume that there is a list of acceptable triples, a triple is of the form (p, t, c) indicating that professor p can teach course c with student t as a TA.

f. Assign students to courses to minimize the total distance that each student must travel. Assume that each student has a list of courses he or she will take, and the courses have already been assigned rooms and time slots. A student's list has a set of options, e.g., ((Math1-9am, Math1-10am), (Phys1-10am, Phys1-11am)), which would indicate the student wants to take Math 1 at 9 or 10, and Physics 1 at 10 or 11. A student may take at most one course in each time slot.

g. Assign lab courses to laboratories. All students in a course must be assigned to the same lab, but several courses may occupy the same lab subject to the constraint that the number of students in the courses is less than the capacity of the lab. Consider the following cases:

i. The number of students in a course is polynomial in the number of courses.

ii. The number of students in a course is exponential in the number of courses.

iii. The number of students in a course is polynomial and there are only two labs.

iv. At most two courses are allowed to occupy a lab and all labs are the same size.

v. (Optional[1]) At most two courses are allowed to occupy a lab and labs do not necessarily have the same size.

[1] I'm not sure of the answer on this one. It's certainly not worth spending a holiday weekend to solve it, but if you do figure it out, let me know.