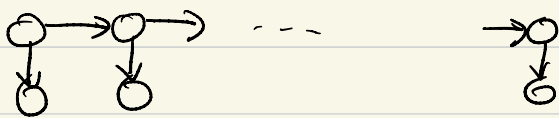
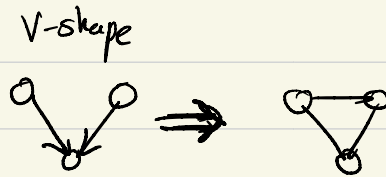
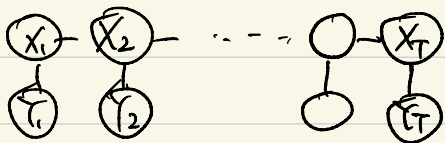


① Moralize BN to get MRF.  
 Compute moral graph of BN.



HMM has no V-shape. ← it is a tree



② Write the factors.

$$P(x) = \frac{1}{Z} \prod_{t \in \{1, \dots, T\}} f_{X_t, X_{t+1}}(x_t, x_{t+1}) = x_{t+1} \left[ \begin{array}{c} x_t \\ | \\ - \\ | \\ - \end{array} \right]$$

$$\begin{cases} P(Y_t | X_t) \cdot P(X_{t+1} | X_t) & \text{if } t < T-1 \\ P(Y_{T-1} | X_{T-1}) P(X_T | X_{T-1}) P(Y_T | X_T) & \text{if } t = T-1 \end{cases}$$

③ Write the BP update.

$$m_{t \rightarrow t+1}(x_{t+1}) = \sum_{x_t} f_{x_t, x_{t+1}}(x_t, x_{t+1}) m_{t+1 \rightarrow t}(x_t).$$

$$m_{1 \rightarrow 2}(x_2) = \sum_{x_1} f_{x_1, x_2}(x_1, x_2)$$

$$m_{T \rightarrow T-1}(x_{T-1}) = \sum_{x_T} f_{x_{T-1}, x_T}(x_{T-1}, x_T)$$

$$m_{t+1 \rightarrow t}(x_t) = \sum_{x_{t+1}} f_{x_t, x_{t+1}}(x_t, x_{t+1}) m_{t+2 \rightarrow t+1}(x_{t+1}).$$

In this example

- we had a BN model  $\rightarrow$  MRF
- we had observation  $Y \rightarrow P(x)$  for inference