Natural Language Processing (CSE 517): Cotext Models

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Thanks to David Mimno for comments.

Quick Review

A language model is a probability distribution over $\mathcal{V}^\dagger$.

Typically p decomposes into probabilities $p(x_i \mid \boldsymbol{h}_i)$.

- We considered n-gram, class-based, log-linear, and neural language models.

Today: probabilistic models that relate a word and its **cotext** (the linguistic environment of the word).

- This might help us learn to represent words, contexts, or both.

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1. the document containing i (a moderate-to-large collection of other words) $\longrightarrow$ topic models
2. the words that occur within a small "window" around i (e.g., x_{i-2}, x_{i-1}, x_{i+1}, x_{i+2}, or maybe the sentence containing i) $\longrightarrow$ distributional semantics
3. a sentence known to be a translation of the one containing i $\longrightarrow$ translation models

Topic Models

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  - Usually these are not named or characterized in advance; they are just k different values with no *a priori* meaning.
- We'll start with a classical topic model, then turn to probabilistic ones.

The Term-Document Matrix

Let $\mathbf{A} \in \mathbb{R}^{V \times C}$ contain statistics of association between words in $\mathcal{V}$ and C documents.

N is the total number of word tokens.

Tiny example, three documents:

- `yes , we have no bananas`
- `say yes for bananas`
- `no bananas , we say`

	1	2	3
`,`	1	0	1
`bananas`	1	1	1
`for`	0	1	0
`have`	1	0	0
`no`	1	0	1
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Count matrix: $[\mathbf{A}]_{v,c} = c_{\boldsymbol{x}_c}(v)$

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Intuition: consider the ratio of *observed* frequency ($c_{\boldsymbol{x}_c}(v)$) to "chance" under independence ($\frac{c_{\boldsymbol{x}_{1:C}}(v)}{N} \cdot \ell_c$).

Pointwise Mutual Information

A common starting point is positive **pointwise mutual information**:

$$[\mathbf{A}]_{v,c} = \left[\log \frac{c_{\boldsymbol{x}_c}(v)}{\frac{c_{\boldsymbol{x}_{1:C}}(v)}{N} \cdot \ell_c}\right]_+ = \left[\log \frac{N \cdot c_{\boldsymbol{x}_c}(v)}{c_{\boldsymbol{x}_{1:C}}(v) \cdot \ell_c}\right]_+$$

From our example:

$$[\mathbf{A}]_{\text{bananas},1} = \log \frac{15 \cdot 1}{3 \cdot 6} \approx -0.18 \to 0$$

$$[\mathbf{A}]_{\text{for},2} = \log \frac{15 \cdot 1}{1 \cdot 4} \approx 1.32$$

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A Nod to Information Theory

Pointwise mutual information for two random variables A and B:

$$\begin{aligned}
\mathsf{PMI}(a, b) &= \log \frac{p(A = a, B = b)}{p(A = a) \cdot p(B = b)} \\
&= \log \frac{p(A = a \mid B = b)}{p(A = a)} \\
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The **average mutual information** is given by:

$$\text{MI}(A, B) = \sum_{a,b} p(A = a, B = b) \cdot \mathrm{PMI}(a, b)$$

This comes from information theory; it is the amount of information each r.v. offers about the other.

(Recall Shannon entropy; that's the amount of information in a single random variable.)

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Notes:

- If a word v appears with nearly the same frequency in every document, its row $[\mathbf{A}]_{v,*}$ will be all nearly zero.

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- One way to think about PMI: it's telling us where a unigram model is most wrong.
- We could use $\mathbf{A}$ as $\mathbf{M}$ (though d is usually much smaller than C) ...

Topic Models: Latent Semantic Indexing/Analysis
(Deerwester et al., 1990)

LSI/A seeks to solve:

$$\underset{V \times C}{\mathbf{A}} \approx \hat{\mathbf{A}} = \underset{V \times d}{\mathbf{M}} \times \underset{d \times d}{\operatorname{diag}(\mathbf{s})} \times \underset{d \times C}{\mathbf{C}}^\top$$

where $\mathbf{M}$ contains embeddings of words, $\mathbf{C}$ contains embeddings of documents.

$$[\mathbf{A}]_{v,c} \approx \sum_{i=1}^{d} [\mathbf{v}_v]_i \cdot [\mathbf{s}]_i \cdot [\mathbf{c}_c]_i$$

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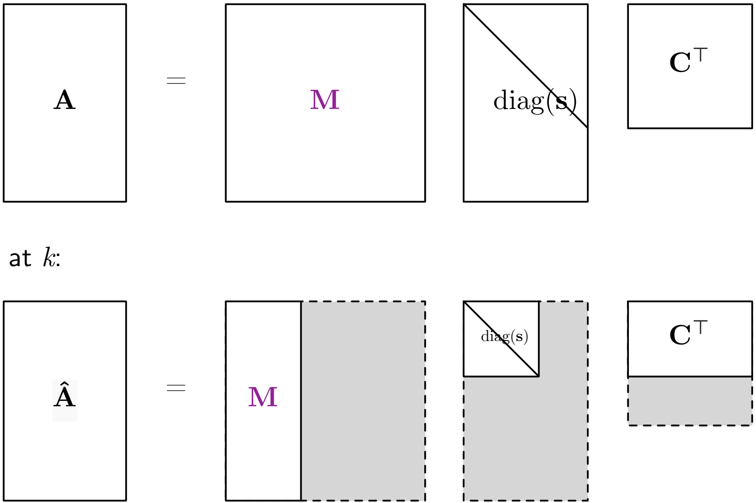
This can be solved by applying singular value decomposition to $\mathbf{A}$, then truncating to d dimensions.

- $\mathbf{M}$ contains left singular vectors of $\mathbf{A}$
- $\mathbf{C}$ contains right singular vectors of $\mathbf{A}$
- $\mathbf{s}$ are singular values of $\mathbf{A}$; they are nonnegative and conventionally organized in decreasing order.

Truncated Singular Value Decomposition

SVD:

truncated at k:



A Nod to Linear Algebra

For (not truncated) singular value decomposition $\mathbf{A} = \mathbf{M} \times \mathrm{diag}(\mathbf{s}) \times \mathbf{C}^\top$:

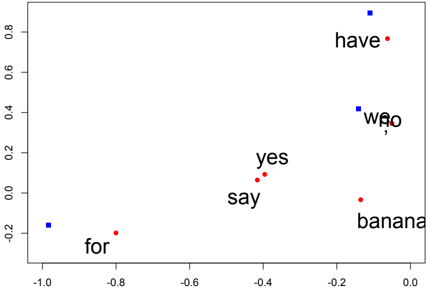
- The columns of $\mathbf{M}$ form an orthonormal basis, $\mathbf{M}$ are eigenvectors of $\mathbf{A}\mathbf{A}^\top$, with eigenvalues $\mathbf{s}^2$.
- The columns of $\mathbf{C}$ form an orthonormal basis, $\mathbf{C}$ are eigenvectors of $\mathbf{A}^\top\mathbf{A}$, with eigenvalues $\mathbf{s}^2$.

If some elements of $\mathbf{s}$ are zero, then $\mathbf{A}$ is "low rank."

Approximating $\mathbf{A}$ by truncating $\mathbf{s}$ equates to a "low rank approximation."

LSI/A Example

$d = 2$



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Words and documents in two dimensions.

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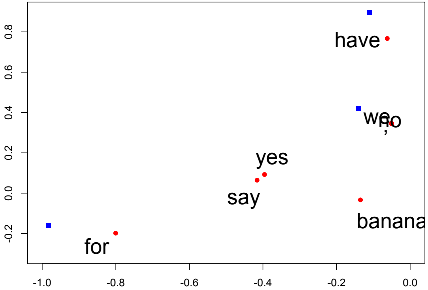

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Words and documents in two dimensions.

Note how `no`, `we`, and `,` are all in the exact same spot. Why?

Understanding LSI/A

- Mapping words and documents into the same k-dimensional space.
- Bag of words assumption (Salton et al., 1975): a document is nothing more than the distribution of words it contains.
- Distributional hypothesis (Harris, 1954; Firth, 1957): words are nothing more than the distribution of contexts (here, documents) they occur in. Words that occur in similar contexts have similar meanings.
- $\mathbf{A}$ is sparse and noisy; LSI/A "fills in" the zeroes and tries to eliminate the noise.
  - It finds the best rank-k approximation to $\mathbf{A}$.

Probabilistic Topic Models

As a language model, LSI/A is kind of broken.

- It assumes the elements of $\mathbf{A}$ are the result of Gaussian noise.

Hofmann (1999) proposed instead to model the probability distribution $p(\boldsymbol{X}_c = \boldsymbol{x}_c \mid c)$, for each document c in the corpus $\mathcal{C}$.

- This is a particular kind of *conditional* language model.

Probabilistic Latent Semantic Analysis

(Hofmann, 1999)

Given a corpus $\mathcal{C}$, for every $c \in \mathcal{C}$:

$$p(\boldsymbol{x} \mid c) = \sum_{\boldsymbol{z} \in \{1,\ldots,k\}^\ell} p(\boldsymbol{x}, \boldsymbol{z} \mid c)$$

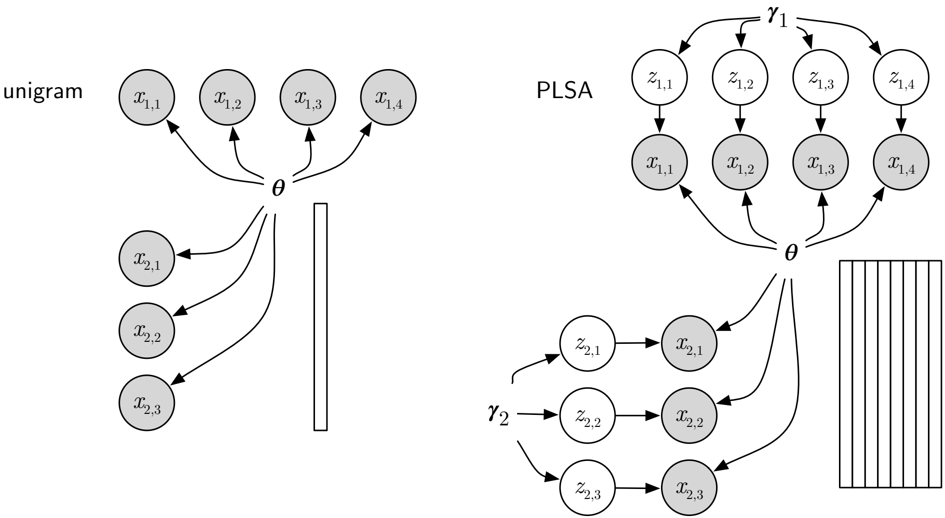
$$\begin{aligned} p(\boldsymbol{x}, \boldsymbol{z} \mid c) &= \prod_{i=1}^{\ell} p(z_i \mid c) \cdot p(x_i \mid z_i) \\ &= \prod_{i=1}^{\ell} \gamma_{z_i|c} \, \theta_{x_i|z_i} \end{aligned}$$

Parameters:

- $\gamma_{z|c}$, $\forall z \in \{1, \ldots, k\}, \forall c \in \mathcal{C}$
- $\theta_{v|z}$, $\forall v \in \mathcal{V}, \forall z \in \{1, \ldots, k\}$

There is no closed form for the MLE!

“Graphical Model” Depiction of PLSA



A Chicken/Egg Problem

If we knew which topic each word token belonged to (i.e., which unigram distribution generated it), we could use relative frequency estimation.

If we knew the parameters γ and $\boldsymbol{\theta}$, we could infer the topic of each word (i.e., which unigram distribution generated it).

"Soft Counts"

Assume for the moment a single document c of length ℓ.

When we estimated unigram language models, everything relied on *counts* of words.

Here, if we knew the counts of every word in every topic in every document, then we'd have a closed form MLE.

$$\hat{\gamma}_{z|c} = \frac{c(z,*)}{\ell}$$

$$\hat{\theta}_{v|z} = \frac{c(z,v)}{c(z,*)}$$

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Instead, we will replace counts with "soft counts."

$$\hat{\gamma}_{z|c} = \frac{\tilde{c}(z,*)}{\ell}$$

$$\hat{\theta}_{v|z} = \frac{\tilde{c}(z,v)}{\tilde{c}(z,*)}$$

Expectation Maximization

Many ways to understand it. Today, we'll stick with a simple one.

Start with arbitrary (e.g., random) parameter values. Alternate between two steps:

- E step: calculate the posterior distribution over each latent variable.
- M step: treat the posteriors as soft counts, and re-estimate the model.

Doing this is a kind of hill-climbing on the likelihood of the *observed* data.

PLSA: M Step

Each word x_i is fractionally assigned to every topic z with value $\tilde{c}_c(z, x_i)$.

$$\hat{\gamma}_{z|c} = \frac{\tilde{c}_c(z)}{\ell_c} = \frac{\sum_{v \in \mathcal{V}} \tilde{c}_c(z, v)}{\ell_c}$$

$$\hat{\theta}_{v|z} = \frac{\sum_{c \in \mathcal{C}} \tilde{c}_c(z, v)}{\sum_{c \in \mathcal{C}} \tilde{c}_c(z)} = \frac{\sum_{c \in \mathcal{C}} \tilde{c}_c(z, v)}{\sum_{c \in \mathcal{C}} \sum_{v \in \mathcal{V}} \tilde{c}_c(z, v)}$$

Note that the $\boldsymbol{\theta}$ parameters are shared across $\mathcal{C}$; all of the documents influence our beliefs about the others through $\boldsymbol{\theta}$.

PLSA: E Step

Assume we have the parameters:

- $\gamma_{z|c}$, $\forall z \in \{1, \ldots, k\}, \forall c \in \mathcal{C}$
- $\theta_{v|z}$, $\forall v \in \mathcal{V}, \forall z \in \{1, \ldots, k\}$

Calculate, for every $c \in \mathcal{C}$, for every word x_i in c, its "membership" to every topic:

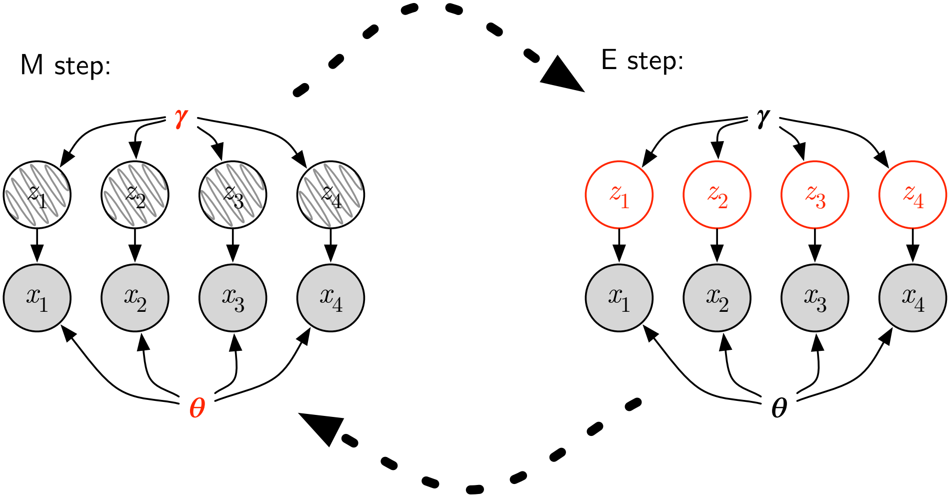
$$
\begin{aligned}
p(Z_i = z \mid x_i, c) &= \frac{p(x_i, z \mid c)}{\sum_{z'} p(x_i, z' \mid c)} \\
&= \frac{p(z \mid c) \cdot p(x_i \mid z)}{\sum_{z'} p(z' \mid c) \cdot p(x_i \mid z')} \\
&= \frac{\gamma_{z|c} \cdot \theta_{x_i|z}}{\sum_{z'} \gamma_{z'|c} \cdot \theta_{x_i|z'}}
\end{aligned}
$$

Each word gets to vote on topics; it can spread its vote fractionally across $\mathcal{Z}$, but the votes sum to 1.

These get summed into soft counts:

$$
\tilde{c}_c(z, v) = \sum_{i : x_i = v} p(Z_i = z \mid x_i, c)
$$

EM for PLSA



Red indicates what is operated on in each step; everything else is held fixed.

Expectation Maximization

Very general technique for learning with *incomplete data*. It's been invented over and over in different fields.

Requires that you specify a generative model with two kinds of variables: **observed** (here, documents and words in each document), and **latent** (here, topic for each word).

Like gradient ascent for neural networks, we are (usually) optimizing a non-convex function. Many tricks exist to try to cope with that.

In NLP, often associated with unsupervised learning. We will see it again!

Remarks

- Like LSI/A, PLSA "squeezes" the relationship between words and contexts (documents) through topics.
- A document is now characterized as a *mixture* of corpus-universal topics (each of which is a unigram model).
- Topic mixtures can be incorporated into language models; see Iyer and Ostendorf (1999), for example.
- Compared to LSI/A: PLSA is more interpretable (e.g., LSI/A can give negative values!).
- PLSA cannot assign probability to a text not in $\mathcal{C}$; it only defines conditional distributions over words given texts in $\mathcal{C}$.
- The next model overcomes this problem by adding another level of randomness: γ becomes a random variable, not a parameter.

Latent Dirichlet Allocation

(Blei et al., 2003)

Widely used today.

$$p(\boldsymbol{x}) = \int_{\boldsymbol{\gamma}} \sum_{\boldsymbol{z} \in \{1,\ldots,k\}^{\ell}} p(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{\gamma}) \; d\boldsymbol{\gamma}$$

$$p(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{\gamma}) = \mathrm{Dir}_{\boldsymbol{\alpha}}(\boldsymbol{\gamma}) \prod_{i=1}^{\ell} \gamma_{z_i} \; \theta_{x_i|z_i}$$

Parameters:

- $\boldsymbol{\alpha} \in \mathbb{R}^k_{>0}$
- $\boldsymbol{\theta}_{*|z} \in \triangle^V$, $\forall z \in \{1, \ldots, k\}$

There is no closed form for the MLE!

"Being Bayesian"

This is another topic that could warrant an entire quarter (e.g., `http://homepages.inf.ed.ac.uk/scohen/bayesian`)

A summary of the Bayesian philosophy in NLP:

- Because we have finite data, we should be uncertain about every estimated model parameter.
- Bayes' rule gives us a way to manage that uncertainty, if we can define a **prior** distribution over model parameters.
- Inference is a "simple matter" of estimating posterior distributions.
  - But exact inference is almost never tractable, so we need approximations.
  - There are many of these, and they tend to be expensive.
  - Some of them look like EM, some don't.

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