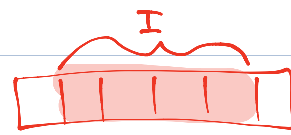
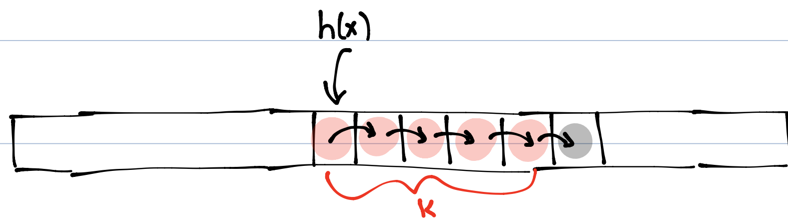
Linear Probing

If it takes a lot of time, then $\exists$ very long full intervals

$$L_I = \left| \{ x \in S \mid h(x) \in I \} \right|$$



L_I: # of items that hash here



some interval of length $\geq k$ must be full

$$E(L_I) = \frac{|I|}{2} \quad \text{if } m = 2n$$

Chernoff Bound

$X = \sum_{i=1}^{n} X_i$ $\quad$ X_i mutually indep indicator r.v.'s

$p_i := \Pr(X_i = 1) \implies E(X) = \sum p_i = \mu$

$$\Pr\left(X > (1+\delta)\mu\right) < \left[\frac{e^{\delta}}{(1+\delta)^{1+\delta}}\right]^{\mu} \quad \forall\ \delta > 0$$

$$\Pr(L_I > 2E(L_I)) \le \left(\frac{e}{4}\right)^{E(L_I)}$$ take $\delta = 1$ in Chernoff

$$E(\text{insertion time}) = \sum_{k=1}^{n} k \underbrace{\Pr(\text{insertion time } k)}_{\le \sum_{j=k}^{n} \underbrace{\Pr(L_I \text{ is full})}_{\le \left(\frac{e}{4}\right)^{j/2}}}$$

$\sqrt{\frac{e}{4}} \approx 0.82$

$$\le \frac{0.82^k}{1-0.82}$$

$$= O(1)$$

Used

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \qquad 0 \le x < 1$$

$$\sum_{k=1}^{\infty} k x^{k-1} = \frac{1}{(1-x)^2}$$

$O(1)$ expected insertion time for 5-wise indep

but not 4-wise