University of Washington | February 20, 2000
Department of Computer Science and Engineering
CSE 521, Winter 2000

Homework 7, **Due Wednesday, February 23, 2000**

Problem 1.:

Page 599, Exercises 27.2-4 and 27.2-7.

Problem 2.:

Page 600, Exercise 27.2-8.

Problem 3. B-matching:

The B-matching problem is a generalization of the matching problem which allows vertices to be matched with a number of edges.
Given a graph $G = (V, E)$, and integers $b_1, \ldots, b_n$, determine if there exists $E' \subseteq E$, such that v_i is adjacent to exactly b_i edges in E'.
Give an efficient algorithm for B-matching in a bipartite graph.

Problem 4. Hall's theorem:

Page 604, Exercise 27.3-4. Hint - the maxflow - mincut theorem is useful for one direction. You may use infinite capacity edges between vertices of L and R in Figure 27.9. (This is a correction from listing 27.3-5 on the assignment.)

Problem 5. Minimum flow problem:

The minimum flow problem is: given a graph where each edge has upper and lower bounds on its edges, find the minimum feasible flow for the network. An edge e has upperbound $u(e)$ and lowerbound $l(e)$. The flow assigned to edge e must satisfy $l(e) \leq f(e) \leq u(e)$. Thus, you must find a flow which satisfies all of the edge constraints, and minimizes the amount of flow leaving the source.

Show that this problem can be solved with a network flow algorithm.

Hint: First find a feasible flow in the network, then reduce the flow to a minimum flow. A more detailed hint will be on the course web.

Detailed hint:
The first thing you need to do is find a feasible flow in the network with upper and lower bounds on the edges. To handle the lower bounds on the edges, you introduce additional sources and sinks. If an edge has lower bound K, its neighboring vertices get connected to sources and sinks with edges of capacity K. (I'm not going to spell out all the details). Its also convenient to get rid of the orginal source and sink by adding an infinite capacity edge from t back to s.

The second part is to convert the feasible flow to a minimum flow. Use the residual graph. Find a maximum flow from the sink to the source.

Problem 6. Optimal coverage of political campaign events:

A group of reporters want to cover a set of campaign events. The events are held in several locations throughout the country. We know the starting time of each event, its duration, and the location where it is held. We are also given the travel times between different locations. We want to determine the least number of reporters required to cover the campaign. How do you solve this problem? (Important hint: the previous problem . . .)