Natural Language Processing (CSE 517): Cotext Models

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Thanks to David Mimno for comments.
Quick Review

A language model is a probability distribution over $\mathcal{V}^\dagger$.

Typically $p$ decomposes into probabilities $p(x_i | h_i)$.

- We considered n-gram, class-based, log-linear, and neural language models.

Today: probabilistic models that relate a word and its **cotext** (the linguistic environment of the word).

- This might help us learn to represent words, contexts, or both.
Three Kinds of Cotext

If we consider a word token at a particular position $i$ in text to be the observed value of a random variable $X_i$, what other random variables are predictive of/related to $X_i$?

1. the document containing $i$ (a moderate-to-large collection of other words)
2. the words that occur within a small “window” around $i$ (e.g., $x_{i-2}$, $x_{i-1}$, $x_{i+1}$, or maybe the sentence containing $i$)
3. a sentence known to be a translation of the one containing $i$
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1. the document containing $i$ (a moderate-to-large collection of other words) $\rightarrow$ topic models

2. the words that occur within a small “window” around $i$ (e.g., $x_{i-2}$, $x_{i-1}$, $x_{i+1}$, $x_{i+2}$, or maybe the sentence containing $i$) $\rightarrow$ distributional semantics

3. a sentence known to be a translation of the one containing $i$ $\rightarrow$ translation models
Words are not IID!

- Predictable given history: n-gram/Markov models
- Predictable given other words in the document: topic models
Topic Models

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- Let $\mathcal{Z} = \{1, \ldots, k\}$ be a set of “topics” or “themes” that will help us capture the interdependence of words in a document.
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  - Usually these are not named or characterized in advance; they are just $k$ different values with no \textit{a priori} meaning.
- We’ll start with a classical topic model, then turn to probabilistic ones.
The Term-Document Matrix

Let $A \in \mathbb{R}^{V \times C}$ contain statistics of association between words in $V$ and $C$ documents. $N$ is the total number of word tokens.

Tiny example, three documents:

- yes, we have no bananas
- say yes for bananas
- no bananas, we say

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Count matrix: $[A]_{v,c} = c_{x_c}(v)$
What we really want here is some way to get at “surprise.”
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Intuition: consider the ratio of observed frequency ($c_{\mathbf{x}_{c}}(v)$) to “chance” under independence ($\frac{c_{\mathbf{x}_{1:C}}(v)}{N} \cdot \ell_c$).
Pointwise Mutual Information

A common starting point is positive **pointwise mutual information**:

\[
[A]_{v,c} = \log \frac{c_{\mathbf{x}_c}(v)}{\frac{c_{\mathbf{x}_1:C}(v)}{N} \cdot \ell_c} = \log \frac{N \cdot c_{\mathbf{x}_c}(v)}{c_{\mathbf{x}_1:C}(v) \cdot \ell_c} + \]

From our example:

\[
[A]_{\text{bananas,1}} = \log \frac{15 \cdot 1}{3 \cdot 6} \approx -0.18 \rightarrow 0
\]

\[
[A]_{\text{for,2}} = \log \frac{15 \cdot 1}{1 \cdot 4} \approx 1.32
\]

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A Nod to Information Theory

Pointwise mutual information for two random variables $A$ and $B$:

$$\text{PMI}(a, b) = \log \frac{p(A = a, B = b)}{p(A = a) \cdot p(B = b)}$$

$$= \log \frac{p(A = a | B = b)}{p(A = a)}$$

$$= \log \frac{p(B = b | A = a)}{p(B = b)}$$
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The **average mutual information** is given by:

$$\text{MI}(A, B) = \sum_{a,b} p(A = a, B = b) \cdot \text{PMI}(a, b)$$

This comes from information theory; it is the amount of information each r.v. offers about the other.

(Recall Shannon entropy; that’s the amount of information in a single random variable.)
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$$[A]_{v,c} = \left[ \log \frac{c_{x_c}(v)}{c_{x_1:C}(v) \cdot \ell_c} \right] = \left[ \log \frac{N \cdot c_{x_c}(v)}{c_{x_1:C}(v) \cdot \ell_c} \right]$$

Notes:

▶ If a word $v$ appears with nearly the same frequency in every document, its row $[A]_{v,*}$ will be all nearly zero.
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- If a word \( v \) occurs *only* in document \( c \), PMI will be large and positive.
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[A]_{v,c} = \left[ \log \frac{c_{v,c}(v)}{c_{1:C}(v) \cdot l_c} \right]_+ = \left[ \log \frac{N \cdot c_{v,c}(v)}{c_{1:C}(v) \cdot l_c} \right]_+
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▶ One way to think about PMI: it’s telling us where a unigram model is most wrong.

▶ We could use $A$ as $M$ (though $d$ is usually much smaller than $C'$) . . .
LSI/A seeks to solve:

\[ \mathbf{A} \approx \hat{\mathbf{A}} = \mathbf{M} \times \text{diag}(\mathbf{s}) \times \mathbf{C}^\top \]

where \( \mathbf{M} \) contains embeddings of words, \( \mathbf{C} \) contains embeddings of documents.

\[ [\mathbf{A}]_{v,c} \approx \sum_{i=1}^{d} [\mathbf{v}]_i \cdot [\mathbf{s}]_i \cdot [\mathbf{c}]_i \]
Topic Models: Latent Semantic Indexing/Analysis

(Deerwester et al., 1990)

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\[
[A]_{v,c} \approx \sum_{i=1}^{d} [v]_i \cdot [s]_i \cdot [c]_i
\]

This can be solved by applying singular value decomposition to \( \mathbf{A} \), then truncating to \( d \) dimensions.

- \( \mathbf{M} \) contains left singular vectors of \( \mathbf{A} \)
- \( \mathbf{C} \) contains right singular vectors of \( \mathbf{A} \)
- \( \mathbf{s} \) are singular values of \( \mathbf{A} \); they are nonnegative and conventionally organized in decreasing order.
Truncated Singular Value Decomposition

\[
\text{SVD: } \quad \begin{bmatrix} A \end{bmatrix} = \begin{bmatrix} M \end{bmatrix} \begin{bmatrix} \text{diag}(s) \end{bmatrix} \begin{bmatrix} C^T \end{bmatrix}
\]

truncated at \(k\): 

\[
\hat{A} = \hat{M} \begin{bmatrix} \text{diag}(s) \end{bmatrix} \begin{bmatrix} C^T \end{bmatrix}
\]
A Nod to Linear Algebra

For (not truncated) singular value decomposition $A = M \times \text{diag}(s) \times C^T$:

- The columns of $M$ form an orthonormal basis, $M$ are eigenvectors of $AA^T$, with eigenvalues $s^2$.
- The columns of $C$ form an orthonormal basis, $C$ are eigenvectors of $A^TA$, with eigenvalues $s^2$.

If some elements of $s$ are zero, then $A$ is “low rank.”

Approximating $A$ by truncating $s$ equates to a “low rank approximation.”
Words and documents in two dimensions.
Words and documents in two dimensions.
Note how no, we, and , are all in the exact same spot. Why?
Mapping words and documents into the same $k$-dimensional space.

Bag of words assumption (Salton et al., 1975): a document is nothing more than the distribution of words it contains.

Distributional hypothesis (Harris, 1954; Firth, 1957): words are nothing more than the distribution of contexts (here, documents) they occur in. Words that occur in similar contexts have similar meanings.

$A$ is sparse and noisy; LSI/A “fills in” the zeroes and tries to eliminate the noise.

- It finds the best rank-$k$ approximation to $A$. 
Probabilistic Topic Models

As a language model, LSI/A is kind of broken.
- It assumes the elements of $\mathbf{A}$ are the result of Gaussian noise.

Hofmann (1999) proposed instead to model the probability distribution
\[ p(\mathbf{X}_c = \mathbf{x}_c \mid c), \]
for each document $c$ in the corpus $C$.
- This is a particular kind of *conditional* language model.
Probabilistic Latent Semantic Analysis
(Hofmann, 1999)

Given a corpus \( C \), for every \( c \in C \):

\[
p(x \mid c) = \sum_{z \in \{1, \ldots, k\}^\ell} p(x, z \mid c)
\]

\[
p(x, z \mid c) = \prod_{i=1}^{\ell} p(z_i \mid c) \cdot p(x_i \mid z_i)
\]

Parameters:

- \( \gamma_{z \mid c}, \forall z \in \{1, \ldots, k\}, \forall c \in C \)
- \( \theta_{v \mid z}, \forall v \in V, \forall z \in \{1, \ldots, k\} \)

There is no closed form for the MLE!
“Graphical Model” Depiction of PLSA

unigram \( x_{1,1}, x_{1,2}, x_{1,3}, x_{1,4} \)

PLSA \( z_{1,1}, z_{1,2}, z_{1,3}, z_{1,4} \)

\( \theta \)

\( \gamma_1, \gamma_2 \)
A Chicken/Egg Problem

If we knew which topic each word token belonged to (i.e., which unigram distribution generated it), we could use relative frequency estimation.

If we knew the parameters $\gamma$ and $\theta$, we could infer the topic of each word (i.e., which unigram distribution generated it).
soft counts

Assume for the moment a single document $c$ of length $\ell$.

When we estimated unigram language models, everything relied on counts of words.

Here, if we knew the counts of every word in every topic in every document, then we’d have a closed form MLE.

\[
\hat{\gamma}_{z|c} = \frac{c(z,*)}{\ell} \\
\hat{\theta}_{v|z} = \frac{c(z,v)}{c(z,*)}
\]
“Soft Counts”

Assume for the moment a single document \( c \) of length \( \ell \).
When we estimated unigram language models, everything relied on \textit{counts} of words.
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\[
\hat{\gamma}_z|c = \frac{c(z, *)}{\ell} \\
\hat{\theta}_{v|z} = \frac{c(z, v)}{c(z, *)}
\]

Instead, we will replace counts with “soft counts.”

\[
\hat{\gamma}_z|c = \frac{\tilde{c}(z, *)}{\ell} \\
\hat{\theta}_{v|z} = \frac{\tilde{c}(z, v)}{\tilde{c}(z, *)}
\]
Expectation Maximization

Many ways to understand it. Today, we’ll stick with a simple one.

Start with arbitrary (e.g., random) parameter values. Alternate between two steps:

- **E step**: calculate the posterior distribution over each latent variable.
- **M step**: treat the posteriors as soft counts, and re-estimate the model.

Doing this is a kind of hill-climbing on the likelihood of the *observed* data.
Each word \( x_i \) is fractionally assigned to every topic \( z \) with value \( \tilde{c}_c(z, x_i) \).

\[
\hat{\gamma}_{z|c} = \frac{\tilde{c}_c(z)}{\ell_c} = \frac{\sum_{v \in V} \tilde{c}_c(z, v)}{\ell_c}
\]

\[
\hat{\theta}_{v|z} = \frac{\sum_{c \in C} \tilde{c}_c(z, v)}{\sum_{c \in C} \tilde{c}_c(z)} = \frac{\sum_{c \in C} \tilde{c}_c(z, v)}{\sum_{c \in C} \sum_{v \in V} \tilde{c}_c(z, v)}
\]

Note that the \( \theta \) parameters are shared across \( C \); all of the documents influence our beliefs about the others through \( \theta \).
PLSA: E Step

Assume we have the parameters:

- $\gamma_{z|c}, \forall z \in \{1, \ldots, k\}, \forall c \in C$
- $\theta_{v|z}, \forall v \in V, \forall z \in \{1, \ldots, k\}$

Calculate, for every $c \in C$, for every word $x_i$ in $c$, its “membership” to every topic:

$$p(Z_i = z \mid x_i, c) = \frac{p(x_i, z \mid c)}{\sum_{z'} p(x_i, z' \mid c)}$$

$$= \frac{p(z \mid c) \cdot p(x_i \mid z)}{\sum_{z'} p(z' \mid c) \cdot p(x_i \mid z')}$$

$$= \frac{\gamma_{z|c} \cdot \theta_{x_i|z}}{\sum_{z'} \gamma_{z'|c} \cdot \theta_{x_i|z'}}$$

Each word gets to vote on topics; it can spread its vote fractionally across $Z$, but the votes sum to 1.

These get summed into soft counts:

$$\tilde{c}_c(z, v) = \sum_{i : x_i = v} p(Z_i = z \mid x_i, c)$$
EM for PLSA

M step: $\theta$

E step: $\gamma$

Red indicates what is operated on in each step; everything else is held fixed.
Expectation Maximization

Very general technique for learning with *incomplete data*. It’s been invented over and over in different fields.

Requires that you specify a generative model with two kinds of variables: *observed* (here, documents and words in each document), and *latent* (here, topic for each word).

Like gradient ascent for neural networks, we are (usually) optimizing a non-convex function. Many tricks exist to try to cope with that.

In NLP, often associated with unsupervised learning. We will see it again!
Remarks

- Like LSI/A, PLSA “squeezes” the relationship between words and contexts (documents) through topics.
- A document is now characterized as a *mixture* of corpus-universal topics (each of which is a unigram model).
- Topic mixtures can be incorporated into language models; see Iyer and Ostendorf (1999), for example.
- Compared to LSI/A: PLSA is more interpretable (e.g., LSI/A can give negative values!).
- PLSA cannot assign probability to a text not in $C$; it only defines conditional distributions over words given texts in $C$.
- The next model overcomes this problem by adding another level of randomness: $\gamma$ becomes a random variable, not a parameter.
Latent Dirichlet Allocation

(Blei et al., 2003)

Widely used today.

\[
p(x) = \int \gamma \sum_{z \in \{1, \ldots, k\}} p(x, z, \gamma) \, d\gamma
\]

\[
p(x, z, \gamma) = \text{Dir} \alpha(\gamma) \prod_{i=1}^{\ell} \gamma_{z_i} \theta_{x_i|z_i}
\]

Parameters:

- \( \alpha \in \mathbb{R}_{>0}^k \)
- \( \theta_{*|z} \in \Delta^V, \forall z \in \{1, \ldots, k\} \)

There is no closed form for the MLE!
“Being Bayesian”

This is another topic that could warrant an entire quarter (e.g., http://homepages.inf.ed.ac.uk/sciocen/bayesian)

A summary of the Bayesian philosophy in NLP:

▶ Because we have finite data, we should be uncertain about every estimated model parameter.
▶ Bayes’ rule gives us a way to manage that uncertainty, if we can define a prior distribution over model parameters.
▶ Inference is a “simple matter” of estimating posterior distributions.
  ▶ But exact inference is almost never tractable, so we need approximations.
  ▶ There are many of these, and they tend to be expensive.
  ▶ Some of them look like EM, some don’t.


