CSE 521: Design and Analysis of Algorithms **Fall 2025**

Problem Set 1

Deadline: Oct 08 (at 11:59 PM) in gradescope

Instructions

- You should think about each problem by yourself for at least an hour before choosing to collaborate with others.
- You are allowed to collaborate with fellow students taking the class in solving the problems. But you **must** write your solution on your own.
- You are not allowed to search for answers or hints on the web. You are encouraged to contact the instructor or the TAs for a possible hint.
- You cannot collaborate on Extra credit problems
- Solutions typeset in LATEX are preferred.
- Feel free to use the Discussion Board or email the instructor or the TAs if you have any questions or would like any clarifications about the problems.
- Please upload your solutions to Gradescope.

In solving these assignments and any future assignment, feel free to use these approximations:

$$1 - x \approx e^{-x}, \qquad \sqrt{1-x} \approx 1 - x/2, \qquad n! \approx (n/e)^n, \qquad \left(\frac{n}{k}\right)^k \le \binom{n}{k} \le \left(\frac{en}{k}\right)^k$$

1) Given a graph $G = (V, E)$ with $n = |V|$ vertices, a cut is an α-approximate min cut, if the number of its edges is at most α times the minimum cut of G. Use an extension of the contraction algorithm to show that any graph G and any integer $\alpha \ge 1$, G has at most $O(n^{2\alpha})$ many α-approximate min cuts. **Hint:** Run Karger's algorithm until 2α nodes remain.

2) Given a graph $G = (V, E)$ such that $d(v) \ge k$ for all v. Design a (randomized) polynomial time algorithm that outputs a set S of size $|S| \le O(\frac{n}{k} \log k)$ such that every vertex $v \in \overline{S}$ has at least one neighbor in S. Recall that u is a neighbor of v if $(u, v) \in E$.