University of Washington
Department of Computer Science and Engineering
CSE 521, Spring 1998

April 7, 1998

Homework 2, **Due April 15, 1998**

Problem 1:

Let

$$T(n) = \sum_{i=1}^{k} a_i T\left(\left\lfloor \frac{n}{b_i} \right\rfloor\right) + cn$$

with

$$\sum_{i=1}^{k} \frac{a_i}{b_i} = x < 1.$$

Prove that $T(n)$ is $O(n)$.

How good a bound can you get on the constant factor?

Problem 2:

Consider the following algorithm for sorting. The call *SlowSort*(A, i, j) sorts the items $A[i] \ldots A[j]$.

```
SlowSort(A, i, j)
  if i ≥ j then return
  m := ⌊(i + j)/2⌋;
  SlowSort(A, i, m);
  SlowSort(A, m + 1, j);
  if A[m] > A[j] then
    exchange A[j], A[m];
  SlowSort(A, i, j − 1);
```

a) Give an informal proof that the algorithm sorts correctly.

b) Give a recurrence relation for the run time $T(n)$ to sort n items.

c) Show that $T(n)$ is $O(n^{(\log n)/2})$ and $T(n)$ is $\Omega(n^{(\log n)/4})$.

(This algorithm is due to Broder and Stolfi [SIGACT News, Fall, 1984]).

Problem 3:

Meansort is a version of quick sort that chooses the closest element to the arithmetic mean of the keys as the splitting element. Prove that in the worst case, Meansort takes $\Omega(n^2)$ time.

Problem 4:

Alice and Bob are bugs who live on the convex polygons P and Q respectively. Given an algorithm that Alice and Bob can use to count the number of intersections of the polygons, which runs in time linear in the number of edges of P and Q. Alice and Bob do not have access to the coordinates of the vertices, but given three vertices p, q and r, they can determine whether r is to the left or the right of the line $\overline{pq}$.

Problem 5:

Let $H = \{h_1, \ldots, h_n\}$ be a set of n closed half-planes in the plane.

Give an $O(n \log n)$ algorithm for computing the intersection of the planes in H.

Problem 6:

(Optional) CLR, Page 134, problem 6-3.

Problem 7:

(Optional) CLR, Page 914, problem 35-3.