Computer-Aided Reasoning for Software

CSE507

Finite Model Finding

Today

Last lecture

- The DPPL(T) framework for deciding quantifier-free SMT formulas

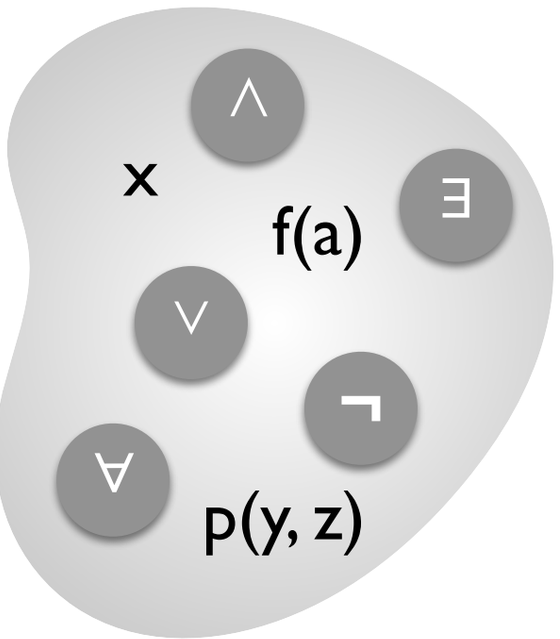
Today

- Finite model finding for quantified FOL and beyond

topics

Finite model finding

a finite universe U



a formula F in FOL

finite model finder

- custom search (SEM, CVC4)
- reduction to SAT (Paradox, Kodkod)

- a **model** ⟨U, I⟩ ⊨ F if one exists
- a minimal unsat core (or "unsat") otherwise

Some applications of finite model finding

Proving theorems in finite algebras (Finder, SEM, MACE)

Checking lightweight formal specifications (Alloy, ProB, ExUML)

Counterexamples to tentative theorems in interactive proof assistants (Nitpick/Isabelle)

Bounded verification of code and memory models (Forge, Miniatur, TACO, MemSAT)

Declarative configuration and execution (ConfigAssure, Margrave, Squander, PBnJ)

Nitpick

alloy

ProB

TACO

MemSAT

FORGE

SQUANDER

Some applications of finite model finding

Checking lightweight formal specifications (Alloy, ProB, ExUML)

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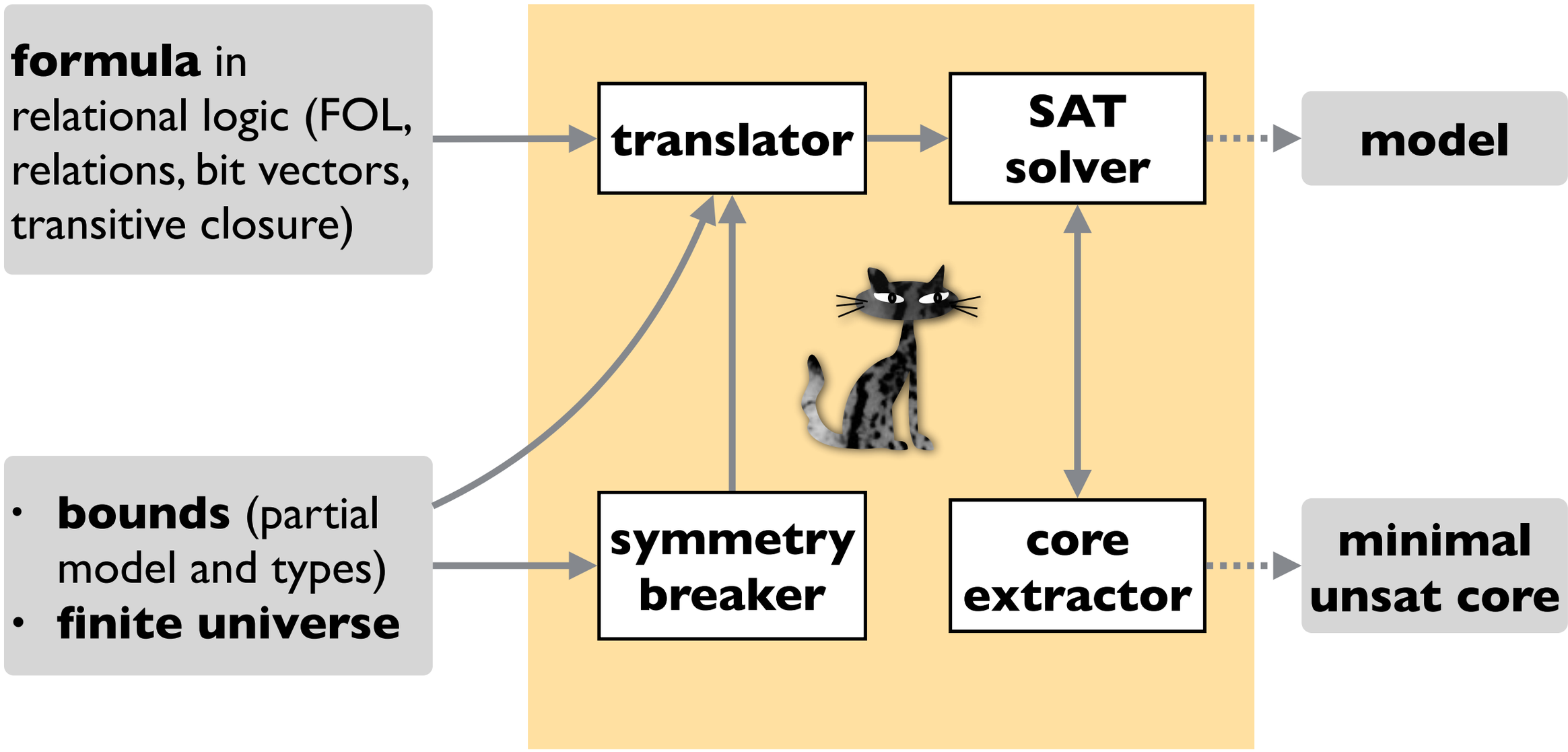
KODKOD

Overview of Kodkod



Relational logic by example

a minimalistic formal specification of a filesystem

- The root of a filesystem hierarchy is a directory.
- Directories may contain files or directories.
- All directories and files are reachable from the root.
- The contents relation is acyclic.

Bounded relational logic by example

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

{ **R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$ }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D**$_1$⟩, ⟨**D**$_2$⟩}

{} ⊆ File ⊆ {⟨**F**$_1$⟩, ⟨**F**$_2$⟩}

{} ⊆ contents ⊆ {**R**, **D**$_1$, **D**$_2$} × {**R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$}

Finite universe of interpretation.

Bounds for each relation:

- Tuples it *must* contain (partial model).
- Tuples it *may* contain (type).

Bounded relational logic by example

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

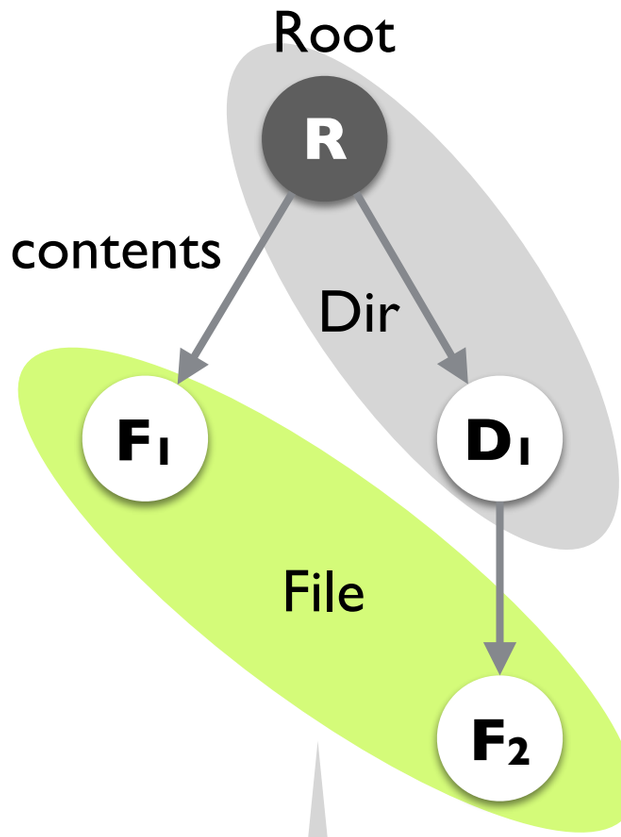
{ **R**, **D1**, **D2**, **F1**, **F2** }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D1**⟩, ⟨**D2**⟩}

{} ⊆ File ⊆ {⟨**F1**⟩, ⟨**F2**⟩}

{} ⊆ contents ⊆ {**R**, **D1**, **D2**} × {**R**, **D1**, **D2**, **F1**, **F2**}



Root = {⟨**R**⟩}

Dir = {⟨**R**⟩, ⟨**D1**⟩}

File = {⟨**F1**⟩, ⟨**F2**⟩}

contents = {⟨**R**, **F1**⟩, ⟨**R**, **D1**⟩, ⟨**D1**, **F2**⟩}

Translation by example

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

{ **R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$ }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D**$_1$⟩, ⟨**D**$_2$⟩}

{} ⊆ File ⊆ {⟨**F**$_1$⟩, ⟨**F**$_2$⟩}

{} ⊆ contents ⊆ {**R**, **D**$_1$, **D**$_2$} × {**R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$}

Encode

- relational constants as boolean matrices
- relational expressions as matrix operations
- formulas as constraints over matrix entries

Relational constants as boolean matrices

	R	**D_1**	**D_2**	**F_1**	**F_2**
	1	0	0	0	0
	d_0	d_1	d_2	0	0
	0	0	0	f_0	f_1
R	c_0	c_1	c_2	c_3	c_4
D_1	c_5	c_6	c_7	c_8	c_9
D_2	c_{10}	c_{11}	c_{12}	c_{13}	c_{14}
F_1	0	0	0	0	0
F_2	0	0	0	0	0

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

$\{\} \subseteq \text{File} \subseteq \{\langle \mathbf{F_1} \rangle, \langle \mathbf{F_2} \rangle\}$

$\{\} \subseteq \text{contents} \subseteq \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}\} \times \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\}$

Relational expressions as matrix operations

File

0	0	0	f_0	f_1

$\vee$

Dir

d_0	d_1	d_2	0	0

=

File ∪ Dir

d_0	d_1	d_2	f_0	f_1

Dir

d_0
d_1
d_2
0
0

×

File ∪ Dir

d_0	d_1	d_2	f_0	f_1

=

Dir × (File ∪ Dir)

$d_0 \wedge d_0$	$d_0 \wedge d_1$	$d_0 \wedge d_2$	$d_0 \wedge f_0$	$d_0 \wedge f_1$
$d_1 \wedge d_0$	$d_1 \wedge d_1$	$d_1 \wedge d_2$	$d_1 \wedge f_0$	$d_1 \wedge f_1$
$d_2 \wedge d_0$	$d_2 \wedge d_1$	$d_2 \wedge d_2$	$d_2 \wedge f_0$	$d_2 \wedge f_1$
0	0	0	0	0
0	0	0	0	0

Formulas as constraints over matrix entries

contents

c_0	c_1	c_2	c_3	c_4
c_5	c_6	c_7	c_8	c_9
c_{10}	c_{11}	c_{12}	c_{13}	c_{14}
0	0	0	0	0
0	0	0	0	0

$\rightarrow$

Dir × (File ∪ Dir)

$d_0 \wedge d_0$	$d_0 \wedge d_1$	$d_0 \wedge d_2$	$d_0 \wedge f_0$	$d_0 \wedge f_1$
$d_1 \wedge d_0$	$d_1 \wedge d_1$	$d_1 \wedge d_2$	$d_1 \wedge f_0$	$d_1 \wedge f_1$
$d_2 \wedge d_0$	$d_2 \wedge d_1$	$d_2 \wedge d_2$	$d_2 \wedge f_0$	$d_2 \wedge f_1$
0	0	0	0	0
0	0	0	0	0

=

contents ⊆ Dir × (File ∪ Dir)

$$
\begin{aligned}
&(c_0 \rightarrow d_0 \wedge d_0) \wedge \\
&(c_1 \rightarrow d_0 \wedge d_1) \wedge \\
&(c_2 \rightarrow d_0 \wedge d_2) \wedge \\
&(c_3 \rightarrow d_0 \wedge f_0) \wedge \\
&(c_4 \rightarrow d_0 \wedge f_1) \wedge \\
&(c_5 \rightarrow d_1 \wedge d_0) \wedge \\
&\ldots \\
&(c_{14} \rightarrow d_2 \wedge f_1)
\end{aligned}
$$

Dealing with sparseness and redundancy

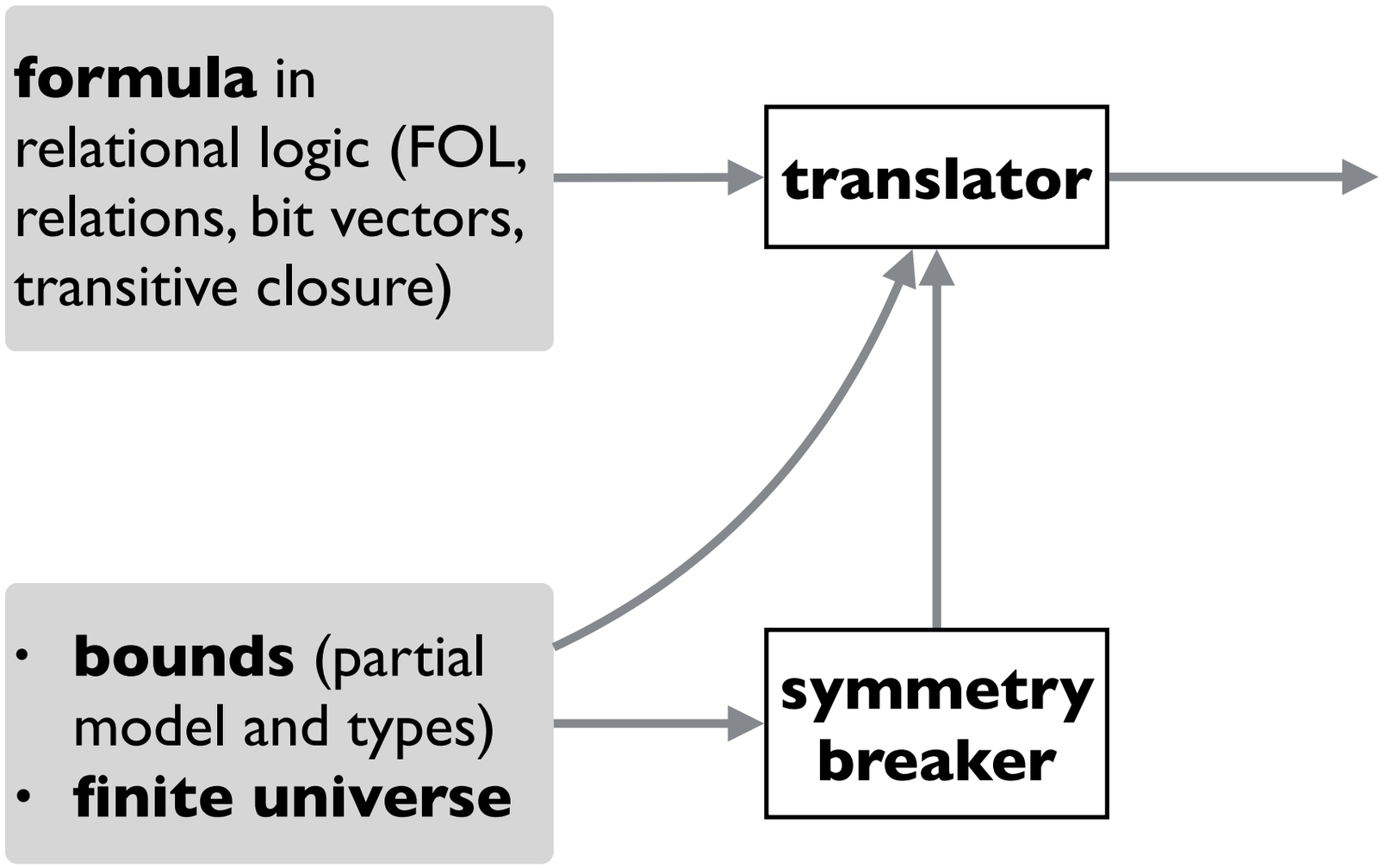
Compact Boolean Circuits (CBCs).

Dir × (File ∪ Dir)

$d_0 \wedge d_0$	$d_0 \wedge d_1$	$d_0 \wedge d_2$	$d_0 \wedge f_0$	$d_0 \wedge f_1$
$d_1 \wedge d_0$	$d_1 \wedge d_1$	$d_1 \wedge d_2$	$d_1 \wedge f_0$	$d_1 \wedge f_1$
$d_2 \wedge d_0$	$d_2 \wedge d_1$	$d_2 \wedge d_2$	$d_2 \wedge f_0$	$d_2 \wedge f_1$
0	0	0	0	0
0	0	0	0	0

Sparse matrices represented as interval trees.

Overview of Kodkod



Symmetry by example

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

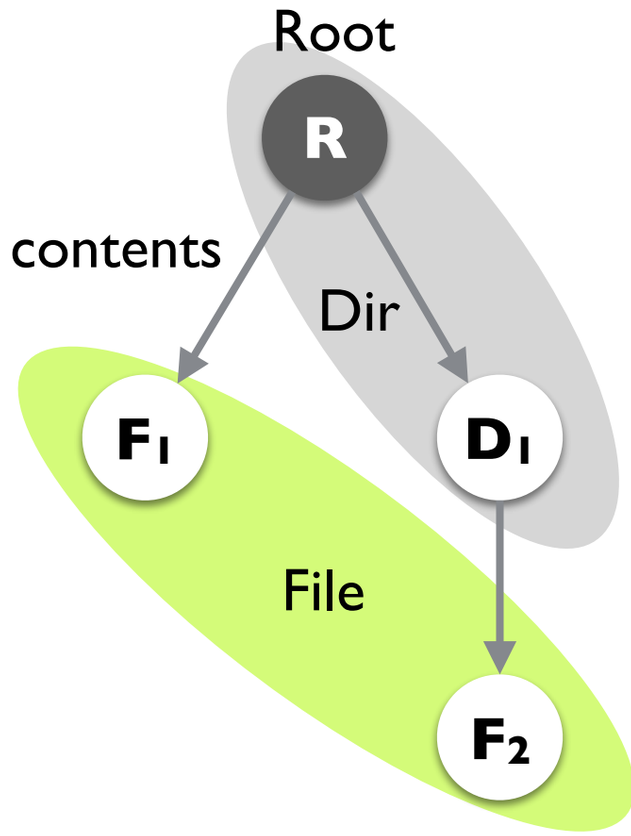
{ **R**, **D**, **D₂**, **F₁**, **F₂** }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D₁**⟩, ⟨**D₂**⟩}

{} ⊆ File ⊆ {⟨**F₁**⟩, ⟨**F₂**⟩}

{} ⊆ contents ⊆ {**R**, **D₁**, **D₂**} × {**R**, **D₁**, **D₂**, **F₁**, **F₂**}



Symmetries between models

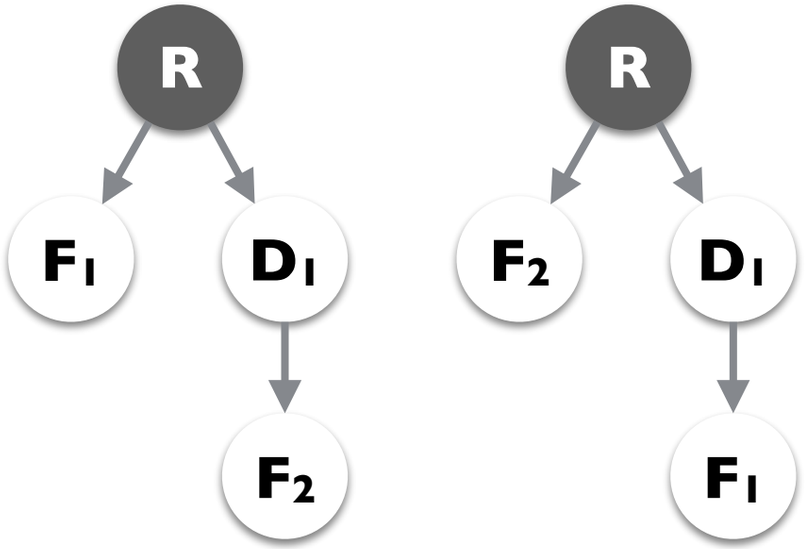
Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

{ **R**, **D1**, **D2**, **F1**, **F2** }

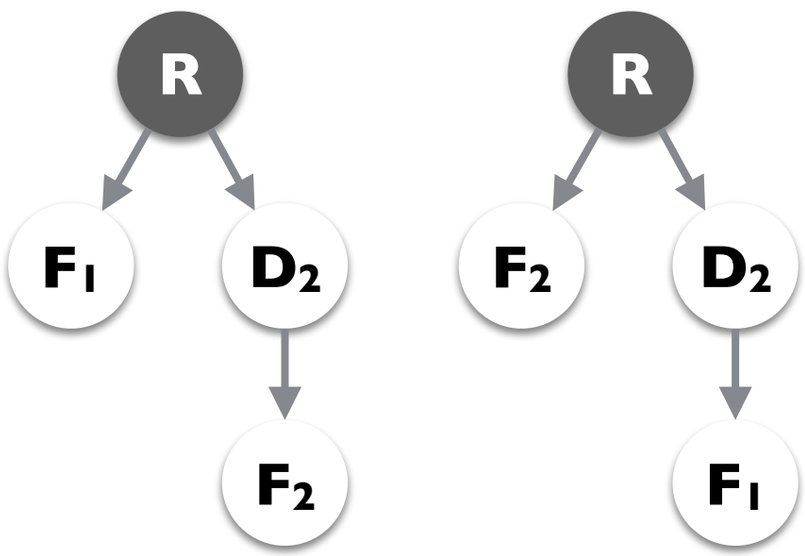


{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D1**⟩, ⟨**D2**⟩}

{} ⊆ File ⊆ {⟨**F1**⟩, ⟨**F2**⟩}

{} ⊆ contents ⊆ {**R**, **D1**, **D2**} × {**R**, **D1**, **D2**, **F1**, **F2**}



Symmetries between non-models

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

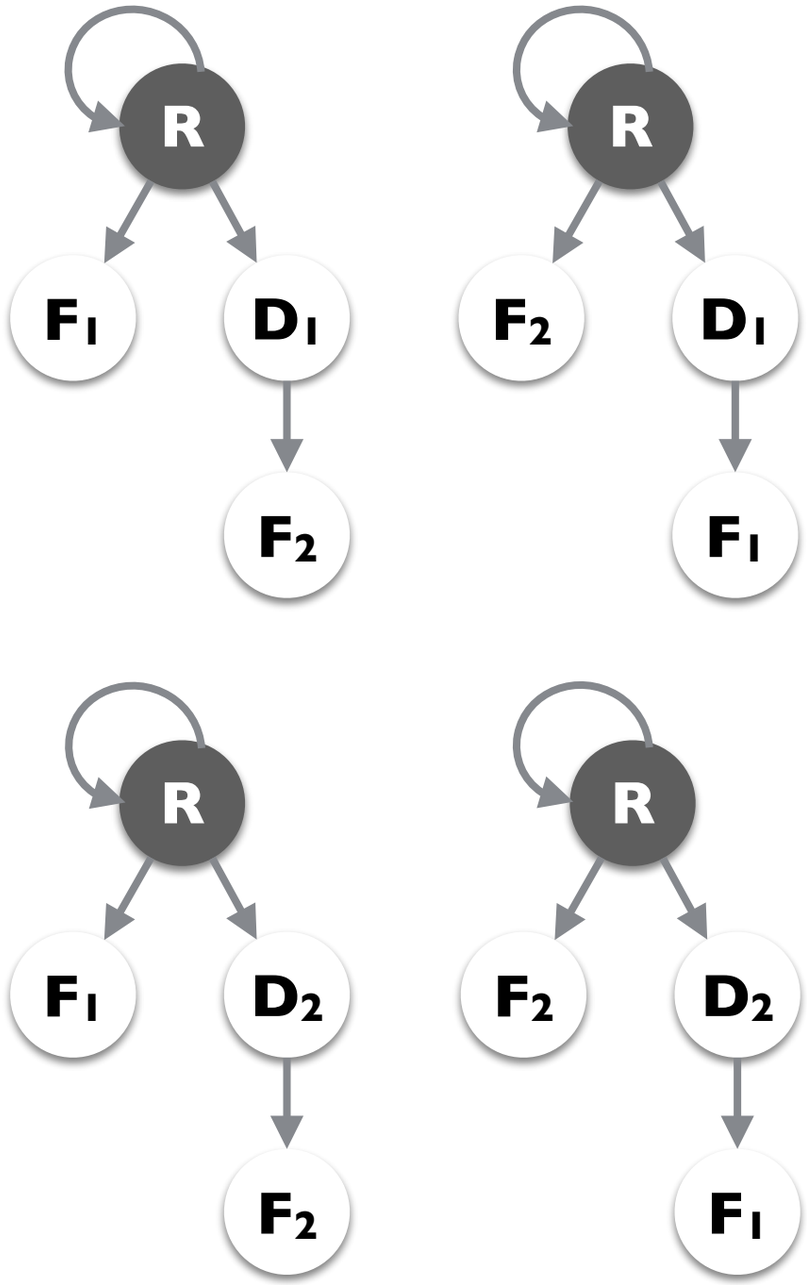
{ **R**, **D1**, **D2**, **F1**, **F2** }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D1**⟩, ⟨**D2**⟩}

{} ⊆ File ⊆ {⟨**F1**⟩, ⟨**F2**⟩}

{} ⊆ contents ⊆ {**R**, **D1**, **D2**} × {**R**, **D1**, **D2**, **F1**, **F2**}



Symmetries induce equivalence classes

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

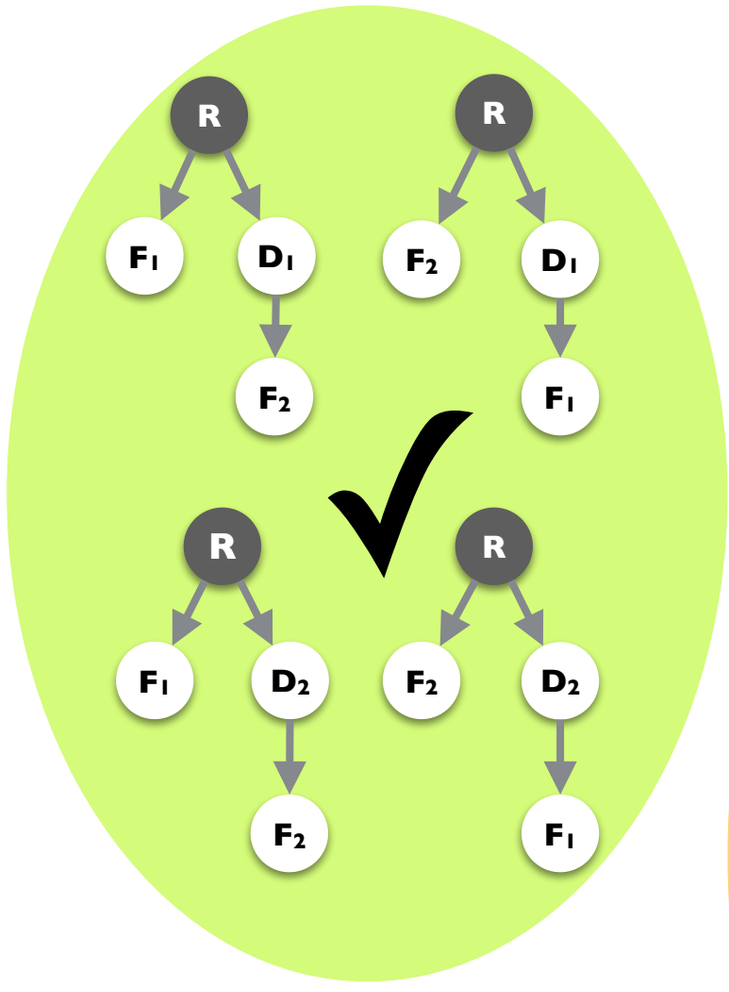
{ **R**, **D1**, **D2**, **F1**, **F2** }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

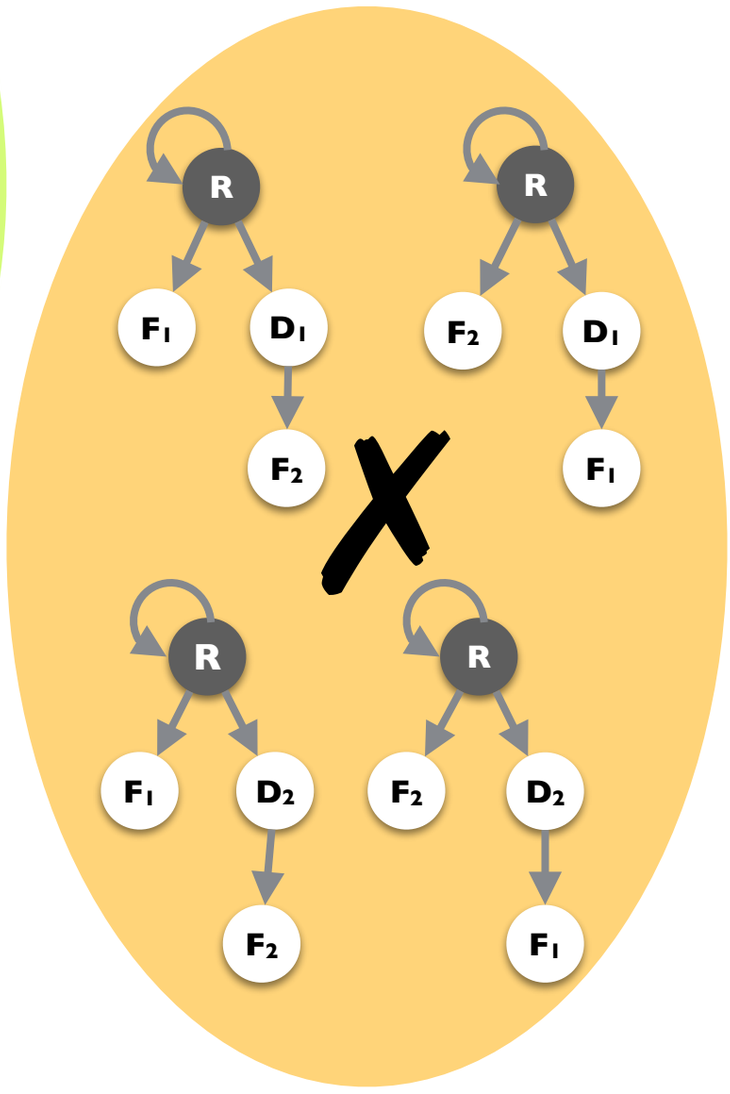
{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D1**⟩, ⟨**D2**⟩}

{} ⊆ File ⊆ {⟨**F1**⟩, ⟨**F2**⟩}

{} ⊆ contents ⊆ {**R**, **D1**, **D2**} × {**R**, **D1**, **D2**, **F1**, **F2**}



Sufficient to check one interpretation per equivalence class.



Symmetry detection

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

{ **R**, **D1**, **D2**, **F1**, **F2** }

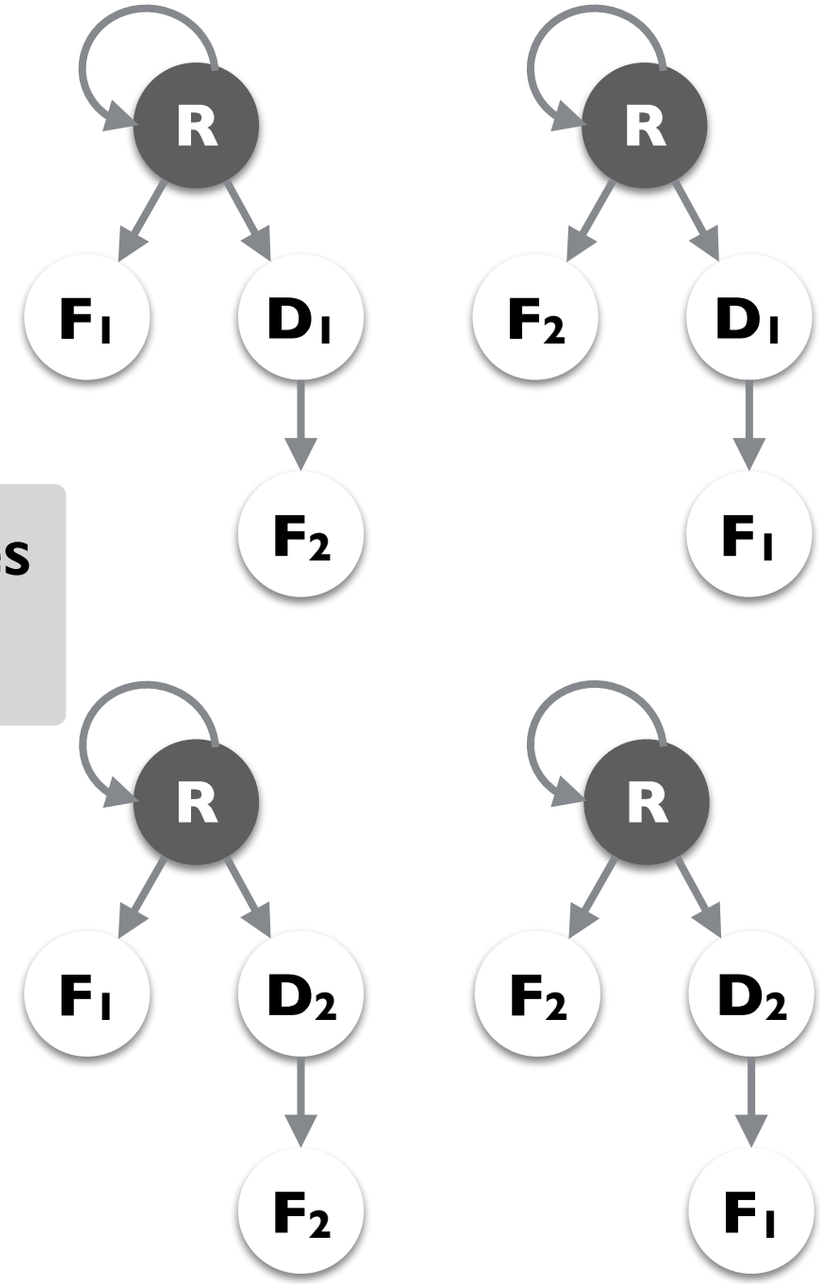
Interpretation symmetries = bound symmetries

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D1**⟩, ⟨**D2**⟩}

{} ⊆ File ⊆ {⟨**F1**⟩, ⟨**F2**⟩}

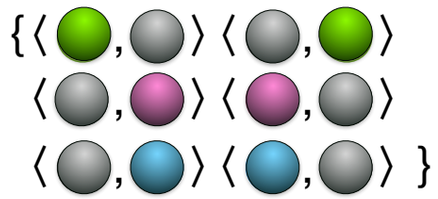
{} ⊆ contents ⊆ {**R**, **D1**, **D2**} × {**R**, **D1**, **D2**, **F1**, **F2**}

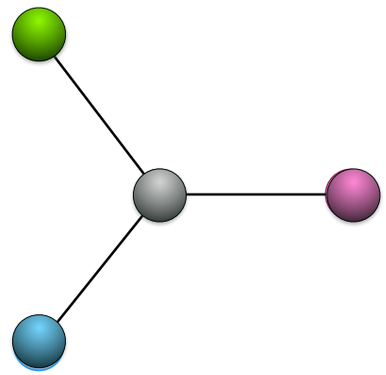


Detecting symmetries is hard ...

Interpretation symmetries
= bound symmetries

Graph automorphism
detection





But only a few symmetries needed in practice

Greedy algorithm that partitions the universe into equivalence classes

Graph automorphism detection

Base partitioning: practical symmetry detection

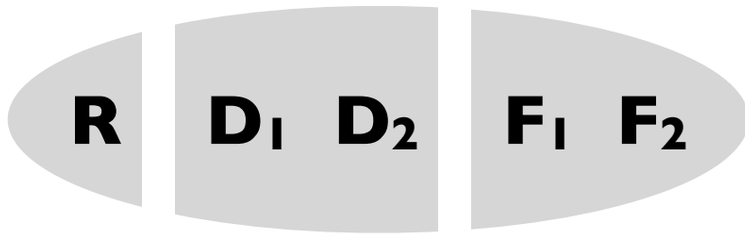
{ **R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$ }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D**$_1$⟩, ⟨**D**$_2$⟩}

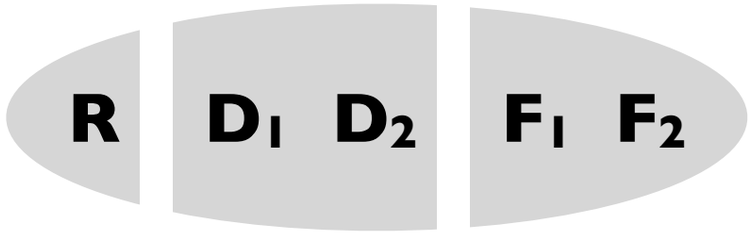
{} ⊆ File ⊆ {⟨**F**$_1$⟩, ⟨**F**$_2$⟩}

{} ⊆ contents ⊆ {**R**, **D**$_1$, **D**$_2$} × {**R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$}



The coarsest partition of the universe such that each non-empty bound is expressible as a union of products of parts.

Finding the base partitioning



$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

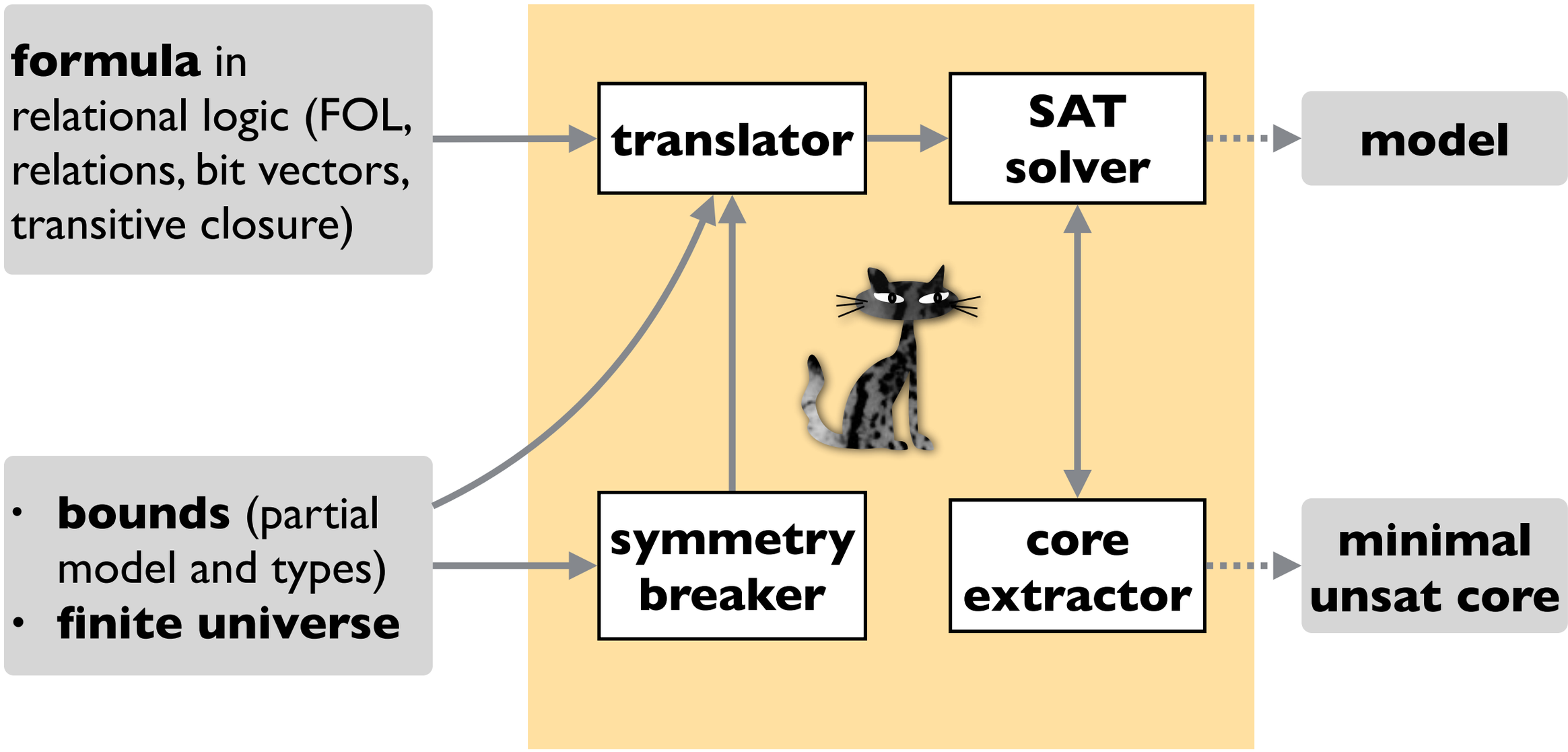
$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

$\{\} \subseteq \text{File} \subseteq \{\langle \mathbf{F_1} \rangle, \langle \mathbf{F_2} \rangle\}$

$\{\} \subseteq \text{contents} \subseteq \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}\} \times \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\}$

start with a single partition
and refine minimally for
each non-empty lower and
upper bound

Overview of Kodkod



A bug in the tiny filesystem

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

{ **R**, **D1**, **D2**, **F1**, **F2** }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D1**⟩, ⟨**D2**⟩}

{} ⊆ File ⊆ {⟨**F1**⟩, ⟨**F2**⟩}

{} ⊆ contents ⊆ {**R**, **D1**, **D2**} × {**R**, **D1**, **D2**, **F1**, **F2**}

Fixing the tiny filesystem

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

∀ f: File | one contents.f

∀ d: Dir | one contents.d

Minimal unsatisfiable core: an unsatisfiable subset of a formula that becomes satisfiable if any of its members are removed.

{ **R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$ }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D**$_1$⟩, ⟨**D**$_2$⟩}

{} ⊆ File ⊆ {⟨**F**$_1$⟩, ⟨**F**$_2$⟩}

{} ⊆ contents ⊆ {**R**, **D**$_1$, **D**$_2$} × {**R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$}

Resolution-based core extraction

Root ⊆ Dir

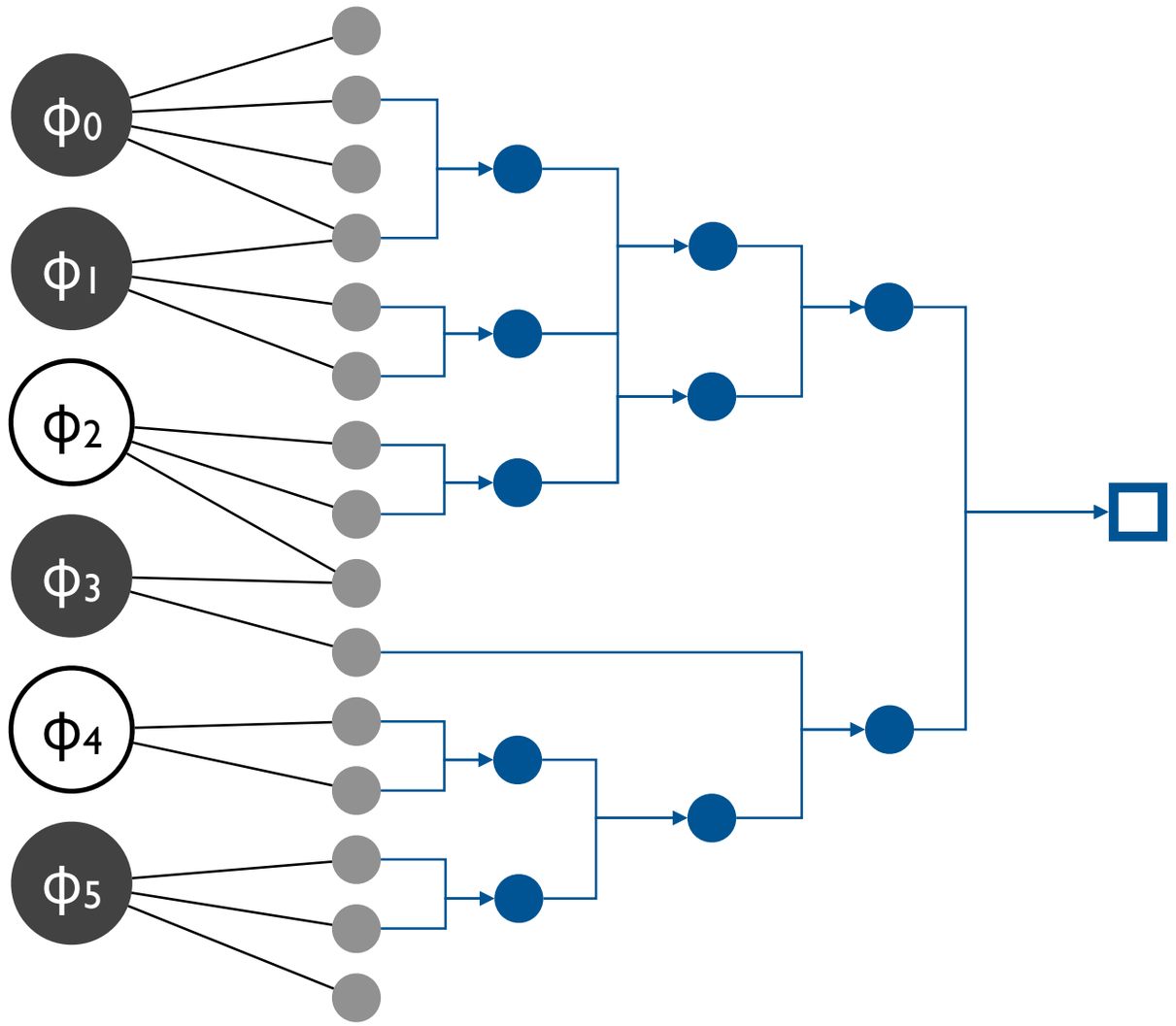
contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

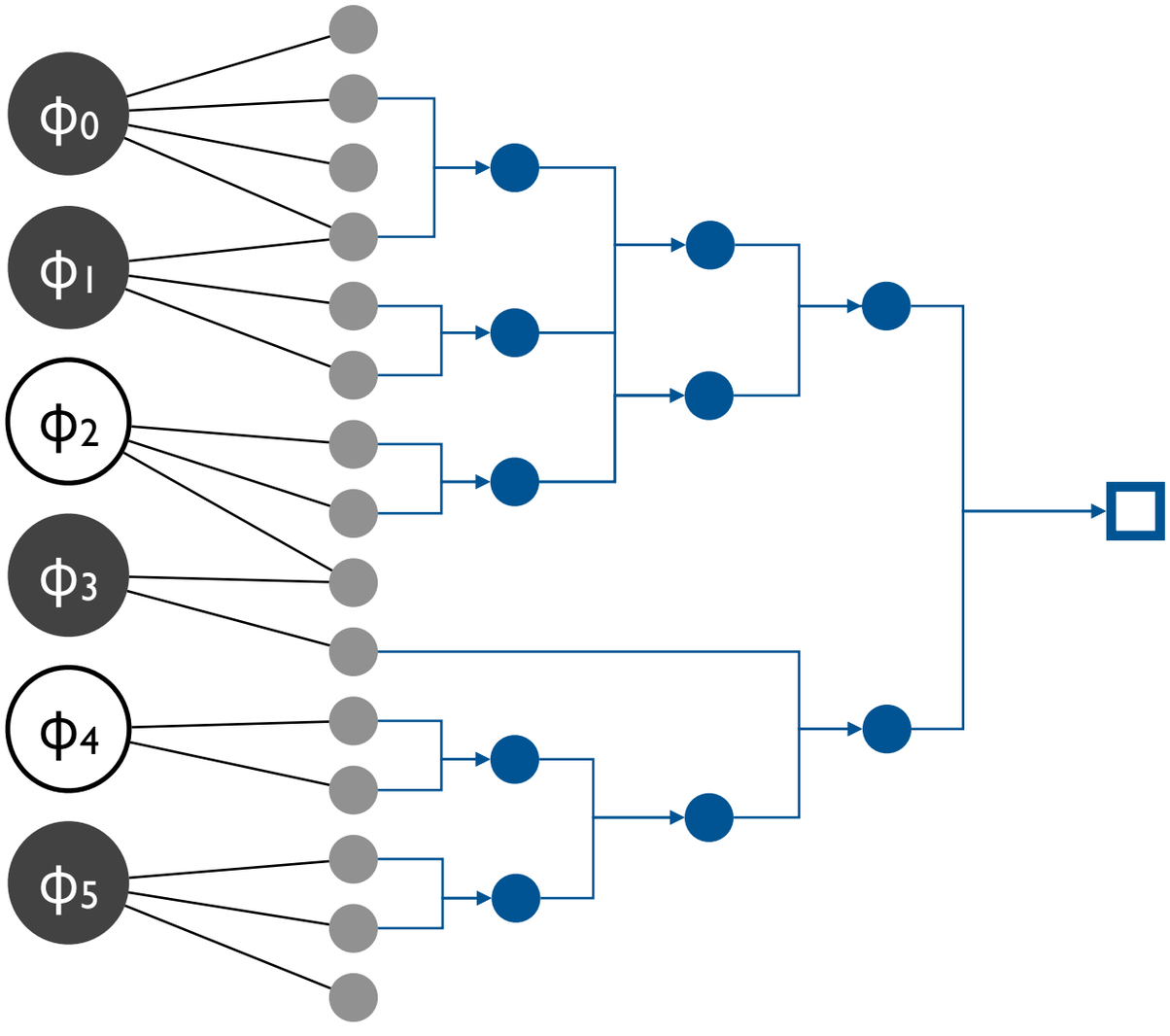
∀ d: Dir | ¬ (d ⊆ d.^contents)

∀ f: File | one contents.f

∀ d: Dir | one contents.d



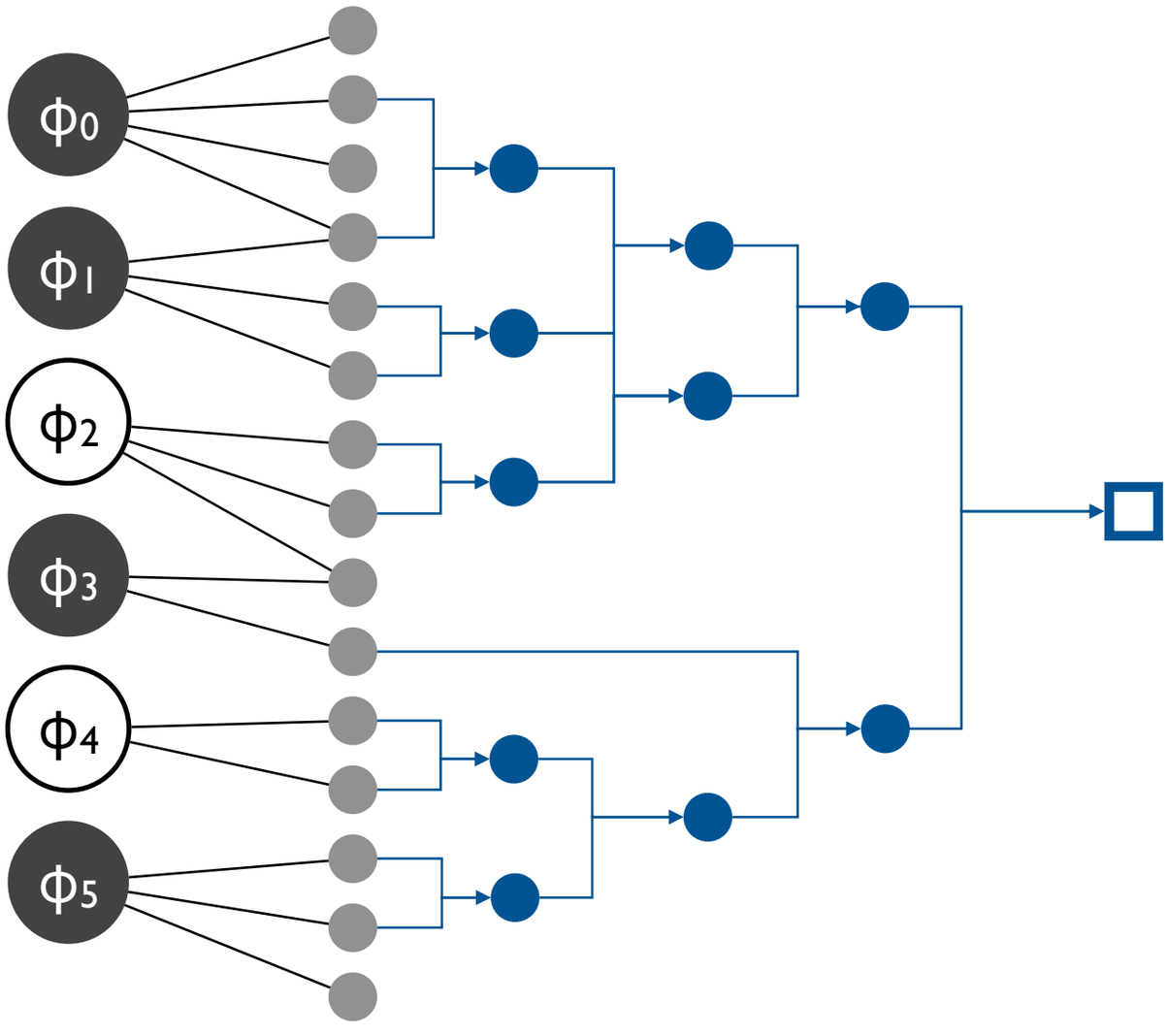
High-level minimal cores from low-level proofs



How to use the proof at the SAT level to find a minimal core at the specification level when

- SAT proof is not minimal
- minimal SAT core may map to a large specification core?

Recycling core extraction



Key idea: minimize core by removing constraints at the specification level but re-use valid resolvents from the previous step so that the solver doesn't have to re-derive them.

Summary

Today

- Finite model finding for first-order logic with quantifiers, relations, and transitive closure

Next lecture

- Reasoning about program correctness