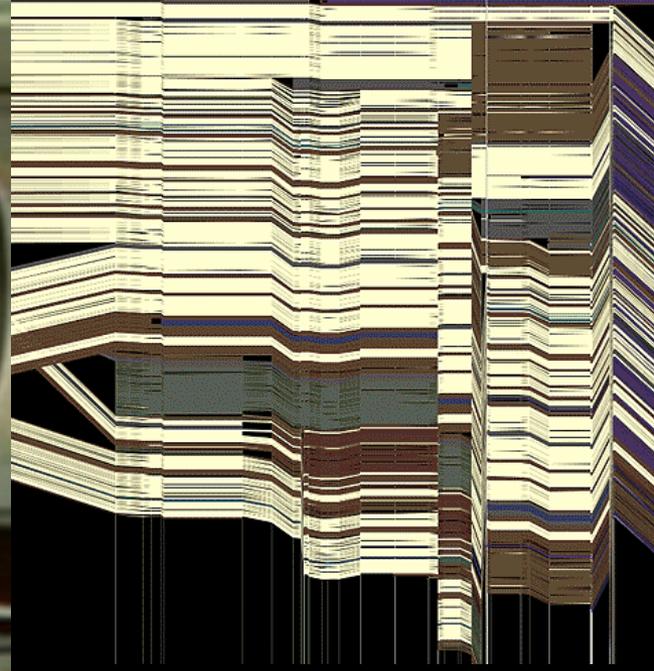
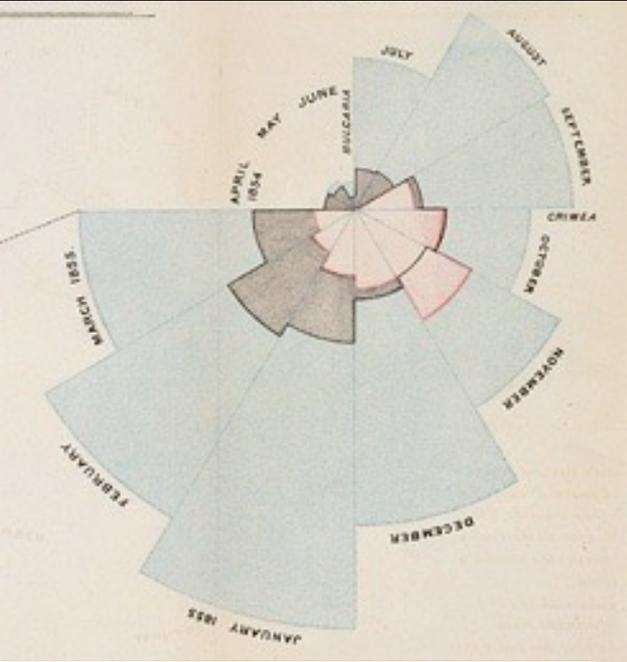


CSE 512 - Data Visualization

# Networks



Jeffrey Heer University of Washington

# Topics

Visualizing Trees

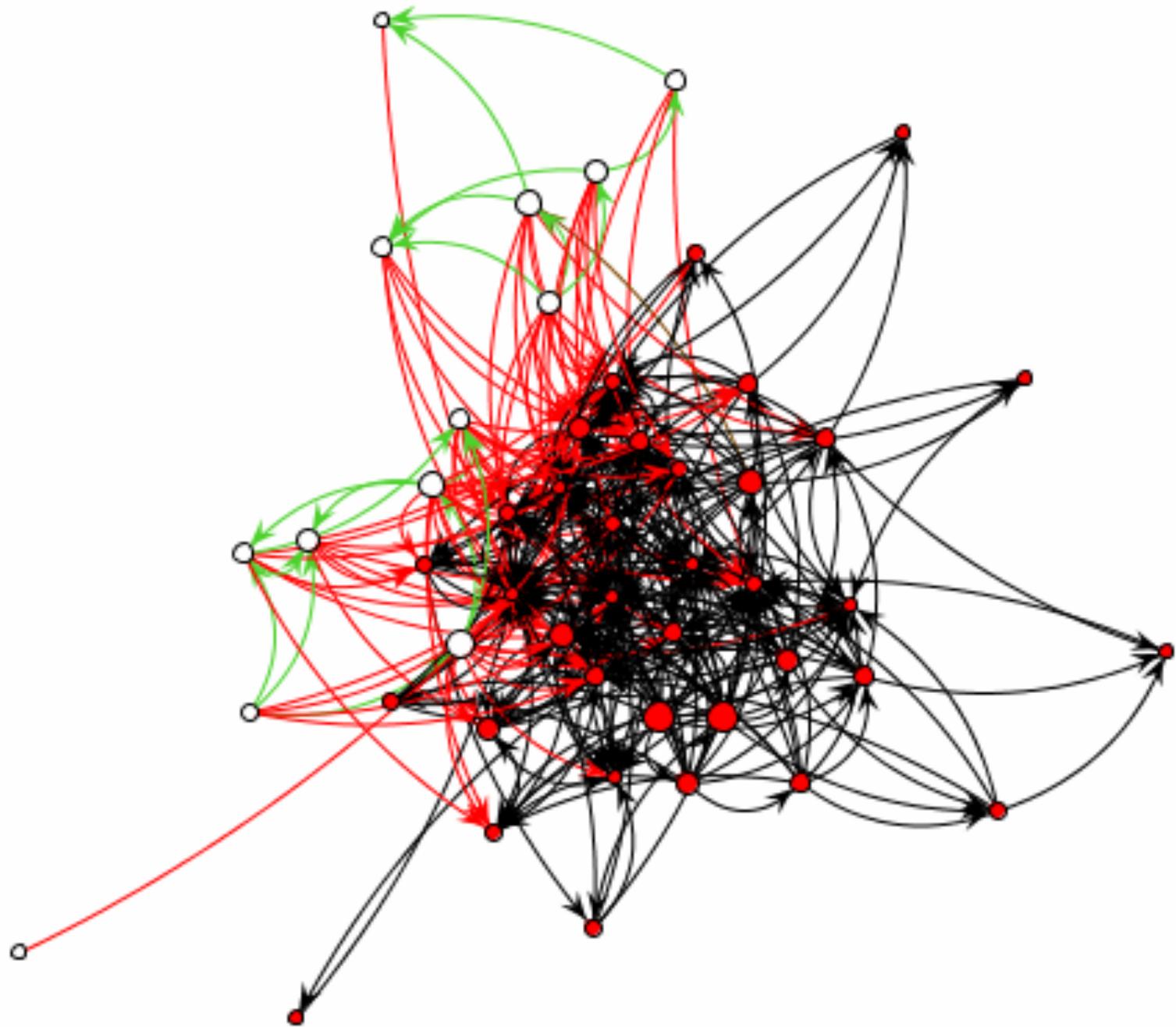
Visualizing Graphs

## Goals

Overview of layout approaches

Assess strengths and weaknesses

Insight into implementation techniques

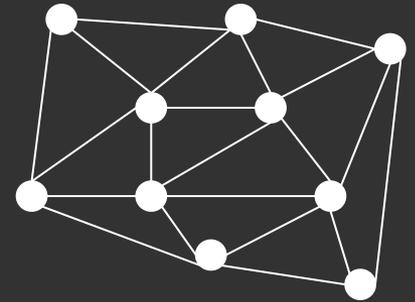


# Graphs and Trees

## Graphs

Model relations among data

*Nodes and edges*

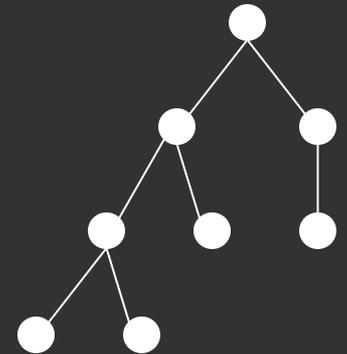


## Trees

Graphs with hierarchical structure

- Connected graph with  $N-1$  edges

Nodes as *parents and children*



# Spatial Layout

A primary concern of graph drawing is the spatial arrangement of nodes and edges.

Often (but not always) the goal is to effectively depict the graph structure:

- Connectivity, path-following
- Network distance
- Clustering
- Ordering (e.g., hierarchy level)

# Applications

Tournaments

Organization Charts

Genealogy

Diagramming (e.g., Visio)

Biological Interactions (Genes, Proteins)

Computer Networks

Social Networks

Simulation and Modeling

Integrated Circuit Design

# Tree Layout

# Tree Visualization

## Indentation

Linear list, indentation encodes depth



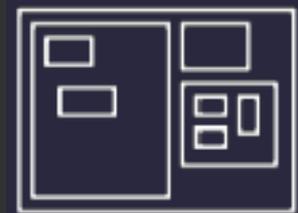
## Node-Link diagrams

Nodes connected by lines/curves



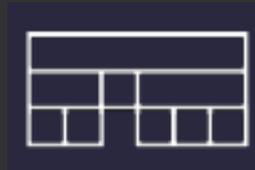
## Enclosure diagrams

Represent hierarchy by enclosure



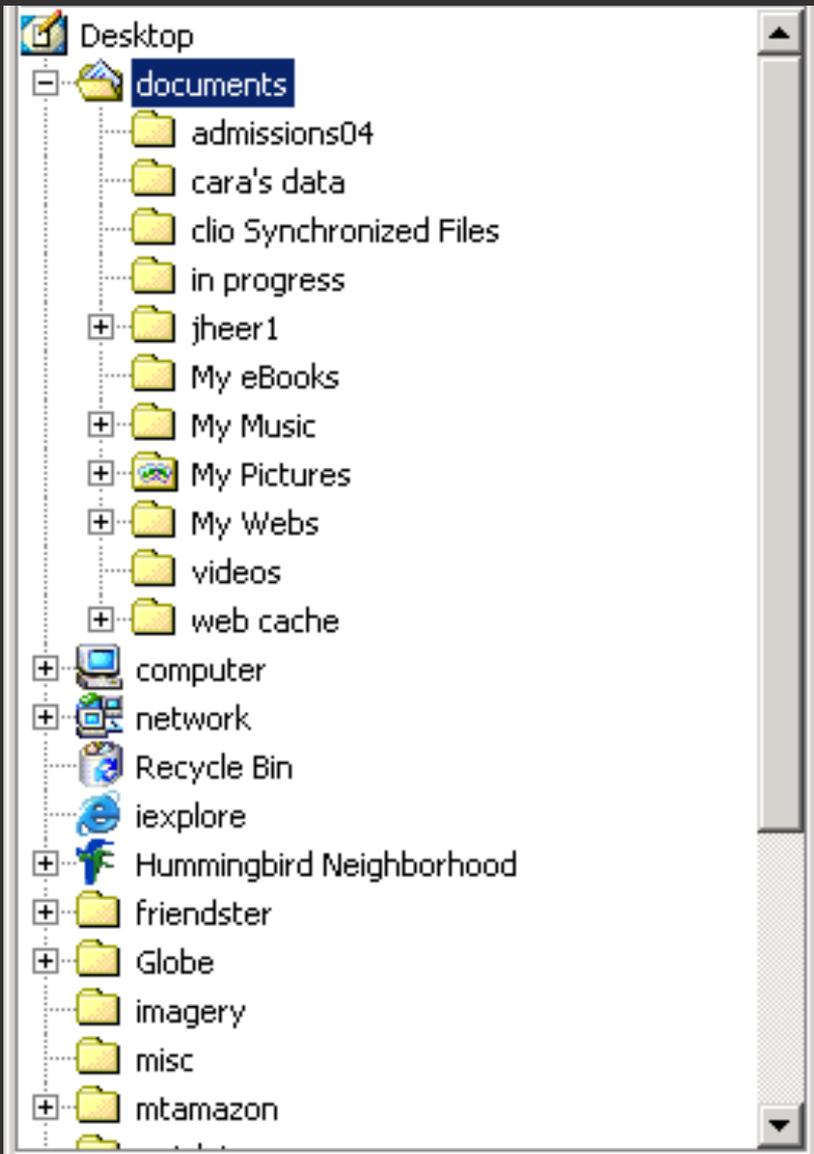
## Layering

Relative position and alignment



*Fast:*  $O(n)$  or  $O(n \log n)$ , interactive layout

# Indentation



Places all items along vertically spaced rows

Indentation used to show parent/child relationships

Commonly used as a component in an interface

Breadth and depth contend for space

Often requires a great deal of scrolling



# Node-Link Diagram

Nodes are distributed in space, connected by straight or curved lines

Typical approach is to use 2D space to break apart breadth and depth

Often space is used to communicate hierarchical orientation (e.g., towards authority or generality)

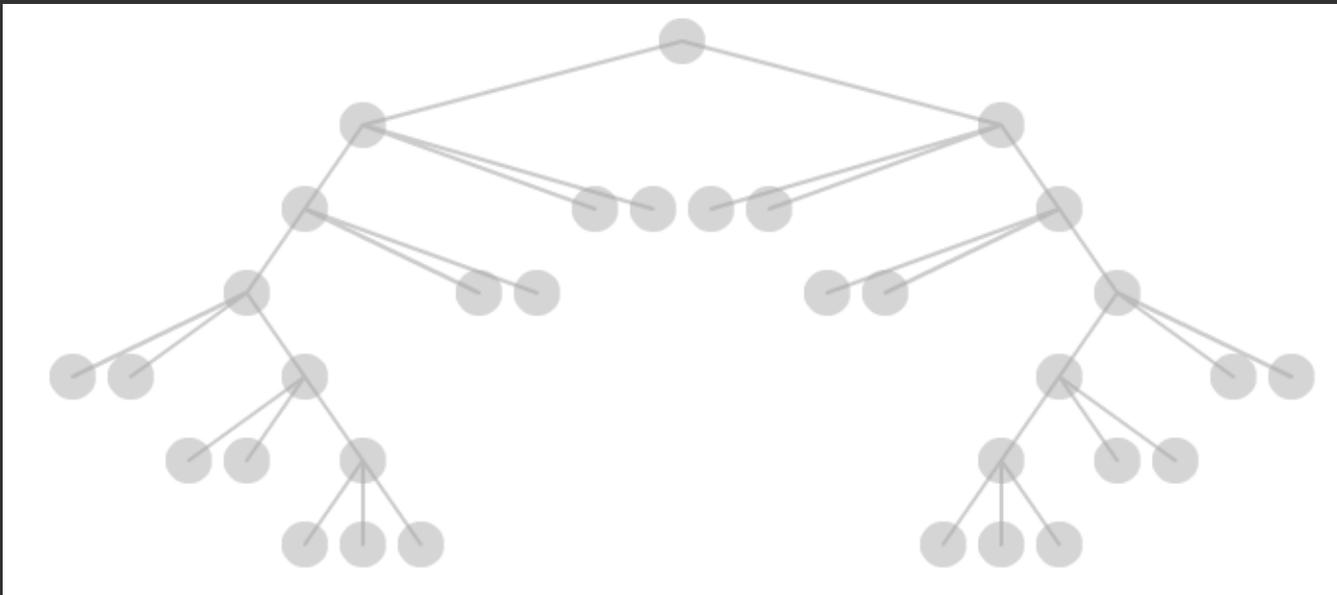


# Basic Recursive Approach

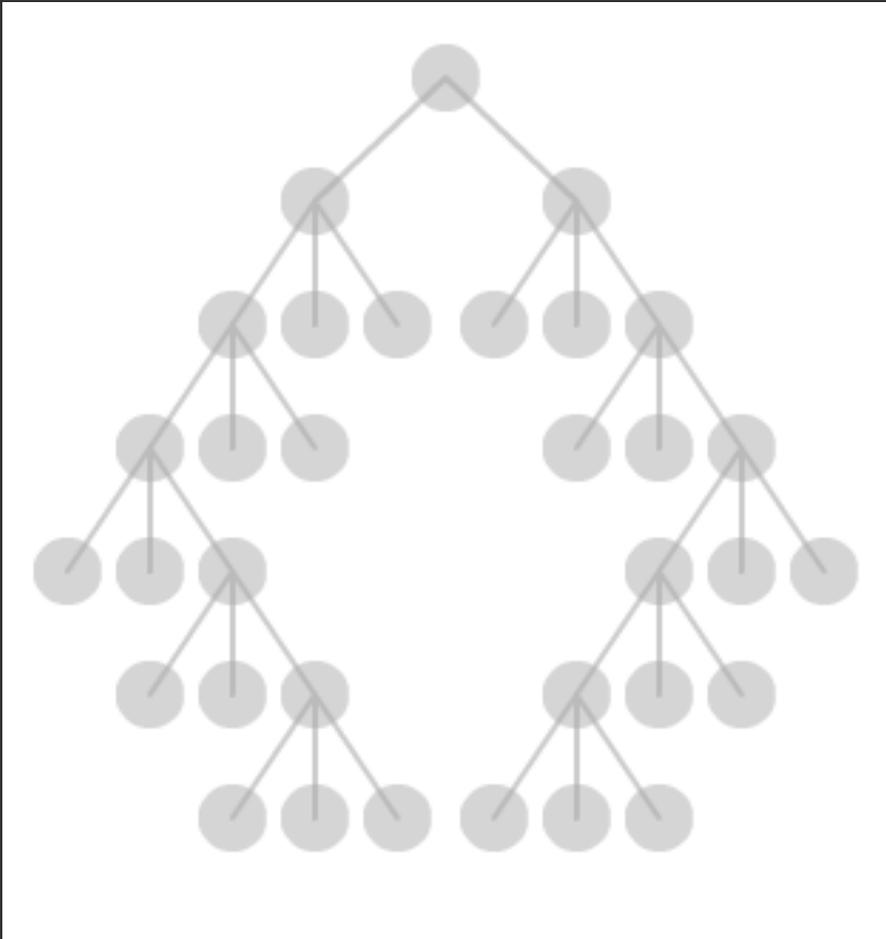
Repeatedly divide space for subtrees by leaf count

- Breadth of tree along one dimension
- Depth along the other dimension

Problem: exponential growth of breadth



# Reingold & Tilford's "Tidy" Layout



Goal: make smarter use of space, maximize density and symmetry.

Originally binary trees, extended by Walker to cover general case.

Corrected by Buchheim et al. to achieve a linear time algorithm.

# Reingold-Tilford Layout

## Design considerations

Clearly encode depth level

No edge crossings

Isomorphic subtrees drawn identically

Ordering and symmetry preserved

*Compact layout (don't waste space)*

# Reingold-Tilford Layout

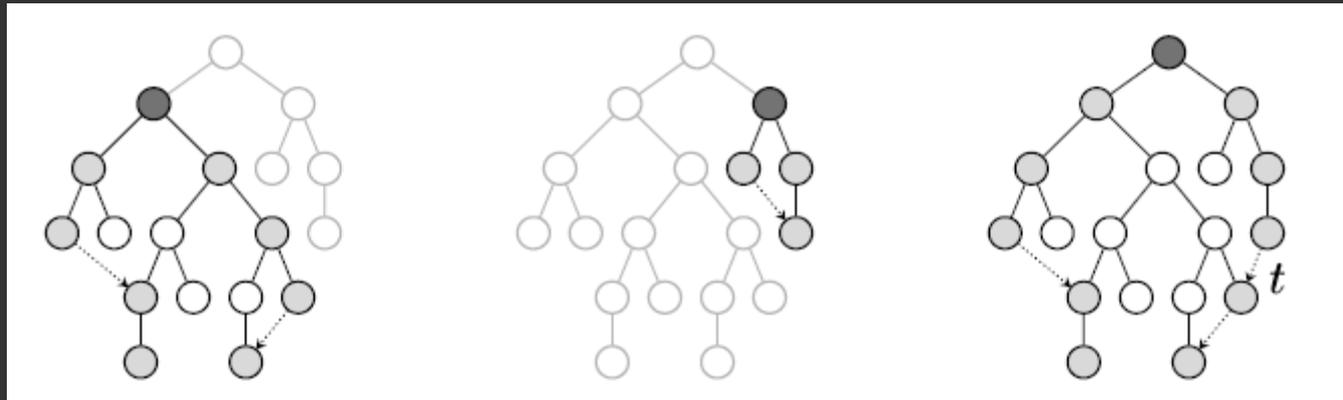
Linear algorithm - starts with bottom-up pass of the tree  
Y-coord by depth, arbitrary starting X-coord

Merge left and right subtrees

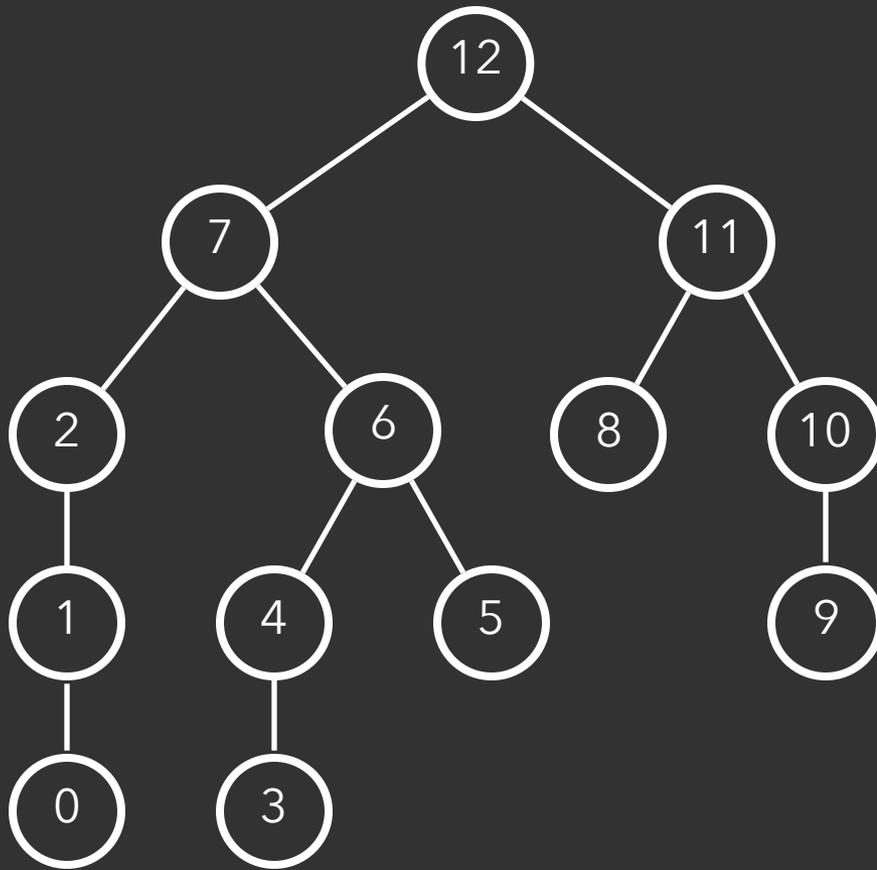
- Shift right as close as possible to left
  - Computed efficiently by maintaining subtree contours
- "Shifts" in position saved for each node as visited
- Parent nodes are centered above their children

Top-down pass for assignment of final positions

- Sum of initial layout and aggregated shifts



# Reingold-Tilford Layout



# Reingold-Tilford Layout

0

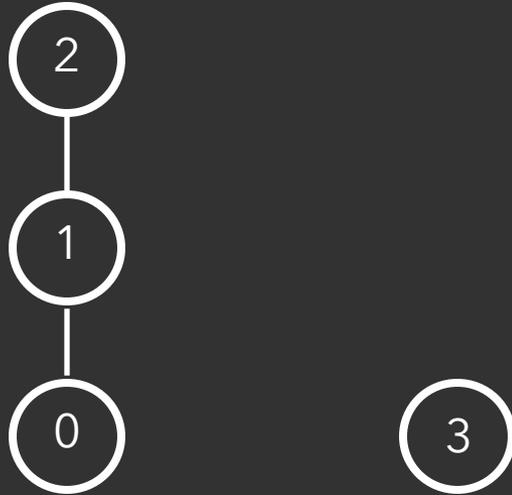
# Reingold-Tilford Layout



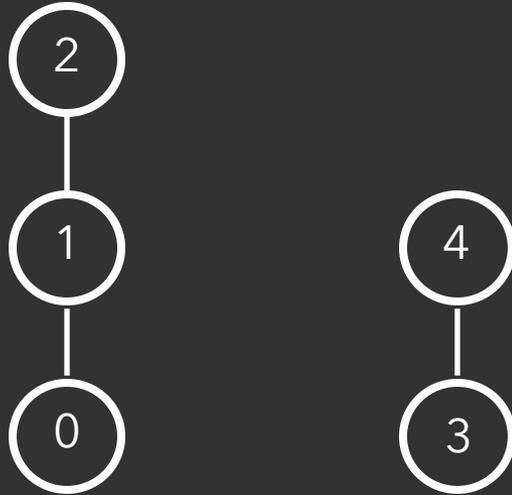
# Reingold-Tilford Layout



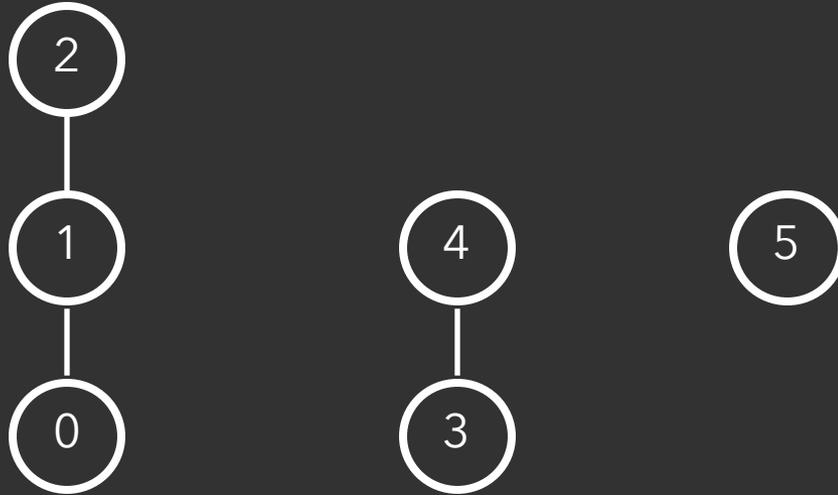
# Reingold-Tilford Layout



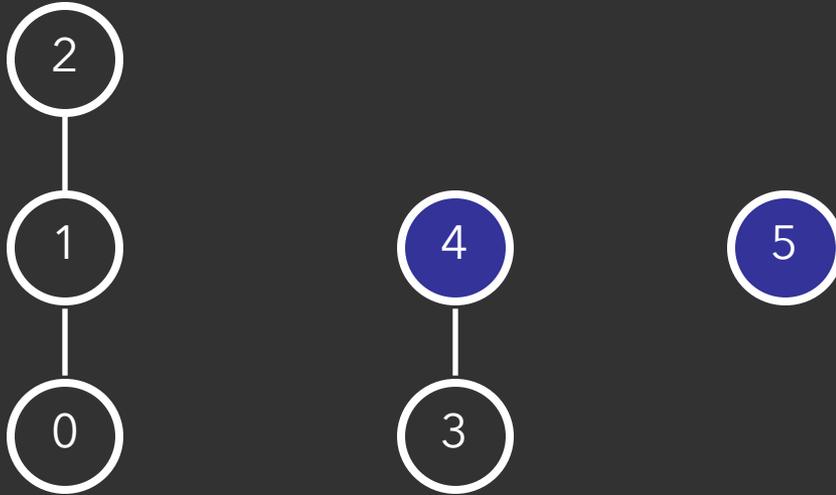
# Reingold-Tilford Layout



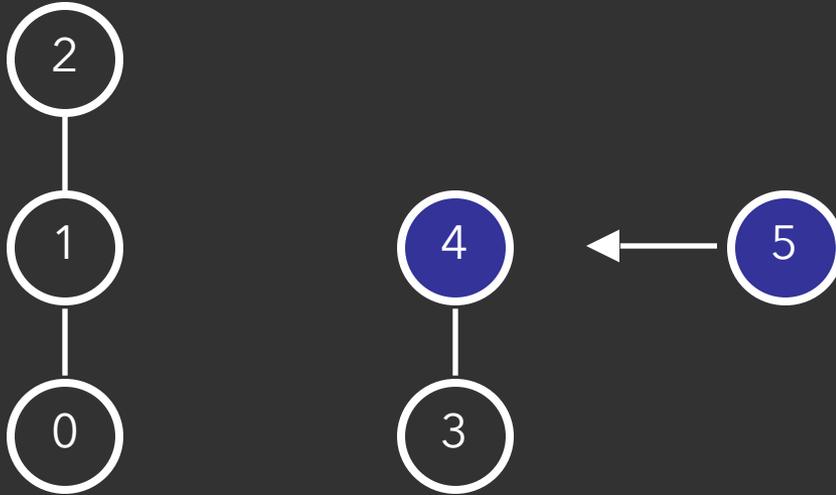
# Reingold-Tilford Layout



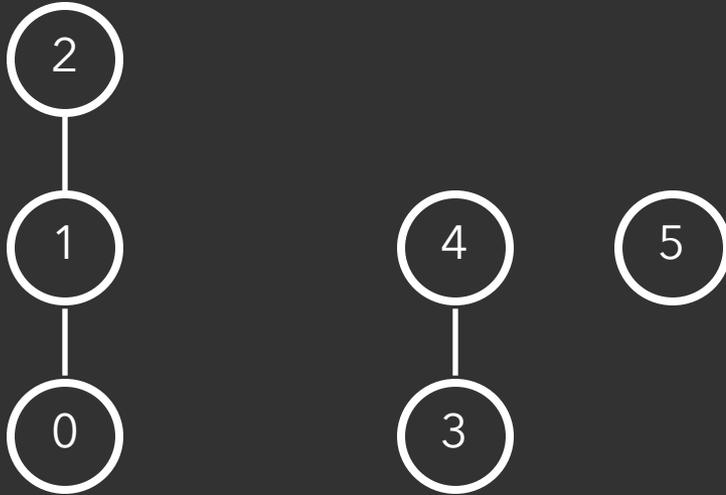
# Reingold-Tilford Layout



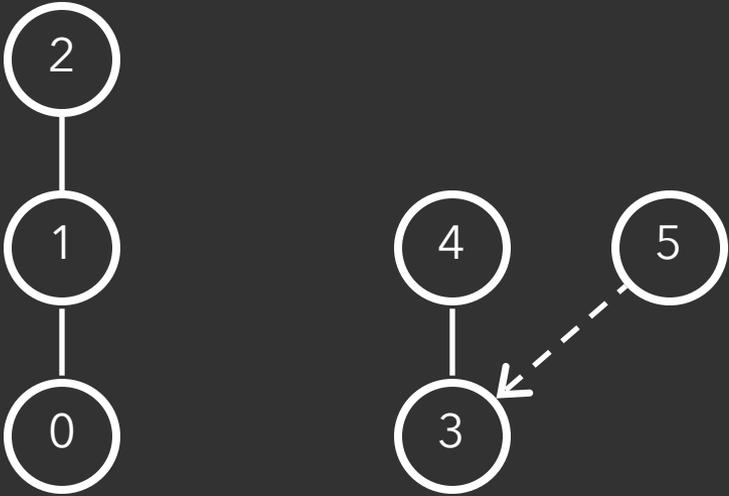
# Reingold-Tilford Layout



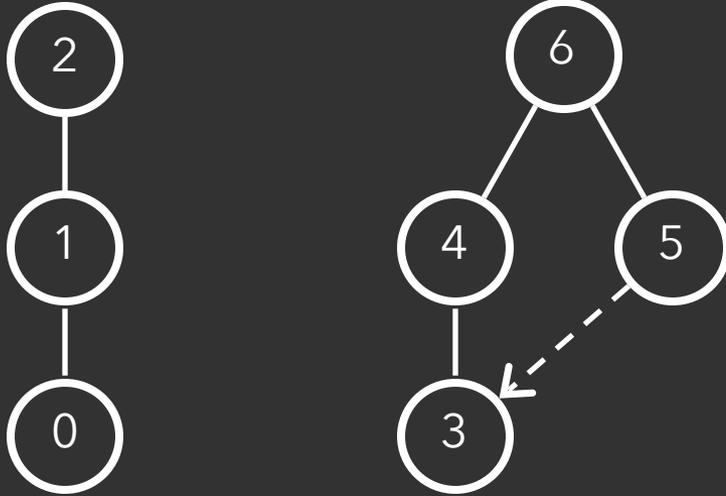
# Reingold-Tilford Layout



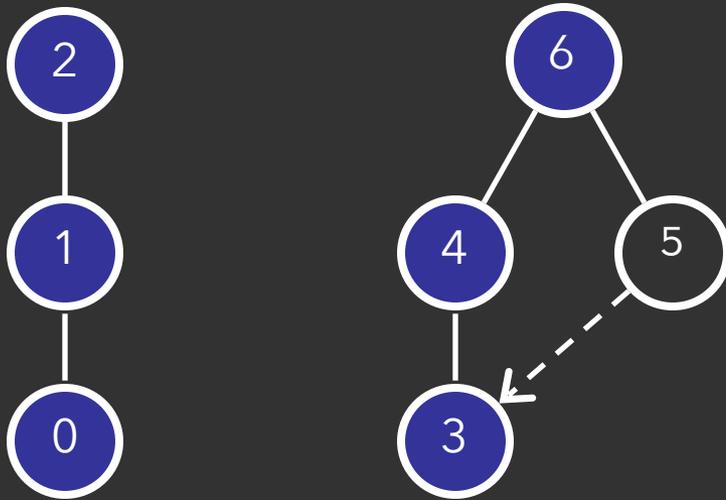
# Reingold-Tilford Layout



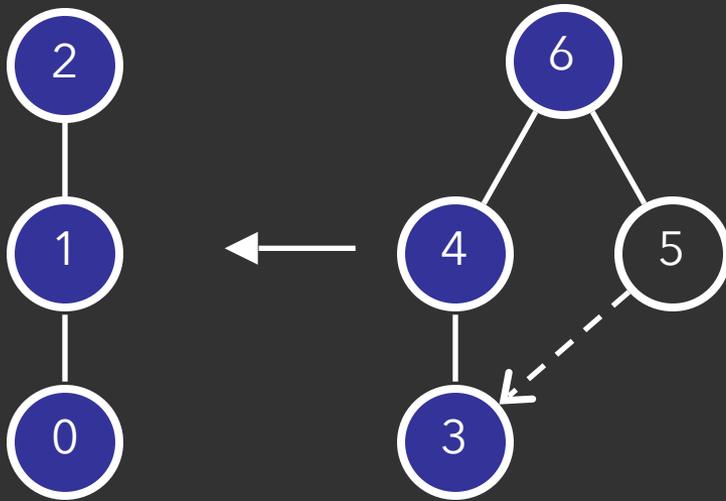
# Reingold-Tilford Layout



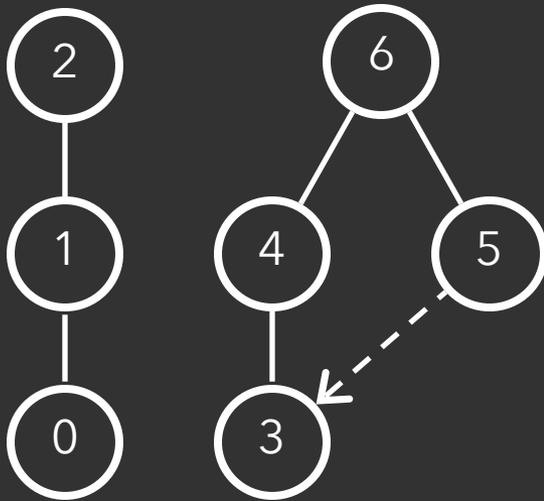
# Reingold-Tilford Layout



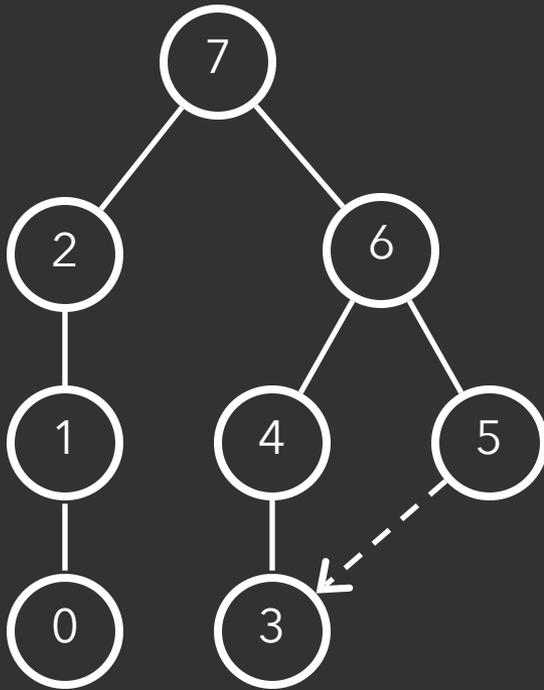
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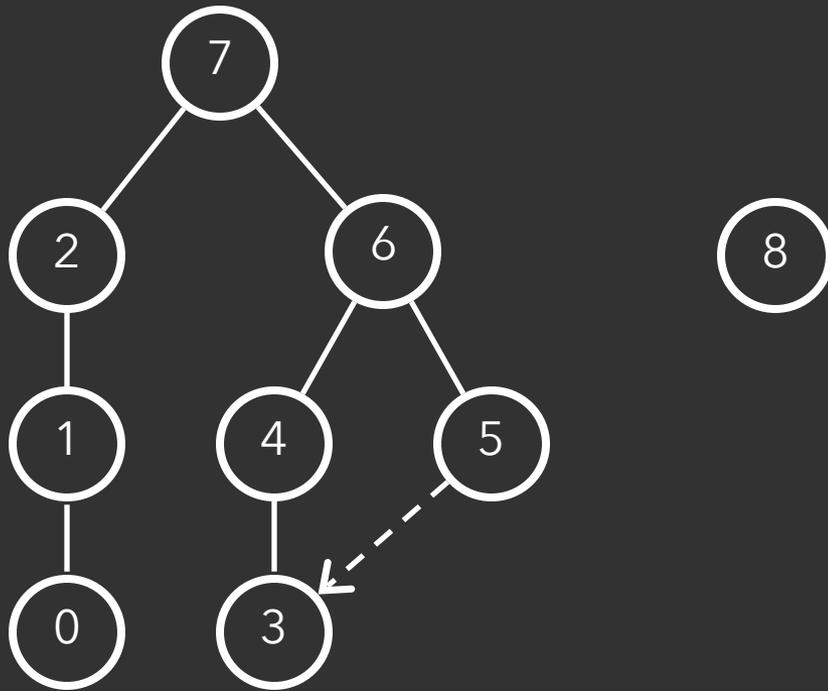
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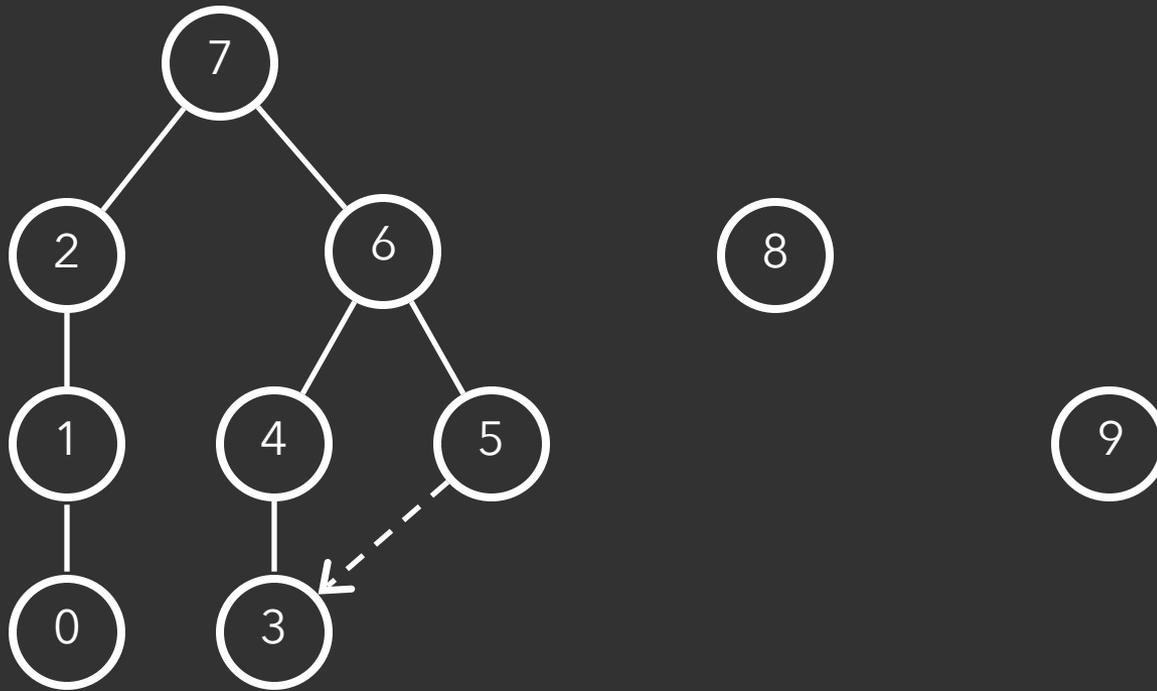
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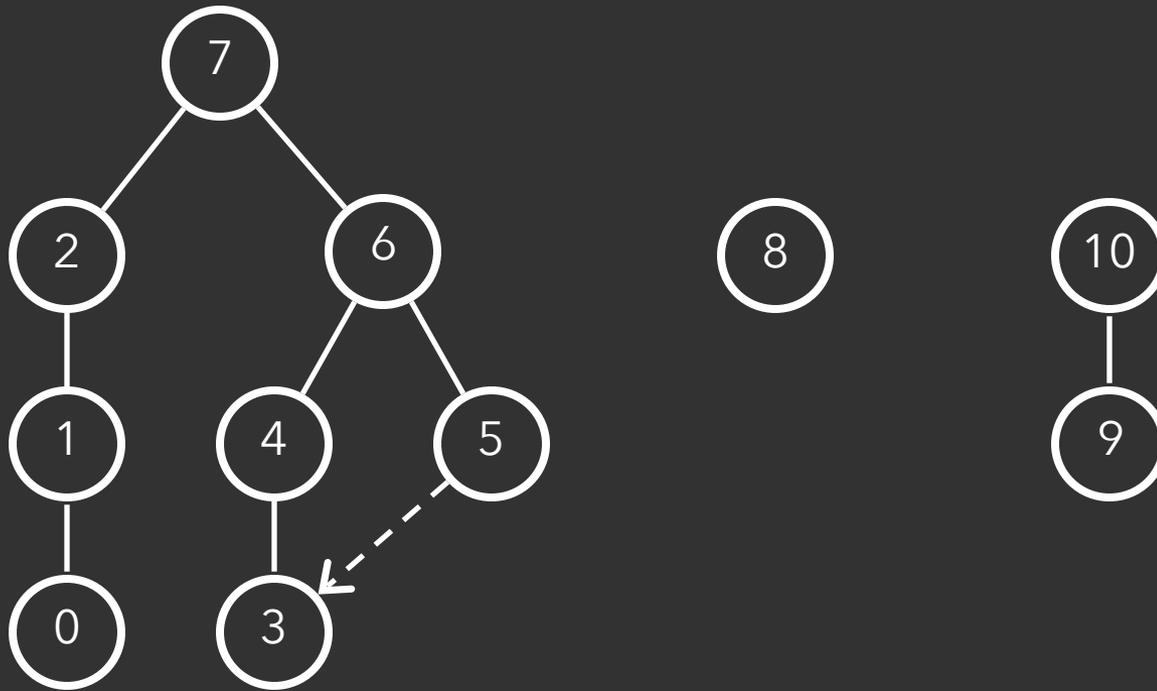
# Reingold-Tilford Layout



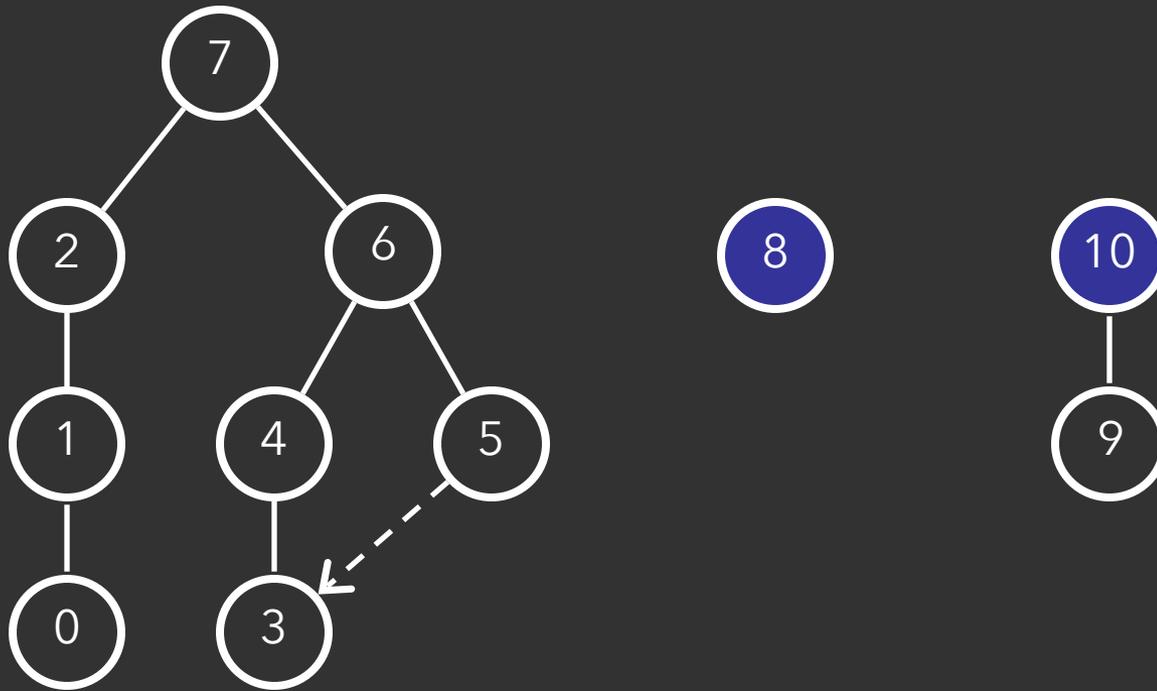
# Reingold-Tilford Layout



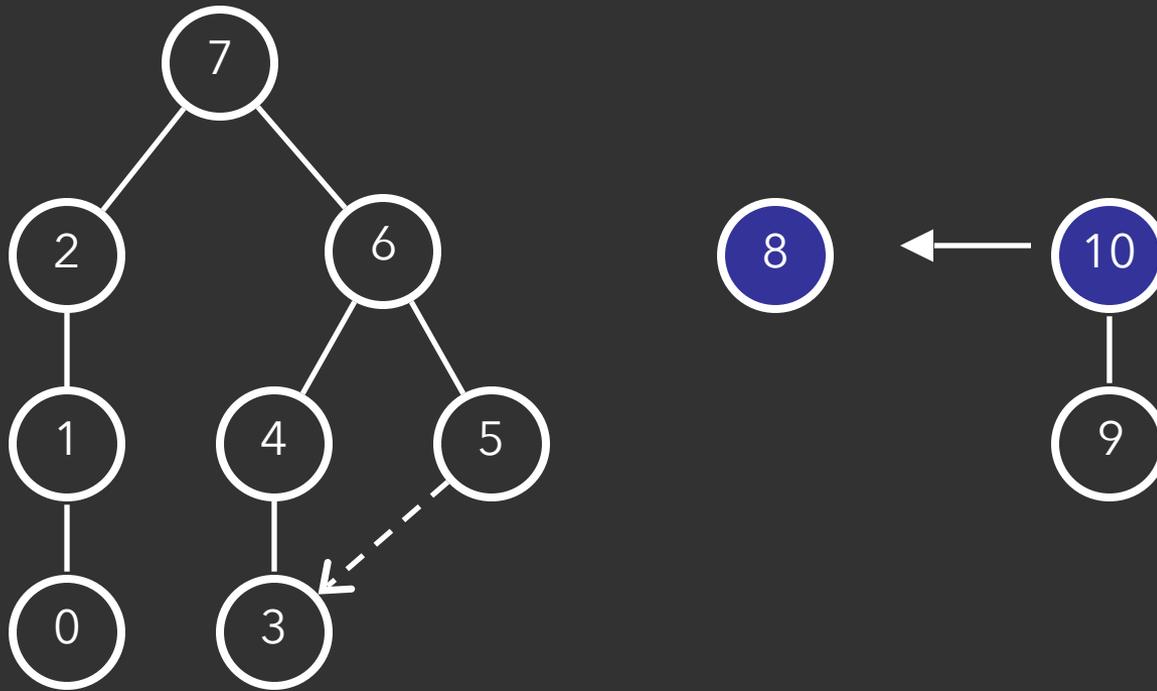
# Reingold-Tilford Layout



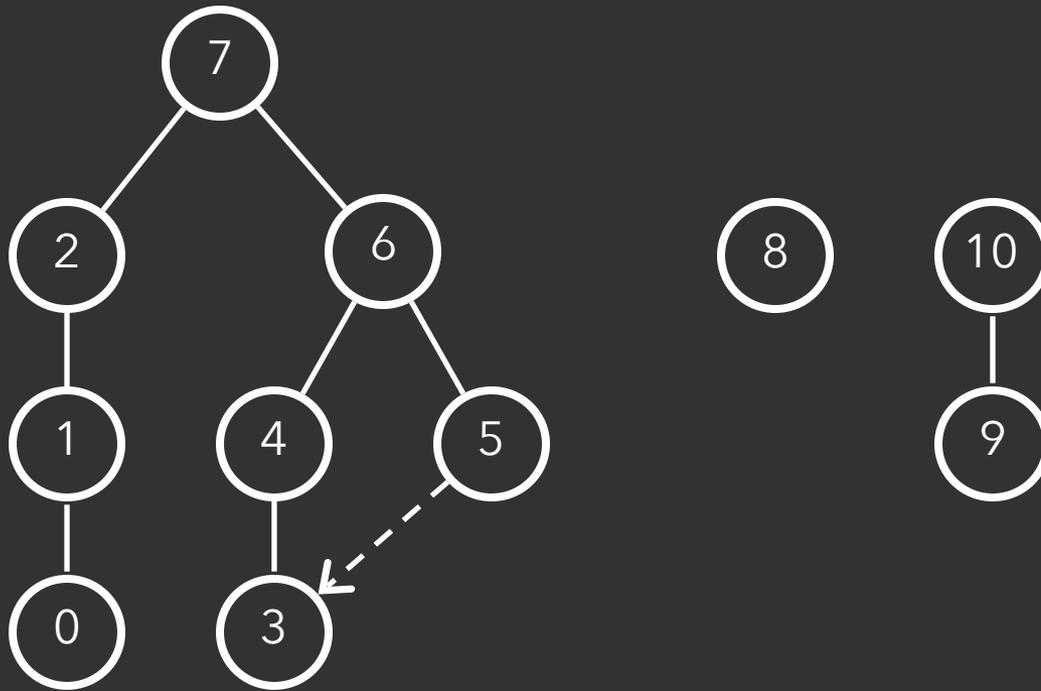
# Reingold-Tilford Layout



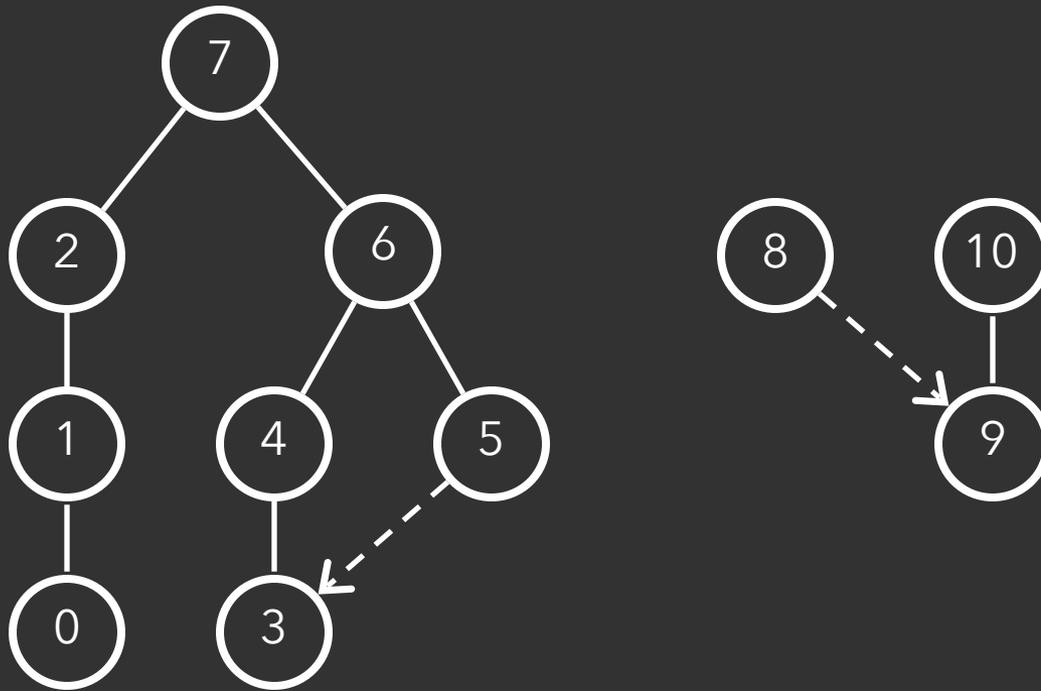
# Reingold-Tilford Layout



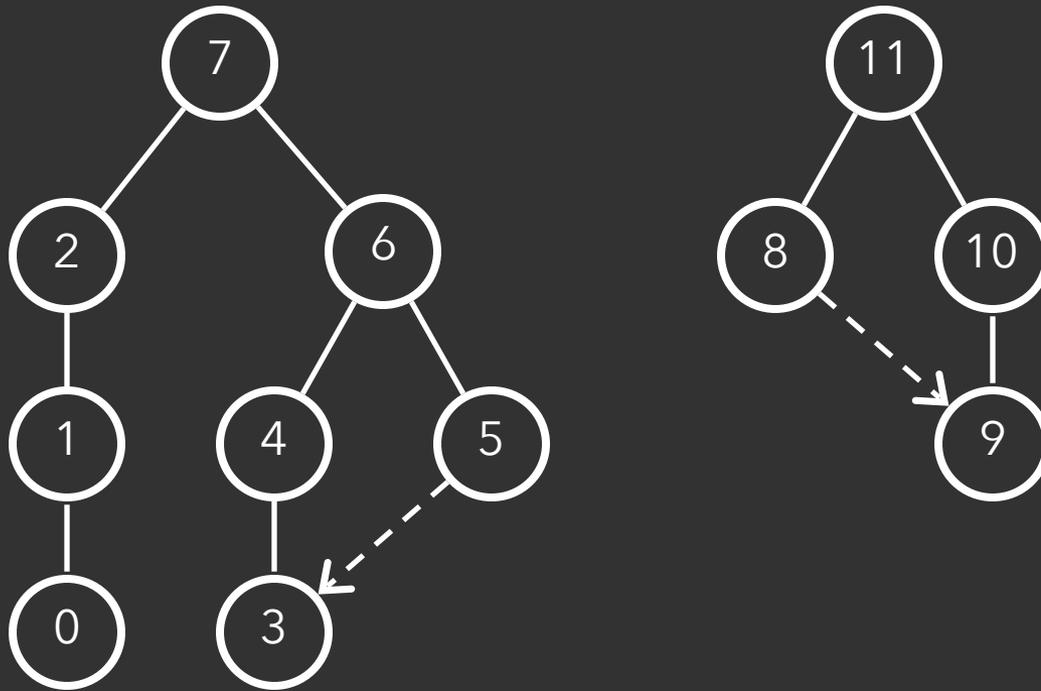
# Reingold-Tilford Layout



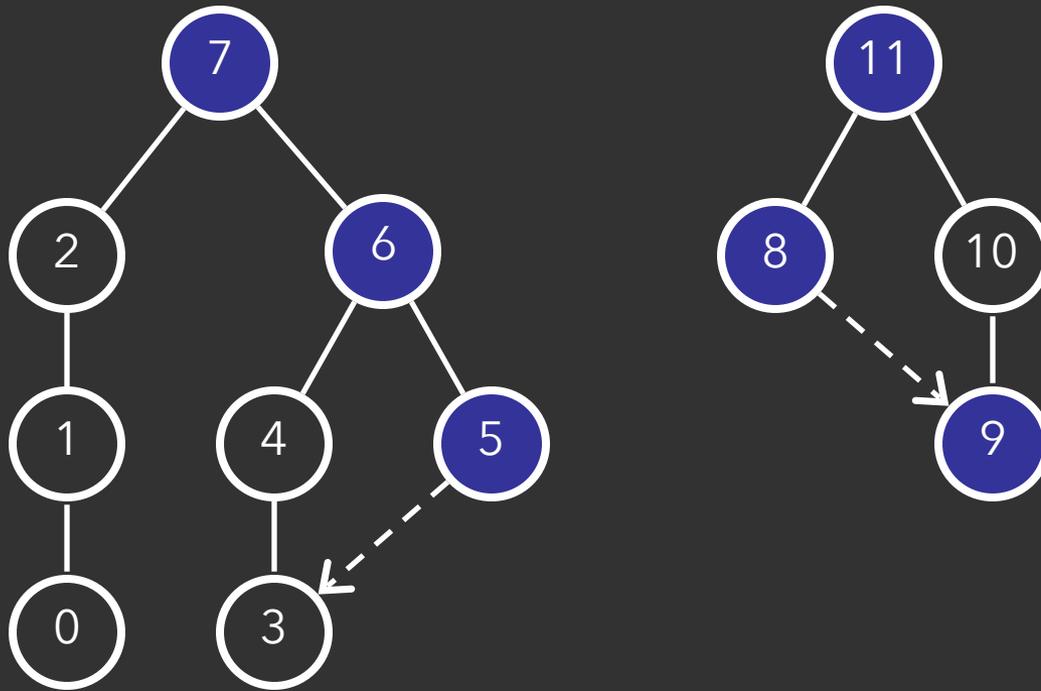
# Reingold-Tilford Layout



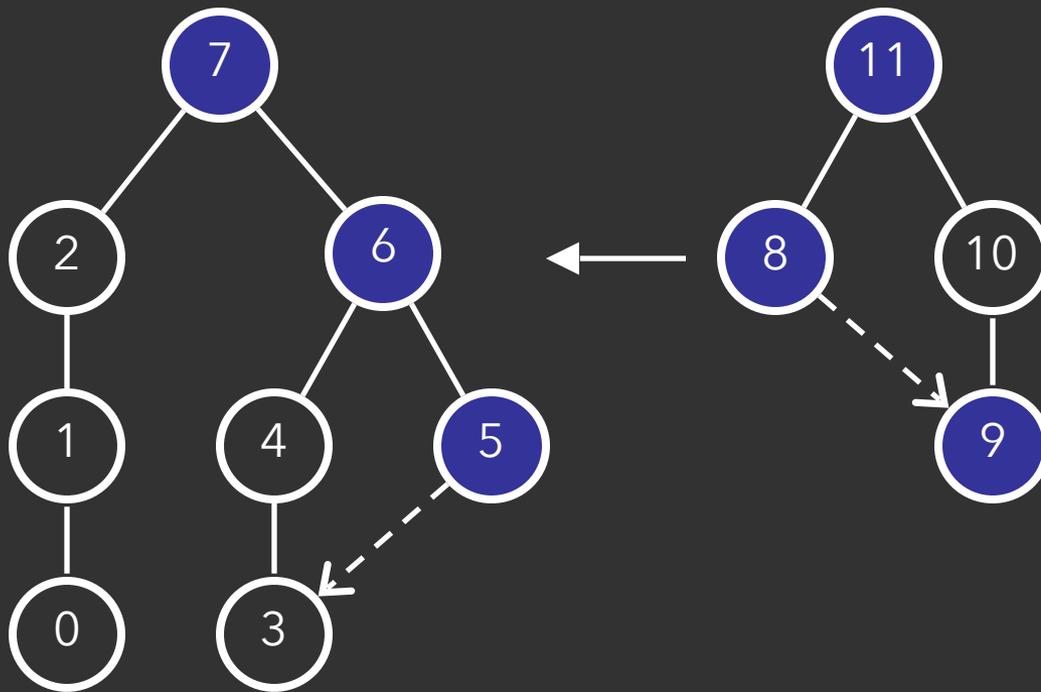
# Reingold-Tilford Layout



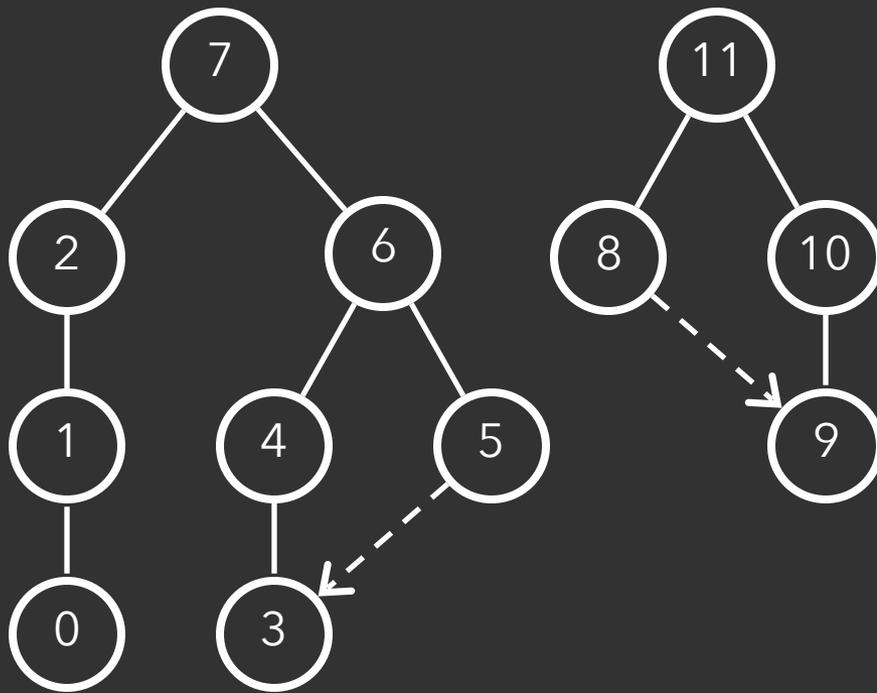
# Reingold-Tilford Layout



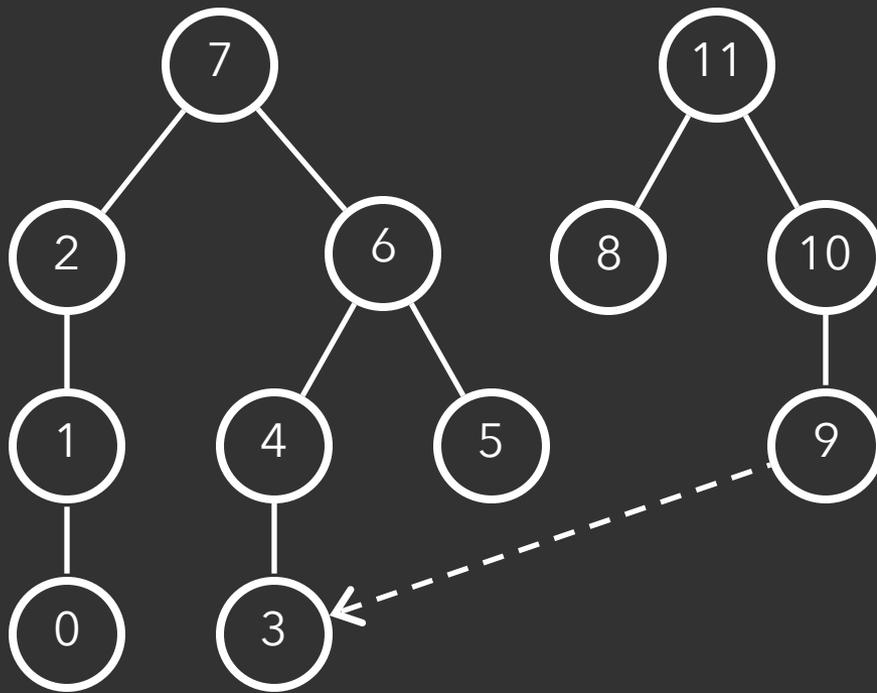
# Reingold-Tilford Layout



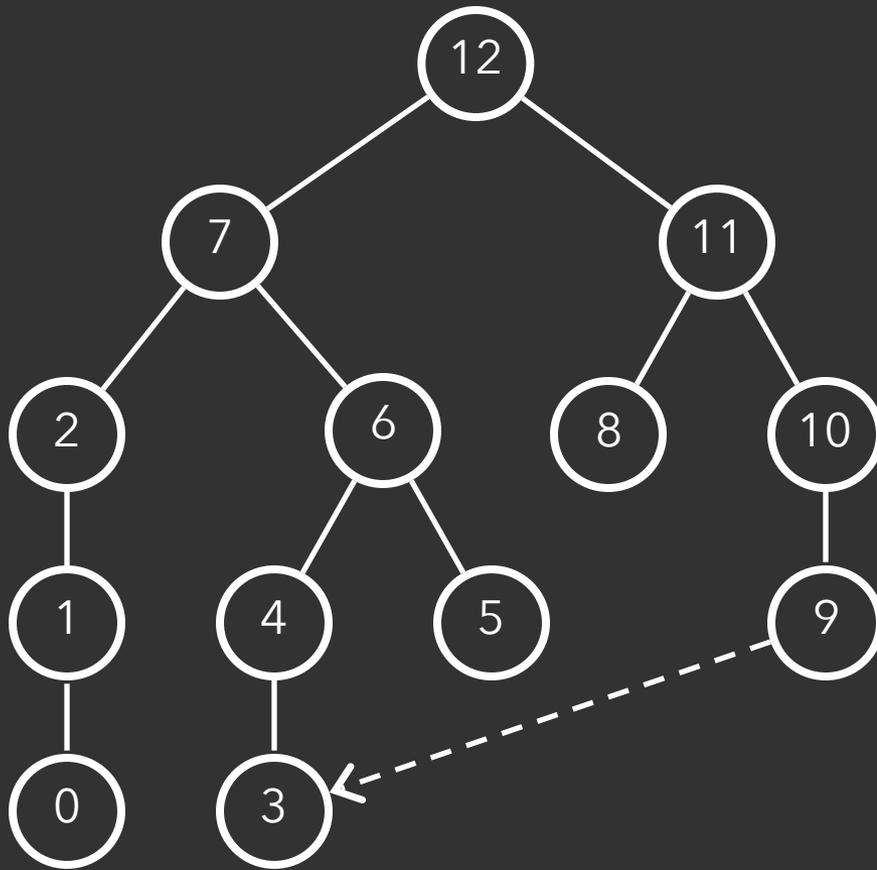
# Reingold-Tilford Layout



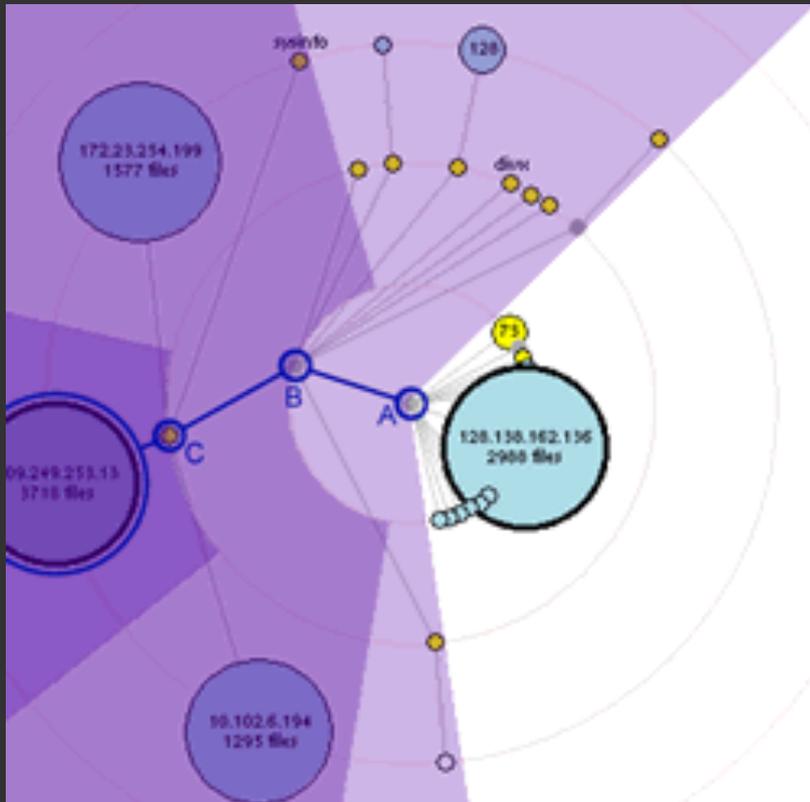
# Reingold-Tilford Layout



# Reingold-Tilford Layout



# Radial Layout



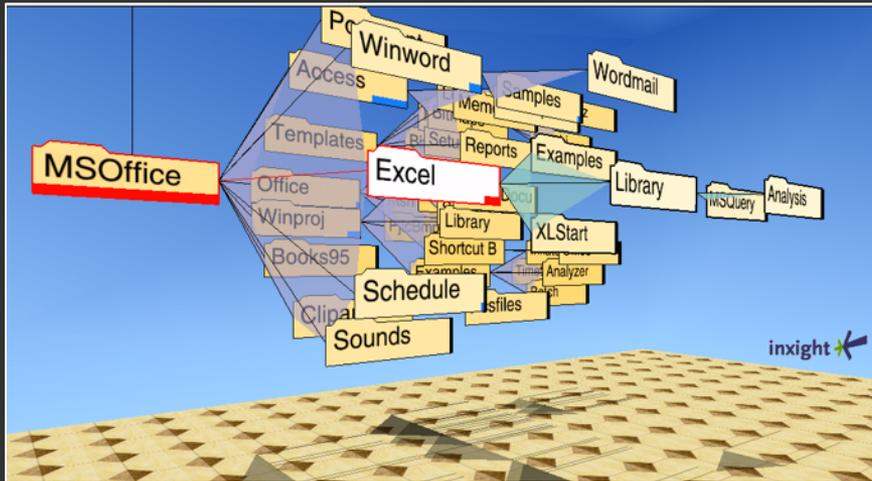
Node-link diagram in polar co-ordinates.

Radius encodes depth, with root in the center.

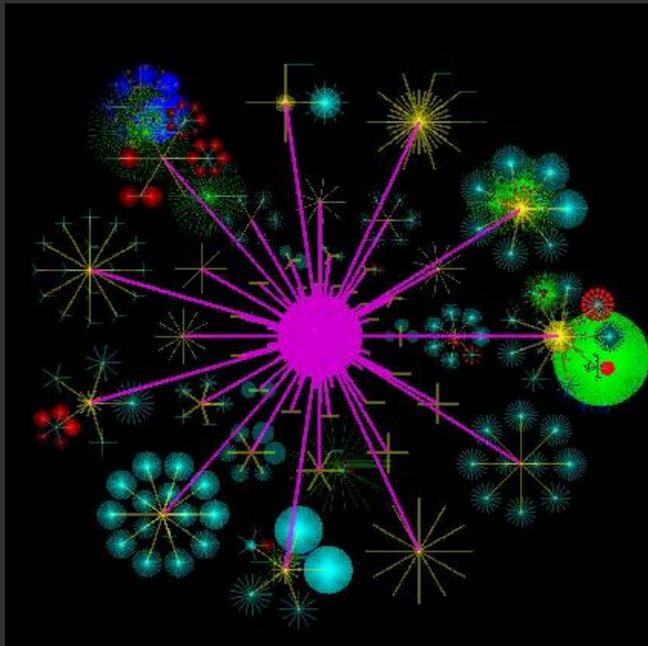
Angular sectors assigned to subtrees (typically uses recursive approach).

Reingold-Tilford method could be applied here.

# Circular Tree Layouts



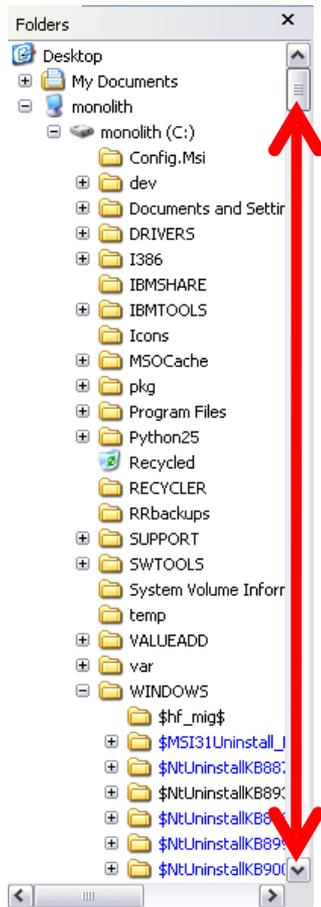
Layout in 3D to form Cone Trees.



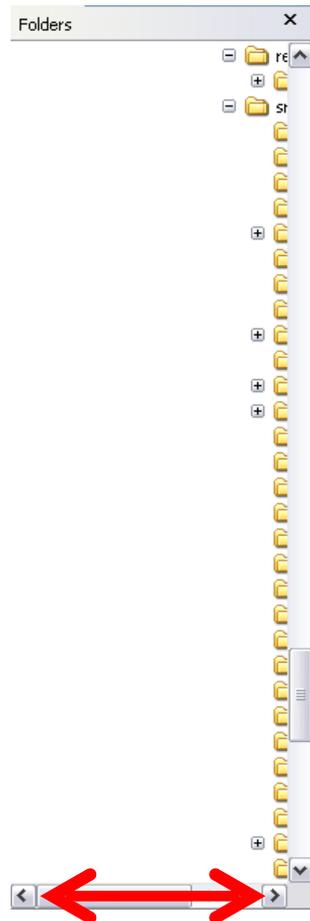
Balloon Trees can be described as a 2D variant of a Cone Tree. Not just a flattening process, as circles must not overlap.

**Focus + Context**

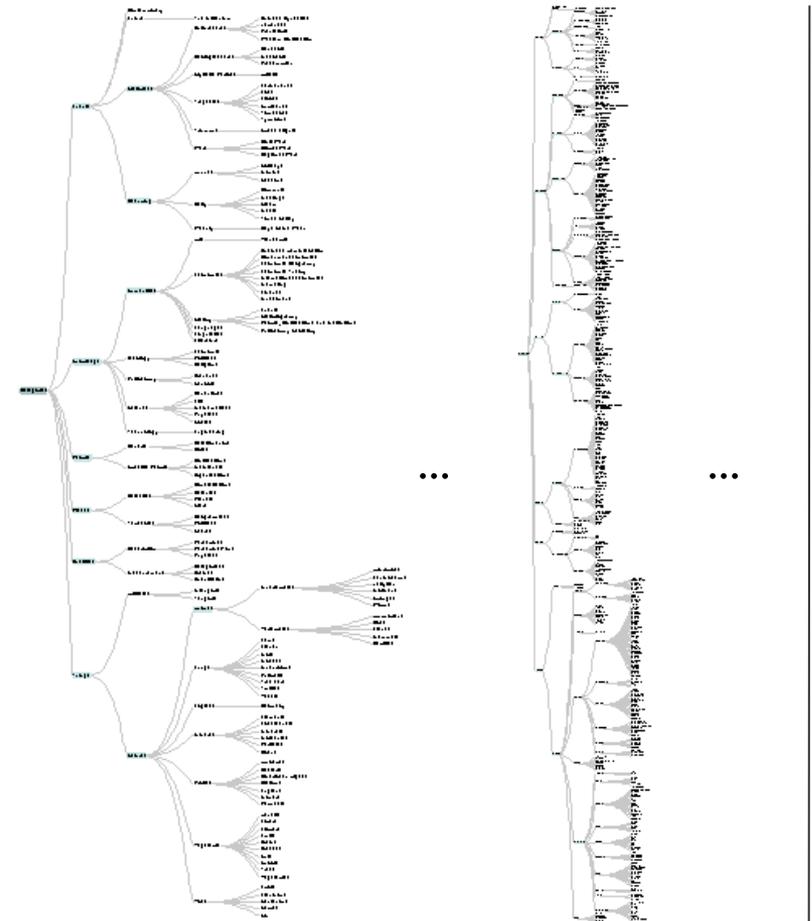
# Visualizing Large Hierarchies



...



...



Indented Layout

Reingold-Tilford Layout

# More Nodes, More Problems...

## Scale

Tree breadth often grows exponentially  
Even with tidy layout, quickly run out of space

## Possible Solutions

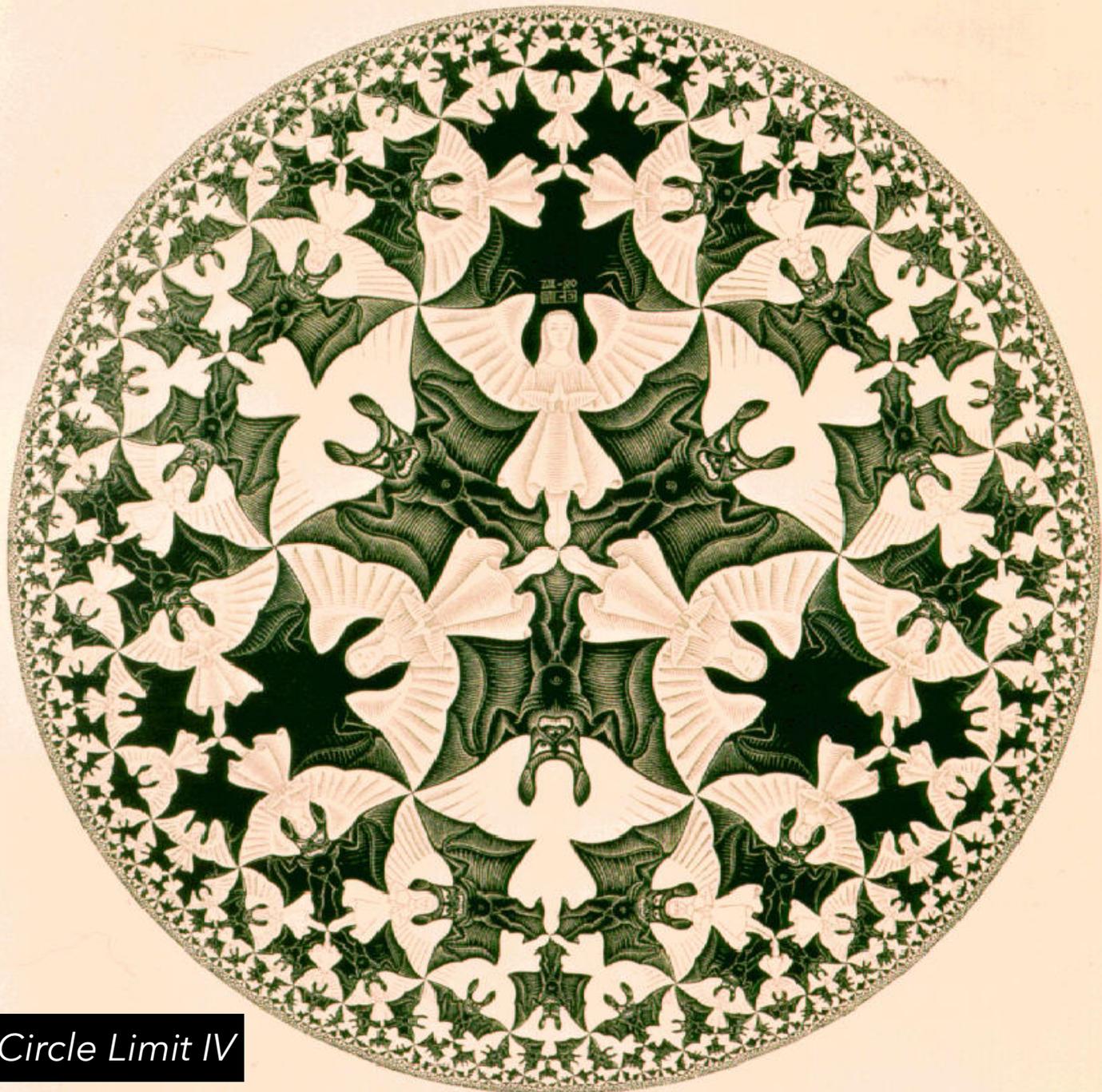
Filtering

Focus+Context

Scrolling or Panning

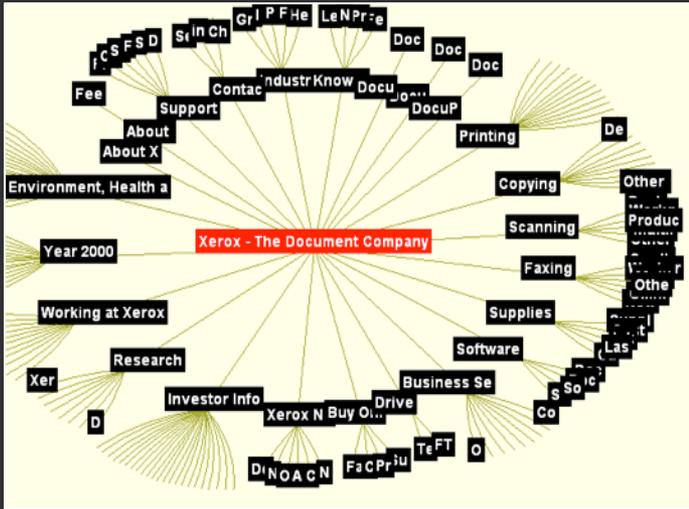
Zooming

Aggregation



MC Escher, *Circle Limit IV*

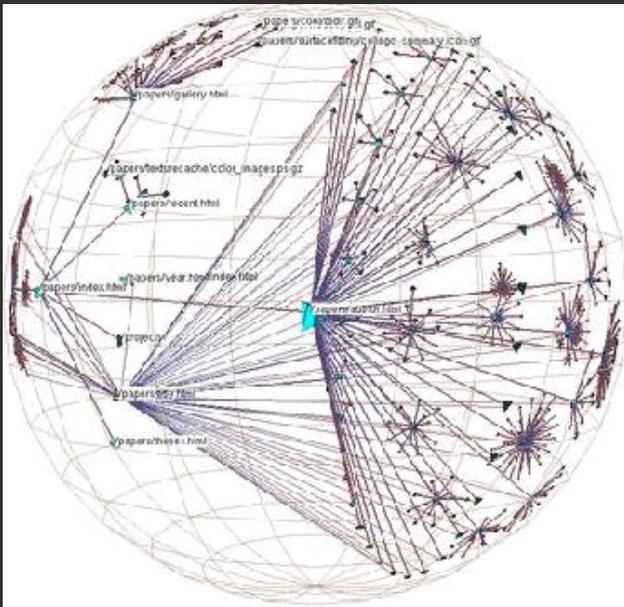
# Hyperbolic Layout



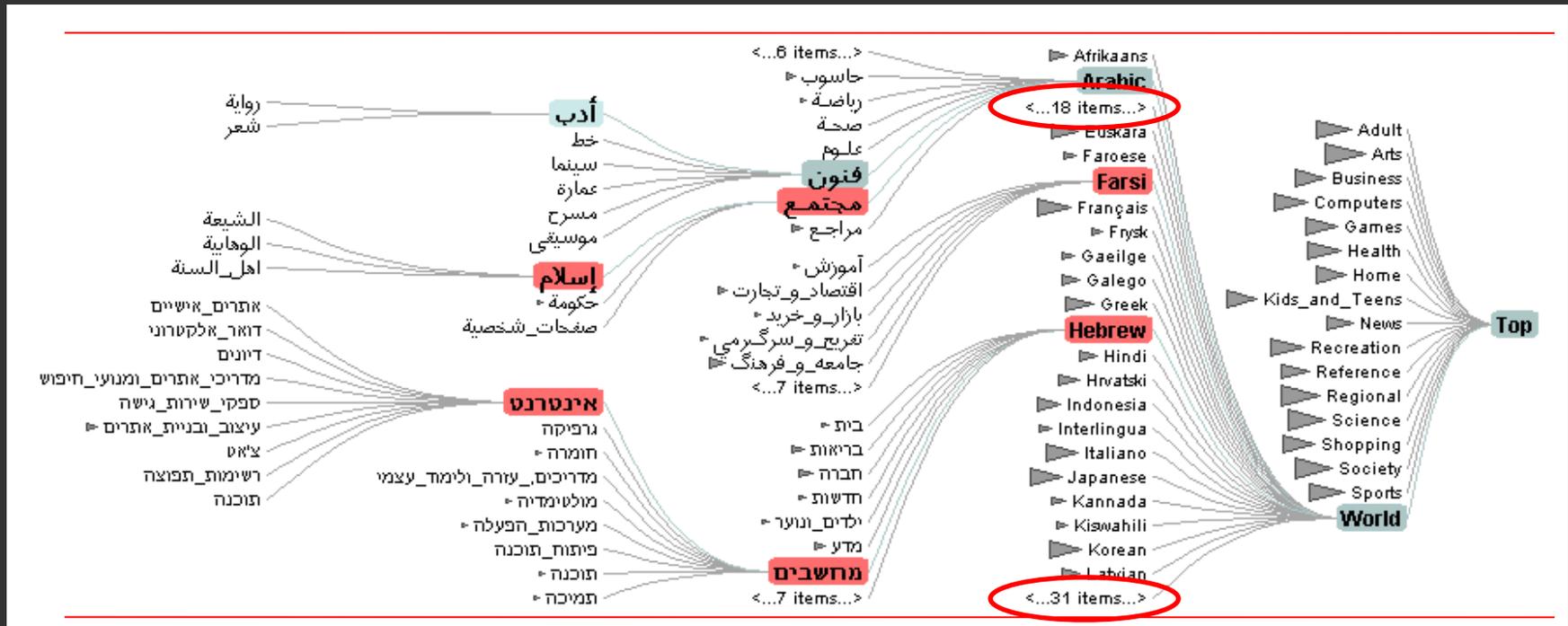
Perform tree layout in hyperbolic geometry, project the result on to the Euclidean plane.

Why? Like tree breadth, the hyperbolic plane expands exponentially!

Also computable in 3D, projected into a sphere.

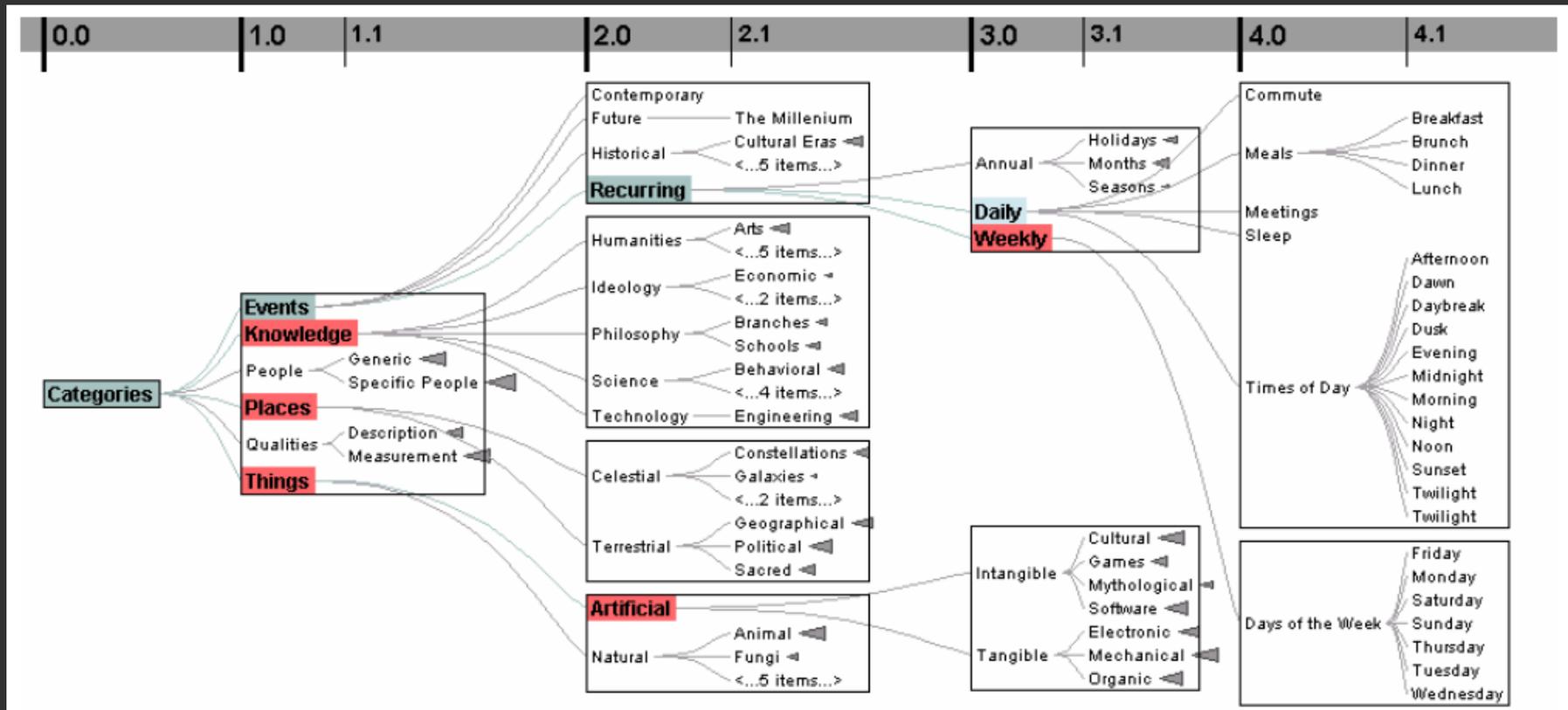


# Degree-of-Interest Trees



Space-constrained, multi-focal tree layout

# Degree-of-Interest Trees

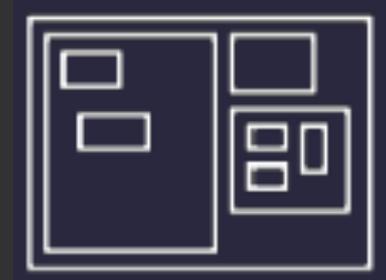


Cull "un-interesting" nodes on a per block basis until all blocks on a level fit within bounds. Attempt to center child blocks beneath parents.

# Enclosure / Layering

# Enclosure Diagrams

Encode structure using **spatial enclosure**  
Popularly known as **treemaps**



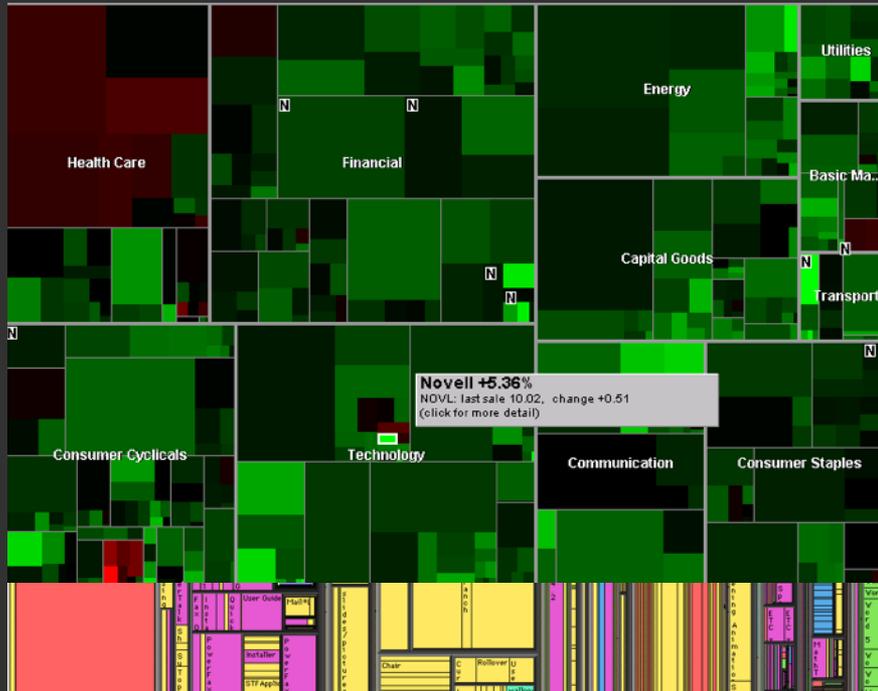
## Benefits

Provides a single view of an entire tree  
Easier to spot large/small nodes

## Problems

Difficult to accurately read structure / depth

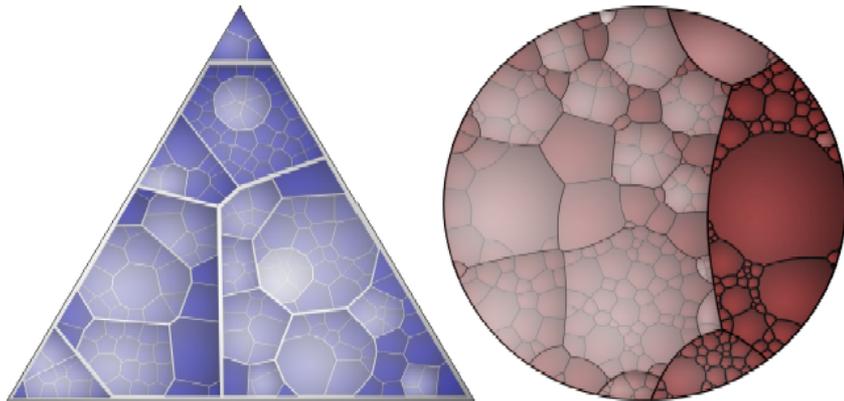
# Treemaps



Recursively fill space.  
Enclosure signifies hierarchy.

Additional measures can  
be taken to control  
aspect ratio of cells.

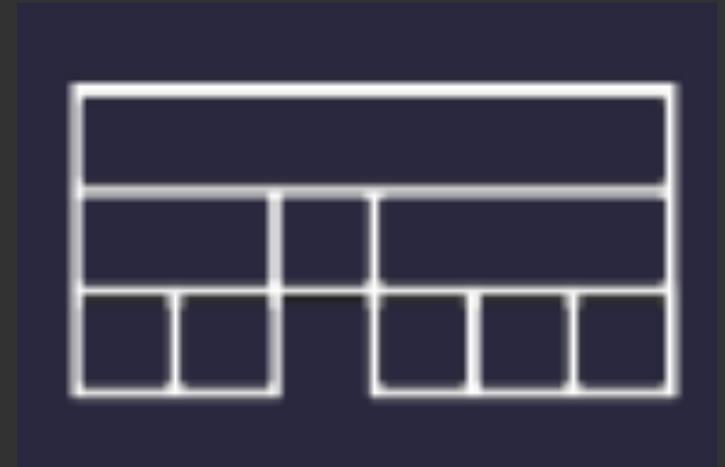
Often uses rectangles,  
but other shapes are  
possible, e.g., iterative  
Voronoi tessellation.



# Layered Diagrams

Signify tree structure using

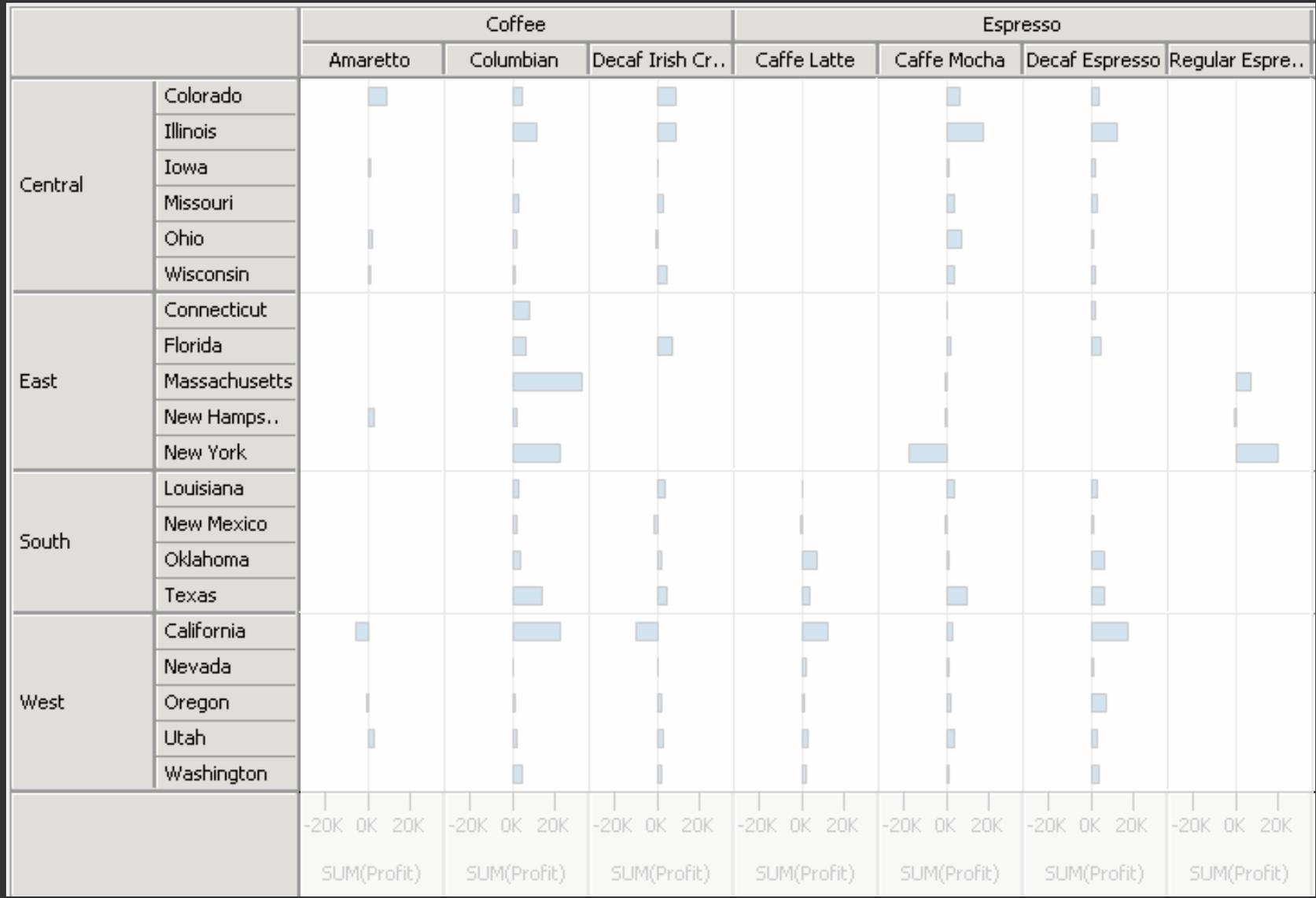
- Layering
- Adjacency
- Alignment



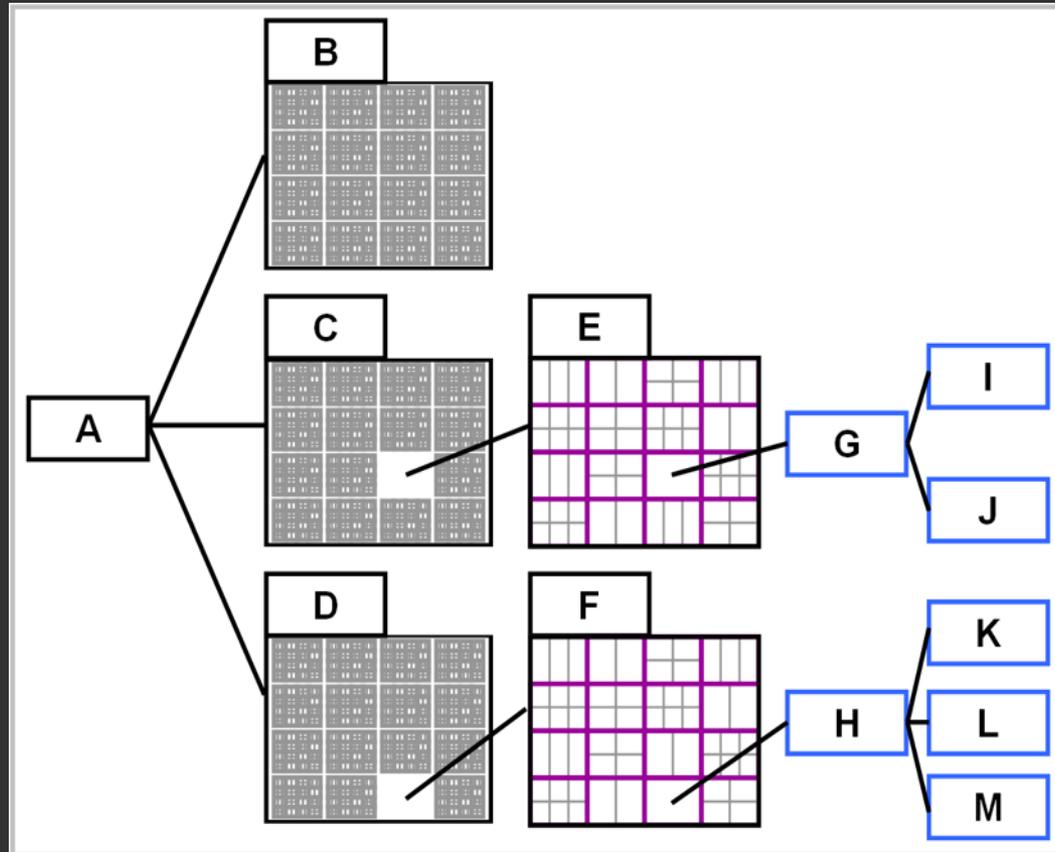
Involves recursive sub-division of space.



# Layered Tree Drawing



# Hybrids are also possible...



“Elastic Hierarchies”  
Node-link diagram  
with treemap nodes.

# Administrivia

# Final Project Schedule

<i>Proposal</i>	<b>Tues, May 10 (5pm)</b>
<i>Presentation</i>	Thur, May 19 (slides: 5/18, 5pm)
<i>Poster &amp; Demo</i>	Tues, Jun 7 (5-8pm)
<i>Final Paper</i>	Thur, Jun 9 (8am)

## **Logistics**

Groups of up to 4 people

Clearly report responsibilities of each member

# Graph Layout

# Approaches to Graph Drawing

## Calculation using Graph Structure

Tree layout on spanning tree

Sugiyama-style (hierarchical) layout

Adjacency matrix layout

## Optimization Methods

Constraint satisfaction

Force-directed layout

## Attribute-Driven Layout

Layout using data attributes, not linkage

# Spanning Tree Layout

# Spanning Tree Layout

**Many graphs have useful spanning trees**

Websites, Social Networks

**Use tree layout on spanning tree of graph**

Trees created by BFS / DFS

Min/max spanning trees

Fast tree layouts allow graph layouts to be recalculated at interactive rates

Heuristics may further improve layout



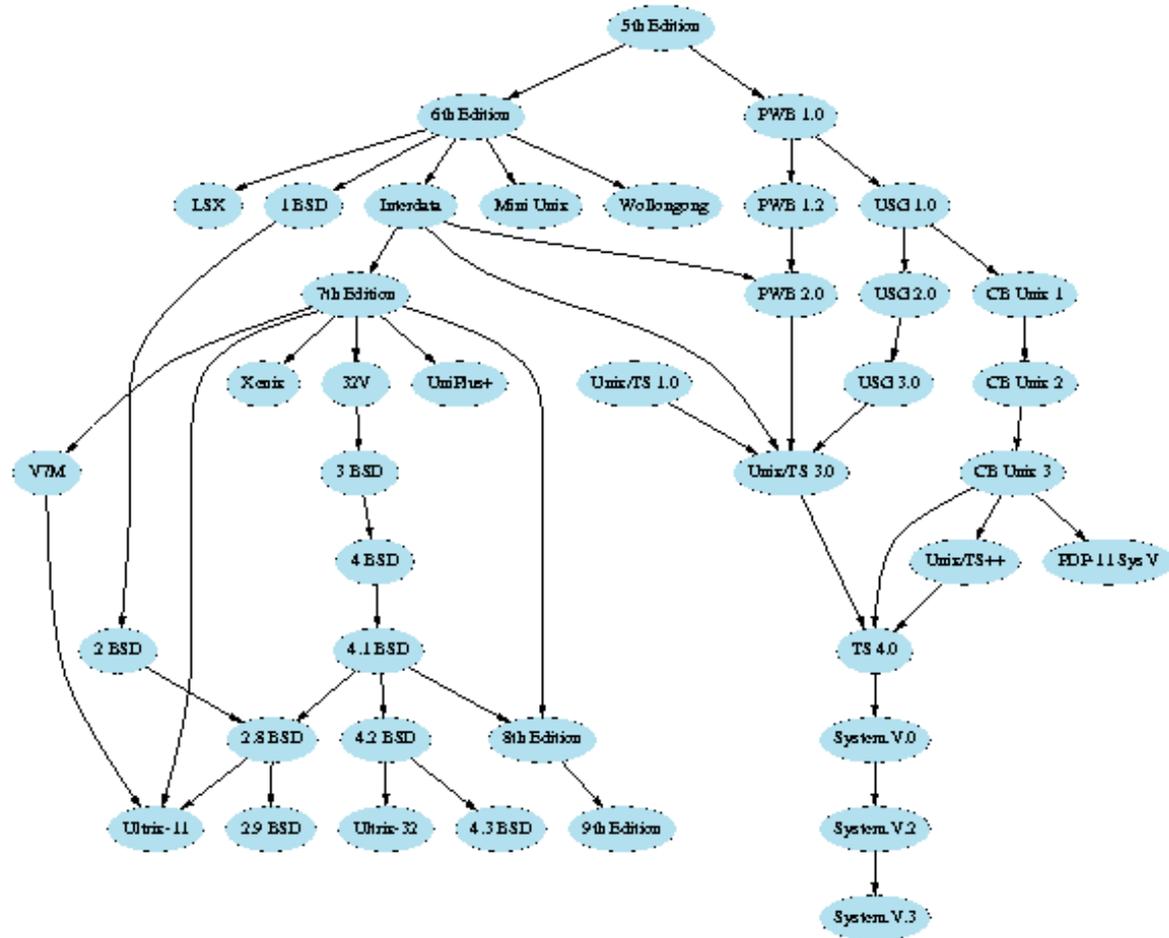
Spanning tree layout may result in arbitrary parent node

# Sugiyama-Style Layout

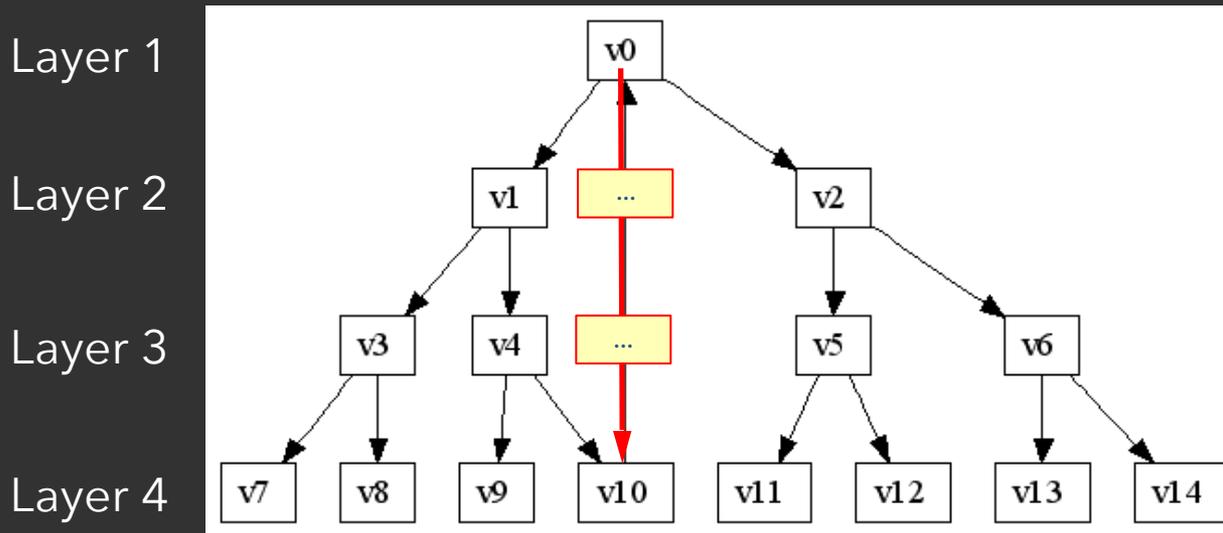
# Sugiyama-style Layout

Evolution of the UNIX operating system

Hierarchical layering based on descent



# Sugiyama-style Layout



Reverse edges to remove cycles

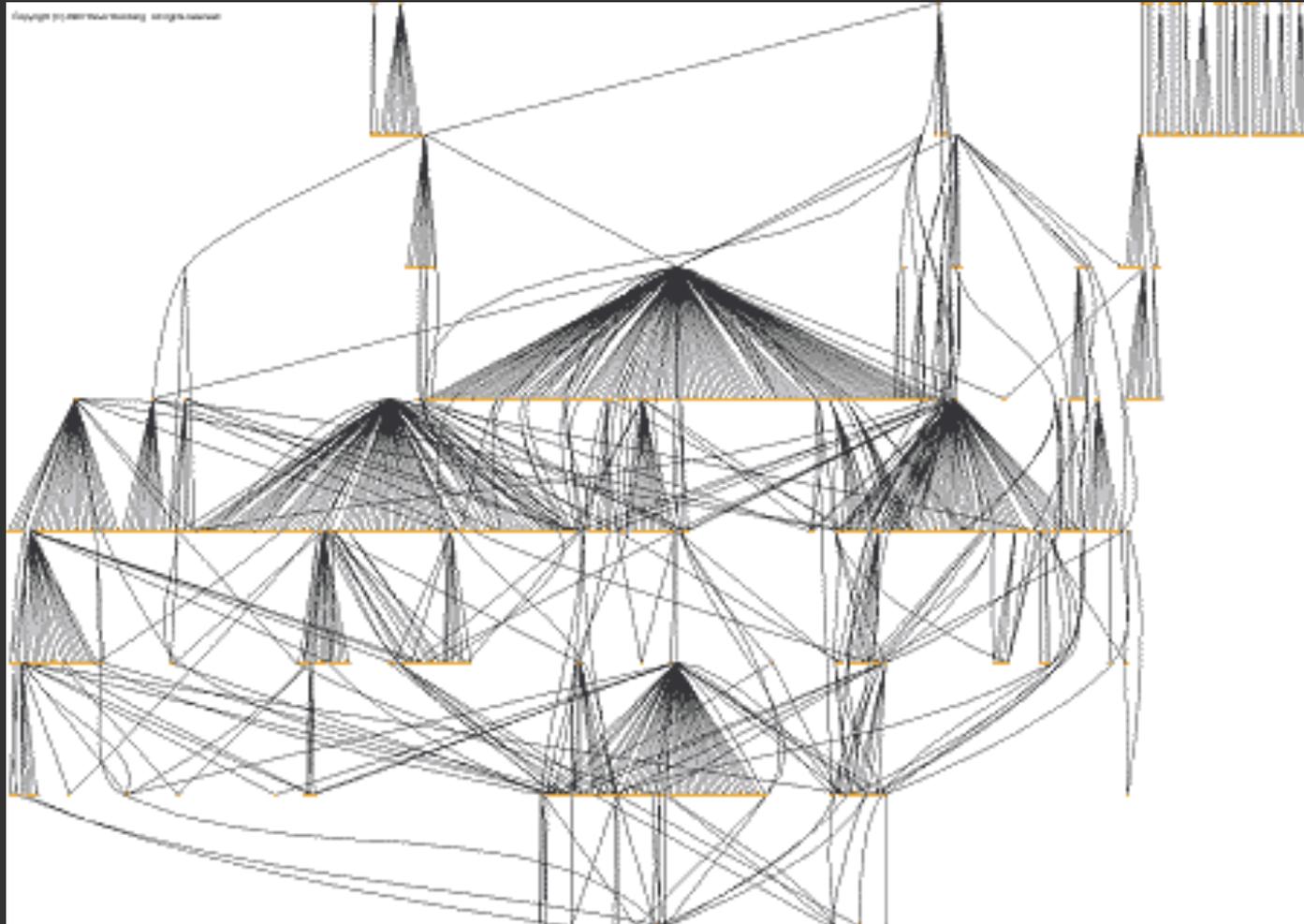
Assign nodes to hierarchy layers

Create dummy nodes to "fill in" missing layers

Arrange nodes within layer, minimize edge crossings

Route edges - layout splines if needed

# Hierarchical Layout



Gnutella network

# Force-Directed Layout

# Optimization Techniques

**Treat layout as an *optimization problem***

Define layout using an *energy model* along with *constraints*: equations the layout should obey.

Use optimization algorithms to solve

**Commonly posed as a physical system**

Charged particles, springs, drag force, ...

**We can introduce directional constraints**

*DiG-CoLa* (Di-Graph Constr Optimization Layout) [Dwyer 05]

Iterative constraint relaxation

# Optimizing Aesthetic Constraints

Minimize edge crossings

Minimize area

Minimize line bends

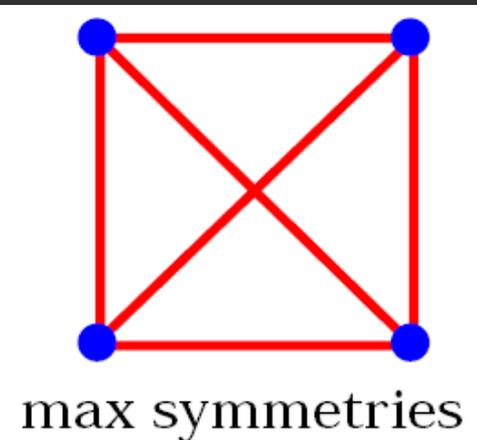
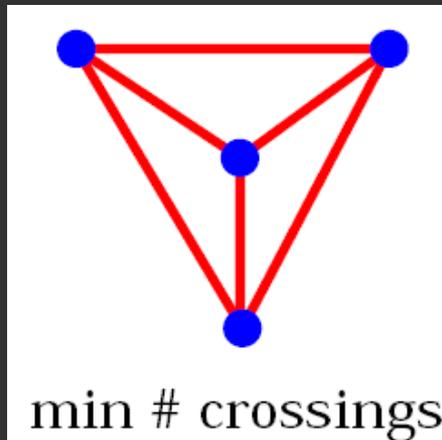
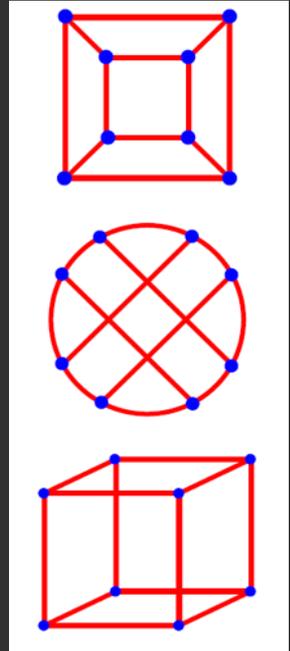
Minimize line slopes

Maximize smallest angle between edges

Maximize symmetry

but, can't do it all.

Optimizing these criteria is often NP-Hard, requiring approximations.



# Force-Directed Layout

Nodes = charged particles  $F = G * m_1 * m_2 / (x_i - x_j)^2$

with air resistance  $F = -b * v_i$

Edges = springs  $F = -k * (x_i - x_j - L)$

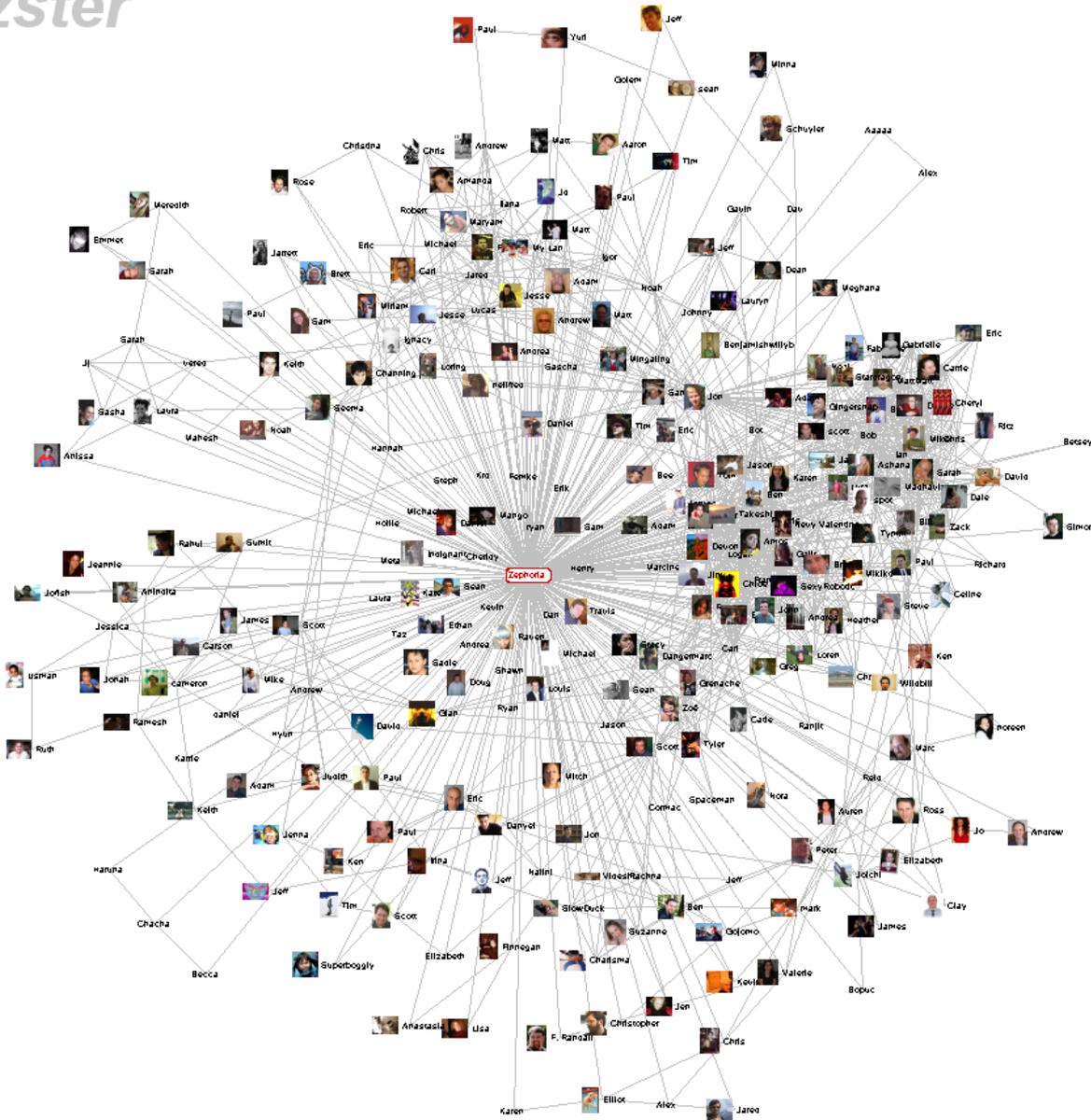
**Iteratively calculate forces, update node positions**

Naïve n-body calculation is  $O(N^2)$

$O(N \log N)$  using quadtree or k-d tree

Numerical integration of forces at each time step

vizster



community >>

Enable

search >>

# Zephoria

<b>User ID</b>	21721
<b>Friends</b>	<input type="checkbox"/> 266
<b>Age</b>	??
<b>Gender</b>	<input type="checkbox"/> Female
<b>Status</b>	<input type="checkbox"/> Single
<b>Location</b>	San Francisco, CA
<b>Hometown</b>	Lancaster, PA
<b>Occupation</b>	researcher: social networks, identity, context
<b>Interests</b>	apophenia, observing people, culture, questioning power, reading, buddhism, ipseity, computer-mediated communication, social networks, technology, anthropology, stomping
<b>Music</b>	psytrance/goa/trance [Infected Mushroom, Son Kite... Iboga/Digital Structures], Ani Difranco, downtempo, Thievery Corporation, Beth Orton, Morcheeba, Ween, White Stripes
<b>Books</b>	Authors: Erving Goffman, Stanley Milgram, Jeanette Winterson, Eric Schlosser, Leslie Feinberg, Dorothy Allison, Italo Calvino, Hermann Hesse
<b>TV Shows</b>	??
<b>Movies</b>	Koyaanisqatsi, Amelie, Waking Life, Tank Girl, The Matrix, Clockwork Orange, American Beauty, Fight Club, Boys Don't Cry
<b>Member Since</b>	??
<b>Last Login</b>	2003-10-21
<b>Last Updated</b>	2003-10-21
<b>About</b>	[Some know me as danah...]

I'm a geek, an activist and an academic, fascinated by people and society. I see life as a very large playground and enjoy exploring its intricacies. I revel in life's chaos, while simultaneously providing my own insane element.

My musings:  
<http://www.zephoria.org/thoughts/>

**Want to Meet** Someone who makes life's complexities seem simply elegant.

# Constrained Optimization

## Minimize stress function

$$\text{stress}(X) = \sum_{i < j} w_{ij} ( \|X_i - X_j\| - d_{ij} )^2$$

$X$ : node positions,  $d$ : optimal edge length,

$w$ : normalization constants

*Says: Try to place nodes  $d_{ij}$  apart*

# Constrained Optimization

## Minimize stress function

$$\text{stress}(X) = \sum_{i < j} w_{ij} ( \|X_i - X_j\| - d_{ij} )^2$$

$X$ : node positions,  $d$ : optimal edge length,  
 $w$ : normalization constants

*Says: Try to place nodes  $d_{ij}$  apart*

## Add hierarchy ordering constraints

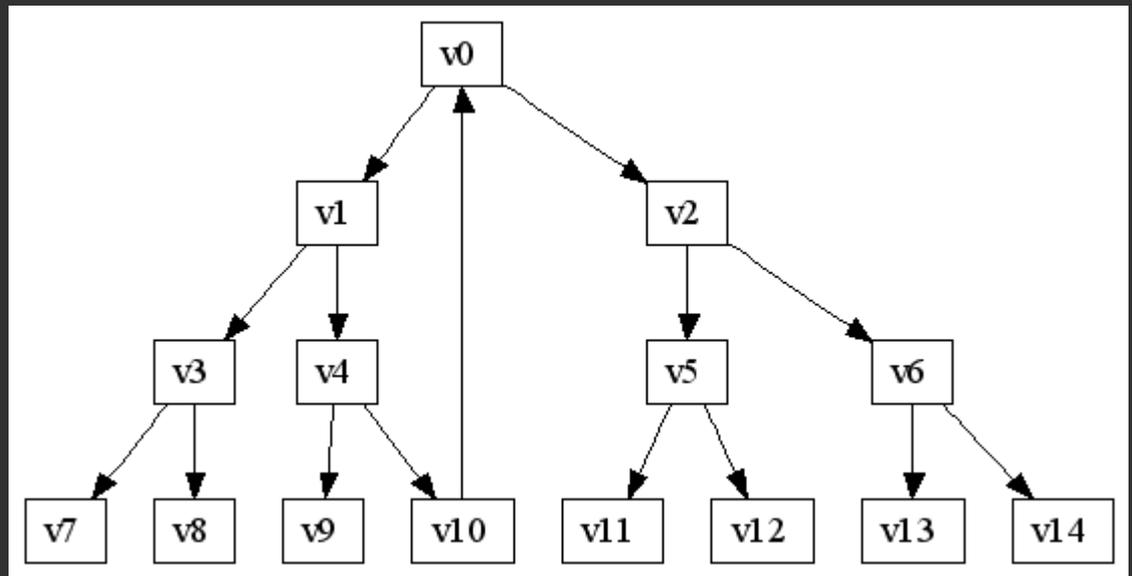
$$E_H(y) = \sum_{(i,j) \in E} ( y_i - y_j - \delta_{ij} )^2$$

$y$ : node  $y$ -coordinates

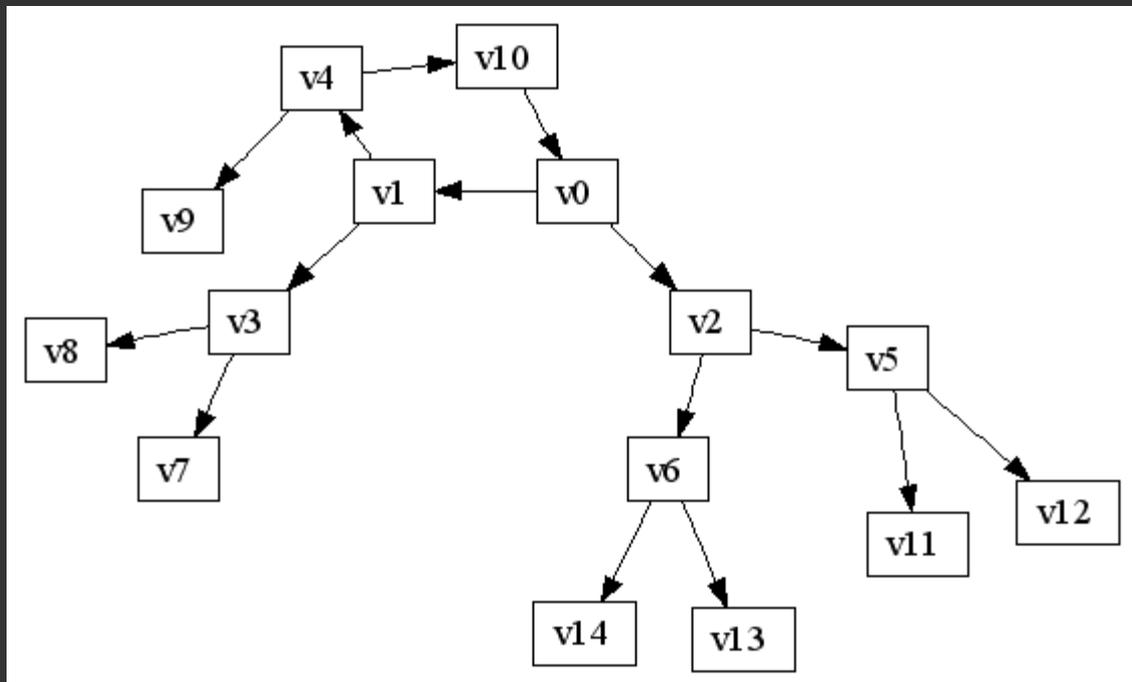
$\delta$ : edge direction (e.g., 1 for  $i \rightarrow j$ , 0 for undirected)

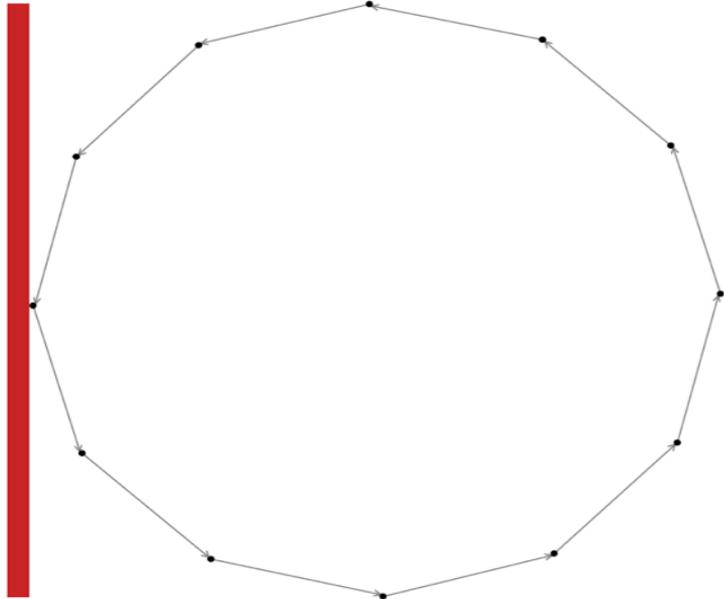
*Says: If  $i$  points to  $j$ , it should have a lower  $y$ -value*

Sugiyama layout (dot)  
Preserve tree structure

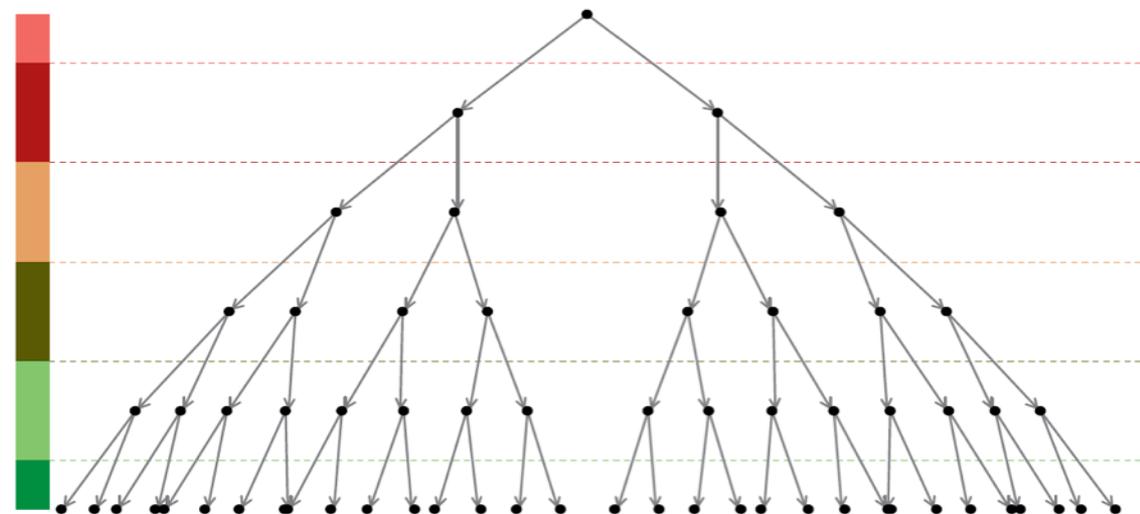


DiG-CoLa method  
Preserve edge lengths

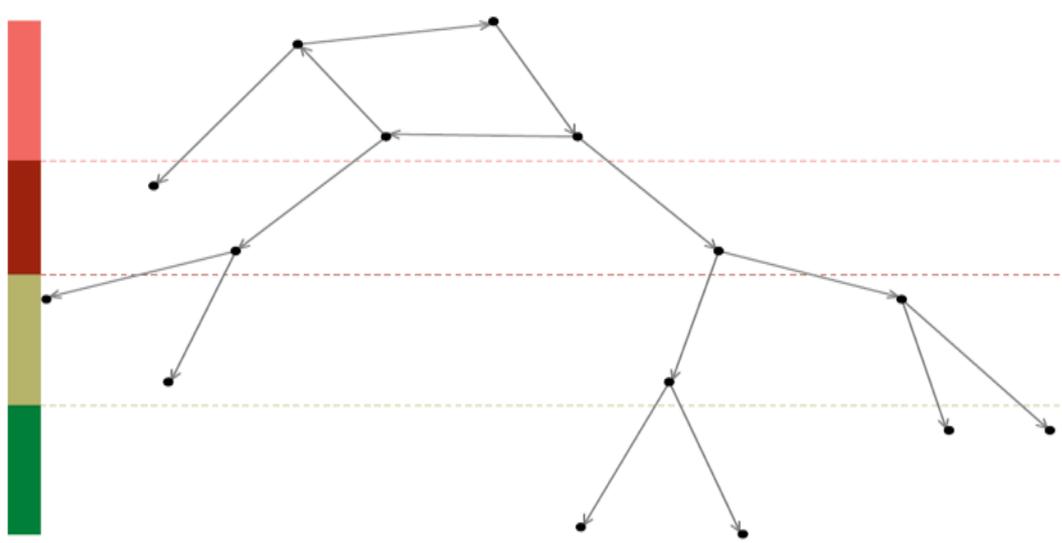




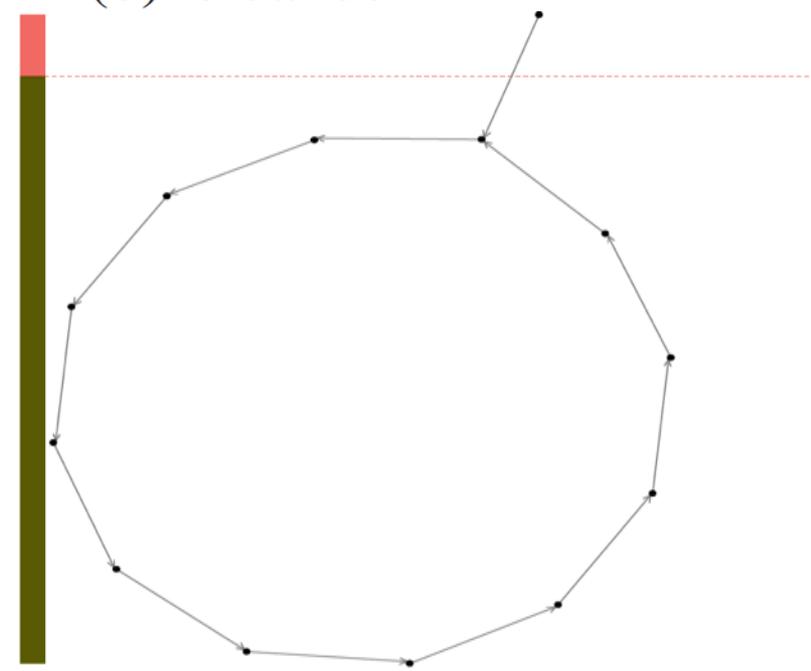
(a) 1 band



(b) 6 bands



(c) 4 bands



(d) 2 bands

# Iterative Constraint Relaxation

Quadratic programming is complex to code and computationally costly. Is there a simpler way?

Iteratively **relax** each constraint [Dwyer 09]

Given a constraint (e.g.,  $|x_i - x_j| = 5$ )

Simply push the nodes to satisfy!

Each relaxation may **clobber** prior results

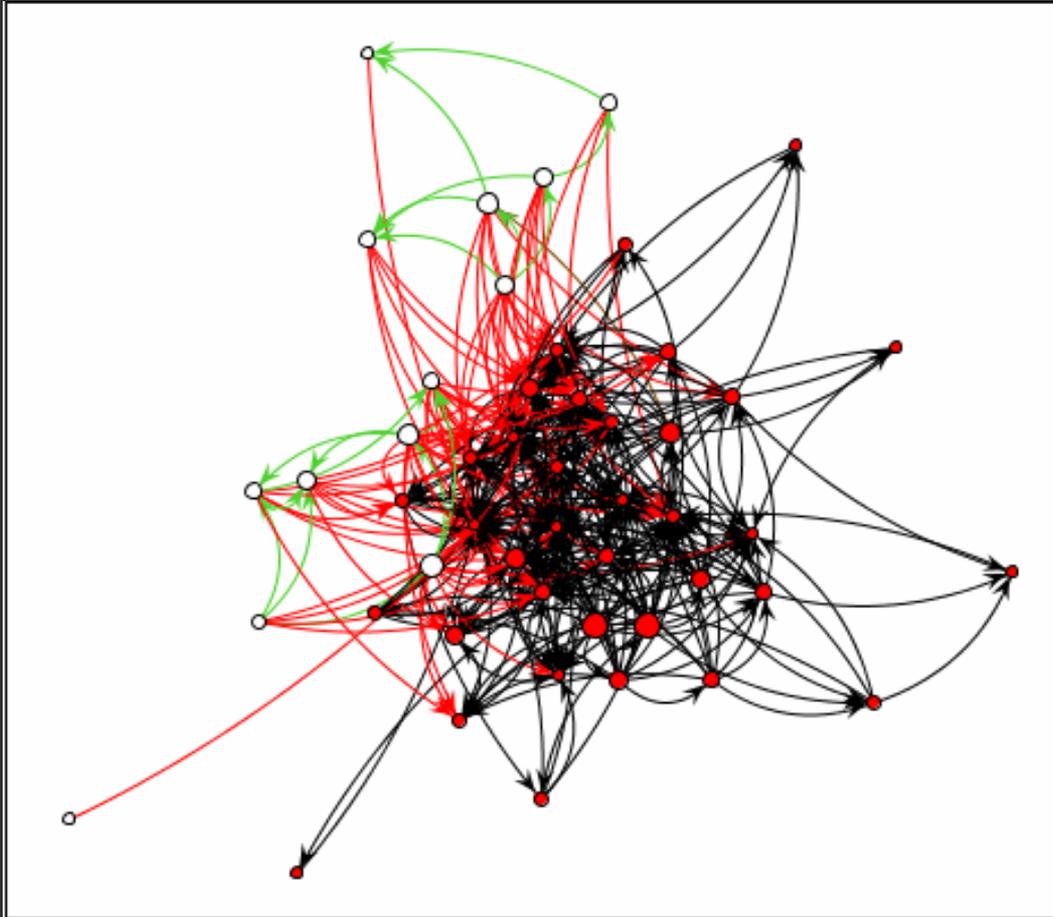
But this typically **converges quickly**

Enables **expressive constraints!**

# Use the Force!

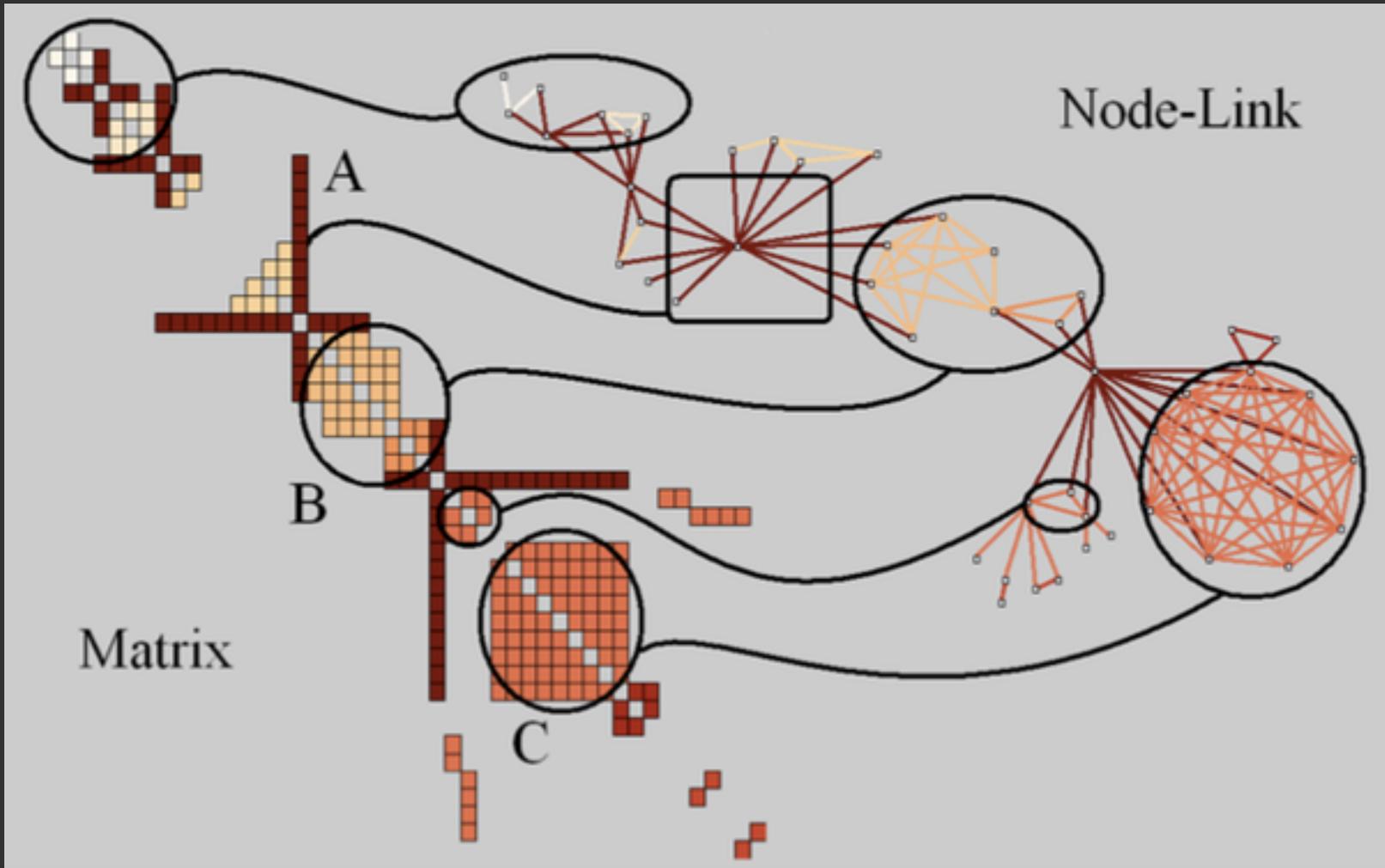
<http://mbostock.github.io/d3/talk/20110921/>

# Limitations of Node-Link Layout

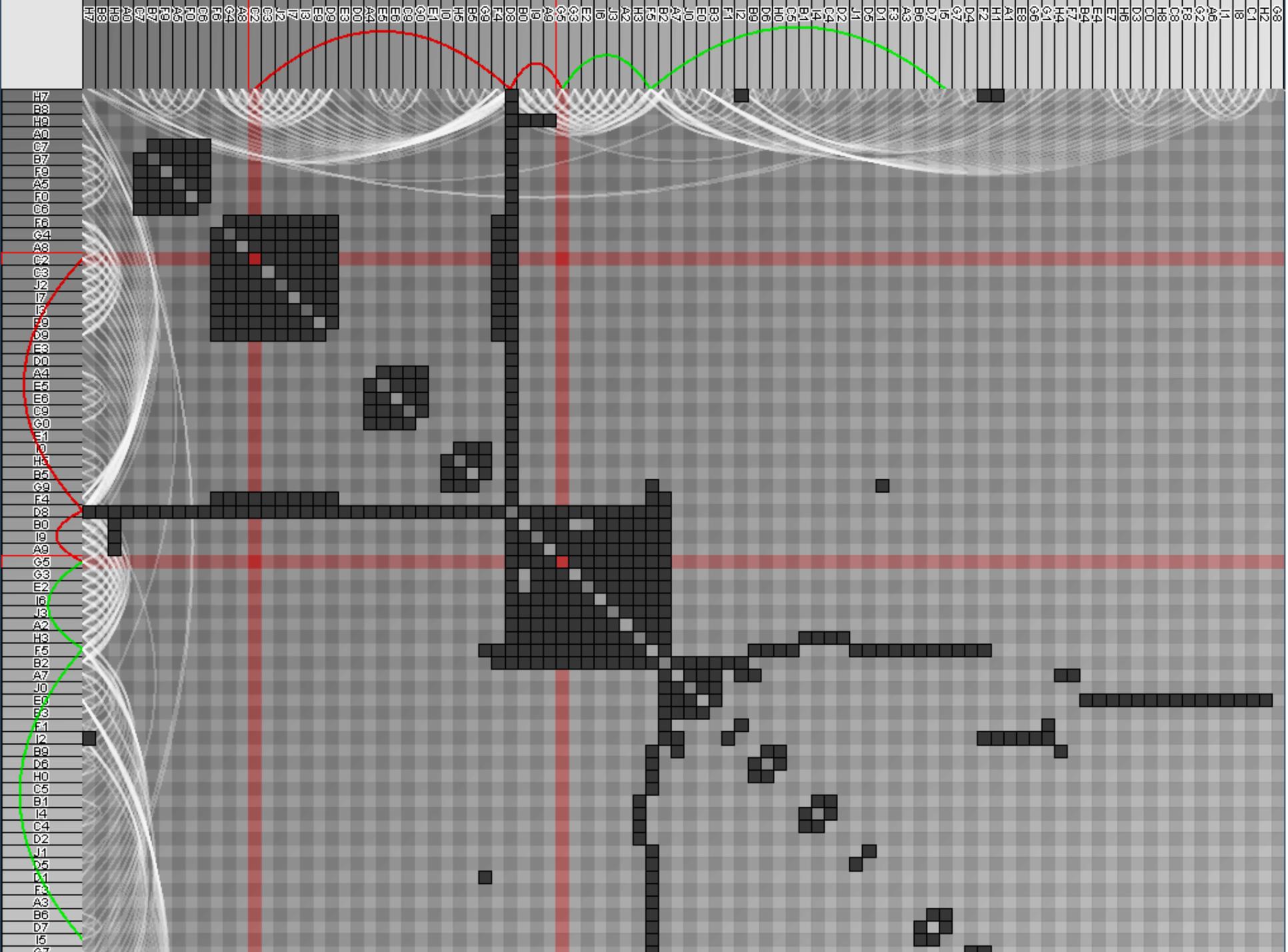


Edge-crossings and occlusion

# Matrix Diagrams



# Adjacency Matrices



### Graph Viewer

Roll-up by:

All

Visualization:

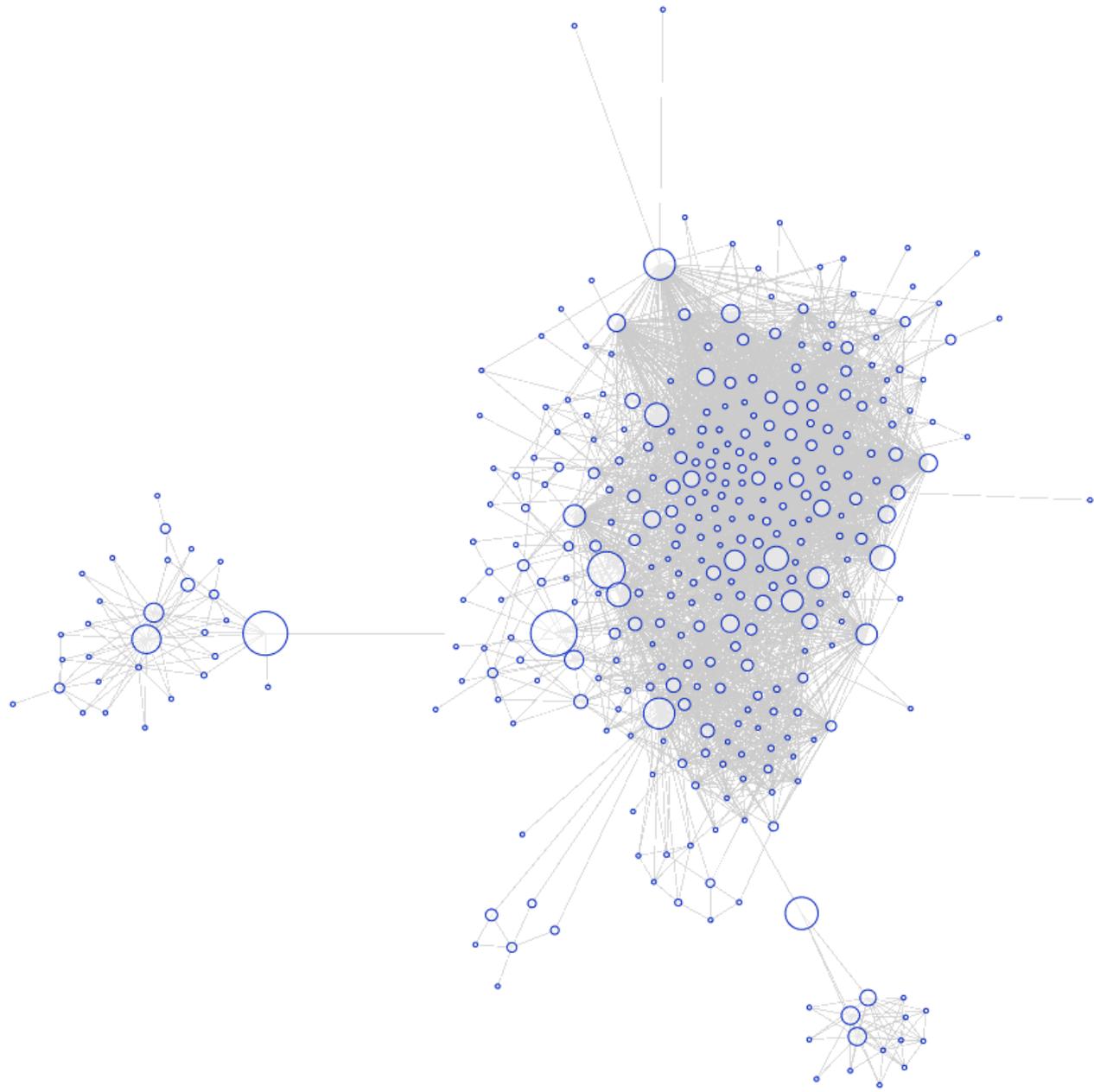
Node-Link

Sort by:

None

Edge centrality filters:

Two horizontal sliders for edge centrality filtering, both currently set to the minimum value.



- Images
- Animate

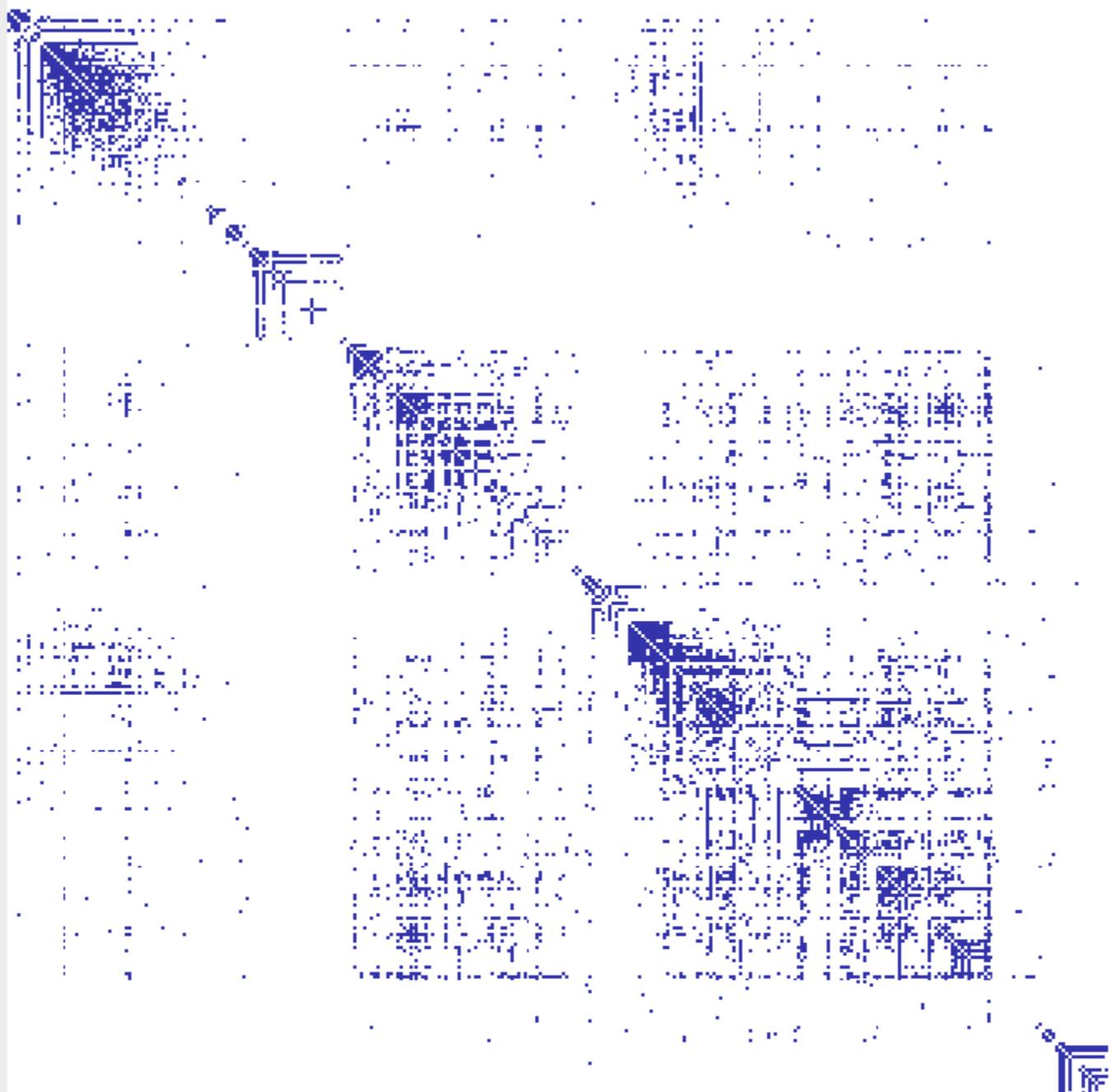
# Graph Viewer

Roll-up by:

Visualization:

Sort by:

Edge centrality filters:

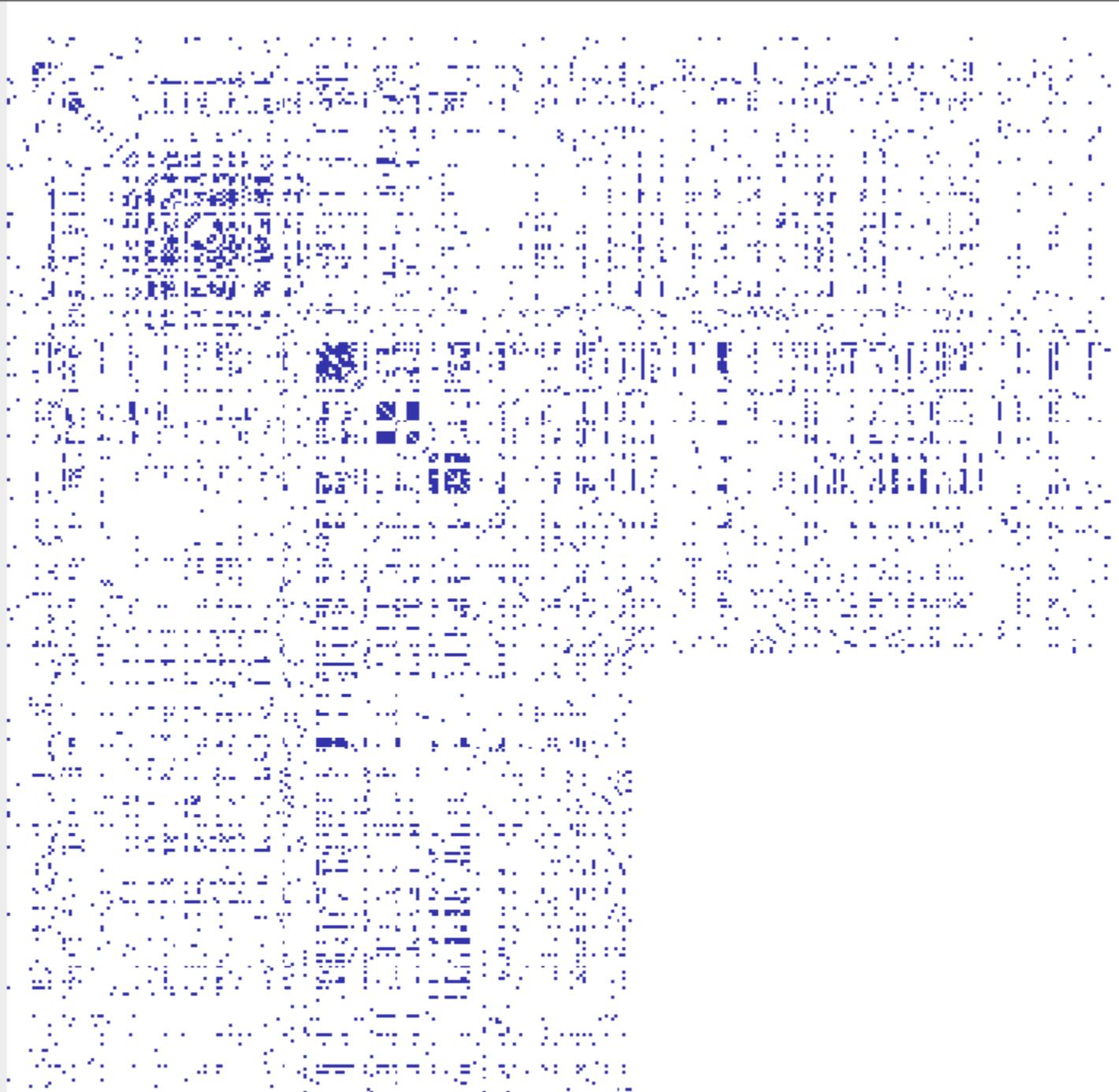
# Graph Viewer

Roll-up by:

Visualization:

Sort by:

Edge centrality filters:



# Attribute-Driven Layout

# Attribute-Driven Layout

Large node-link diagrams **get messy!**

Is there additional structure we can exploit?

*Idea:* Use **data attributes** to perform layout

For example, scatter plot based on node values

Dynamic queries / brushing to explore...

# Attribute-Driven Layout

## The "Skitter" Layout

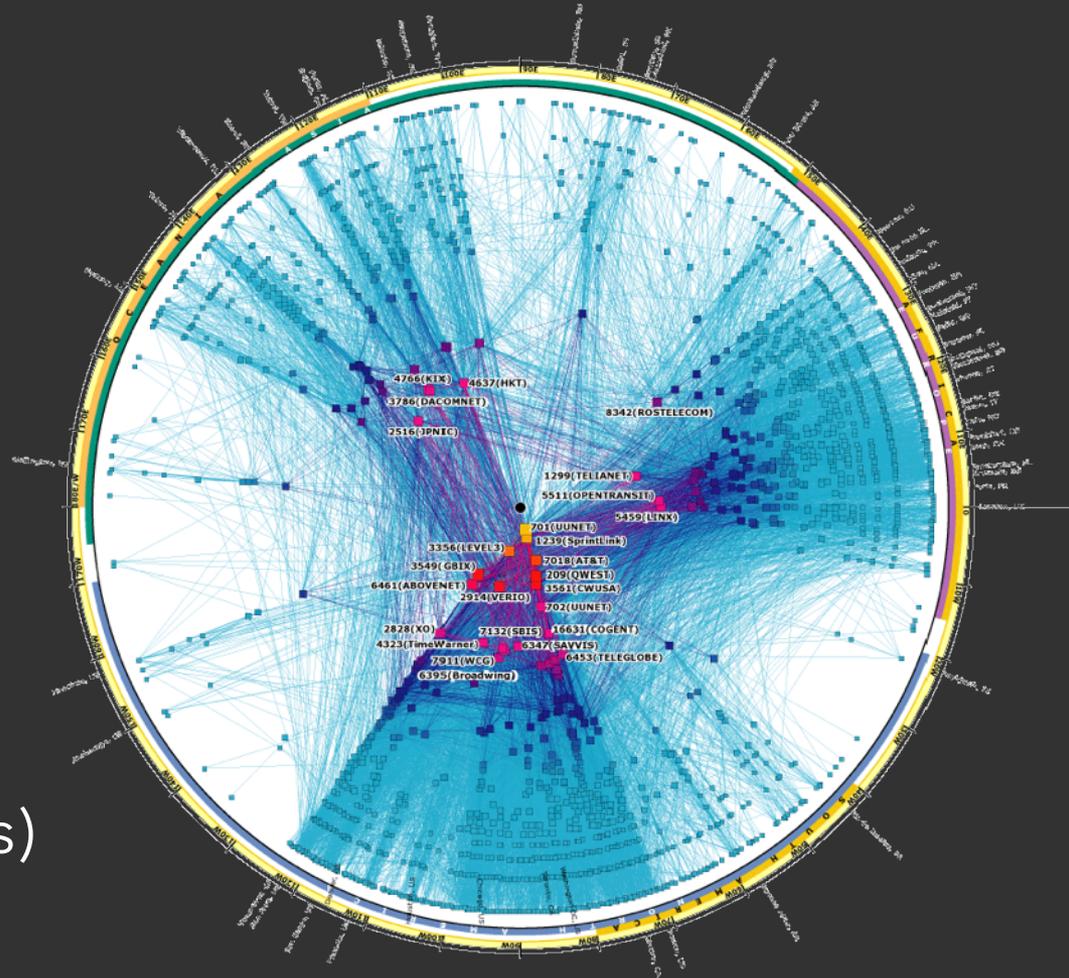
- Internet Connectivity
- Radial Scatterplot

Angle = Longitude

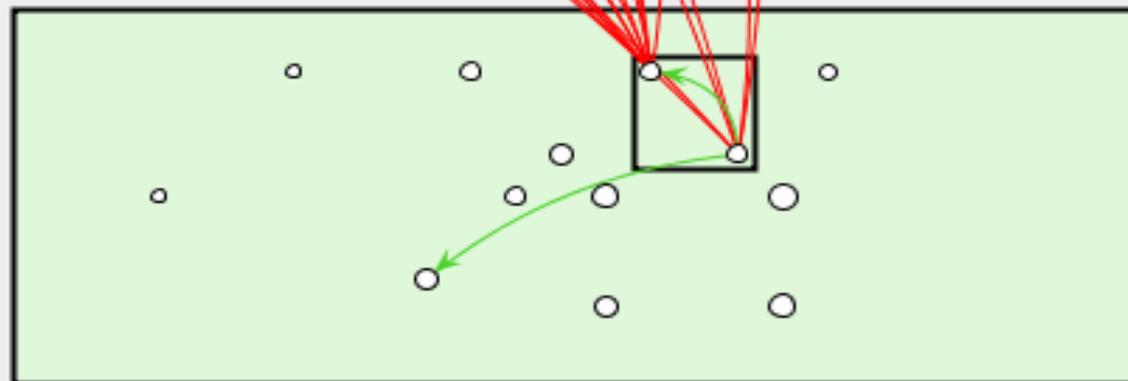
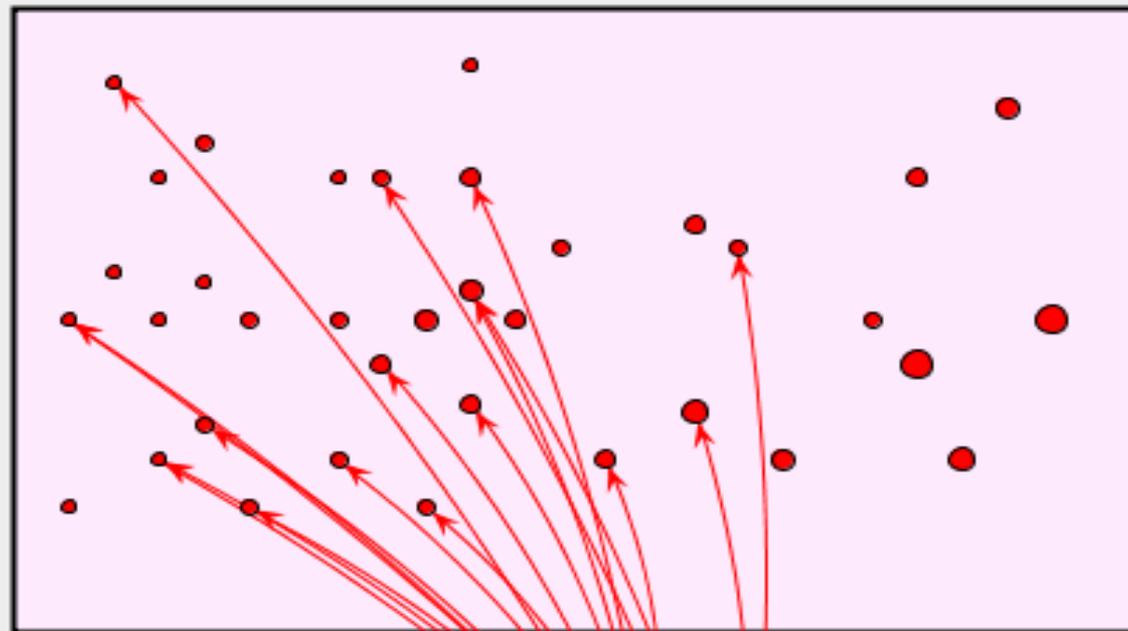
- Geography

Radius = Degree

- # of connections
- (a statistic of the nodes)



Supreme 1982 1987 1992 1998



Circuit 1982 1987 1992 1998

REGIONS

- 36  Supreme
- 13  Circuit

CITES

- 0  Supreme to Supreme
- 0  Supreme to Circuit
- 18   Circuit to Supreme
- 2   Circuit to Circuit

RANGES

Supreme

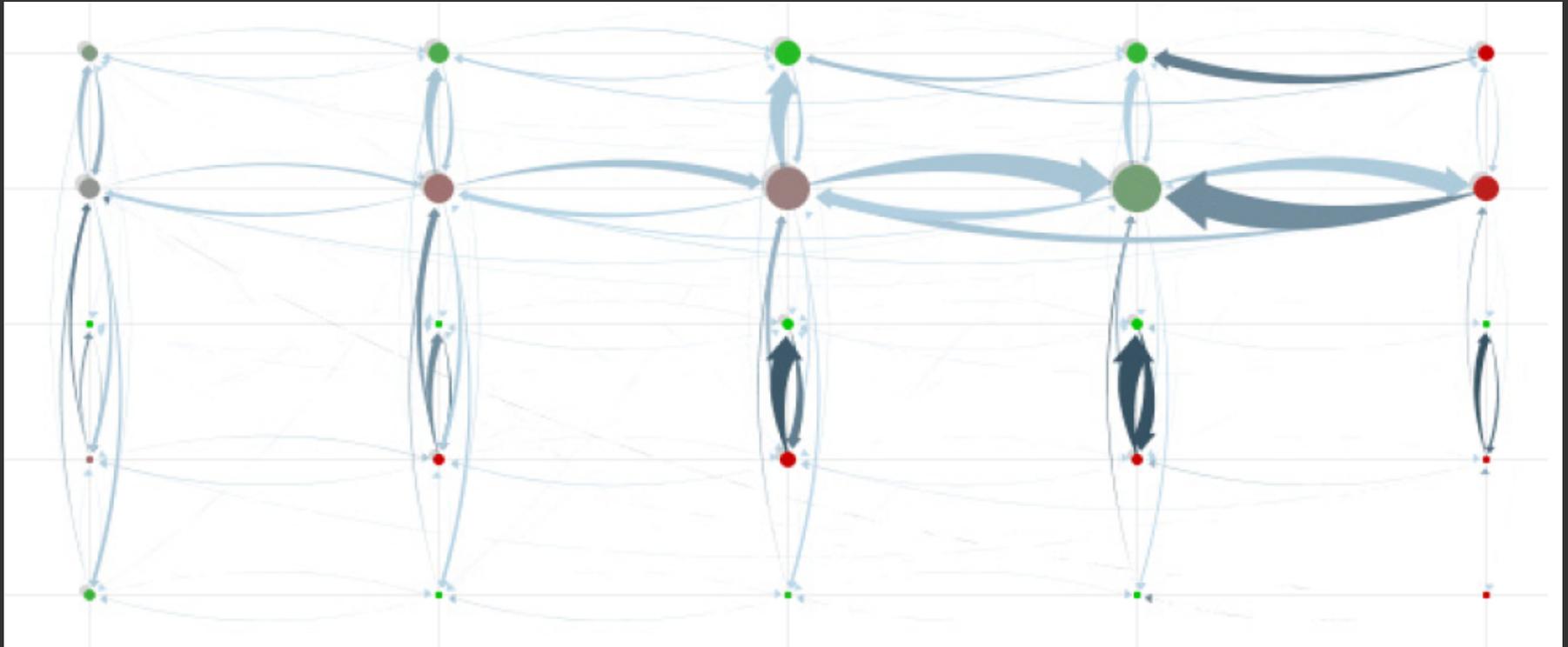
1978 -- 2002

Circuit

1991 -- 1993

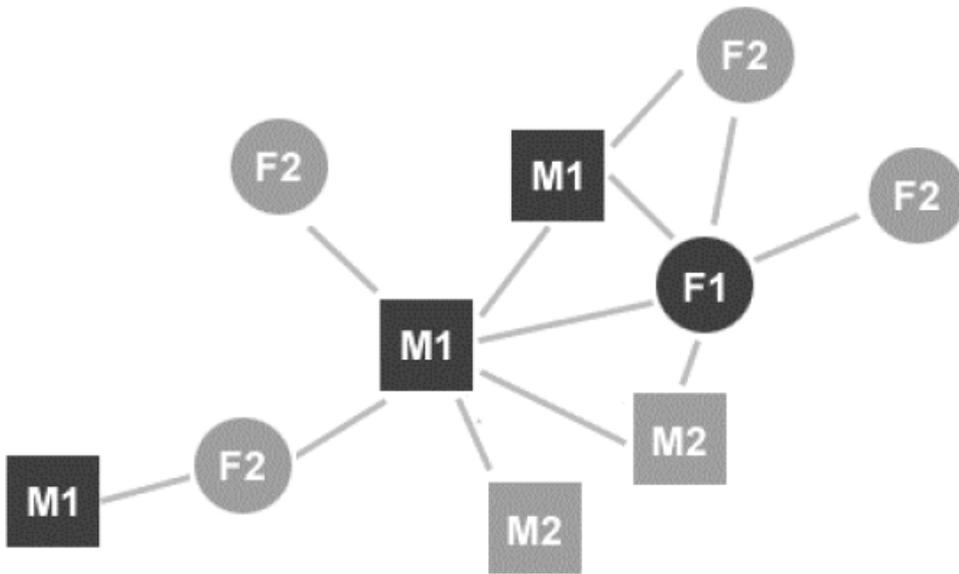


# PivotGraph [Wattenberg'06]

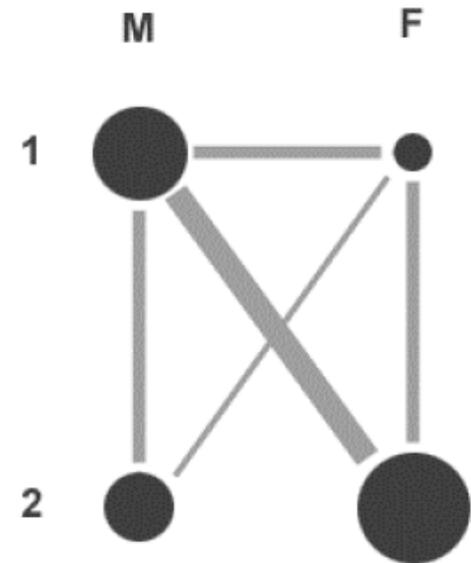


Layout aggregate graphs using node attributes.  
Analogous to pivot tables and trellis display.

# PivotGraph



Node and Link Diagram



PivotGraph Roll-up

X-Axis: Gender

Y-Axis: Location

Flip X/Y Clear

People

- 25
- 13
- 5
- 3

In vs. Out Degree

- 10.0
- 0.0
- 10.0

Relationships

- 50
- 25
- 10
- 5

Rel. / Person

- 10,000
- 5,000
- 2,000
- 1,000

Select:

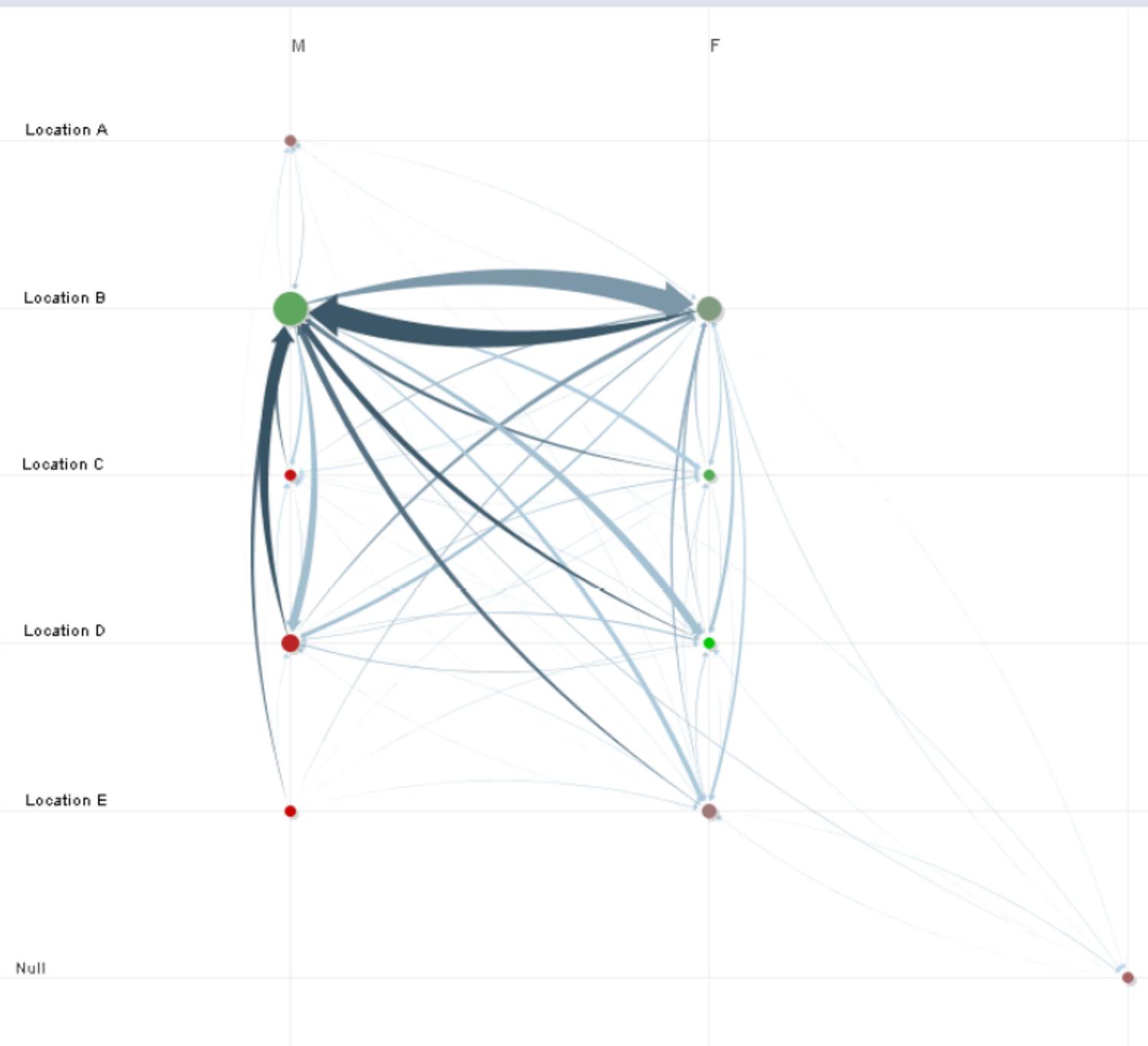
Gender: (All Values)

Legacy: (All Values)

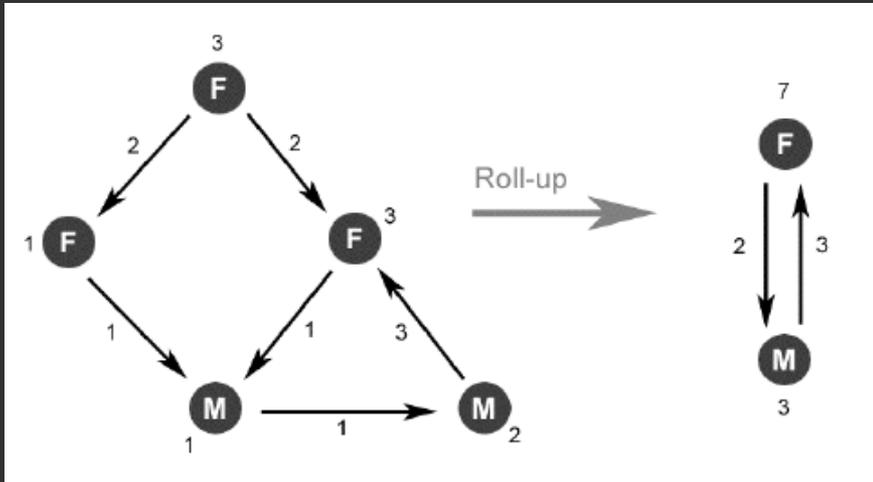
Department: (All Values)

Level: (All Values)

Location: (All Values)

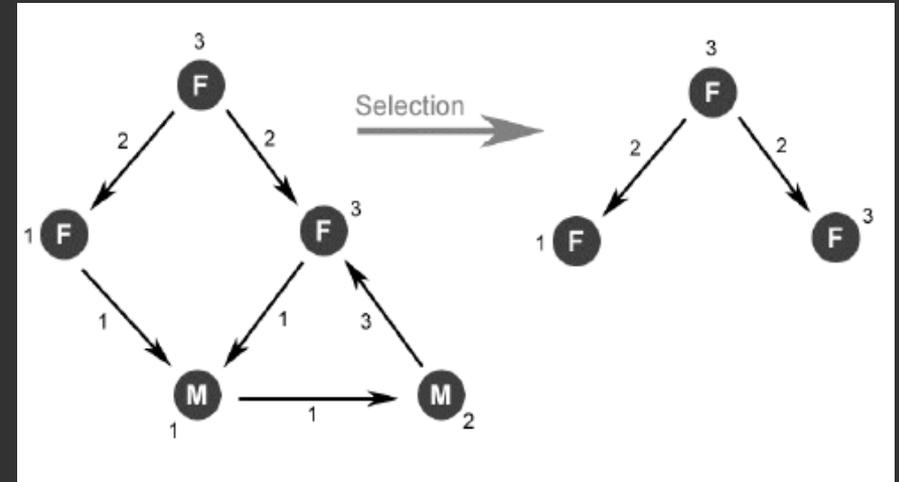


# Operators



## Roll-Up

Aggregate items with matching data values



## Selection

Filter on data values



# Limitations of PivotGraph

Only 2 variables (no nesting as in Tableau)

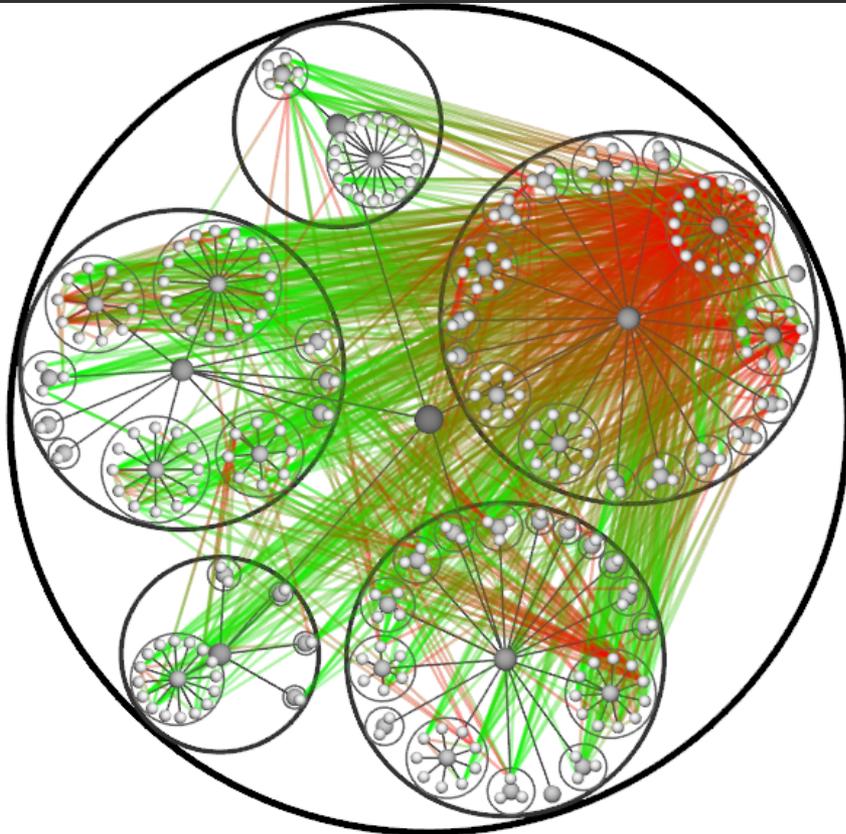
Doesn't support continuous variables

Multivariate edges?

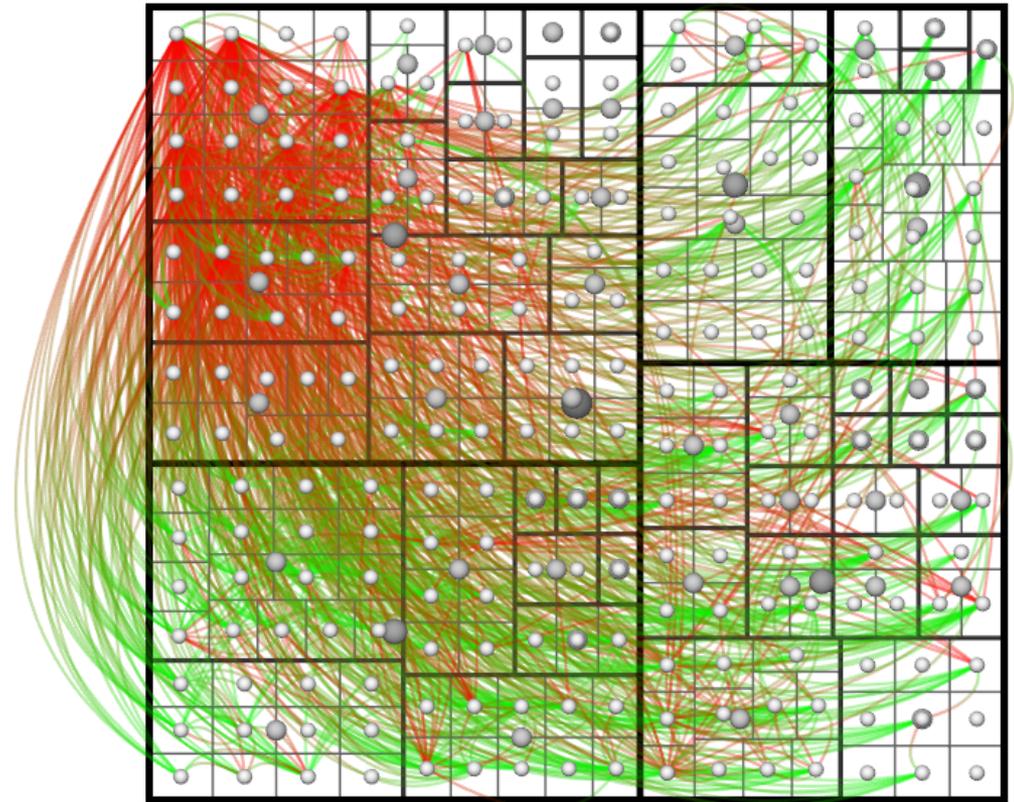
ID	Edge count	Edge-vertex ratio	Edge density	Vertex count	Component count	Component sizes	Duplicate edge count	Degree	Duration (s)	Start	Tower
1	193	0.906	3.875E-3	213	45		18				
2	312	1.156	3.676E-3	270	33		45				
3	430	1.419	3.716E-3	303	19		90				
4	555	1.647	3.736E-3	337	11		132				
5	592	1.711	3.720E-3	346	9		148				
6	568	1.701	3.830E-3	334	6		142				
7	522	1.568	3.627E-3	333	12		121				
8	393	1.264	3.381E-3	311	21		67				
9	289	1.032	3.303E-3	280	42		31				
10	181	0.858	3.837E-3	211	50		11				
11	191	0.88	3.691E-3	217	54		18				
12	284	1.036	3.249E-3	274	44		41				
13	413	1.295	3.332E-3	319	28		75				
14	541	1.587	3.554E-3	341	13		129				
15	567	1.673	3.727E-3	339	13		140				
16	546	1.625	3.669E-3	336	21		133				
17	515	1.58	3.681E-3	326	23		125				
18	404	1.299	3.485E-3	311	26		68				
19	314	1.154	3.568E-3	272	38		51				
20	224	0.982	3.748E-3	228	53		30				
21	190	0.909	3.842E-3	209	56		23				

# Hierarchical Edge Bundling

# Trees with Adjacency Relations

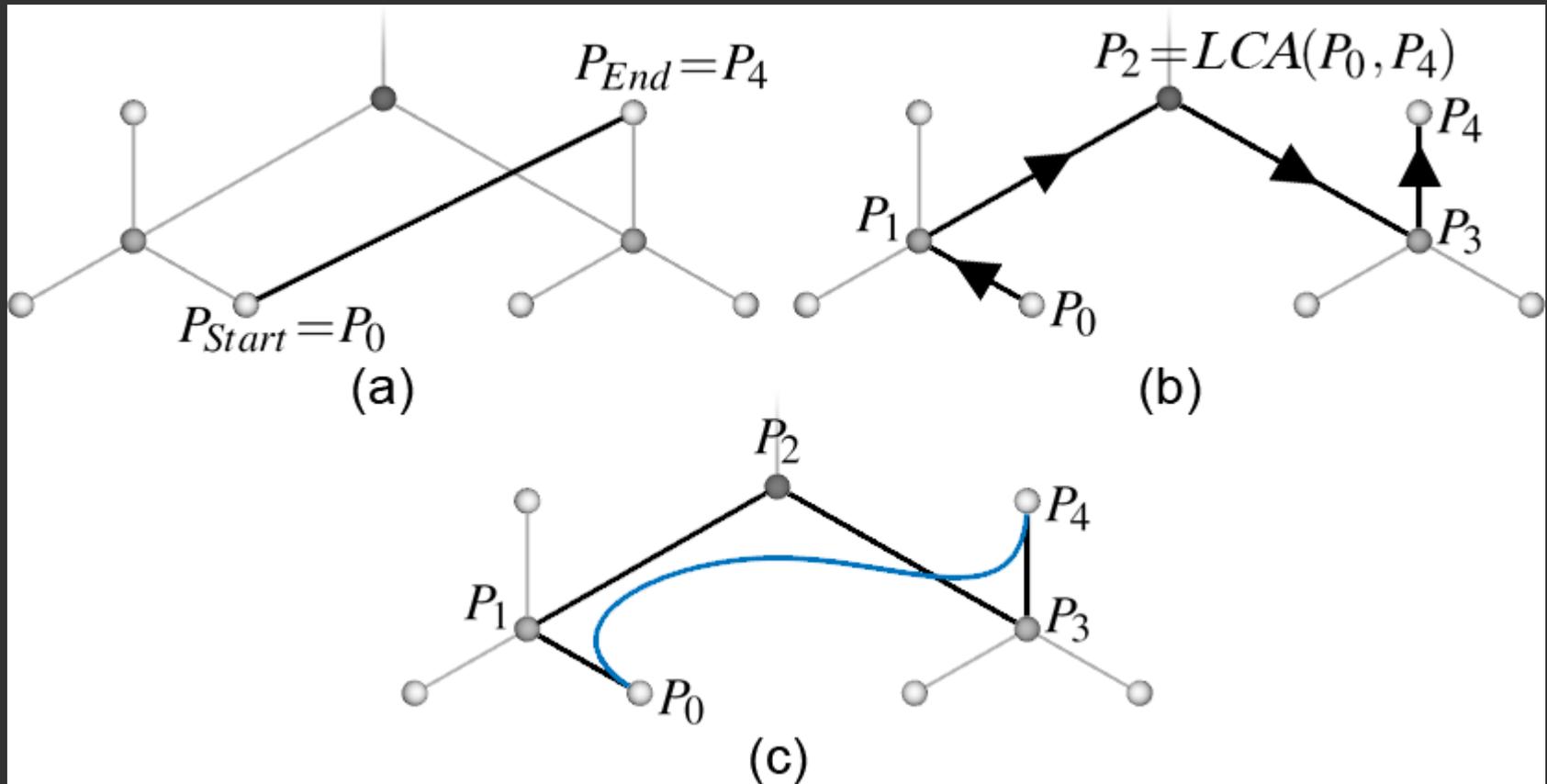


(a)

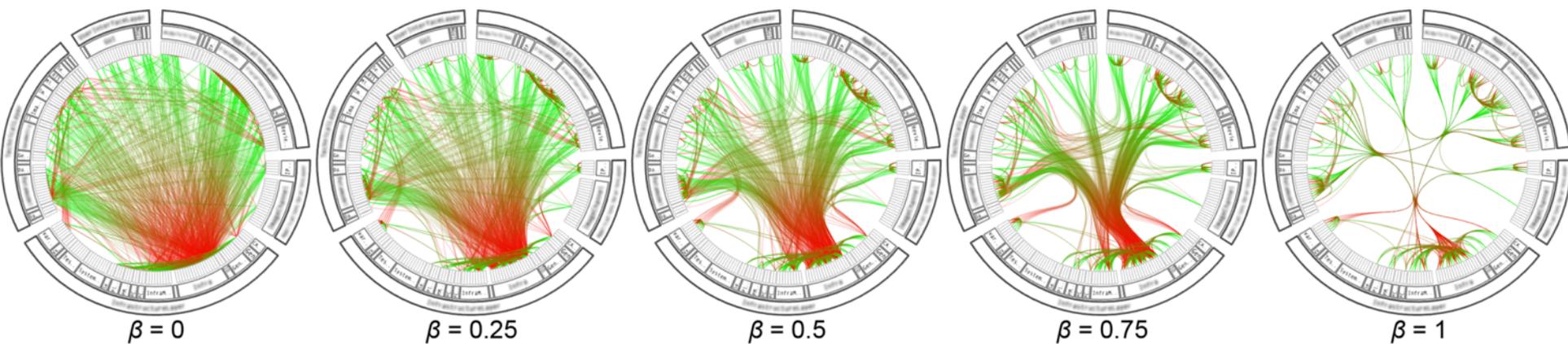
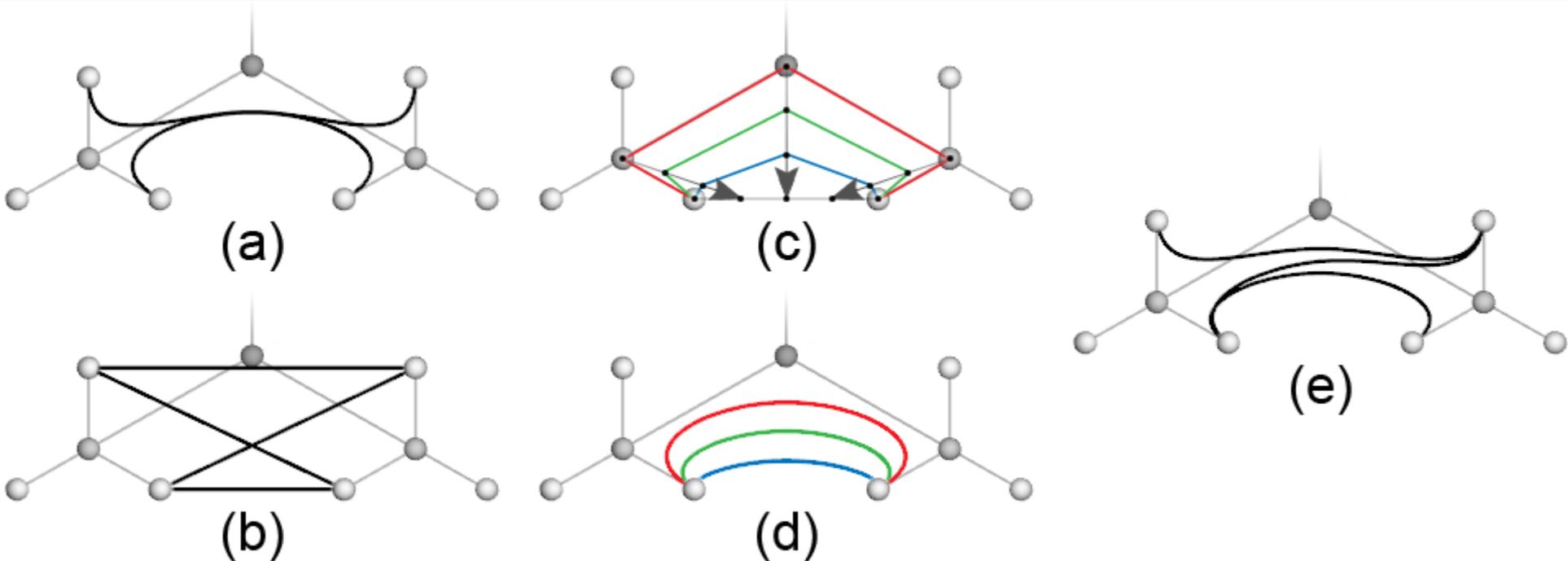


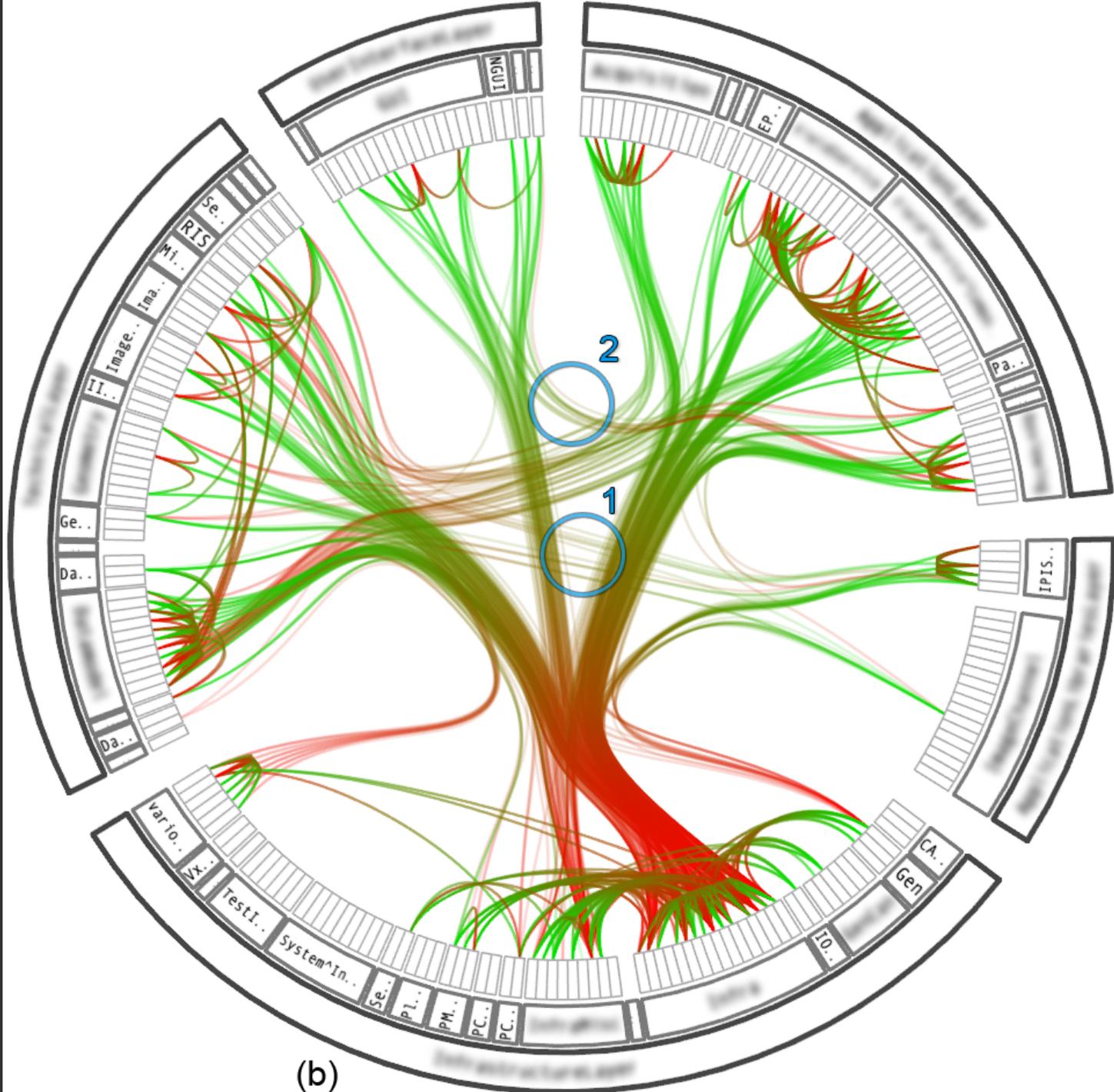
(b)

# Bundle Edges Along Hierarchy



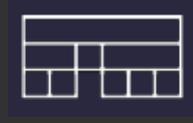
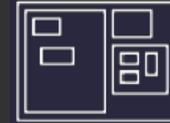
# Configuring Edge Tension





(b)

# Summary



## Tree Layout

Indented / Node-Link / Enclosure / Layers  
Focus+Context techniques for scale

## Graph Layout

Spanning Tree Layout

Hierarchical "Sugiyama" Layout

Optimization (Force-Directed Layout)

Matrix Diagrams

Attribute-Driven Layout