University of Washington January 19, 2000
Department of Computer Science and Engineering
CSE 521, Winter 2000

Homework 3, **Due Wednesday, January 26, 2000**

Reading Read chapter 16 for dynamic programming and chapter 17 for greedy algorithms.

Problem 1. Subset sum:

The subset sum problem is: given a set $S = \{s_1, \ldots, s_n\}$ of integers, and an integer K, determine if there exists a subset $S' \subseteq S$ which sums to exactly K.

Show that the subset sum problem can be solved in $O(nK)$ time using $O(K)$ space (including finding the subset S' if it exists).

Problem 2. From the text:

Page 327, problem 16-5. Note that there can be several edges with the same label leaving a vertex.

Problem 3. From the text:

Page 324, Problem 16-1.

Problem 4. Vector subset sum:

The vector subset sum problem is: given a set $S = \{\vec{s_1}, \ldots, \vec{s_n}\}$ of m dimensional vectors of positive integers, and an m dimensional vector $\vec{b}$, does there exist a subset $S' \subseteq S$ such that $\sum_{s \in S'} s = \vec{b}$.

Show that if the maximum entry of $\vec{b}$ is bounded by B, the problem can be solved in $O(nB^m)$. Thus, if m is a constant and B is polynomial in n, the problem can be solved in polynomial time.

Problem 5. NP-completeness:

Prove that the vector subset sum is NP-complete when m is part of the input, and the maximum entry in $\vec{b}$ is bounded by n. A reduction from SATISFIABILITY is prefered, but not required. (This result actually holds for the case that the vector $\vec{b}$ is the vector $(1, 1, \ldots, 1)$.)

Problem 6. Longest common subsequence of random strings:

(Challenge)
Let A and B be two random binary strings of length n. Investigate the expected length of the Longest Common Sequence. Presumably, the expected length is αN for some α, where $0 \leq \alpha \leq 1$. Can you determine non trivial bounds on α. How does this compare with the length of the longest common substring of A and B? (This question is obviously completely open ended, if you want something concrete to work on, show that $\alpha > \frac{1}{2} + \epsilon$ for some $\epsilon > 0$.)