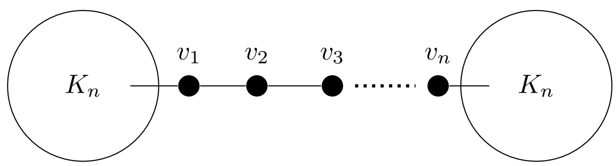
CSE 521: Design and Analysis of Algorithms **Fall 2025**

Problem Set 7

Deadline: Nov 21st in Gradescope

1) In this part implement the following heuristic to find the hidden partition problem: You are given a graph $G = (V, E)$ with adjacency matrix A with $n = 400$ vertices. The graph is stored in the file "hidden1.in". Let D be a degree matrix where $D_{v,v} = d(v)$ is the degree of a vertex v. Then $\tilde{A} := D^{-1/2}AD^{-1/2}$ is called the normalized adjacency matrix of G. Let x be the second largest eigenvector of $\tilde{A}$; let $y = D^{-1/2}x$. find the median of values in y and output vertices below the median as one community and the rest as the other. This graph is constructed with $p = 0.65$ and $q = 0.05$. The true partition is in the file "hidden1.out". Compare your output with the true hidden partition and report how many misplaced vertices are there in your output.

2) a) Let G be a graph with $3n$ vertices that is a union of two disjoint copies the complete graph with n vertices, K_n, connected by a path of length n. Show that $\lambda_2(\tilde{L}_G) \leq O(1/n^3)$, where $\tilde{L}$ is the normalized Laplacian matrix.



   b) Prove that for any unweighted connected graph $G = (V, E)$, $\phi(G) \geq \Omega(1/n^3)$.