

Computer-Aided Reasoning for Software

Reasoning about Programs I

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Overview

Last lecture

- Finite model finding for first-order logic with quantifiers, relations, and transitive closure

This week

- Reasoning about (partial) correctness of programs
 - Hoare Logic (today)
 - Verification with Dafny (next lecture)

Reminders

- HW2 is due tonight.
- HW3 will be released today; start early!

A look ahead (L11–L14)

Classic verification (L11, L12, L13)

- Checking that all (terminating) executions satisfy an FOL property on all inputs

Symbolic execution (L14)

- Systematic checking of FOL properties of all executions of bounded length

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Active research
topic for 45 years

Classic ideas every
computer scientist
should know

Understanding the
ideas can help you
become a better
programmer

A bit of history

1967: *Assigning Meaning to Programs (Floyd)*

- 1978 Turing Award

1969: *An Axiomatic Basis for Computer Programming (Hoare)*

- 1980 Turing Award

1975: *Guarded Commands, Nondeterminacy and Formal Derivation of Programs (Dijkstra)*

- 1972 Turing Award



A tiny Imperative Programming Language (IMP)

Expression E

- $Z \mid V \mid E_1 + E_2 \mid E_1 * E_2$

Conditional C

- $\text{true} \mid \text{false} \mid E_1 = E_2 \mid E_1 \leq E_2$

Statement S

- **skip** (Skip)
- **abort** (Abort)
- $V := E$ (Assignment)
- $S_1; S_2$ (Composition)
- **if C then S₁ else S₂** (If)
- **while C do S** (While)

A minimalist programming language for demonstrating key features of Hoare logic.

Specifying correctness in Hoare logic

{P} S {Q}

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Specifying correctness in Hoare logic

Hoare triple

- S is a program statement (in IMP).
- P and Q are FOL formulas over program variables.
- P is called a ***precondition*** and Q is a ***postcondition***.

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Partial correctness (Hoare triple semantics)

- If S executes from a state satisfying P, and if its execution terminates, then the resulting state satisfies Q.

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Total correctness

- If S executes from a state satisfying P, then its execution terminates and the resulting state satisfies Q.

$$[P] S [Q]$$

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- If S terminates, the resulting state satisfies Q.

Examples of Hoare triples

{false} S {Q}

- Valid for all S and Q.

{P} while (true) do skip {Q}

- Valid for all P and Q.

{true} S {Q}

- If S terminates, the resulting state satisfies Q.

{P} S {true}

Examples of Hoare triples

{false} S {Q}

- Valid for all S and Q.

{P} while (true) do skip {Q}

- Valid for all P and Q.

{true} S {Q}

- If S terminates, the resulting state satisfies Q.

{P} S {true}

- Valid for all P and S.

Proving partial correctness in Hoare logic

Expression E

- $Z \mid V \mid E_1 + E_2 \mid E_1 * E_2$

Conditional C

- $\text{true} \mid \text{false} \mid E_1 = E_2 \mid E_1 \leq E_2$

Statement S

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One inference rule for every statement in the language:

$$\frac{\vdash\{P_1\}S_1\{Q_1\} \dots \vdash\{P_n\}S_n\{Q_n\}}{\vdash\{P\}S\{Q\}}$$

If the Hoare triples $\{P_1\}S_1\{Q_1\} \dots \{P_n\}S_n\{Q_n\}$ are provable, then so is $\{P\}S\{Q\}$.

Hoare logic rules for partial correctness

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$$\frac{\vdash \{P_1\} S \{Q_1\} \quad P \Rightarrow P_1 \quad Q_1 \Rightarrow Q}{\vdash \{P\} S \{Q\}}$$

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$$\frac{\vdash \{P\} S_1 \{R\} \quad \vdash \{R\} S_2 \{Q\}}{\vdash \{P\} S_1; S_2 \{Q\}}$$

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$$\frac{\vdash \{P\} S_1 \{R\} \quad \vdash \{R\} S_2 \{Q\}}{\vdash \{P\} S_1; S_2 \{Q\}}$$

$$\frac{\vdash \{P \wedge C\} S_1 \{Q\} \quad \vdash \{P \wedge \neg C\} S_2 \{Q\}}{\vdash \{P\} \text{ if } C \text{ then } S_1 \text{ else } S_2 \{Q\}}$$

Hoare logic rules for partial correctness

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$$\frac{\vdash \{P\} S_1 \{R\} \quad \vdash \{R\} S_2 \{Q\}}{\vdash \{P\} S_1; S_2 \{Q\}}$$

$$\frac{\vdash \{P \wedge C\} S_1 \{Q\} \quad \vdash \{P \wedge \neg C\} S_2 \{Q\}}{\vdash \{P\} \text{ if } C \text{ then } S_1 \text{ else } S_2 \{Q\}}$$

$$\frac{\vdash \{P \wedge C\} S \{P\}}{\vdash \{P\} \text{ while } C \text{ do } S \{P \wedge \neg C\}}$$

loop invariant

Example: proof outline

$\{x \leq n\}$

while ($x < n$) **do**

$\{x \leq n \wedge x < n\}$

$\{x+1 \leq n\}$

// consequence

$x := x + 1$

$\{x \leq n\}$

// assignment

$\{x \leq n \wedge x \geq n\}$

// while

$\{x = n\}$

// consequence

Example: proof outline with auxiliary variables

$\{x = A \wedge y = B\}$

$\{y = B \wedge x = A\}$

$t := x$

$\{y = B \wedge t = A\}$

$x := y$

$\{x = B \wedge t = A\}$

$y := t$

$\{x = B \wedge y = A\}$

Soundness and relative completeness

Proof rules for Hoare logic are sound

If $\vdash \{P\} S \{Q\}$ then $\models \{P\} S \{Q\}$

Proof rules for Hoare logic are relatively complete

If $\models \{P\} S \{Q\}$ then $\vdash \{P\} S \{Q\}$, assuming an oracle for deciding implications

Summary

Today

- Reasoning about partial correctness of programs
 - Hoare Logic

Next lecture

- Guest lecture by Rustan Leino!
- Verification with Dafny, Boogie, and Z3.

