Computer-Aided Reasoning for Software

CSE507

Finite Model Finding

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Today

Last lecture

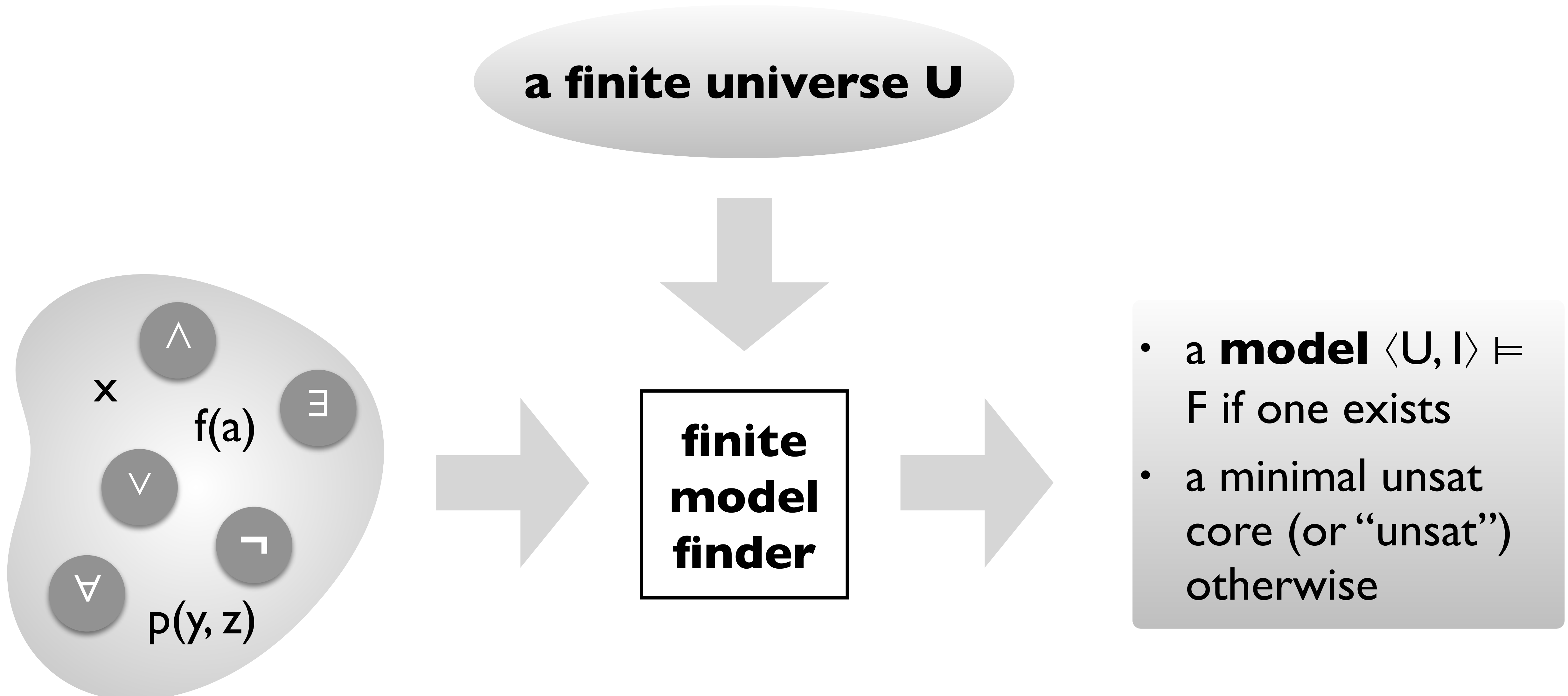
- The DPPL(T) framework for deciding quantifier-free SMT formulas

Today

- Finite model finding for quantified FOL and beyond

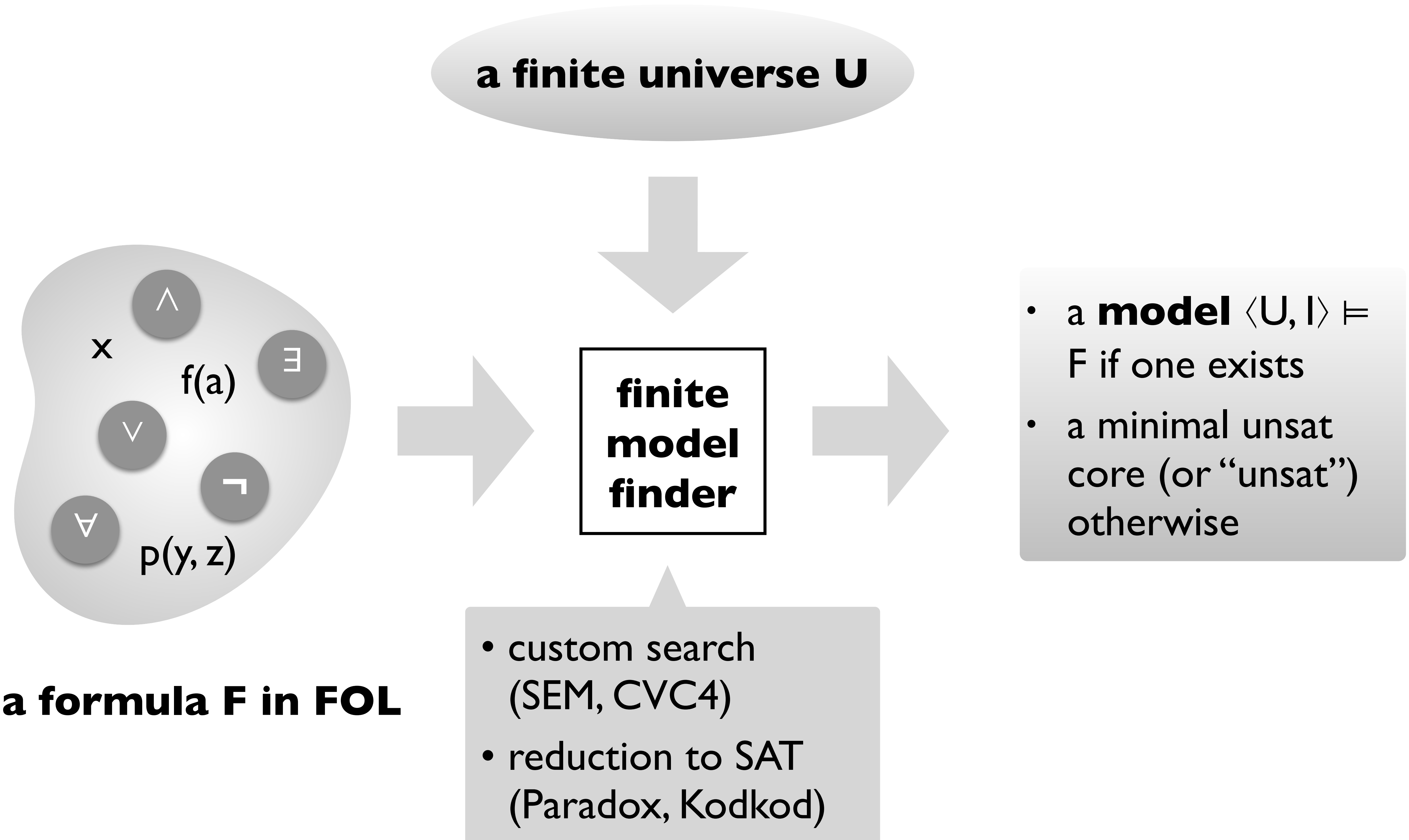
topics

Finite model finding



a formula F in FOL

Finite model finding



Some applications of finite model finding

Proving theorems in finite algebras (Finder, SEM, MACE)

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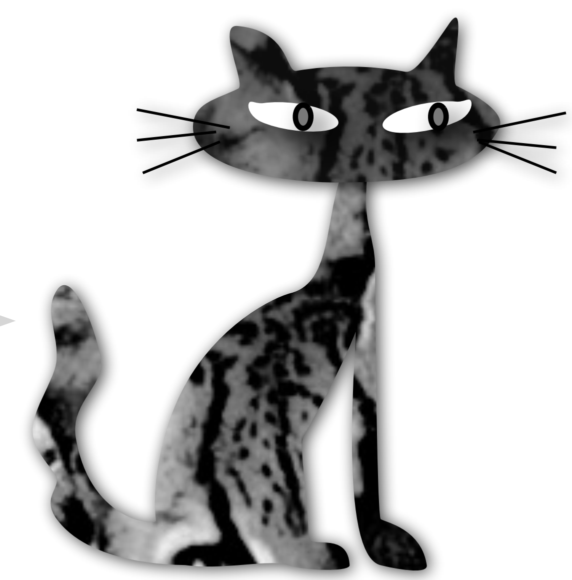
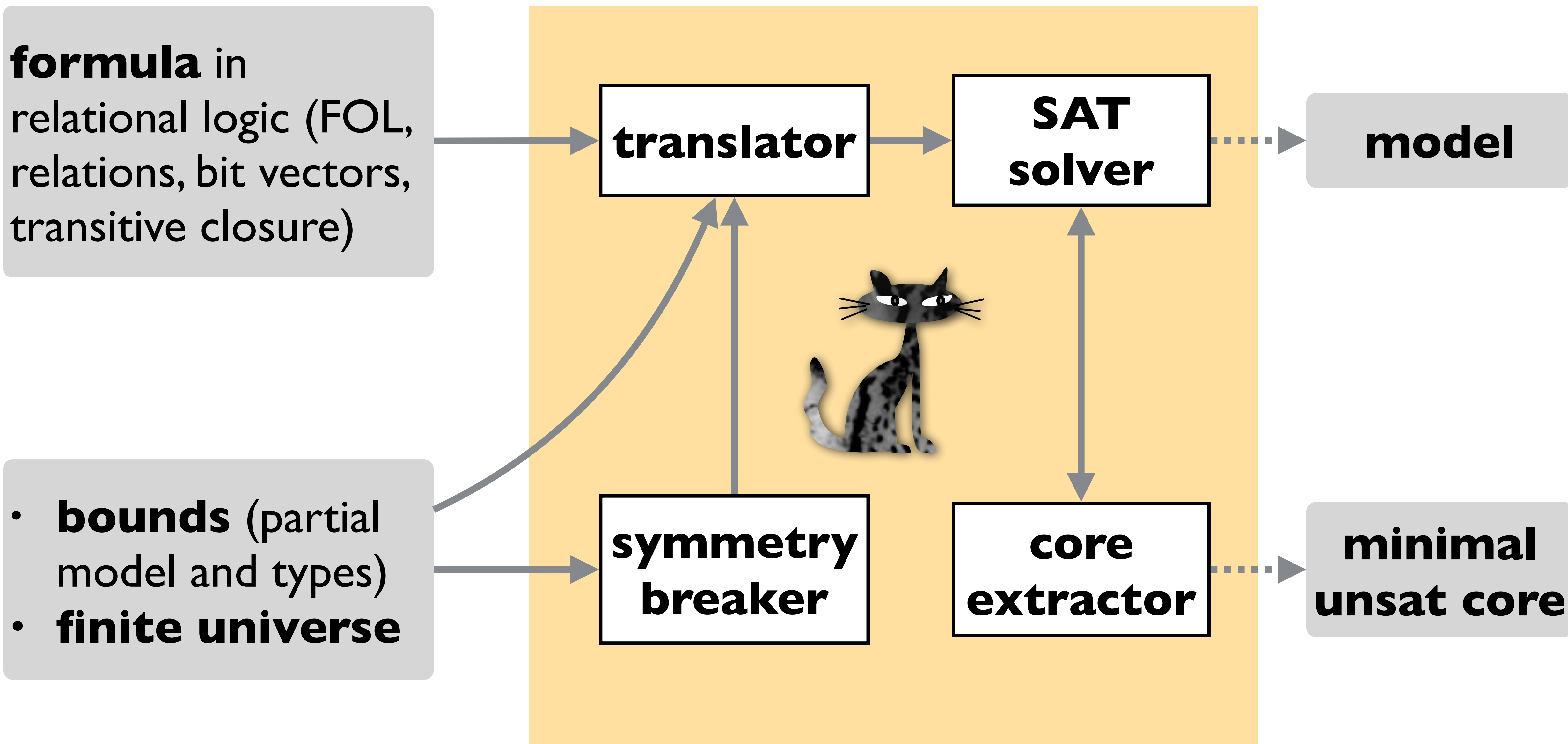


KODKOD

Overview of Kodkod



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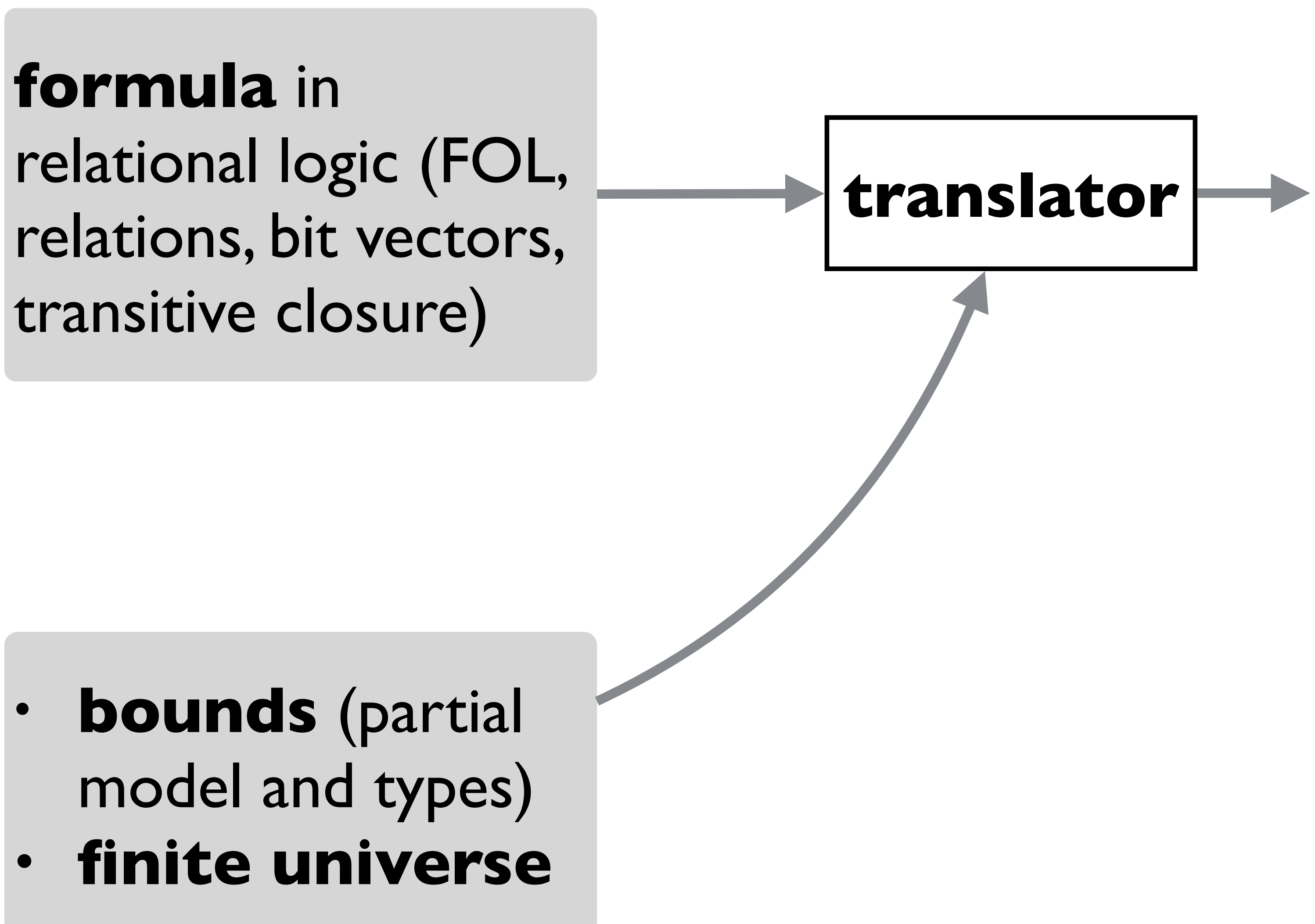

Relational logic by example

a minimalistic formal specification of a filesystem

Relational logic by example

$Root \subseteq Dir$

- The root of a filesystem hierarchy is a directory.

Relational logic by example

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

- The root of a filesystem hierarchy is a directory.
- Directories may contain files or directories.

Relational logic by example

$Root \subseteq Dir$

$contents \subseteq Dir \times (File \cup Dir)$

$(File \cup Dir) \subseteq Root.{*}contents$

- The root of a filesystem hierarchy is a directory.
- Directories may contain files or directories.
- All directories and files are reachable from the root.

Relational logic by example

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$\forall\, d: Dir \mid \neg\, (d \subseteq d.\hat{}contents)$

- The root of a filesystem hierarchy is a directory.
- Directories may contain files or directories.
- All directories and files are reachable from the root.
- The contents relation is acyclic.

Bounded relational logic by example

$Root \subseteq Dir$

$contents \subseteq Dir \times (File \cup Dir)$

$(File \cup Dir) \subseteq Root.{*}contents$

$\forall\, d: Dir \mid \neg\, (d \subseteq d.\hat{}contents)$

$\{\ \mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\ \}$

Finite universe of interpretation.

$\{\langle \mathbf{R} \rangle\} \subseteq Root \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq Dir \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

$\{\} \subseteq File \subseteq \{\langle \mathbf{F_1} \rangle, \langle \mathbf{F_2} \rangle\}$

$\{\} \subseteq contents \subseteq \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}\} \times \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\}$

Bounded relational logic by example

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

{ **R**, $\mathbf{D_1}$, $\mathbf{D_2}$, $\mathbf{F_1}$, $\mathbf{F_2}$ }

Finite universe of interpretation.

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨$\mathbf{D_1}$⟩, ⟨$\mathbf{D_2}$⟩}

{} ⊆ File ⊆ {⟨$\mathbf{F_1}$⟩, ⟨$\mathbf{F_2}$⟩}

{} ⊆ contents ⊆ {**R**, $\mathbf{D_1}$, $\mathbf{D_2}$} × {**R**, $\mathbf{D_1}$, $\mathbf{D_2}$, $\mathbf{F_1}$, $\mathbf{F_2}$}

Bounds for each relation:

- Tuples it *must* contain (partial model).
- Tuples it *may* contain (type).

Bounded relational logic by example

$\text{Root} \subseteq \text{Dir}$

$\text{contents} \subseteq \text{Dir} \times (\text{File} \cup \text{Dir})$

$(\text{File} \cup \text{Dir}) \subseteq \text{Root}.{}^{*}\text{contents}$

$\forall\, d\colon \text{Dir} \mid \neg\, (d \subseteq d.{}^{\wedge}\text{contents})$

$\{\ \mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\ \}$

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

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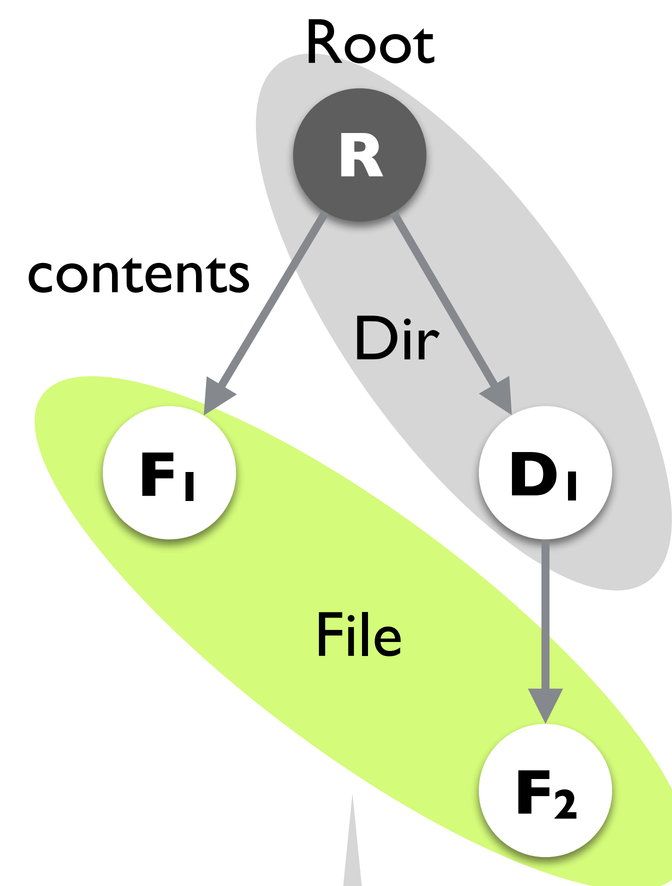
$\{\} \subseteq \text{contents} \subseteq \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}\} \times \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\}$



$\text{Root} = \{\langle \mathbf{R} \rangle\}$

$\text{Dir} = \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle\}$

$\text{File} = \{\langle \mathbf{F_1} \rangle, \langle \mathbf{F_2} \rangle\}$

$\text{contents} = \{\langle \mathbf{R}, \mathbf{F_1} \rangle, \langle \mathbf{R}, \mathbf{D_1} \rangle, \langle \mathbf{D_1}, \mathbf{F_2} \rangle\}$

Translation by example

$\mathrm{Root} \subseteq \mathrm{Dir}$

$\mathrm{contents} \subseteq \mathrm{Dir} \times (\mathrm{File} \cup \mathrm{Dir})$

$(\mathrm{File} \cup \mathrm{Dir}) \subseteq \mathrm{Root}.{}^{*}\mathrm{contents}$

$\forall\, d{:}\, \mathrm{Dir} \mid \neg\, (d \subseteq d.{}^{\wedge}\mathrm{contents})$

$\{\ \mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\ \}$

$\{\langle \mathbf{R} \rangle\} \subseteq \mathrm{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

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Translation by example

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

{ $\mathbf{R}$, $\mathbf{D_1}$, $\mathbf{D_2}$, $\mathbf{F_1}$, $\mathbf{F_2}$ }

{⟨$\mathbf{R}$⟩} ⊆ Root ⊆ {⟨$\mathbf{R}$⟩}

{} ⊆ Dir ⊆ {⟨$\mathbf{R}$⟩, ⟨$\mathbf{D_1}$⟩, ⟨$\mathbf{D_2}$⟩}

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{} ⊆ contents ⊆ {$\mathbf{R}$, $\mathbf{D_1}$, $\mathbf{D_2}$} × {$\mathbf{R}$, $\mathbf{D_1}$, $\mathbf{D_2}$, $\mathbf{F_1}$, $\mathbf{F_2}$}

Encode

- relational constants as boolean matrices
- relational expressions as matrix operations
- formulas as constraints over matrix entries

Relational constants as boolean matrices

Relational constants as boolean matrices

R	**D_1**	**D_2**	**F_1**	**F_2**
1	0	0	0	0

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

Relational constants as boolean matrices

R	$\mathbf{D_1}$	$\mathbf{D_2}$	$\mathbf{F_1}$	$\mathbf{F_2}$
1	0	0	0	0

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

d_0	d_1	d_2	0	0

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

Relational constants as boolean matrices

R	$\mathbf{D_1}$	$\mathbf{D_2}$	$\mathbf{F_1}$	$\mathbf{F_2}$
1	0	0	0	0

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

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$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

0	0	0	f_0	f_1

$\{\} \subseteq \text{File} \subseteq \{\langle \mathbf{F_1} \rangle, \langle \mathbf{F_2} \rangle\}$

Relational constants as boolean matrices

R	D_1	D_2	F_1	F_2
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R	D_1	D_2	F_1	F_2
d_0	d_1	d_2	0	0

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

R	D_1	D_2	F_1	F_2
0	0	0	f_0	f_1

$\{\} \subseteq \text{File} \subseteq \{\langle \mathbf{F_1} \rangle, \langle \mathbf{F_2} \rangle\}$

	R	D_1	D_2	F_1	F_2
R	c_0	c_1	c_2	c_3	c_4
D_1	c_5	c_6	c_7	c_8	c_9
D_2	c_{10}	c_{11}	c_{12}	c_{13}	c_{14}
F_1	0	0	0	0	0
F_2	0	0	0	0	0

$\{\} \subseteq \text{contents} \subseteq \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}\} \times \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\}$

Relational expressions as matrix operations

$$\overset{\text{File}}{\begin{bmatrix}0 & 0 & 0 & f_0 & f_1\end{bmatrix}} \vee \overset{\text{Dir}}{\begin{bmatrix}d_0 & d_1 & d_2 & 0 & 0\end{bmatrix}} = \overset{\text{File} \cup \text{Dir}}{\begin{bmatrix}d_0 & d_1 & d_2 & f_0 & f_1\end{bmatrix}}$$

$$\overset{\text{Dir}}{\begin{bmatrix}d_0 \\ d_1 \\ d_2 \\ 0 \\ 0\end{bmatrix}} \times \overset{\text{File} \cup \text{Dir}}{\begin{bmatrix}d_0 & d_1 & d_2 & f_0 & f_1\end{bmatrix}} = \overset{\text{Dir} \times (\text{File} \cup \text{Dir})}{\begin{bmatrix}d_0 \wedge d_0 & d_0 \wedge d_1 & d_0 \wedge d_2 & d_0 \wedge f_0 & d_0 \wedge f_1 \\ d_1 \wedge d_0 & d_1 \wedge d_1 & d_1 \wedge d_2 & d_1 \wedge f_0 & d_1 \wedge f_1 \\ d_2 \wedge d_0 & d_2 \wedge d_1 & d_2 \wedge d_2 & d_2 \wedge f_0 & d_2 \wedge f_1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0\end{bmatrix}}$$

Formulas as constraints over matrix entries

contents

c_0	c_1	c_2	c_3	c_4
c_5	c_6	c_7	c_8	c_9
c_{10}	c_{11}	c_{12}	c_{13}	c_{14}
0	0	0	0	0
0	0	0	0	0

$\rightarrow$

Dir × (File ∪ Dir)

$d_0 \wedge d_0$	$d_0 \wedge d_1$	$d_0 \wedge d_2$	$d_0 \wedge f_0$	$d_0 \wedge f_1$
$d_1 \wedge d_0$	$d_1 \wedge d_1$	$d_1 \wedge d_2$	$d_1 \wedge f_0$	$d_1 \wedge f_1$
$d_2 \wedge d_0$	$d_2 \wedge d_1$	$d_2 \wedge d_2$	$d_2 \wedge f_0$	$d_2 \wedge f_1$
0	0	0	0	0
0	0	0	0	0

=

contents ⊆ Dir × (File ∪ Dir)

$$
\begin{aligned}
&(c_0 \longrightarrow d_0 \wedge d_0) \wedge \\
&(c_1 \longrightarrow d_0 \wedge d_1) \wedge \\
&(c_2 \longrightarrow d_0 \wedge d_2) \wedge \\
&(c_3 \longrightarrow d_0 \wedge f_0) \wedge \\
&(c_4 \longrightarrow d_0 \wedge f_1) \wedge \\
&(c_5 \longrightarrow d_1 \wedge d_0) \wedge \\
&\ldots \\
&(c_{14} \longrightarrow d_2 \wedge f_1)
\end{aligned}
$$

Dealing with sparseness and redundancy

Dir × (File ∪ Dir)

$d_0 \wedge d_0$	$d_0 \wedge d_1$	$d_0 \wedge d_2$	$d_0 \wedge f_0$	$d_0 \wedge f_1$
$d_1 \wedge d_0$	$d_1 \wedge d_1$	$d_1 \wedge d_2$	$d_1 \wedge f_0$	$d_1 \wedge f_1$
$d_2 \wedge d_0$	$d_2 \wedge d_1$	$d_2 \wedge d_2$	$d_2 \wedge f_0$	$d_2 \wedge f_1$
0	0	0	0	0
0	0	0	0	0

Dealing with sparseness and redundancy

Dir × (File ∪ Dir)

$d_0 \wedge d_0$	$d_0 \wedge d_1$	$d_0 \wedge d_2$	$d_0 \wedge f_0$	$d_0 \wedge f_1$
$d_1 \wedge d_0$	$d_1 \wedge d_1$	$d_1 \wedge d_2$	$d_1 \wedge f_0$	$d_1 \wedge f_1$
$d_2 \wedge d_0$	$d_2 \wedge d_1$	$d_2 \wedge d_2$	$d_2 \wedge f_0$	$d_2 \wedge f_1$
0	0	0	0	0
0	0	0	0	0

Empty regions in matrices (exponential w.r.t. relation arity).

Dealing with sparseness and redundancy

Different circuits for the same formula.

Dir × (File ∪ Dir)

$d_0 \wedge d_0$	$d_0 \wedge d_1$	$d_0 \wedge d_2$	$d_0 \wedge f_0$	$d_0 \wedge f_1$
$d_1 \wedge d_0$	$d_1 \wedge d_1$	$d_1 \wedge d_2$	$d_1 \wedge f_0$	$d_1 \wedge f_1$
$d_2 \wedge d_0$	$d_2 \wedge d_1$	$d_2 \wedge d_2$	$d_2 \wedge f_0$	$d_2 \wedge f_1$
0	0	0	0	0
0	0	0	0	0

Empty regions in matrices (exponential w.r.t. relation arity).

Dealing with sparseness and redundancy

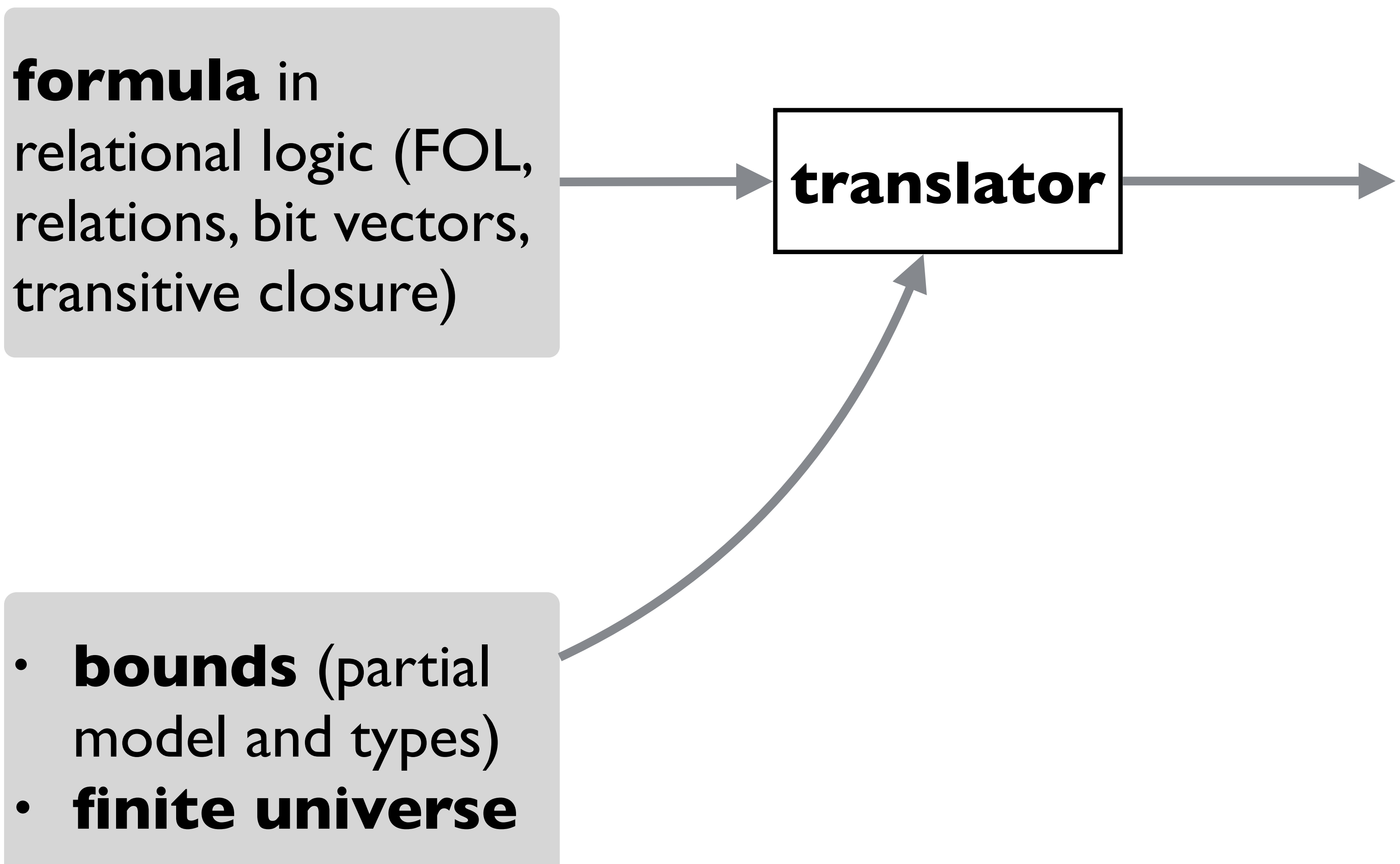
Dir × (File ∪ Dir)

$d_0 \wedge d_0$	$d_0 \wedge d_1$	$d_0 \wedge d_2$	$d_0 \wedge f_0$	$d_0 \wedge f_1$
$d_1 \wedge d_0$	$d_1 \wedge d_1$	$d_1 \wedge d_2$	$d_1 \wedge f_0$	$d_1 \wedge f_1$
$d_2 \wedge d_0$	$d_2 \wedge d_1$	$d_2 \wedge d_2$	$d_2 \wedge f_0$	$d_2 \wedge f_1$
0	0	0	0	0
0	0	0	0	0

Compact Boolean Circuits (CBCs).

Sparse matrices represented as interval trees.

Overview of Kodkod



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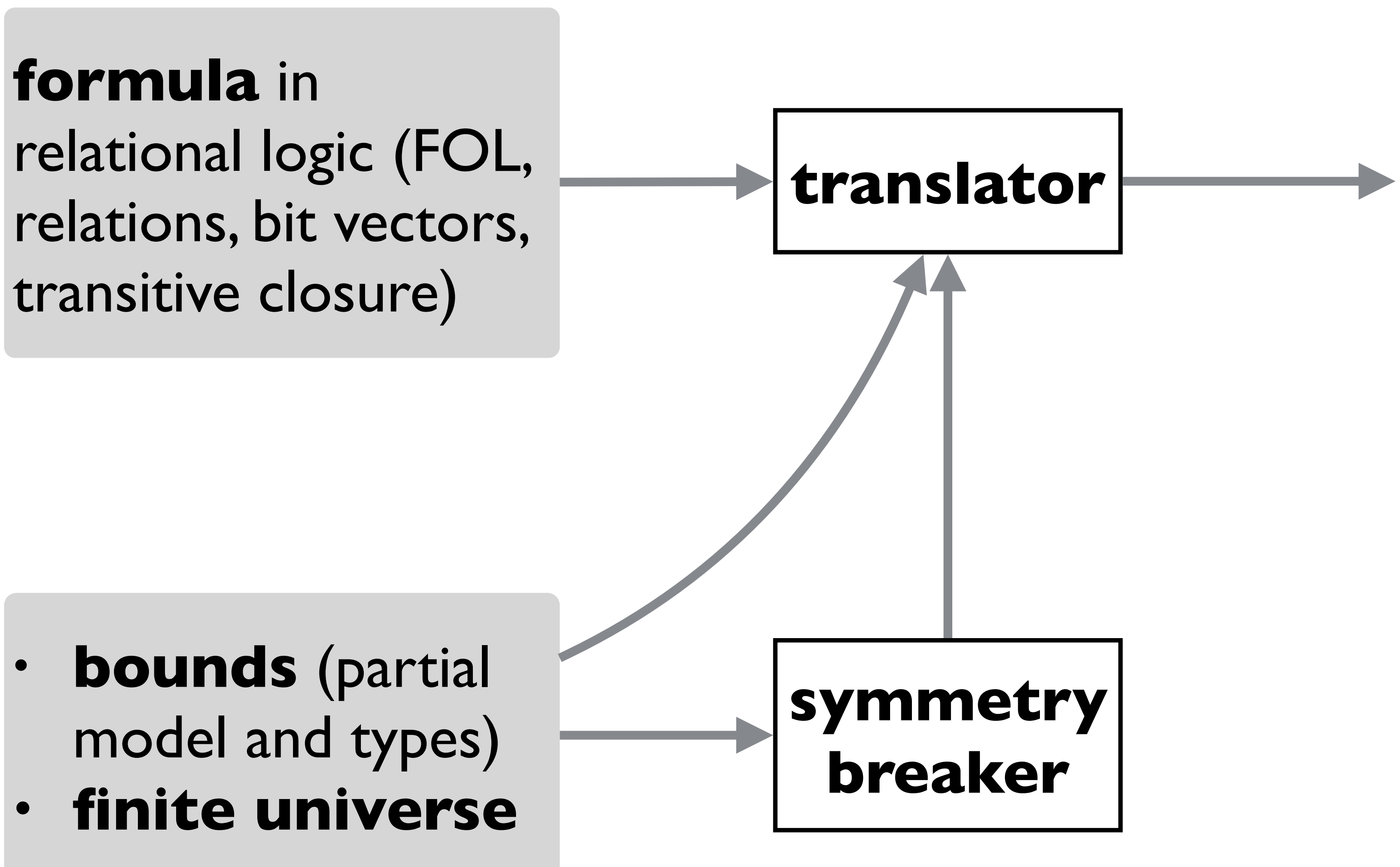

Symmetry by example

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

{ $\mathbf{R}$, $\mathbf{D_1}$, $\mathbf{D_2}$, $\mathbf{F_1}$, $\mathbf{F_2}$ }

{⟨$\mathbf{R}$⟩} ⊆ Root ⊆ {⟨$\mathbf{R}$⟩}

{} ⊆ Dir ⊆ {⟨$\mathbf{R}$⟩, ⟨$\mathbf{D_1}$⟩, ⟨$\mathbf{D_2}$⟩}

{} ⊆ File ⊆ {⟨$\mathbf{F_1}$⟩, ⟨$\mathbf{F_2}$⟩}

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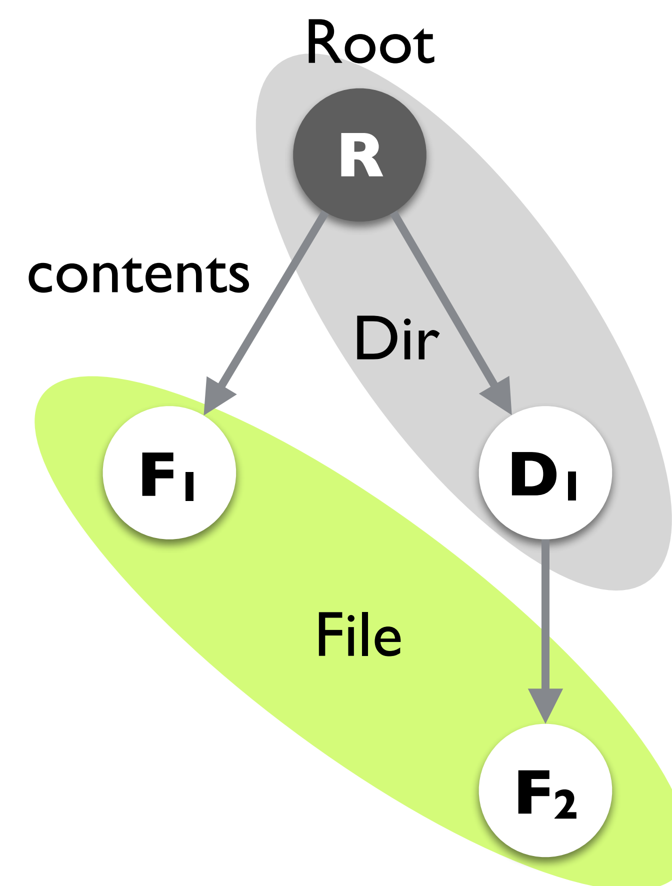

Symmetry by example

$\text{Root} \subseteq \text{Dir}$

$\text{contents} \subseteq \text{Dir} \times (\text{File} \cup \text{Dir})$

$(\text{File} \cup \text{Dir}) \subseteq \text{Root}.^{*}\text{contents}$

$\forall\ d: \text{Dir} \mid \neg\ (d \subseteq d.\hat{}\text{contents})$

$\{\ \mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\ \}$

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

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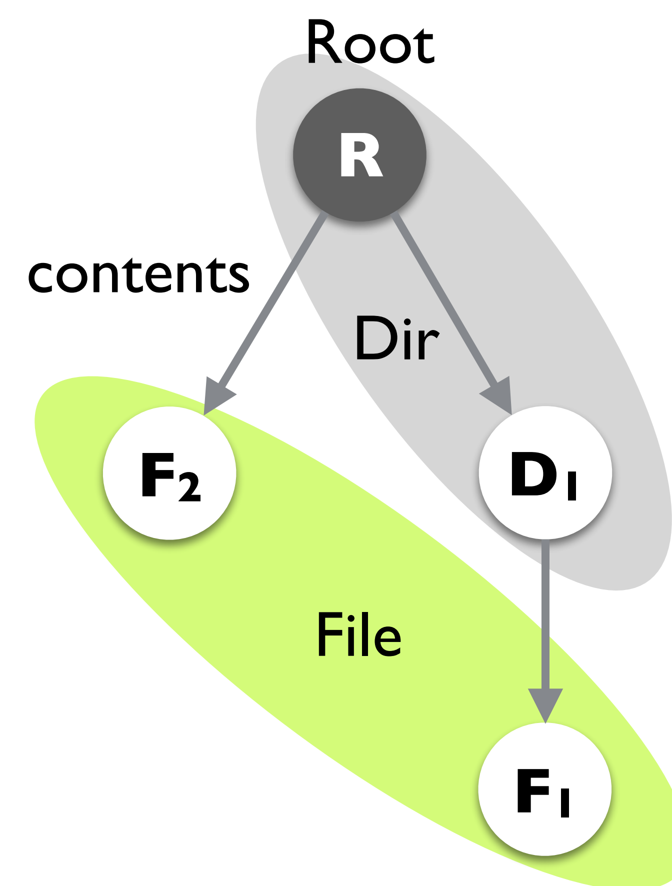

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$\forall\ d: \text{Dir} \mid \neg\ (d \subseteq d.\text{^}\text{contents})$

$\{\ \mathbf{R}, \mathbf{D}\ \mathbf{D_2}\ \mathbf{F_1}, \mathbf{F_2}\ \}$

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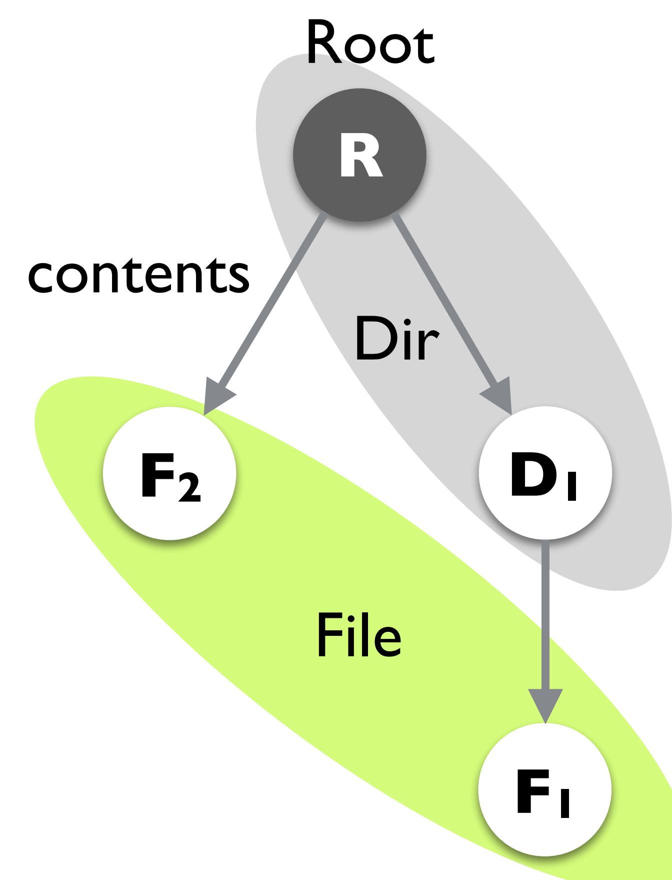

Symmetry by example

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$\forall\ d{:}\ \text{Dir} \mid \neg\ (d \subseteq d.\text{^}\text{contents})$

$\{\ \mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\ \}$

$\{\langle\mathbf{R}\rangle\} \subseteq \text{Root} \subseteq \{\langle\mathbf{R}\rangle\}$

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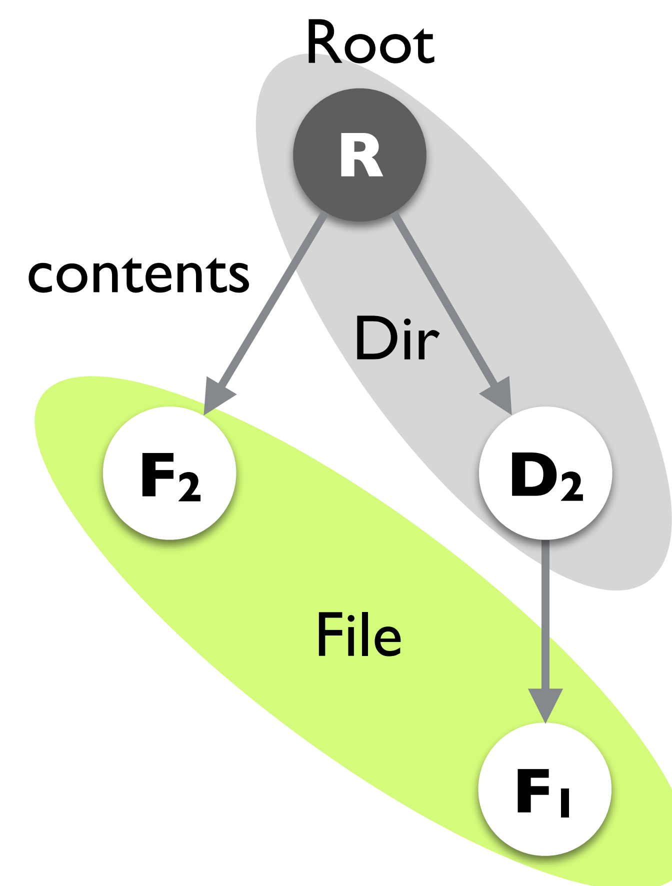


Symmetries between models

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

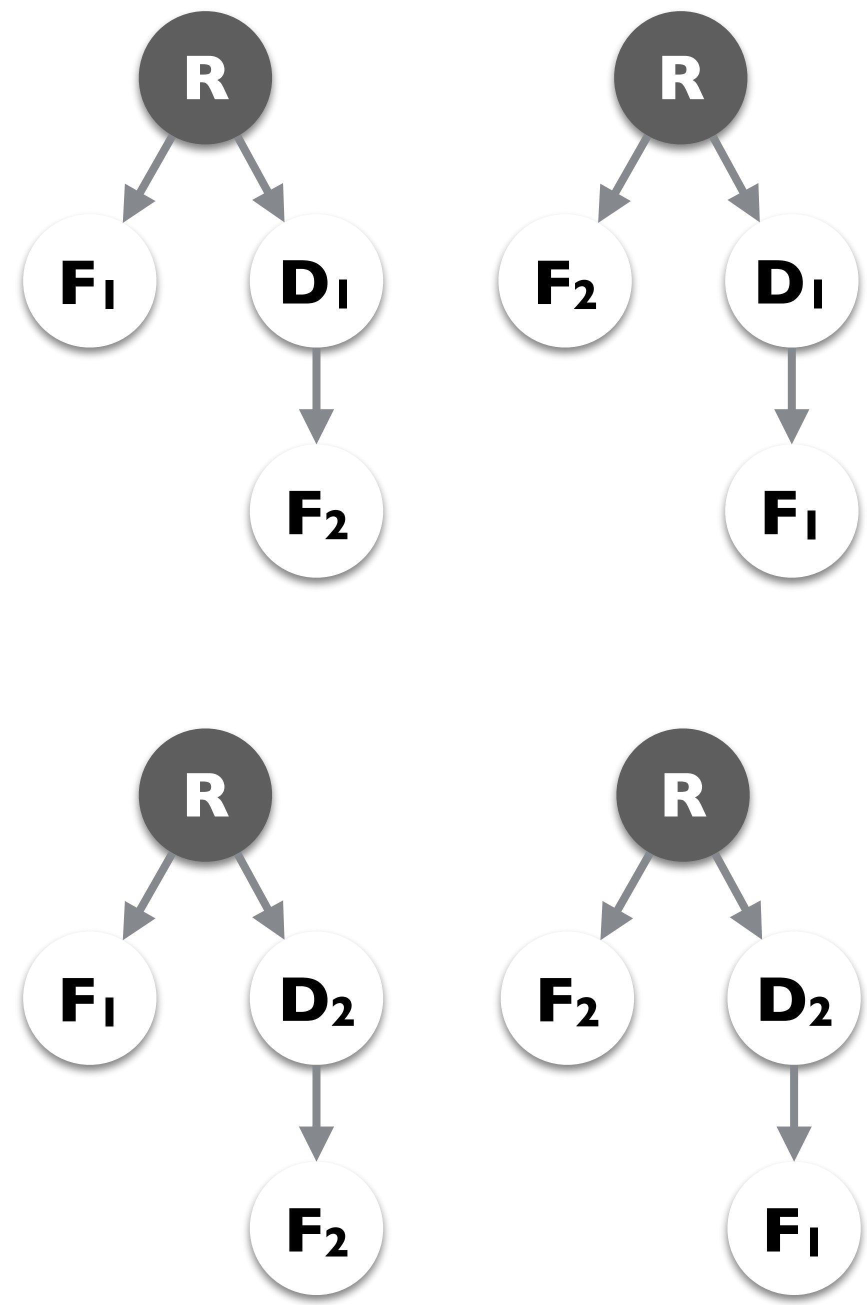
{ **R**, **D₁**, **D₂**, **F₁**, **F₂** }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D₁**⟩, ⟨**D₂**⟩}

{} ⊆ File ⊆ {⟨**F₁**⟩, ⟨**F₂**⟩}

{} ⊆ contents ⊆ {**R**, **D₁**, **D₂**} × {**R**, **D₁**, **D₂**, **F₁**, **F₂**}



Symmetries between non-models

$\text{Root} \subseteq \text{Dir}$

$\text{contents} \subseteq \text{Dir} \times (\text{File} \cup \text{Dir})$

$(\text{File} \cup \text{Dir}) \subseteq \text{Root}.{*}\text{contents}$

$\forall\ d\text{: Dir} \mid \neg\ (d \subseteq d.\hat{}\ \text{contents})$

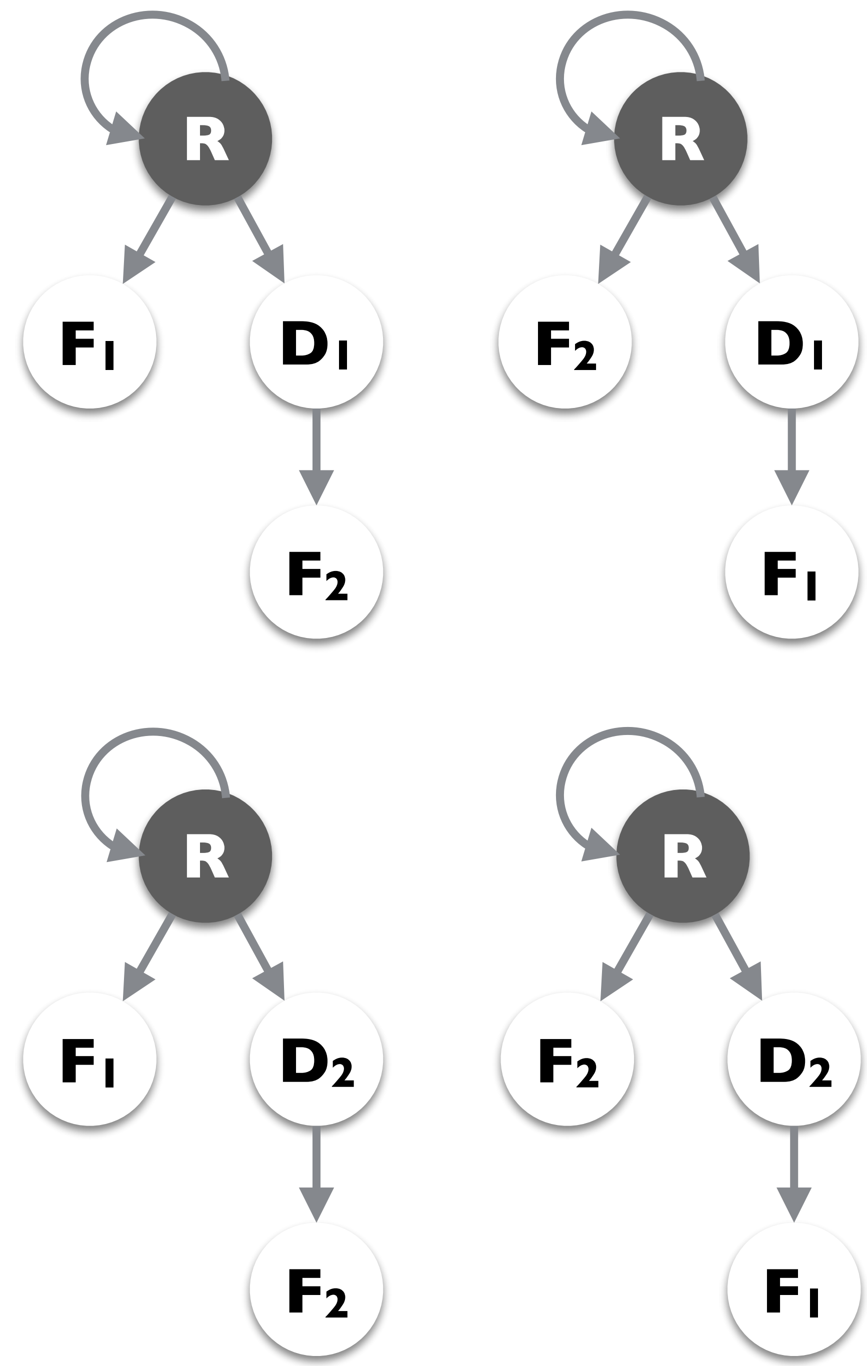
$\{\ \mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\ \}$

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

$\{\} \subseteq \text{File} \subseteq \{\langle \mathbf{F_1} \rangle, \langle \mathbf{F_2} \rangle\}$

$\{\} \subseteq \text{contents} \subseteq \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}\} \times \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\}$



Symmetries induce equivalence classes

Root $\subseteq$ Dir

contents $\subseteq$ Dir $\times$ (File $\cup$ Dir)

(File $\cup$ Dir) $\subseteq$ Root.*contents

$\forall$ d: Dir | $\neg$ (d $\subseteq$ d.^contents)

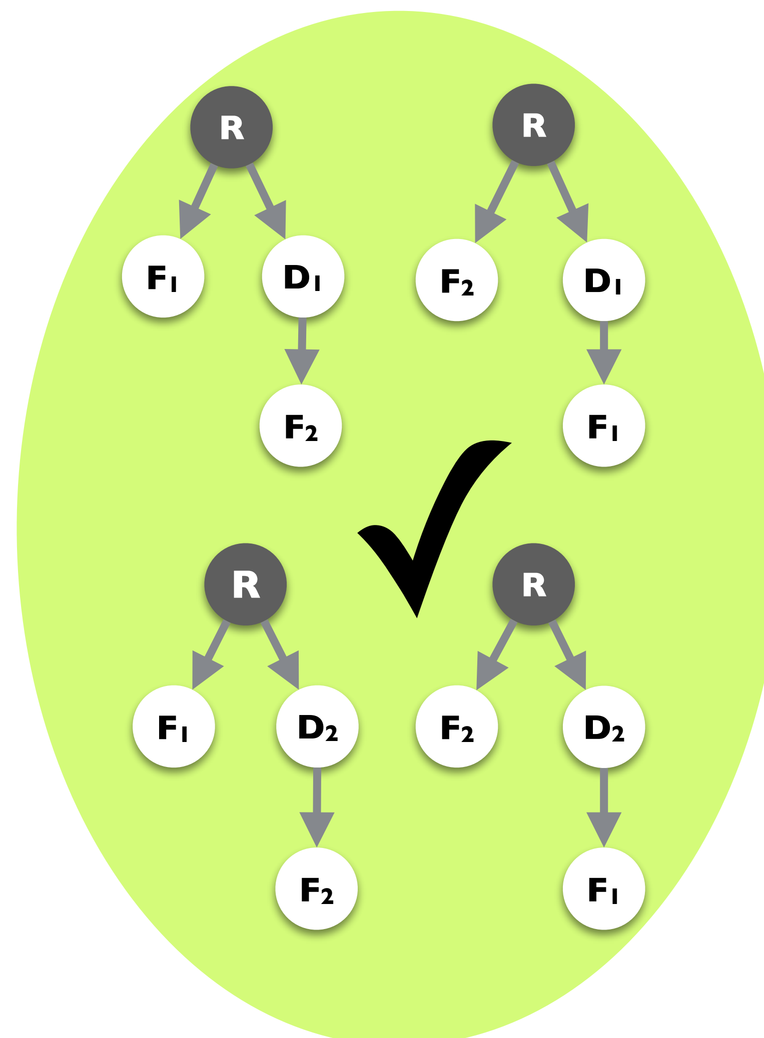
{ **R**, **D$_1$**, **D$_2$**, **F$_1$**, **F$_2$** }

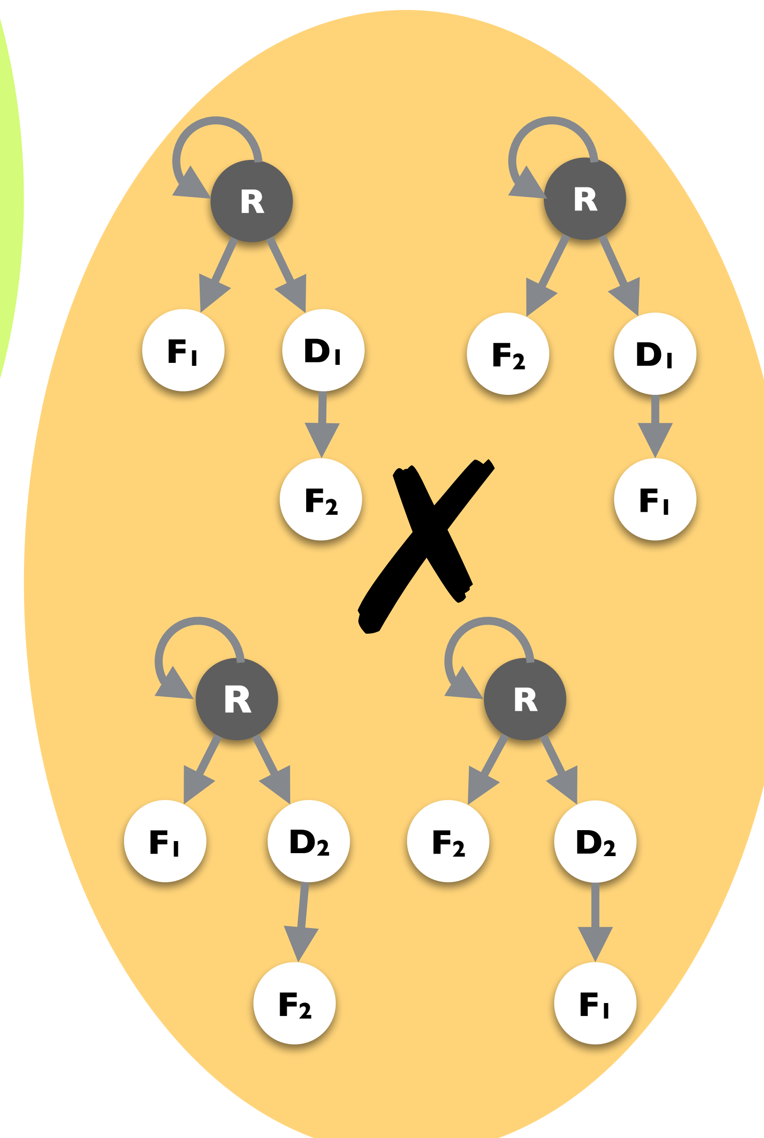
{⟨**R**⟩} $\subseteq$ Root $\subseteq$ {⟨**R**⟩}

{} $\subseteq$ Dir $\subseteq$ {⟨**R**⟩, ⟨**D$_1$**⟩, ⟨**D$_2$**⟩}

{} $\subseteq$ File $\subseteq$ {⟨**F$_1$**⟩, ⟨**F$_2$**⟩}

{} $\subseteq$ contents $\subseteq$ {**R**, **D$_1$**, **D$_2$**} $\times$ {**R**, **D$_1$**, **D$_2$**, **F$_1$**, **F$_2$**}





Symmetries induce equivalence classes

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

$\{\ \mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\ \}$

$\{\langle\mathbf{R}\rangle\} \subseteq \text{Root} \subseteq \{\langle\mathbf{R}\rangle\}$

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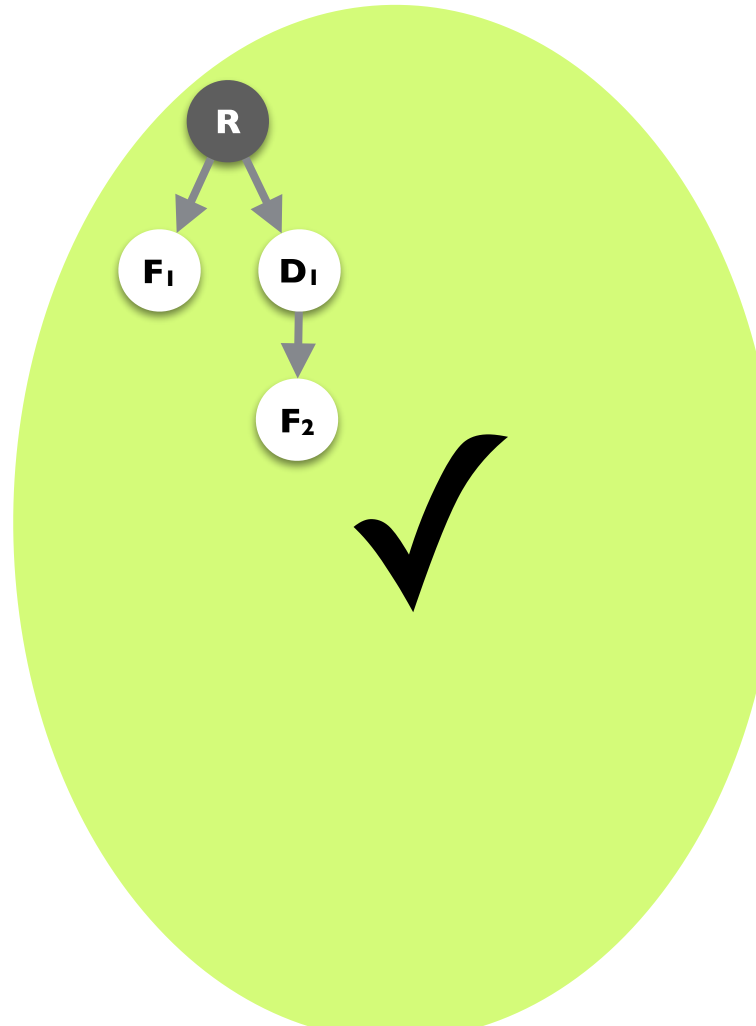
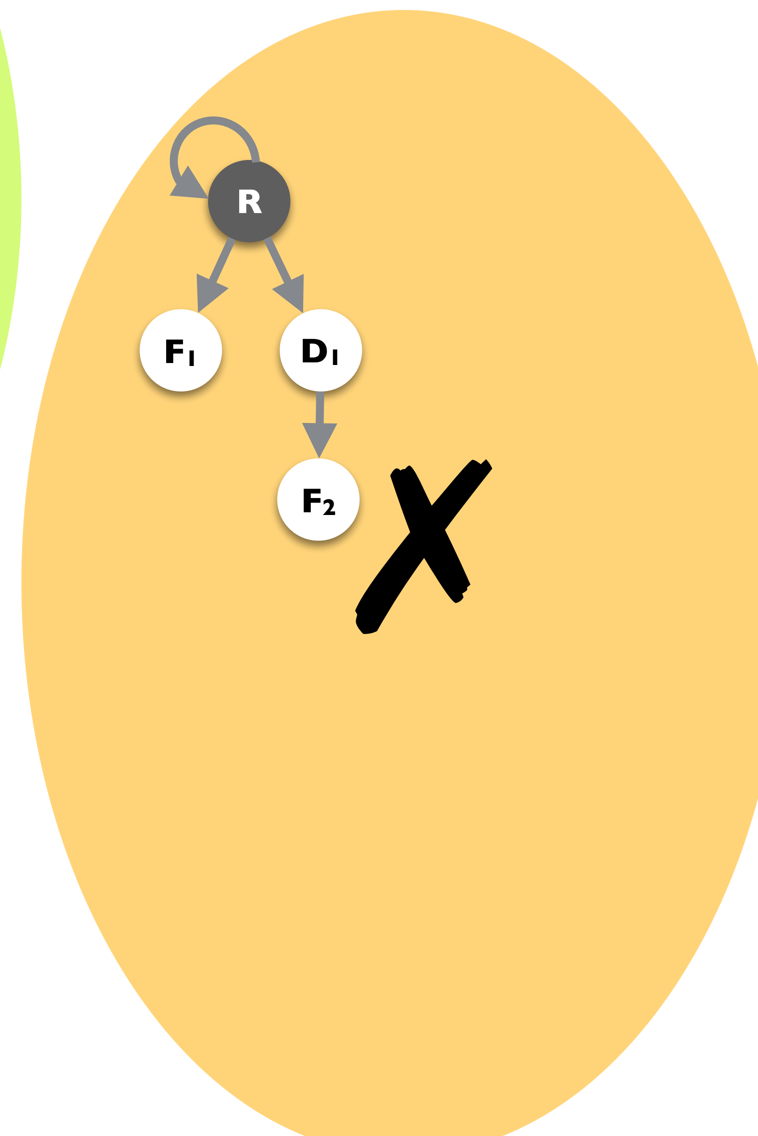


Sufficient to check one interpretation per equivalence class.



Symmetry detection

Root $\subseteq$ Dir

contents $\subseteq$ Dir × (File $\cup$ Dir)

(File $\cup$ Dir) $\subseteq$ Root.*contents

$\forall$ d: Dir | $\neg$ (d $\subseteq$ d.^contents)

{ $\mathbf{R}$, $\mathbf{D_1}$, $\mathbf{D_2}$, $\mathbf{F_1}$, $\mathbf{F_2}$ }

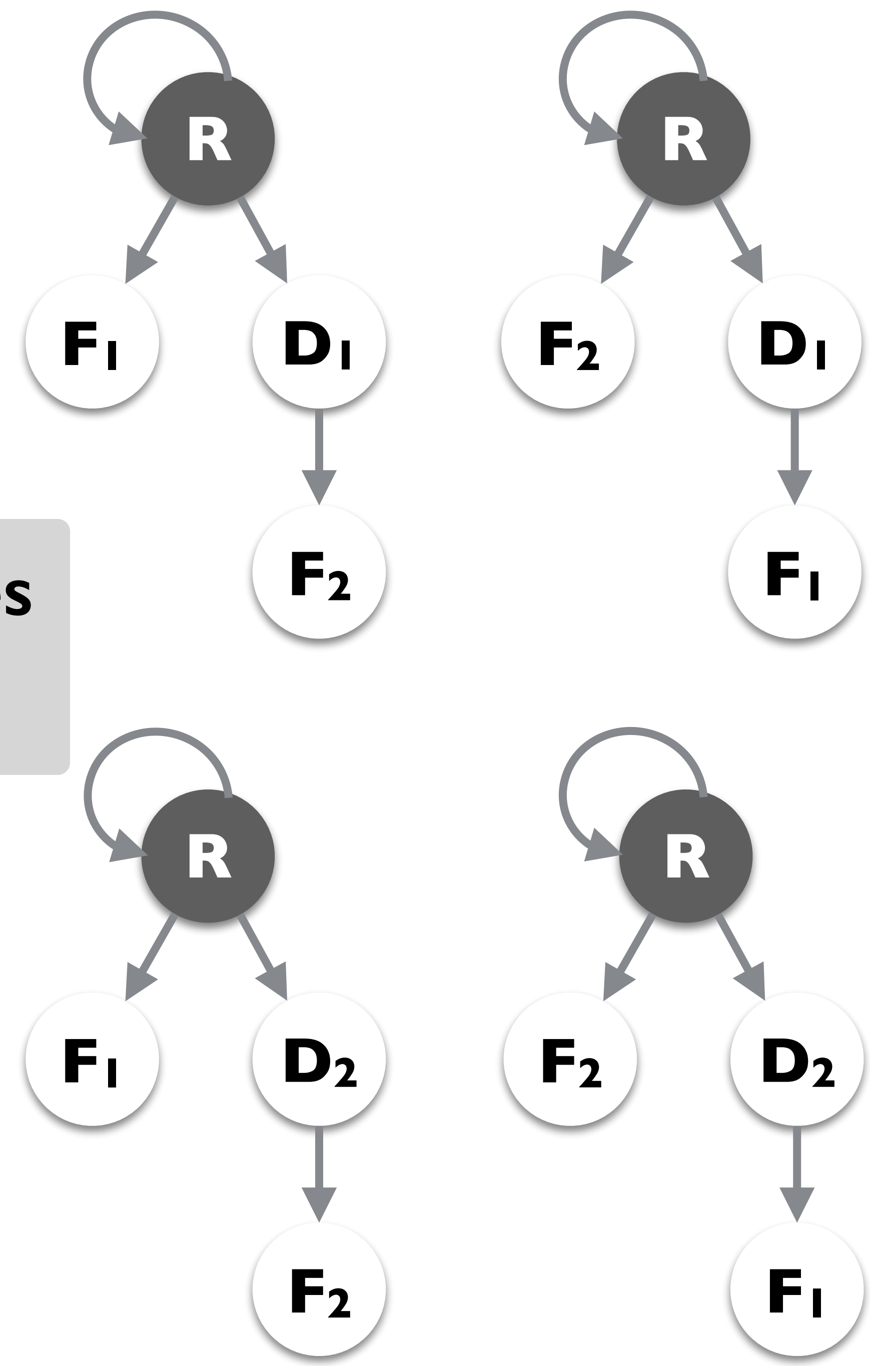
Interpretation symmetries
= bound symmetries

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

$\{\} \subseteq \text{File} \subseteq \{\langle \mathbf{F_1} \rangle, \langle \mathbf{F_2} \rangle\}$

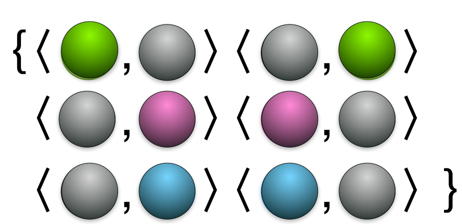
$\{\} \subseteq \text{contents} \subseteq \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}\} \times \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\}$

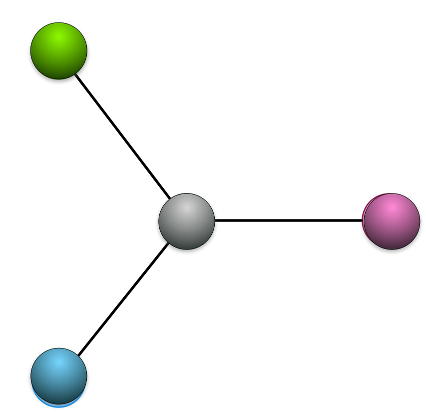


Detecting symmetries is hard …

Interpretation symmetries
= bound symmetries

Graph automorphism
detection





But only a few symmetries needed in practice

Greedy algorithm that partitions the universe into equivalence classes ✓

✗ Graph automorphism detection

Base partitioning: practical symmetry detection

$\{\ \mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\ \}$

$\{\langle\mathbf{R}\rangle\} \subseteq \text{Root} \subseteq \{\langle\mathbf{R}\rangle\}$

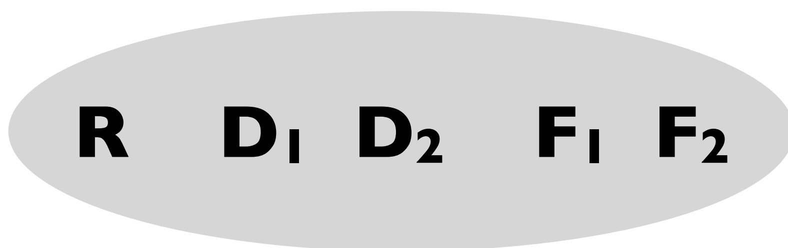
$\{\} \subseteq \text{Dir} \subseteq \{\langle\mathbf{R}\rangle, \langle\mathbf{D_1}\rangle, \langle\mathbf{D_2}\rangle\}$

$\{\} \subseteq \text{File} \subseteq \{\langle\mathbf{F_1}\rangle, \langle\mathbf{F_2}\rangle\}$

$\{\} \subseteq \text{contents} \subseteq \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}\} \times \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\}$

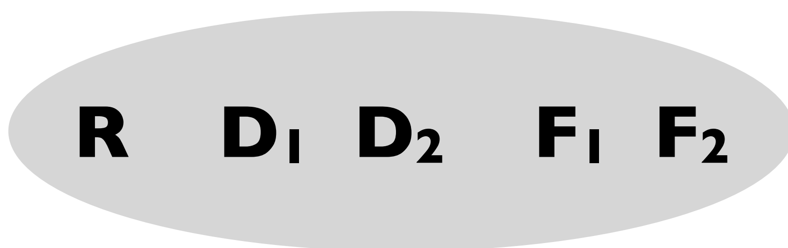
The coarsest partition of the universe such that each non-empty bound is expressible as a union of products of parts.

Finding the base partitioning



start with a single partition and refine minimally for each non-empty lower and upper bound

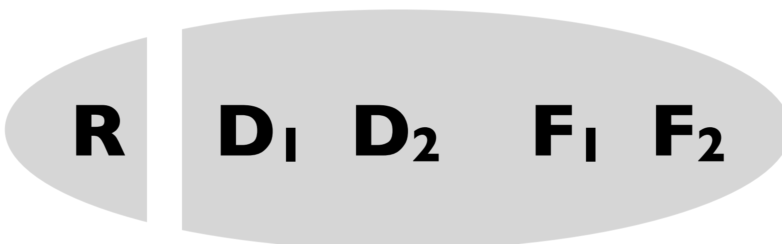
Finding the base partitioning



$$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$$

start with a single partition and refine minimally for each non-empty lower and upper bound

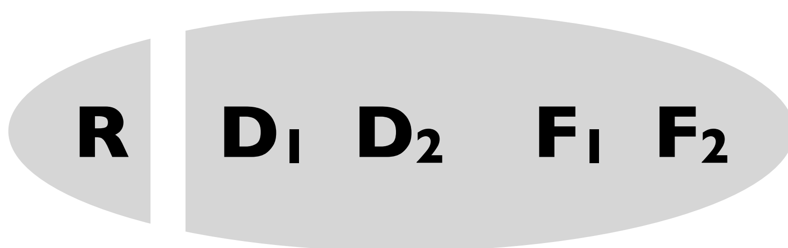
Finding the base partitioning



$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

start with a single partition and refine minimally for each non-empty lower and upper bound

Finding the base partitioning



$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

start with a single partition and refine minimally for each non-empty lower and upper bound

Finding the base partitioning

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Finding the base partitioning

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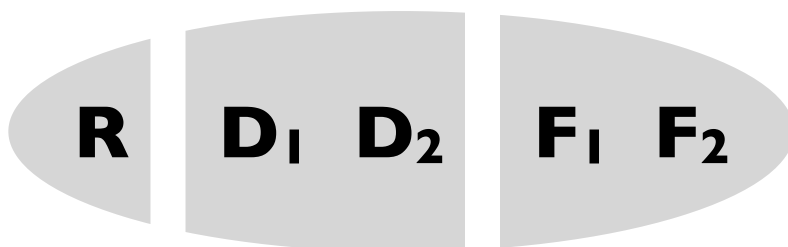
start with a single partition and refine minimally for each non-empty lower and upper bound

Finding the base partitioning



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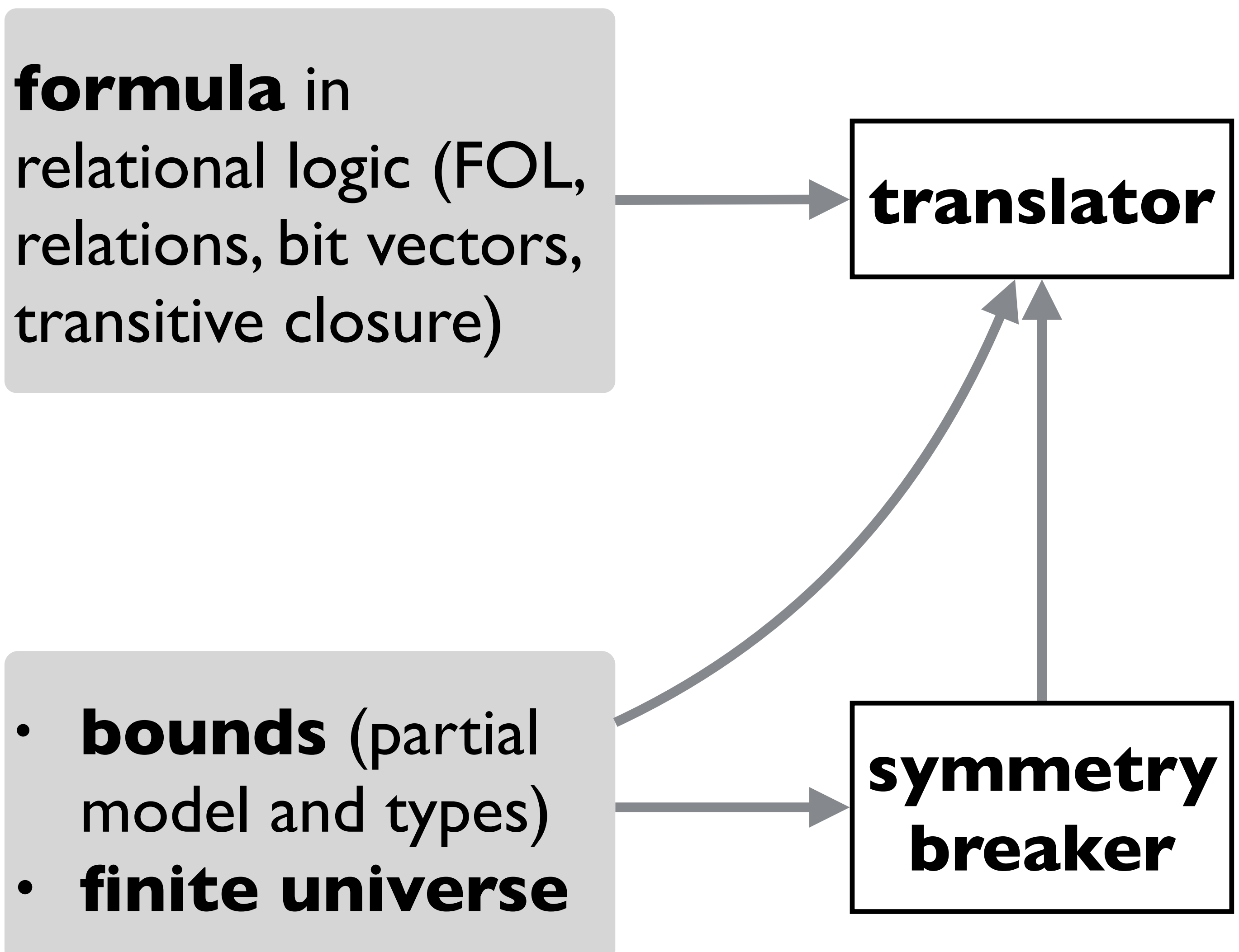
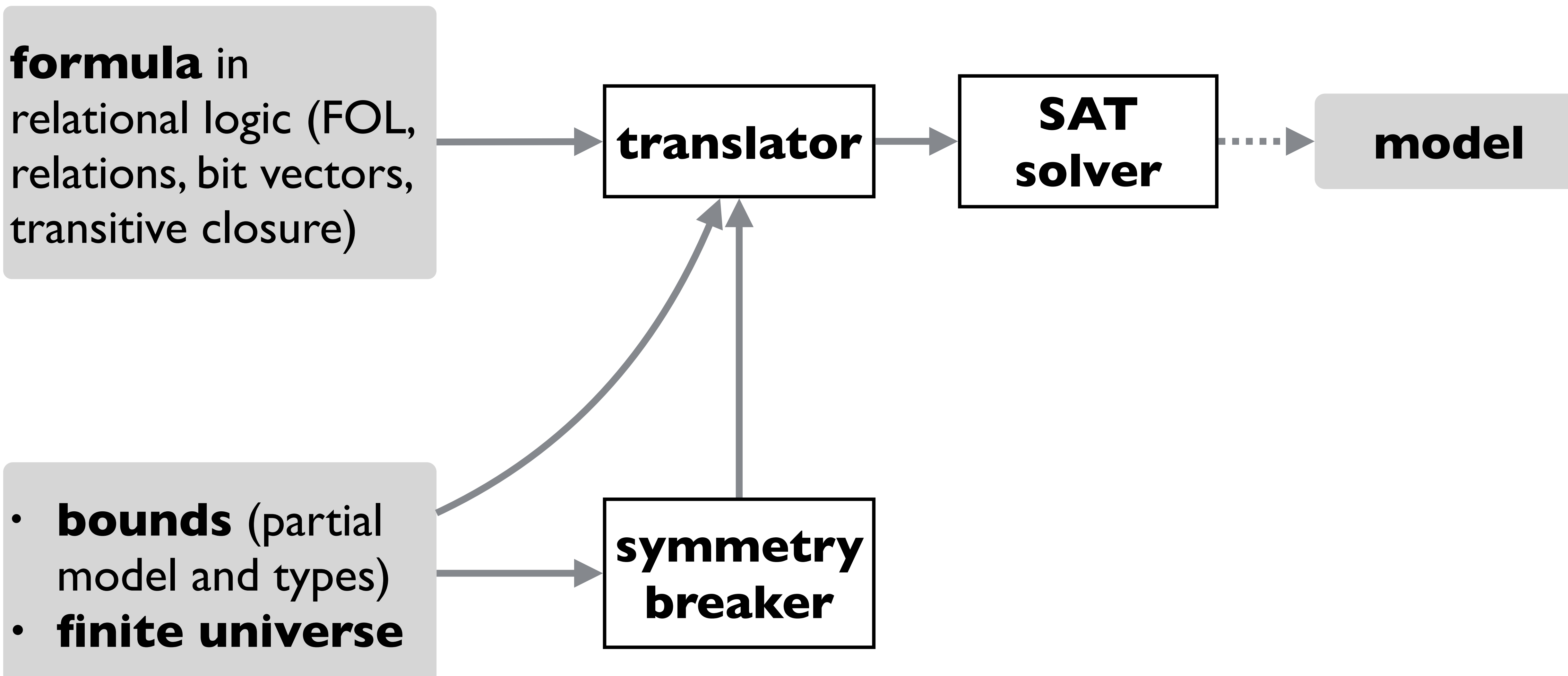
start with a single partition and refine minimally for each non-empty lower and upper bound

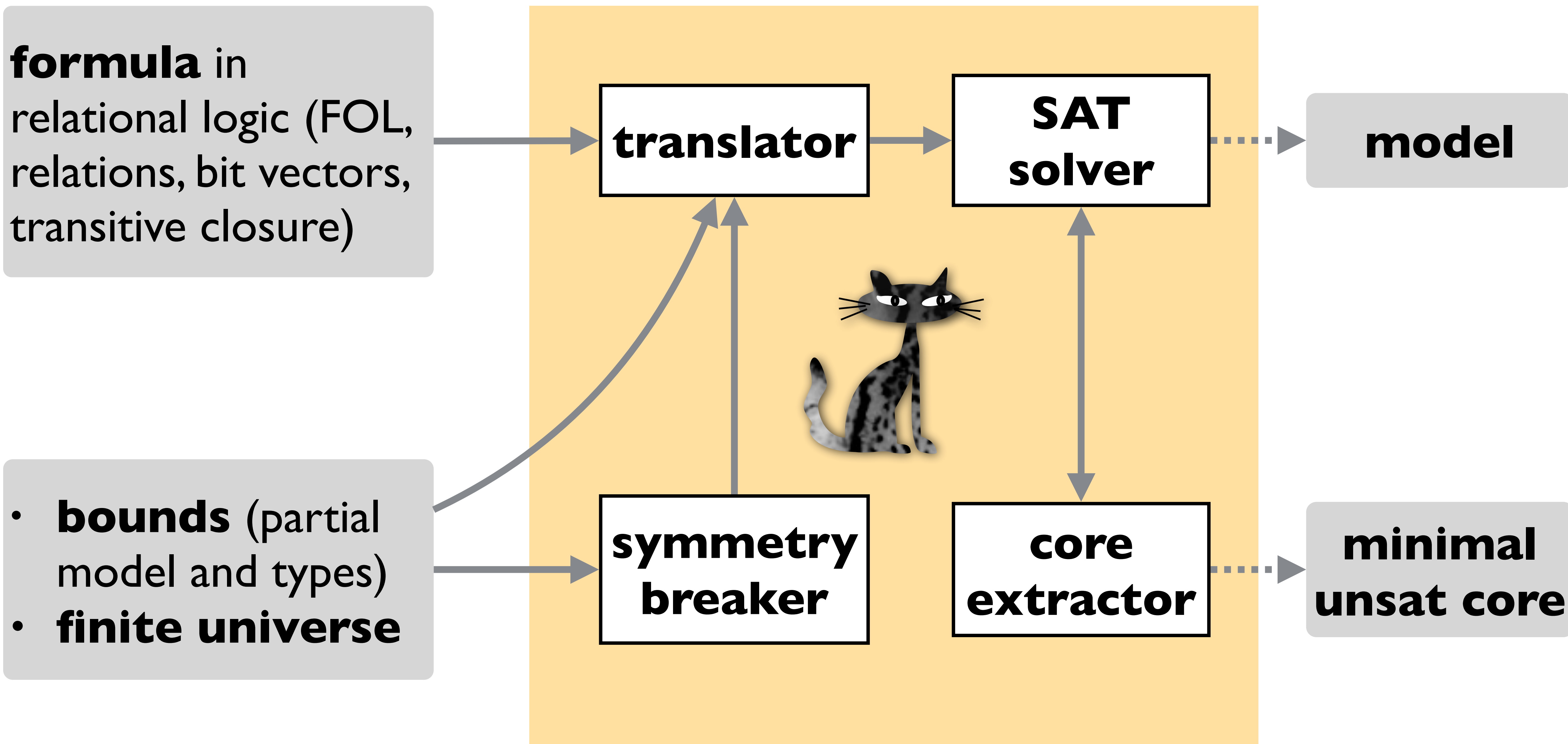
Overview of Kodkod



Overview of Kodkod



Overview of Kodkod



A bug in the tiny filesystem

Root $\subseteq$ Dir

contents $\subseteq$ Dir × (File $\cup$ Dir)

(File $\cup$ Dir) $\subseteq$ Root.*contents

$\forall$ d: Dir | $\neg$ (d $\subseteq$ d.^contents)

{ $\mathbf{R}$, $\mathbf{D_1}$, $\mathbf{D_2}$, $\mathbf{F_1}$, $\mathbf{F_2}$ }

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

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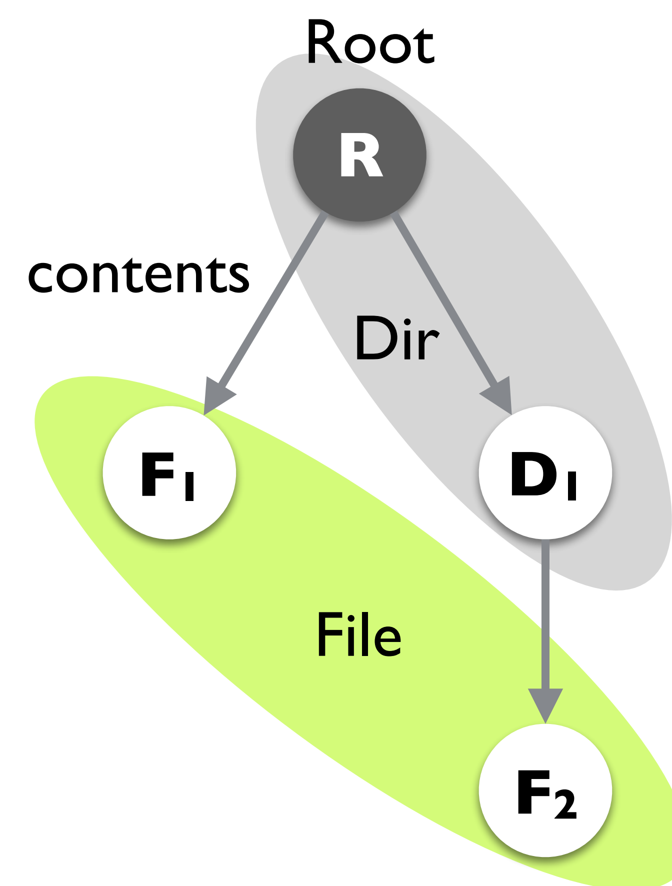

A bug in the tiny filesystem

$\text{Root} \subseteq \text{Dir}$

$\text{contents} \subseteq \text{Dir} \times (\text{File} \cup \text{Dir})$

$(\text{File} \cup \text{Dir}) \subseteq \text{Root.*contents}$

$\forall\ d: \text{Dir} \mid \neg\ (d \subseteq d.\text{^contents})$

$\{\ \mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\ \}$

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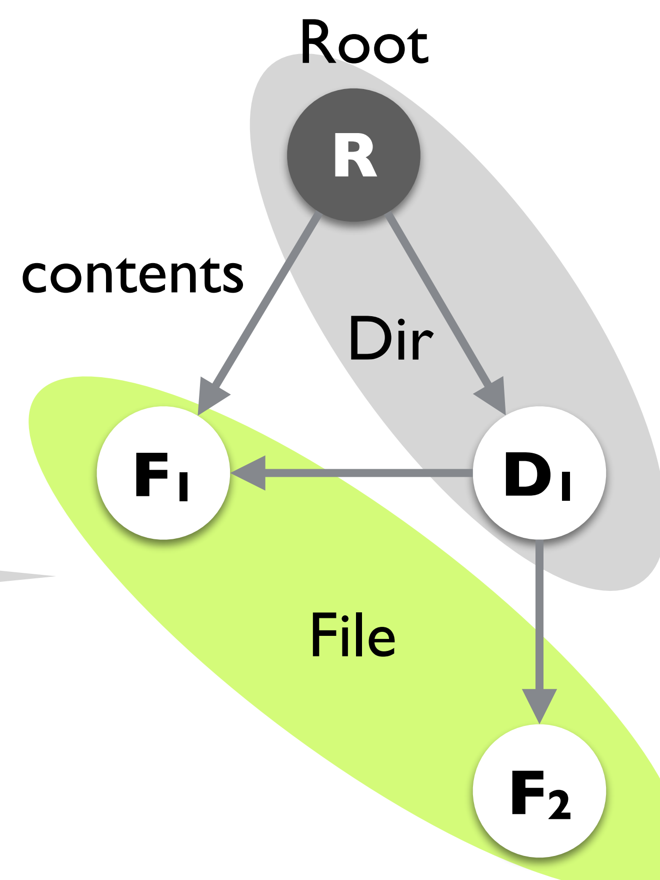
The spec allows multiple parents.



Fixing the tiny filesystem

$Root \subseteq Dir$

$contents \subseteq Dir \times (File \cup Dir)$

$(File \cup Dir) \subseteq Root.{*}contents$

$\forall\ d: Dir \mid \neg\ (d \subseteq d.\hat{}contents)$

$\forall\ f: File \mid one\ contents.f$

$\forall\ d: Dir \mid one\ contents.d$

$\{\ \mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\ \}$

$\{\langle\mathbf{R}\rangle\} \subseteq Root \subseteq \{\langle\mathbf{R}\rangle\}$

$\{\} \subseteq Dir \subseteq \{\langle\mathbf{R}\rangle, \langle\mathbf{D_1}\rangle, \langle\mathbf{D_2}\rangle\}$

$\{\} \subseteq File \subseteq \{\langle\mathbf{F_1}\rangle, \langle\mathbf{F_2}\rangle\}$

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Fixing the tiny filesystem

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$\forall\ f: File \mid one\ contents.f$

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Minimal unsatisfiable core: an unsatisfiable subset of a formula that becomes satisfiable if any of its members are removed.

Resolution-based core extraction

Root ⊆ Dir

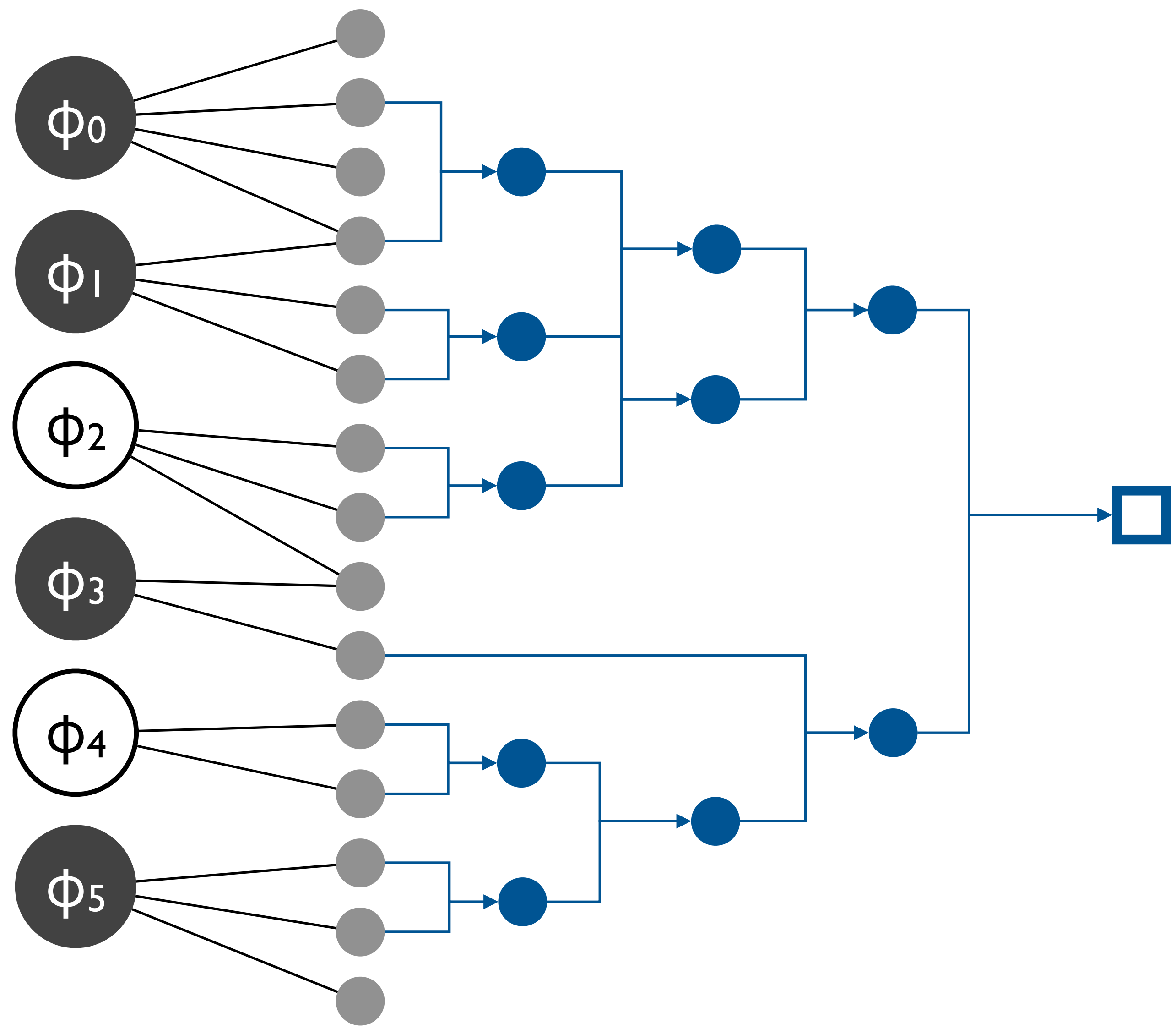
contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

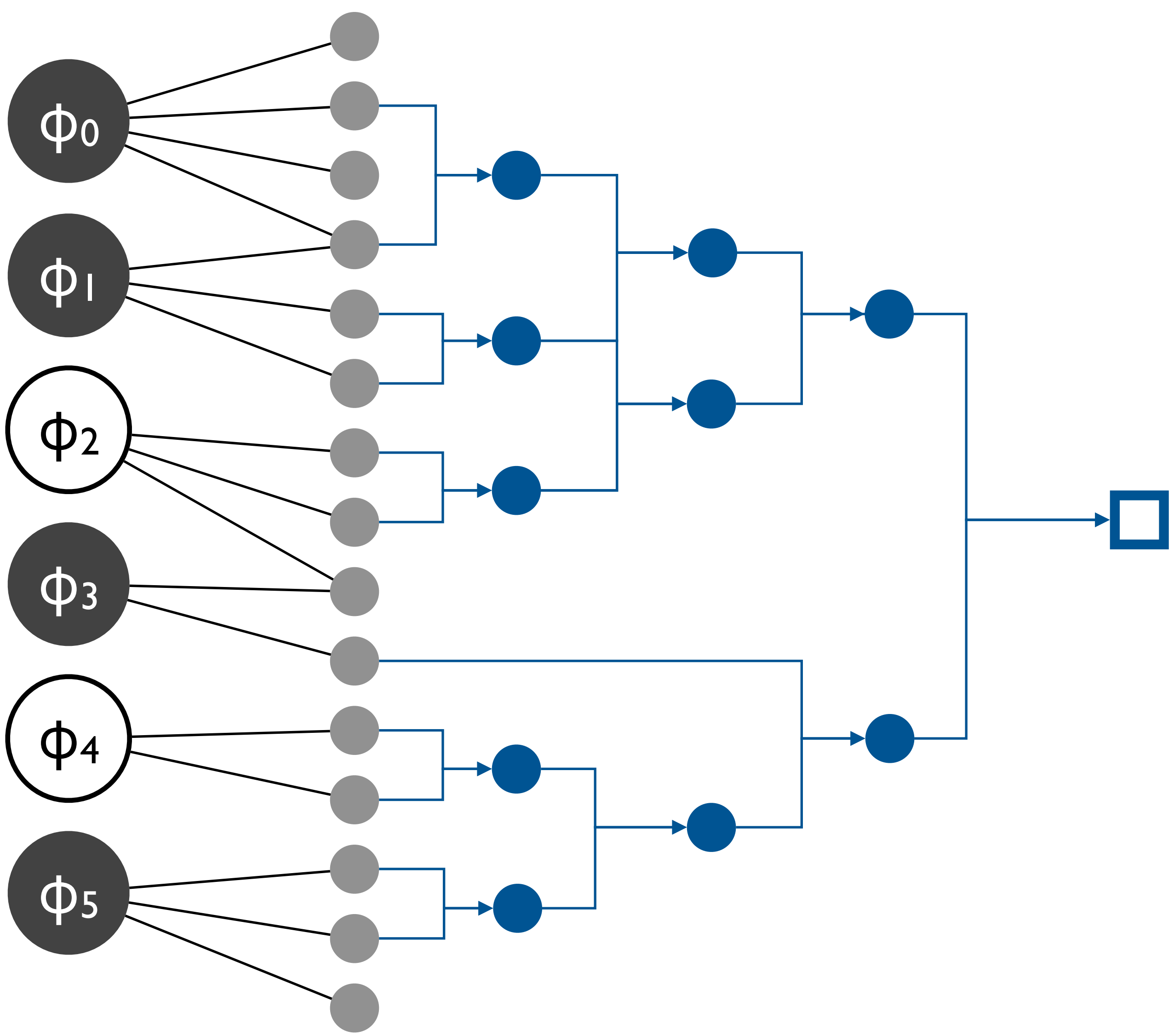
∀ d: Dir | ¬ (d ⊆ d.^contents)

∀ f: File | one contents.f

∀ d: Dir | one contents.d



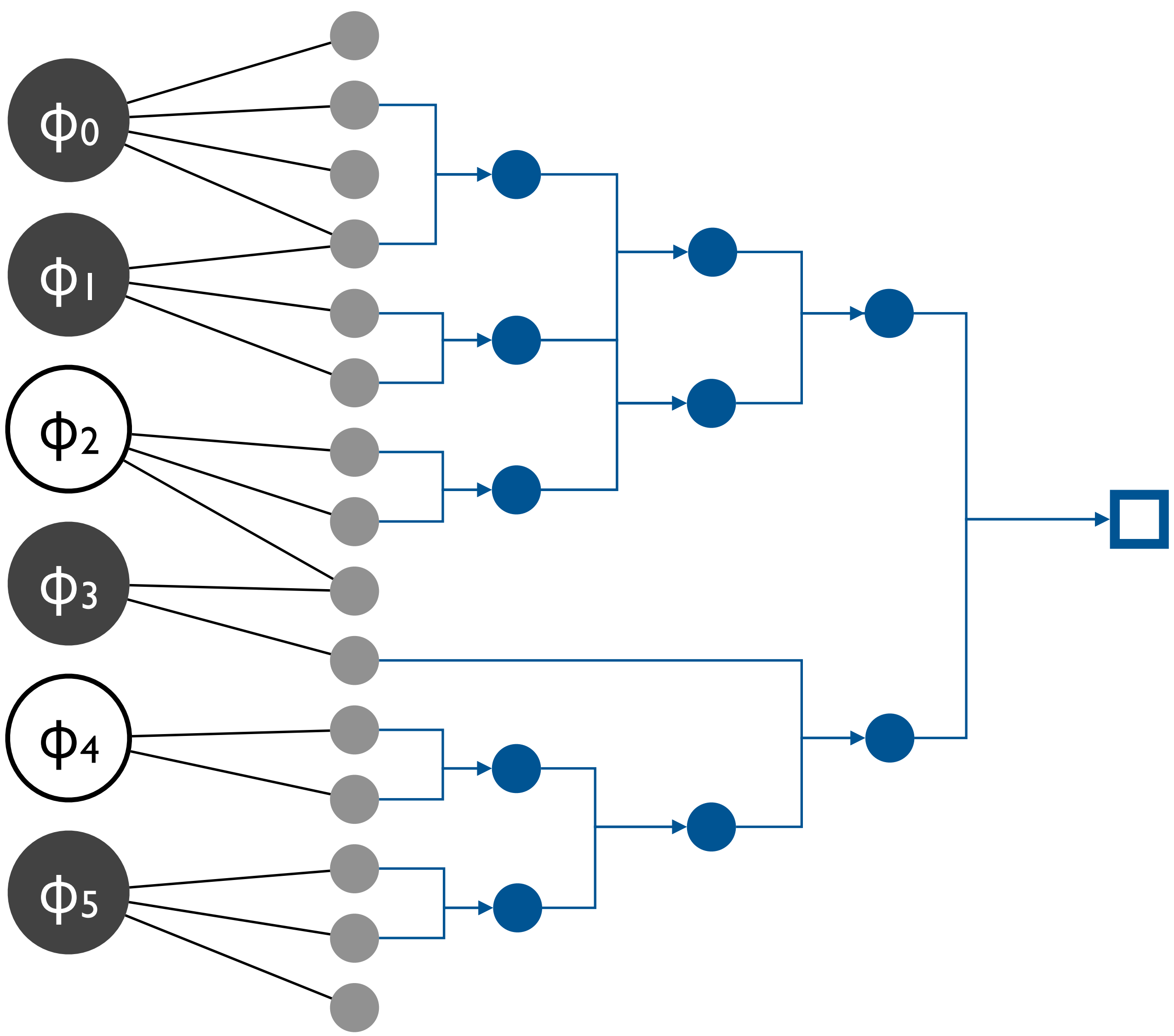
High-level minimal cores from low-level proofs



How to use the proof at the SAT level to find a minimal core at the specification level when

- SAT proof is not minimal
- minimal SAT core may map to a large specification core?

Recycling core extraction



Key idea: minimize core by removing constraints at the specification level but re-use valid resolvents from the previous step so that the solver doesn't have to re-derive them.

Summary

Today

- Finite model finding for first-order logic with quantifiers, relations, and transitive closure

Next lecture

- Reasoning about program correctness