University of Washington May 5, 1998
Department of Computer Science and Engineering
CSE 521, Spring 1998

Homework 6, **Due May 13, 1998**

Problem 1. Buying library shelves:

A library decides to store its books according to their size in order to minimize storage costs.

Suppose the book heights are $H_1, H_2, \ldots, H_n$ with $H_1 < H_2 < \cdots < H_n$, and let L_i be the shelf length required to hold all the books of height H_i. If we order shelving of height H_i, the cost of ordering x_i units is $F_i + C_i x_i$; F_i is a fixed ordering cost (and is independent of the length ordered) and C_i is the cost of the shelf per unit length. Notice that to save the fixed cost of ordering, we might not order shelves of every possible height because we can use a shelf of height H_i to store books of smaller height. We want to determine the length of shelving of each class to minimize the total cost of shelving.

a) Formulate this problem as a shortest paths problem.

b) Suppose that you are only allowed to use K distinct shelf heights, can the problem still be formulated as a shortest paths problem?

Problem 2. Cluster analysis:

Consider a set of n scalar numbers $a_1, a_2, \ldots, a_n$ arranged in non-decreasing order of their values. We wish to partition these numbers into clusters of consecutive elements from the list such that each group has at least p elements. The score of a cluster is the sum of the squared deviation of its numbers from their cluster mean, and we wish to find a partitioning which minimizes the sum of the cluster scores.

Show how to formulate this problem as a shortest paths problem.

Problem 3. Multicommodity flow:

Page 587, Exercise 27.1-9

Problem 4. Bipartite matching in regular graphs:

Page 604, Exercise 27.3-5

Problem 5. Multiple source/sink network flow:

The multiple source/sink network flow problem has sources $s_1, \ldots, s_k$ and sinks $t_1, \ldots, t_j$. The source s_i has a supply c_i, and the sink t_i has demand d_i. The question is does there exist a valid flow where each source s_i introduces c_i units of flow, and each sink t_i recieves d_i units of flow.

Show how you can solve the multiple source/sink network flow problem with a single source/sink maximum flow algorithm.

Problem 6. Minimum flow problem:

The minimum flow problem is: given a graph where each edge has upper and lower bounds on its edges, find the minimum feasible flow for the network. An edge e has upperbound $u(e)$ and lowerbound $l(e)$. The flow assigned to edge e must satisfy $l(e) \leq f(e) \leq u(e)$. Thus, you must find a flow which satisfies all of the edge constraints, and minimizes the amount of flow leaving the source.

Show that this problem can be solved with a network flow algorithm.

Hint: First find a feasible flow in the network, then reduce the flow to a minumum flow. A more detailed hint will be on the course web.

Problem 7. Optimal coverage of sporting events:

A group of reporters want to cover a set of sporting events in an olympiad. The sports events are held in several stadiums throughout a city. We know the starting time of each event, its duration, and the stadium where it is held. We are also given the travel times between different stadiums. We want to determine the least number of reporters required to cover the sporting events. How do you solve this problem?