CSE 521: Design and Analysis of Algorithms **Fall 2025**

Problem Set 8

Deadline: Dec 4th in Gradescope

1) Given an unweighted graph $G+(V,E)$ design a polynomial time algorithm to assign non-negative weights $w : E \to \mathbb{R}_{\geq 0}$ to the edges of G such that

   i) The fractional degree of every vertex is 1, $\sum_{j \sim i} w_{i,j} = 1$.

   ii) Let G_w be the corresponding weighted graph with corresponding adjacency matrix A. The second largest eigenvalue of A is as small as possible.

   Prove that your algorithm is correct.

   Note: In this problem we are finding the best possible expander out of a given graph G. This task has many applications in theory and practice.

2) Consider the following problem that arises in telecommunications networks. The network consists of a cycle on n nodes, numbered 0 through $n-1$ clockwise around the cycle. Some set C of calls is given; each call is a pair (i, j) originating at node i and destined for node j. The call can be routed either clockwise or counterclockwise around the ring. The objective is to route the calls so as to minimize the total load on the network. The load L_i on link $(i, i+1 \bmod n)$ is the number of calls routed through link $(i, i+1 \bmod n)$, and the total load is the $\max_{1 \leq i \leq n} L_i$. Give a linear programming based 2-approximation algorithm for this problem.