Computer-Aided Reasoning for Software

CSE507

Finite Model Finding

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Today

Last lecture

- The DPPL(T) framework for deciding quantifier-free SMT formulas

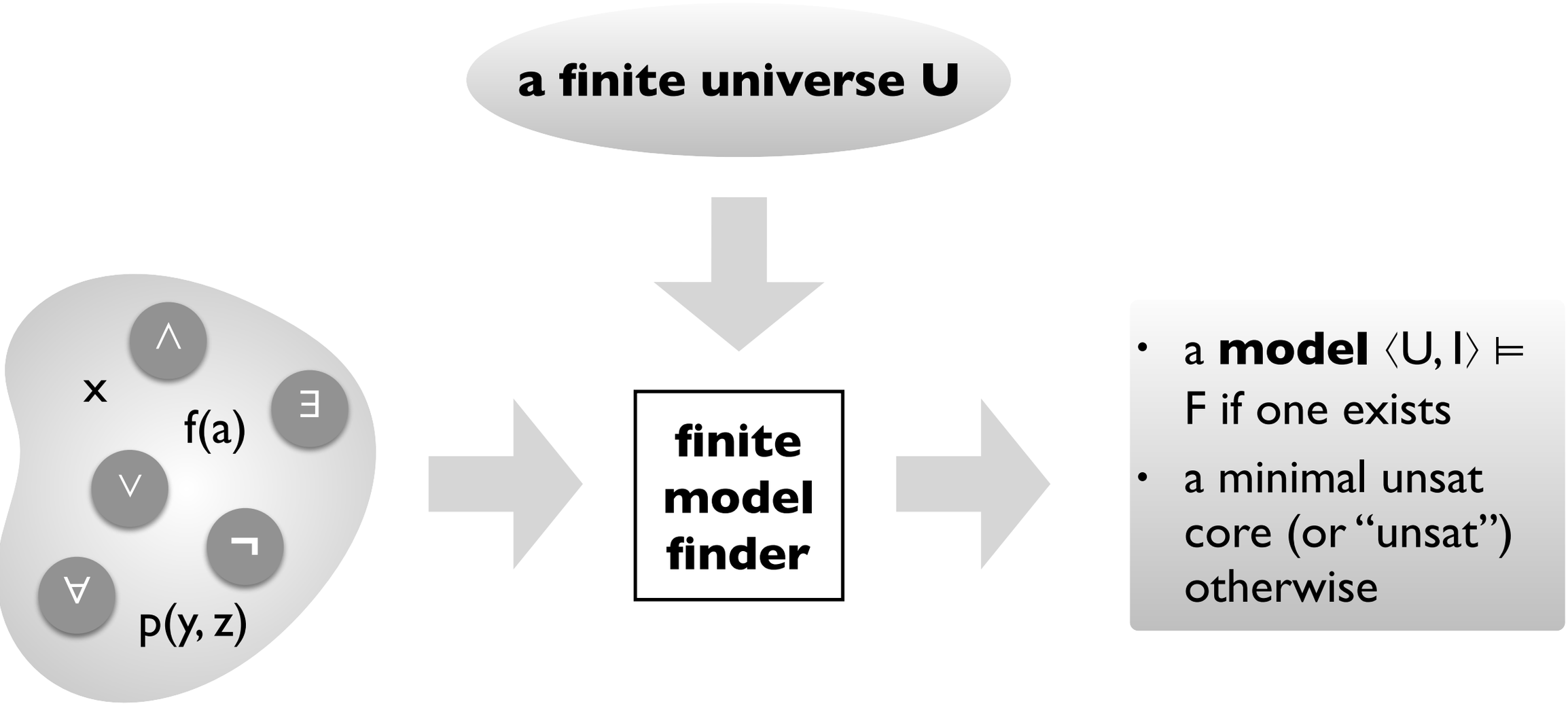
Today

- Finite model finding for quantified FOL and beyond

Reminders

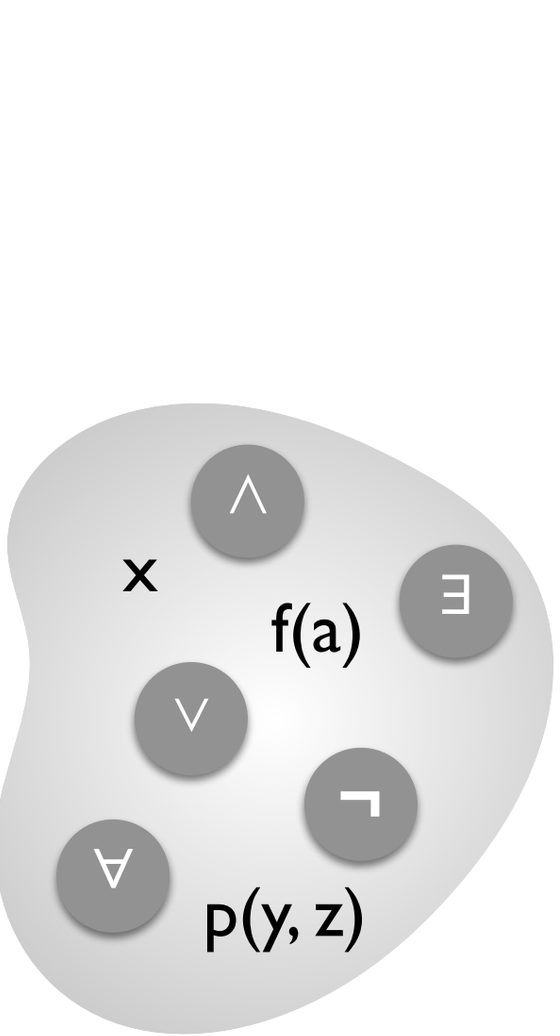
- HW2 is due on Friday!

Finite model finding



a formula F in FOL

Finite model finding



a formula F in FOL

a finite universe U

finite model finder

- custom search (SEM, CVC4)
- reduction to SAT (Paradox, Kodkod)

- a **model** ⟨U, I⟩ ⊨ F if one exists
- a minimal unsat core (or "unsat") otherwise

Some applications of finite model finding

Proving theorems in finite algebras (Finder, SEM, MACE)

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Nitpick

alloy

ProB

TACO

MemSAT

FORGE

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Nitpick alloy

ProB

TACO

MemSAT

FORGE

SQUANDER

Some applications of finite model finding

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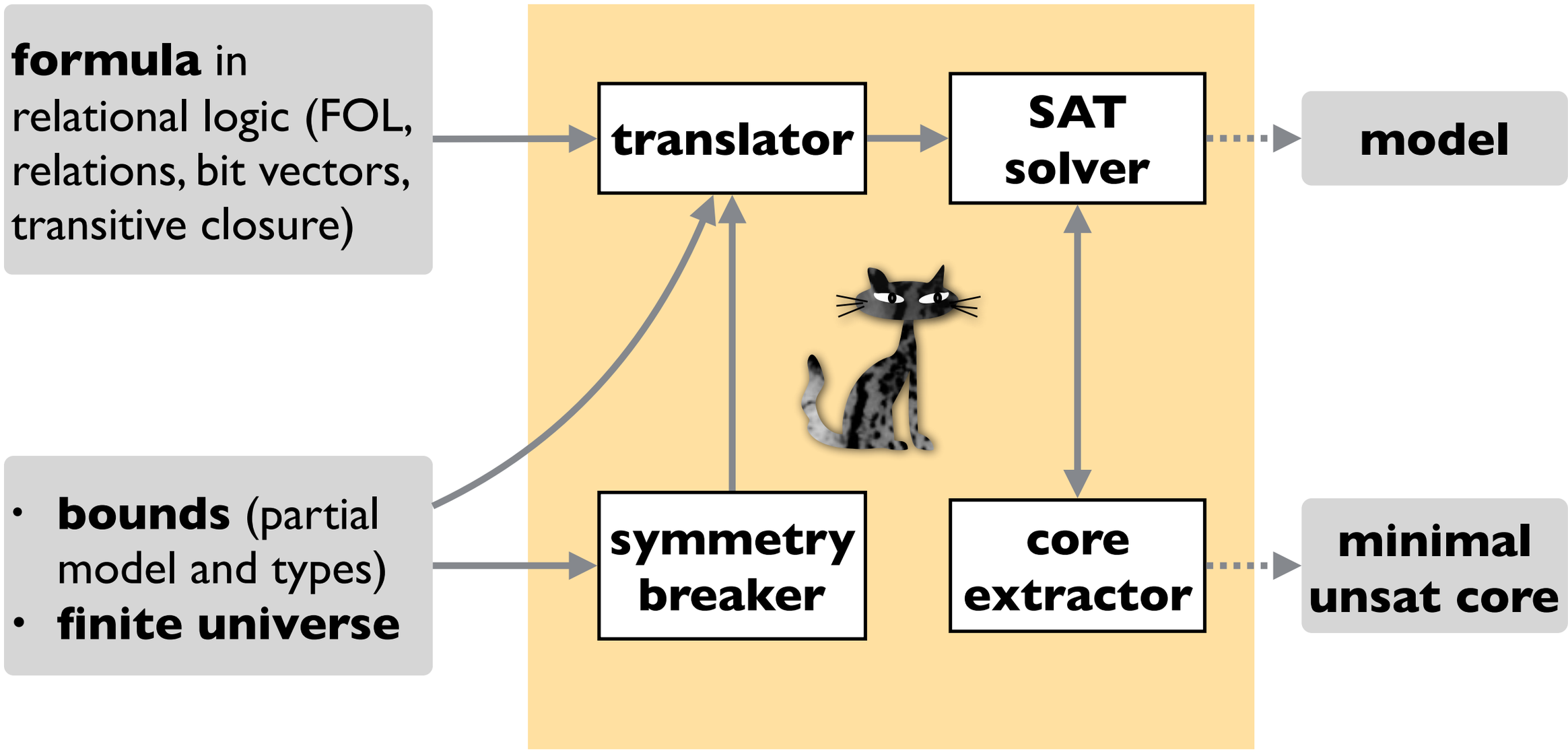
KODKOD

Overview of Kodkod



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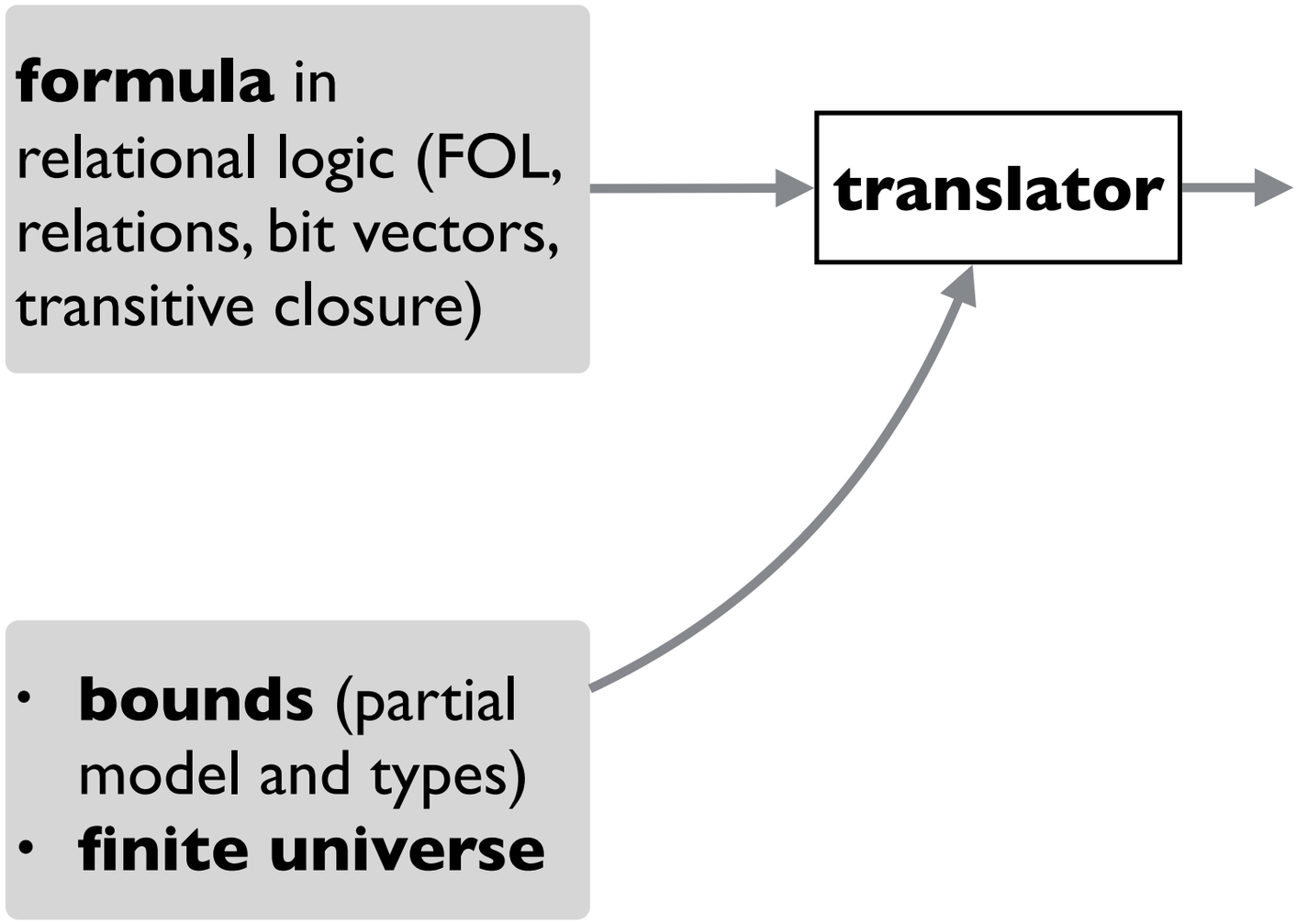

Relational logic by example

a minimalistic formal specification of a filesystem

Relational logic by example

Root ⊆ Dir

- The root of a filesystem hierarchy is a directory.

Relational logic by example

$\text{Root} \subseteq \text{Dir}$

$\text{contents} \subseteq \text{Dir} \times (\text{File} \cup \text{Dir})$

- The root of a filesystem hierarchy is a directory.
- Directories may contain files or directories.

Relational logic by example

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

- The root of a filesystem hierarchy is a directory.
- Directories may contain files or directories.
- All directories and files are reachable from the root.

Relational logic by example

$\text{Root} \subseteq \text{Dir}$

$\text{contents} \subseteq \text{Dir} \times (\text{File} \cup \text{Dir})$

$(\text{File} \cup \text{Dir}) \subseteq \text{Root}.*\text{contents}$

$\forall\ d: \text{Dir} \mid \neg\ (d \subseteq d.\hat{}\text{contents})$

- The root of a filesystem hierarchy is a directory.
- Directories may contain files or directories.
- All directories and files are reachable from the root.
- The contents relation is acyclic.

Bounded relational logic by example

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

{ **R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$ }

Finite universe of interpretation.

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D**$_1$⟩, ⟨**D**$_2$⟩}

{} ⊆ File ⊆ {⟨**F**$_1$⟩, ⟨**F**$_2$⟩}

{} ⊆ contents ⊆ {**R**, **D**$_1$, **D**$_2$} × {**R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$}

Bounded relational logic by example

Root ⊆ Dir

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Finite universe of interpretation.

Bounds for each relation:

- Tuples it *must* contain (partial model).
- Tuples it *may* contain (type).

Bounded relational logic by example

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

{ **R**, **D1**, **D2**, **F1**, **F2** }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D1**⟩, ⟨**D2**⟩}

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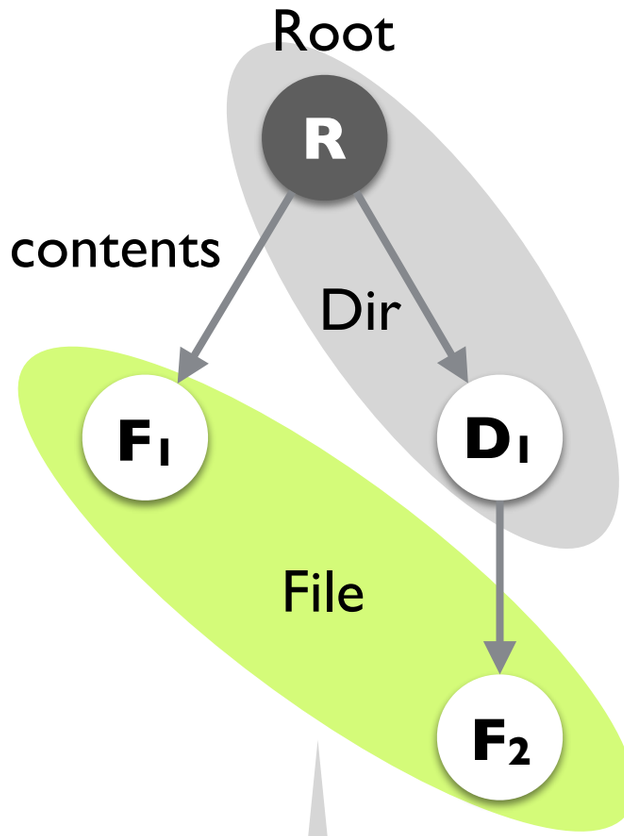

Root = {⟨**R**⟩}

Dir = {⟨**R**⟩, ⟨**D1**⟩}

File = {⟨**F1**⟩, ⟨**F2**⟩}

contents = {⟨**R**, **F1**⟩, ⟨**R**, **D1**⟩, ⟨**D1**, **F2**⟩}

Translation by example

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

{ **R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$ }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D**$_1$⟩, ⟨**D**$_2$⟩}

{} ⊆ File ⊆ {⟨**F**$_1$⟩, ⟨**F**$_2$⟩}

{} ⊆ contents ⊆ {**R**, **D**$_1$, **D**$_2$} × {**R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$}

Translation by example

Root ⊆ Dir

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{ **R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$ }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D**$_1$⟩, ⟨**D**$_2$⟩}

{} ⊆ File ⊆ {⟨**F**$_1$⟩, ⟨**F**$_2$⟩}

{} ⊆ contents ⊆ {**R**, **D**$_1$, **D**$_2$} × {**R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$}

Encode

- relational constants as boolean matrices
- relational expressions as matrix operations
- formulas as constraints over matrix entries

Relational constants as boolean matrices

Relational constants as boolean matrices

R	**D_1**	**D_2**	**F_1**	**F_2**
1	0	0	0	0

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

Relational constants as boolean matrices

R	**D_1**	**D_2**	**F_1**	**F_2**
I	0	0	0	0

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

d_0	d_1	d_2	0	0

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

Relational constants as boolean matrices

R	D_1	D_2	F_1	F_2
1	0	0	0	0
d_0	d_1	d_2	0	0
0	0	0	f_0	f_1

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

$\{\} \subseteq \text{File} \subseteq \{\langle \mathbf{F_1} \rangle, \langle \mathbf{F_2} \rangle\}$

Relational constants as boolean matrices

	R	**D_1**	**D_2**	**F_1**	**F_2**
	1	0	0	0	0
	d_0	d_1	d_2	0	0
	0	0	0	f_0	f_1
R	c_0	c_1	c_2	c_3	c_4
D_1	c_5	c_6	c_7	c_8	c_9
D_2	c_{10}	c_{11}	c_{12}	c_{13}	c_{14}
F_1	0	0	0	0	0
F_2	0	0	0	0	0

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

$\{\} \subseteq \text{File} \subseteq \{\langle \mathbf{F_1} \rangle, \langle \mathbf{F_2} \rangle\}$

$\{\} \subseteq \text{contents} \subseteq \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}\} \times \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\}$

Relational expressions as matrix operations

File

0	0	0	f_0	f_1

$\vee$

Dir

d_0	d_1	d_2	0	0

=

File ∪ Dir

d_0	d_1	d_2	f_0	f_1

Dir

d_0
d_1
d_2
0
0

×

File ∪ Dir

d_0	d_1	d_2	f_0	f_1

=

Dir × (File ∪ Dir)

$d_0 \wedge d_0$	$d_0 \wedge d_1$	$d_0 \wedge d_2$	$d_0 \wedge f_0$	$d_0 \wedge f_1$
$d_1 \wedge d_0$	$d_1 \wedge d_1$	$d_1 \wedge d_2$	$d_1 \wedge f_0$	$d_1 \wedge f_1$
$d_2 \wedge d_0$	$d_2 \wedge d_1$	$d_2 \wedge d_2$	$d_2 \wedge f_0$	$d_2 \wedge f_1$
0	0	0	0	0
0	0	0	0	0

Formulas as constraints over matrix entries

contents

c_0	c_1	c_2	c_3	c_4
c_5	c_6	c_7	c_8	c_9
c_{10}	c_{11}	c_{12}	c_{13}	c_{14}
0	0	0	0	0
0	0	0	0	0

$\rightarrow$

Dir × (File ∪ Dir)

$d_0 \wedge d_0$	$d_0 \wedge d_1$	$d_0 \wedge d_2$	$d_0 \wedge f_0$	$d_0 \wedge f_1$
$d_1 \wedge d_0$	$d_1 \wedge d_1$	$d_1 \wedge d_2$	$d_1 \wedge f_0$	$d_1 \wedge f_1$
$d_2 \wedge d_0$	$d_2 \wedge d_1$	$d_2 \wedge d_2$	$d_2 \wedge f_0$	$d_2 \wedge f_1$
0	0	0	0	0
0	0	0	0	0

=

contents ⊆ Dir × (File ∪ Dir)

$(c_0 \rightarrow d_0 \wedge d_0) \wedge$
$(c_1 \rightarrow d_0 \wedge d_1) \wedge$
$(c_2 \rightarrow d_0 \wedge d_2) \wedge$
$(c_3 \rightarrow d_0 \wedge f_0) \wedge$
$(c_4 \rightarrow d_0 \wedge f_1) \wedge$
$(c_5 \rightarrow d_1 \wedge d_0) \wedge$
…
$(c_{14} \rightarrow d_2 \wedge f_1)$

Dealing with sparseness and redundancy

Dir × (File ∪ Dir)

$d_0 \wedge d_0$	$d_0 \wedge d_1$	$d_0 \wedge d_2$	$d_0 \wedge f_0$	$d_0 \wedge f_1$
$d_1 \wedge d_0$	$d_1 \wedge d_1$	$d_1 \wedge d_2$	$d_1 \wedge f_0$	$d_1 \wedge f_1$
$d_2 \wedge d_0$	$d_2 \wedge d_1$	$d_2 \wedge d_2$	$d_2 \wedge f_0$	$d_2 \wedge f_1$
0	0	0	0	0
0	0	0	0	0

Dealing with sparseness and redundancy

Dir × (File ∪ Dir)

$d_0 \wedge d_0$	$d_0 \wedge d_1$	$d_0 \wedge d_2$	$d_0 \wedge f_0$	$d_0 \wedge f_1$
$d_1 \wedge d_0$	$d_1 \wedge d_1$	$d_1 \wedge d_2$	$d_1 \wedge f_0$	$d_1 \wedge f_1$
$d_2 \wedge d_0$	$d_2 \wedge d_1$	$d_2 \wedge d_2$	$d_2 \wedge f_0$	$d_2 \wedge f_1$
0	0	0	0	0
0	0	0	0	0

Empty regions in matrices (exponential w.r.t. relation arity).

Dealing with sparseness and redundancy

Different circuits for the same formula.

Dir × (File ∪ Dir)

$d_0 \wedge d_0$	$d_0 \wedge d_1$	$d_0 \wedge d_2$	$d_0 \wedge f_0$	$d_0 \wedge f_1$
$d_1 \wedge d_0$	$d_1 \wedge d_1$	$d_1 \wedge d_2$	$d_1 \wedge f_0$	$d_1 \wedge f_1$
$d_2 \wedge d_0$	$d_2 \wedge d_1$	$d_2 \wedge d_2$	$d_2 \wedge f_0$	$d_2 \wedge f_1$
0	0	0	0	0
0	0	0	0	0

Empty regions in matrices (exponential w.r.t. relation arity).

Dealing with sparseness and redundancy

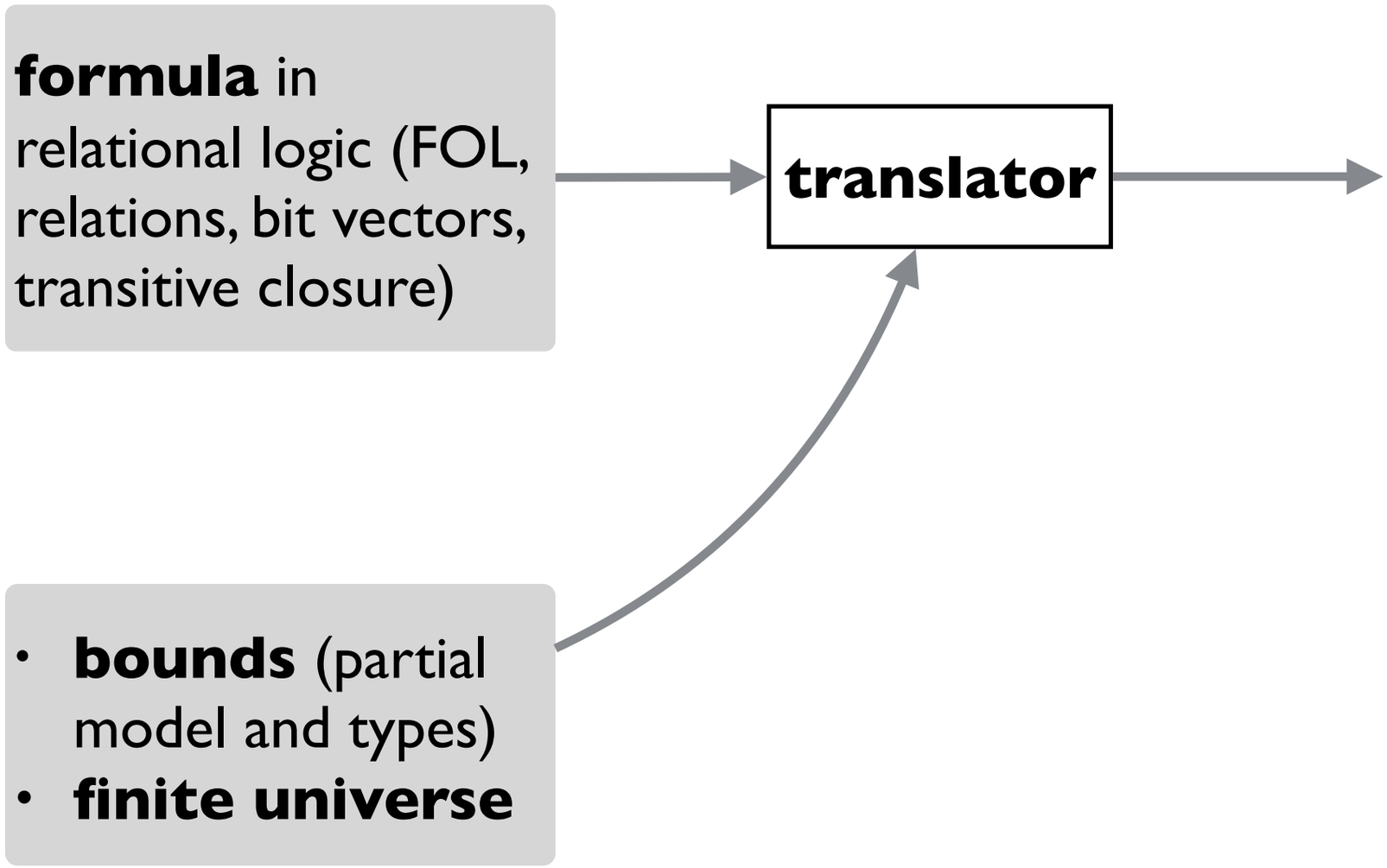
Compact Boolean Circuits (CBCs).

Dir × (File ∪ Dir)

$d_0 \wedge d_0$	$d_0 \wedge d_1$	$d_0 \wedge d_2$	$d_0 \wedge f_0$	$d_0 \wedge f_1$
$d_1 \wedge d_0$	$d_1 \wedge d_1$	$d_1 \wedge d_2$	$d_1 \wedge f_0$	$d_1 \wedge f_1$
$d_2 \wedge d_0$	$d_2 \wedge d_1$	$d_2 \wedge d_2$	$d_2 \wedge f_0$	$d_2 \wedge f_1$
0	0	0	0	0
0	0	0	0	0

Sparse matrices represented as interval trees.

Overview of Kodkod



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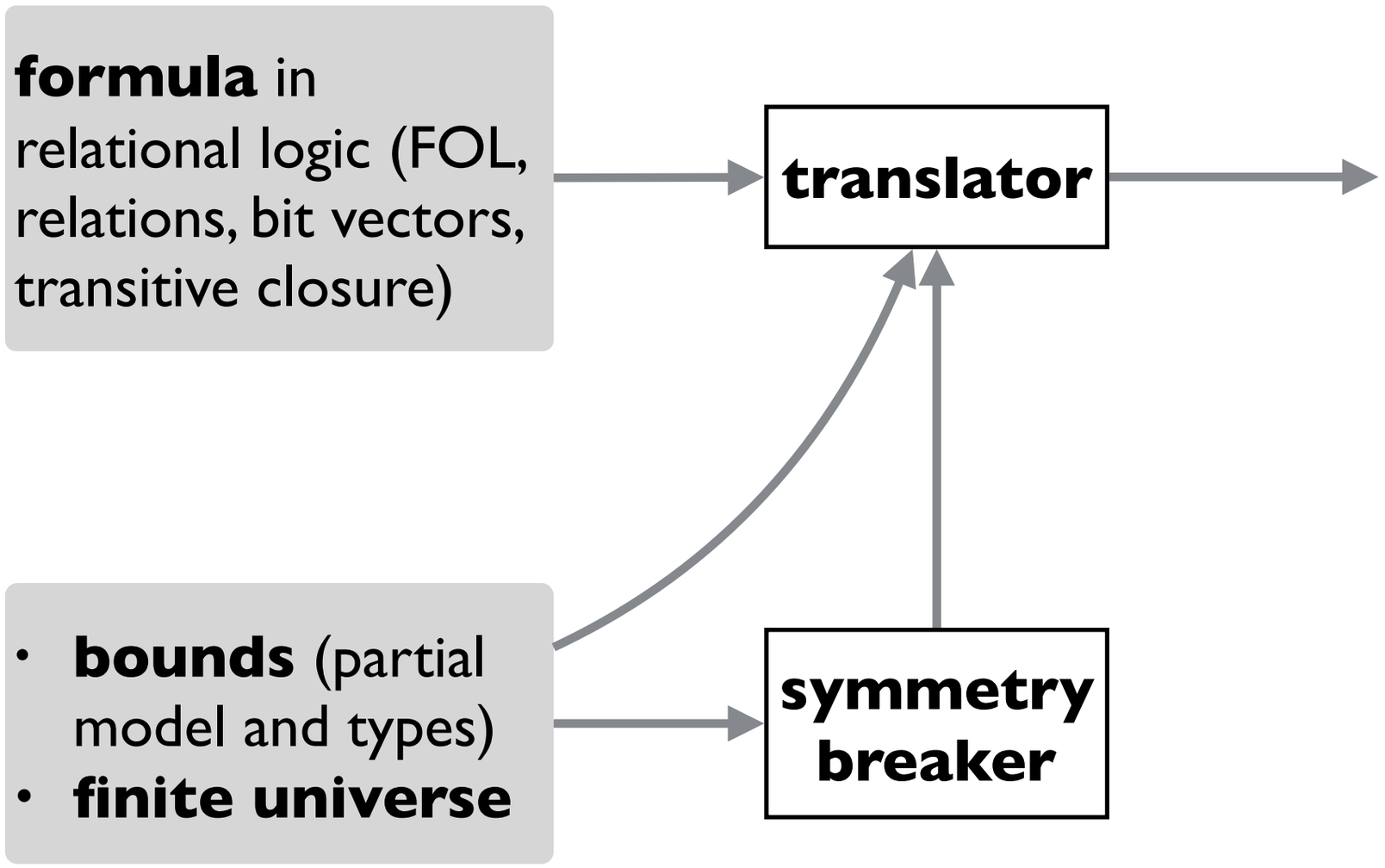


Symmetry by example

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

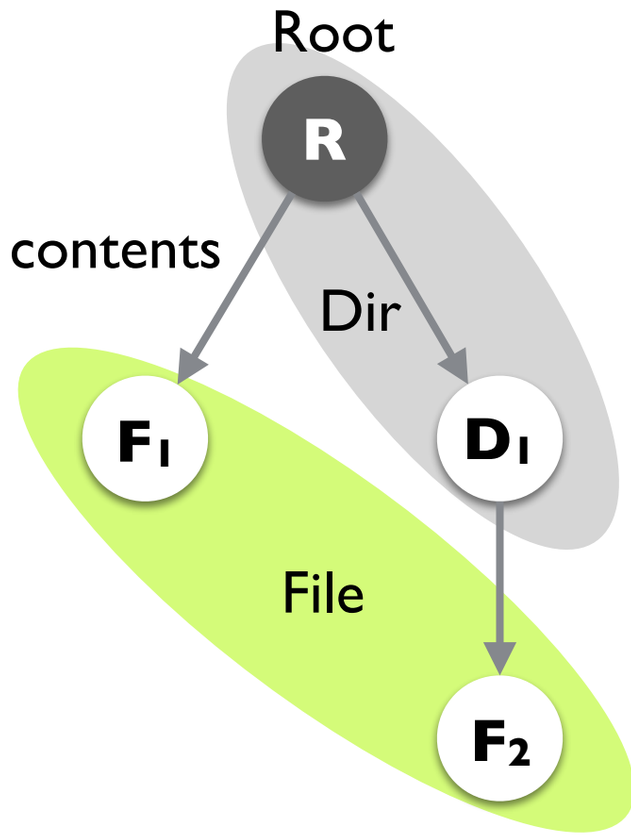
{ **R**, **D1**, **D2**, **F1**, **F2** }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D1**⟩, ⟨**D2**⟩}

{} ⊆ File ⊆ {⟨**F1**⟩, ⟨**F2**⟩}

{} ⊆ contents ⊆ {**R**, **D1**, **D2**} × {**R**, **D1**, **D2**, **F1**, **F2**}



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∀ d: Dir | ¬ (d ⊆ d.^contents)

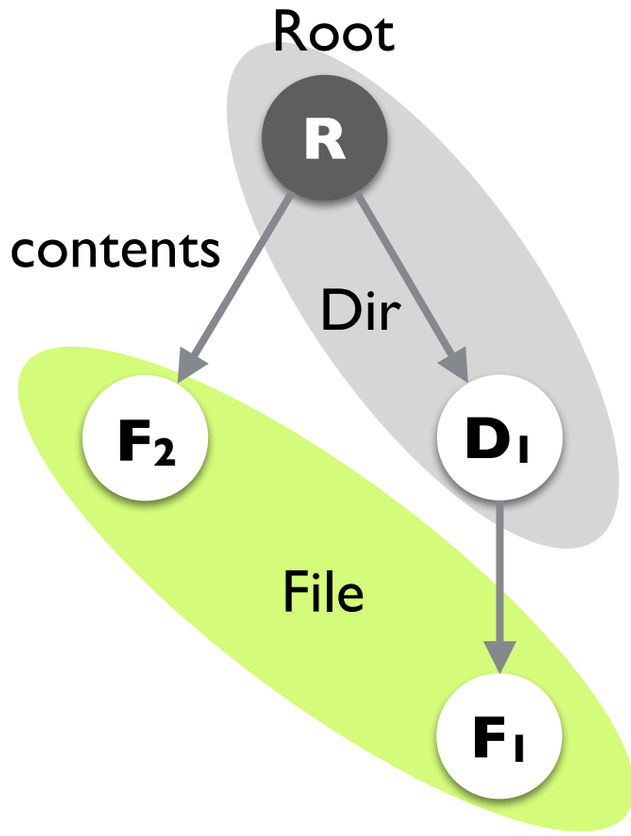
{ **R**, **D1**, **D2**, **F1**, **F2** }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D1**⟩, ⟨**D2**⟩}

{} ⊆ File ⊆ {⟨**F1**⟩, ⟨**F2**⟩}

{} ⊆ contents ⊆ {**R**, **D1**, **D2**} × {**R**, **D1**, **D2**, **F1**, **F2**}



Symmetry by example

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

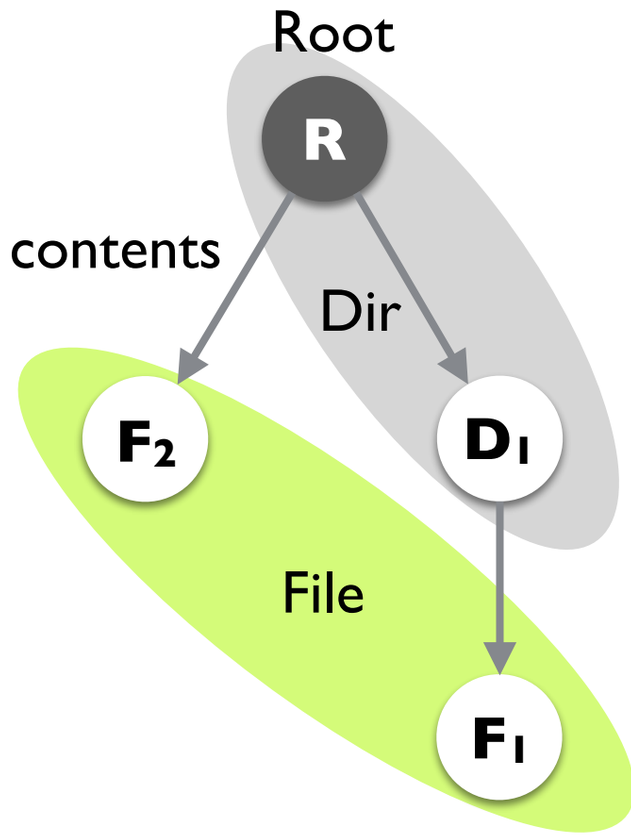
{ **R**, **D**, **D₂**, **F₁**, **F₂** }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D₁**⟩, ⟨**D₂**⟩}

{} ⊆ File ⊆ {⟨**F₁**⟩, ⟨**F₂**⟩}

{} ⊆ contents ⊆ {**R**, **D₁**, **D₂**} × {**R**, **D₁**, **D₂**, **F₁**, **F₂**}



Symmetry by example

Root ⊆ Dir

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{ **R**, **D1**, **D2**, **F1**, **F2** }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D1**⟩, ⟨**D2**⟩}

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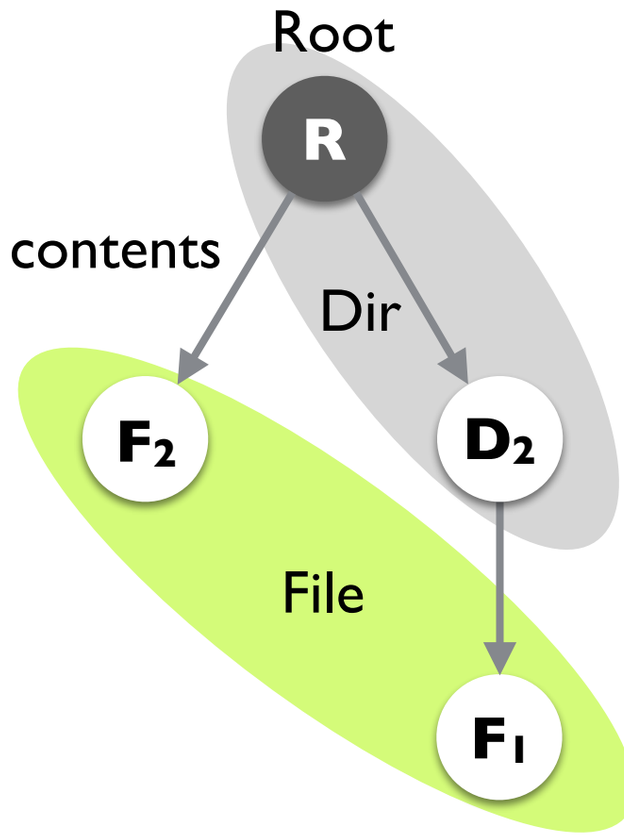

Symmetries between models

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

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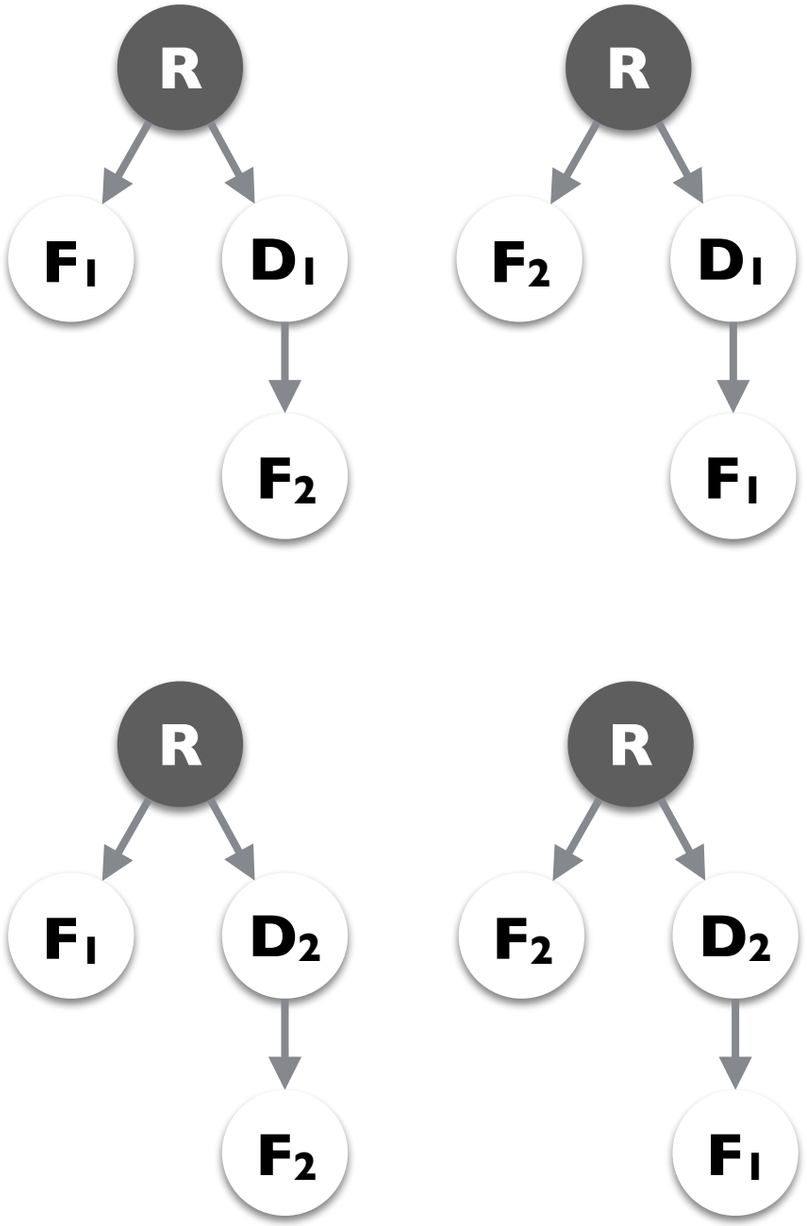
∀ d: Dir | ¬ (d ⊆ d.^contents)

{ **R**, **D1**, **D2**, **F1**, **F2** }



{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D1**⟩, ⟨**D2**⟩}

{} ⊆ File ⊆ {⟨**F1**⟩, ⟨**F2**⟩}

{} ⊆ contents ⊆ {**R**, **D1**, **D2**} × {**R**, **D1**, **D2**, **F1**, **F2**}

Symmetries between non-models

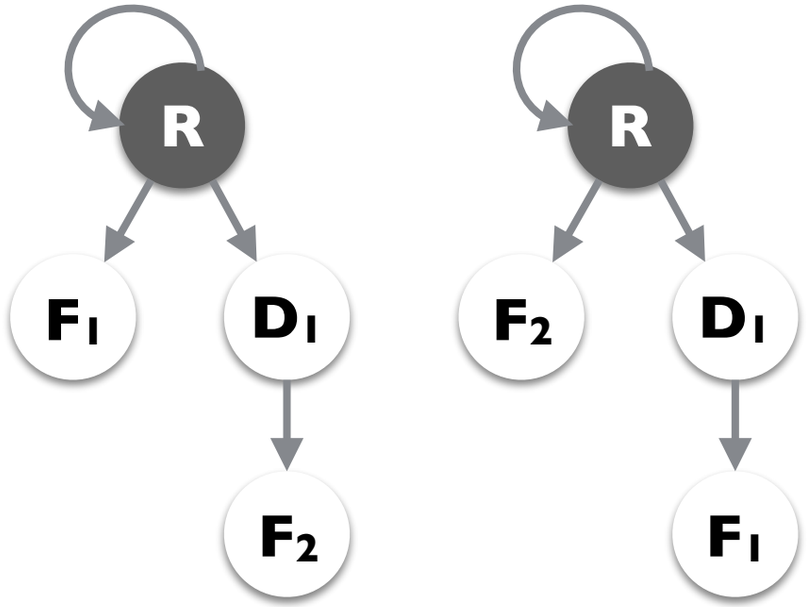
Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

{ **R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$ }

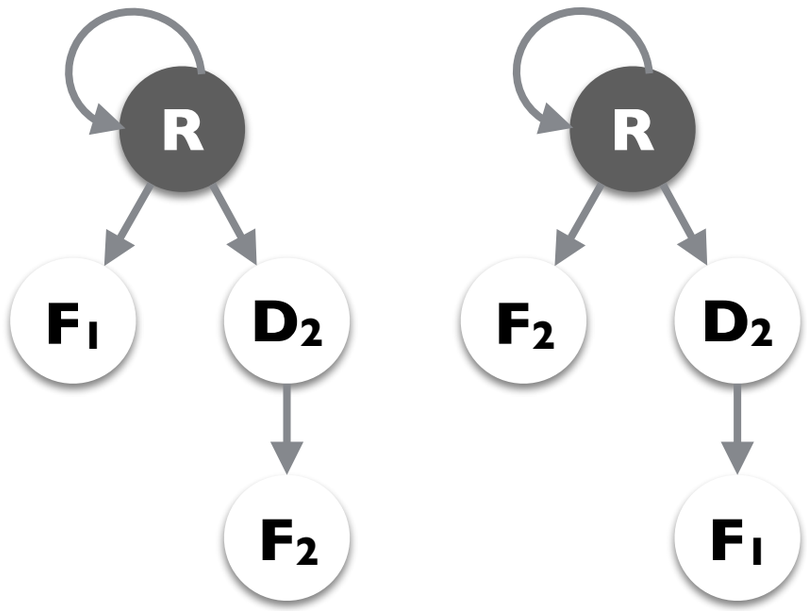


{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D**$_1$⟩, ⟨**D**$_2$⟩}

{} ⊆ File ⊆ {⟨**F**$_1$⟩, ⟨**F**$_2$⟩}

{} ⊆ contents ⊆ {**R**, **D**$_1$, **D**$_2$} × {**R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$}



Symmetries induce equivalence classes

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

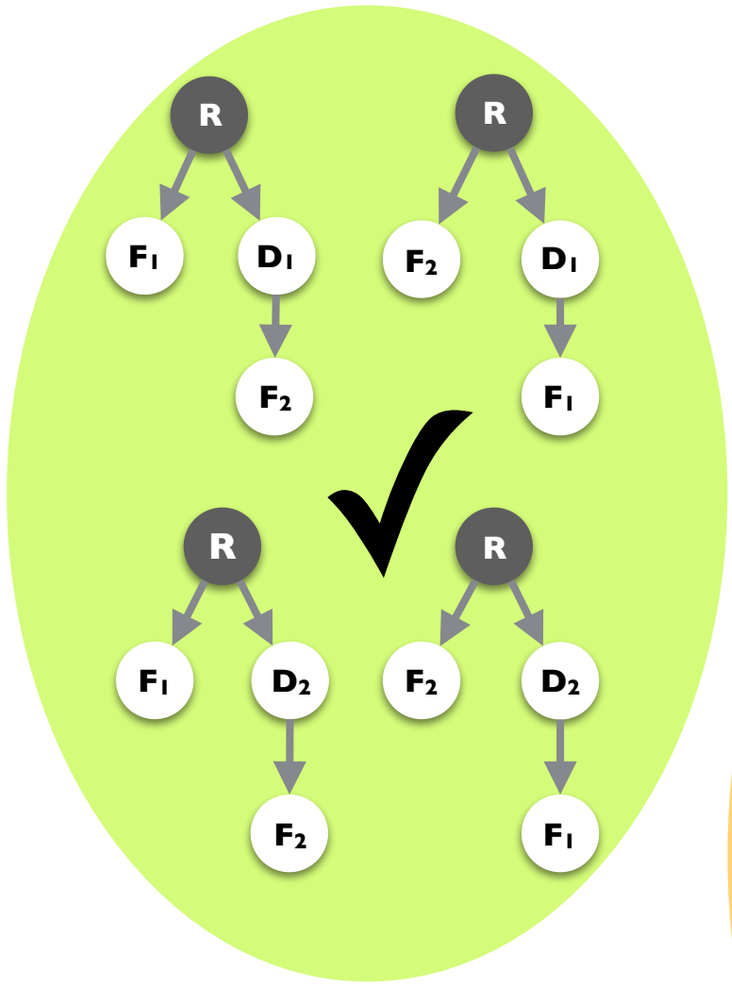
{ **R**, **D1**, **D2**, **F1**, **F2** }

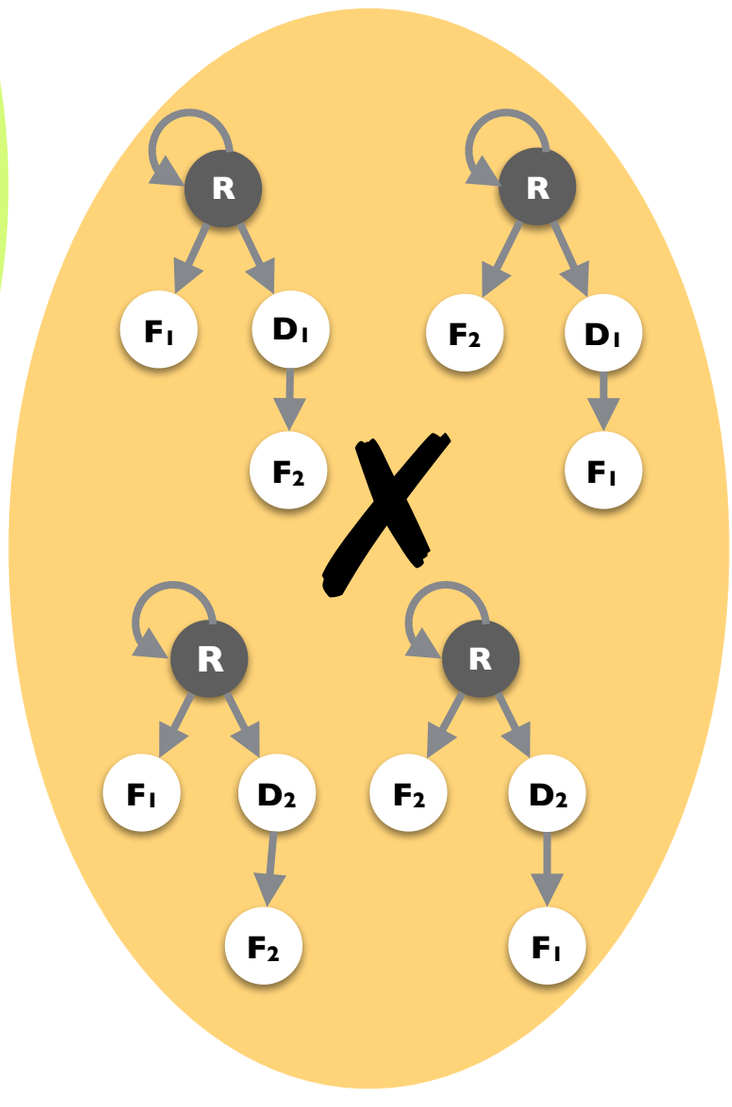
{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D1**⟩, ⟨**D2**⟩}

{} ⊆ File ⊆ {⟨**F1**⟩, ⟨**F2**⟩}

{} ⊆ contents ⊆ {**R**, **D1**, **D2**} × {**R**, **D1**, **D2**, **F1**, **F2**}





Symmetries induce equivalence classes

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

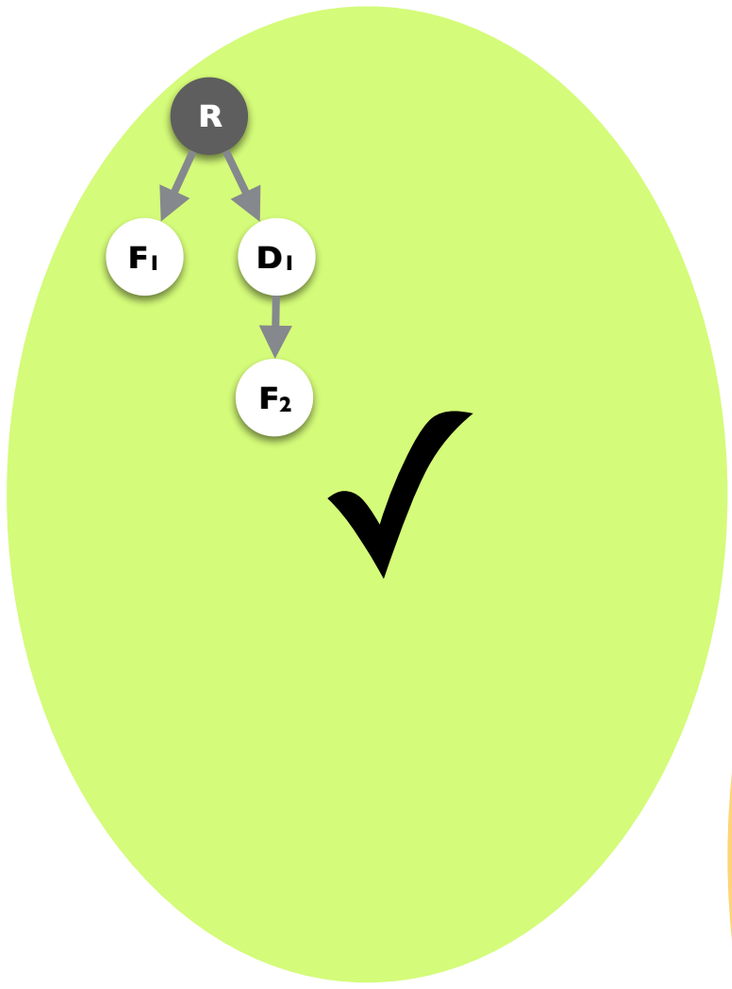
{ **R**, **D₁**, **D₂**, **F₁**, **F₂** }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

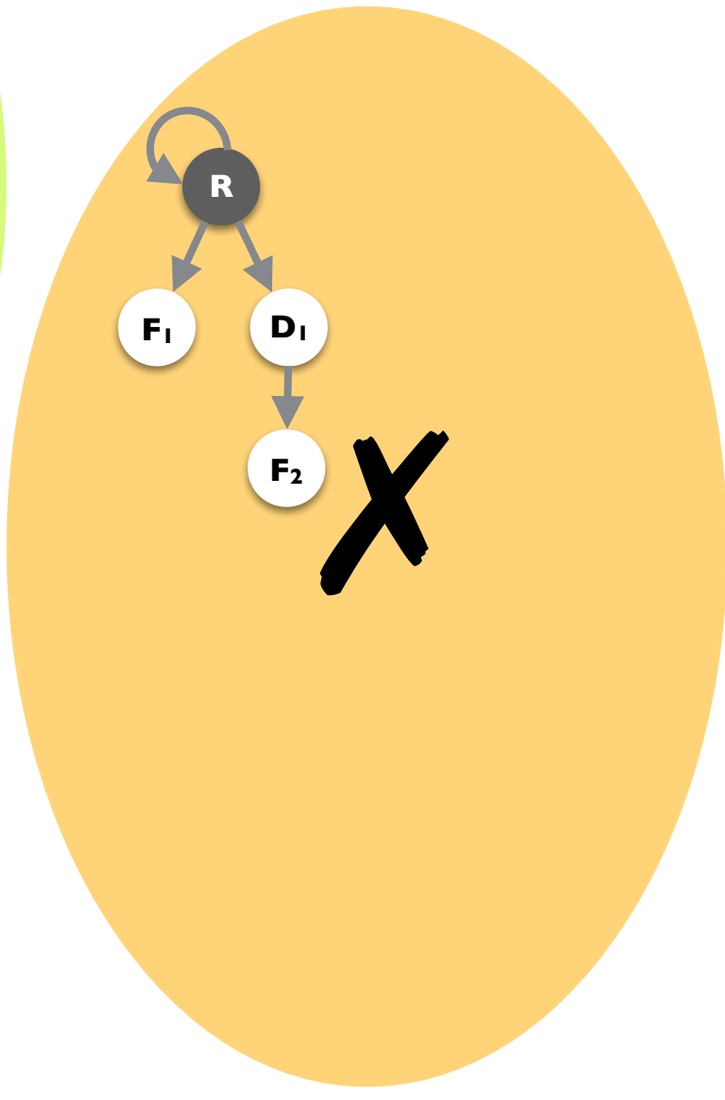
{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D₁**⟩, ⟨**D₂**⟩}

{} ⊆ File ⊆ {⟨**F₁**⟩, ⟨**F₂**⟩}

{} ⊆ contents ⊆ {**R**, **D₁**, **D₂**} × {**R**, **D₁**, **D₂**, **F₁**, **F₂**}



Sufficient to check one interpretation per equivalence class.



Symmetry detection

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

{ **R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$ }

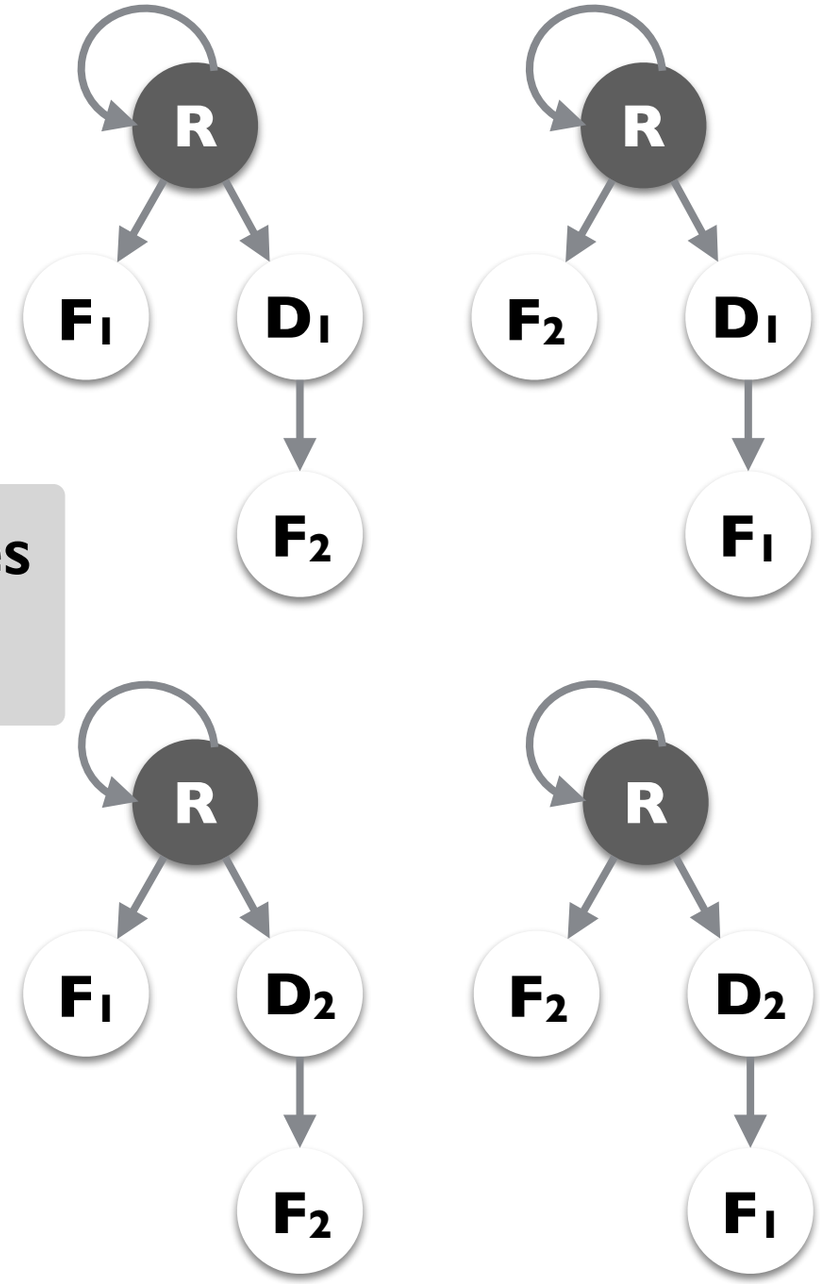
Interpretation symmetries = bound symmetries

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D**$_1$⟩, ⟨**D**$_2$⟩}

{} ⊆ File ⊆ {⟨**F**$_1$⟩, ⟨**F**$_2$⟩}

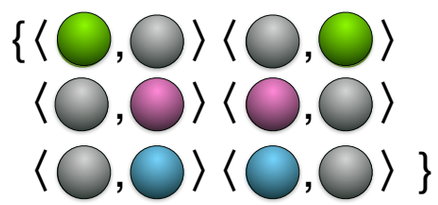
{} ⊆ contents ⊆ {**R**, **D**$_1$, **D**$_2$} × {**R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$}

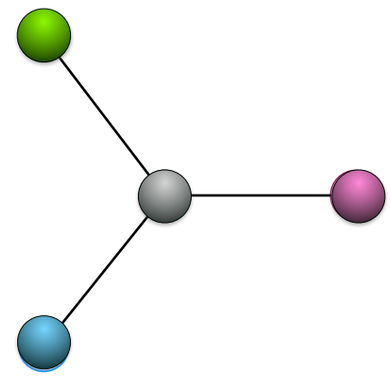


Detecting symmetries is hard ...

Interpretation symmetries
= bound symmetries

Graph automorphism
detection





But only a few symmetries needed in practice

Greedy algorithm that partitions the universe into equivalence classes

Graph automorphism detection

Base partitioning: practical symmetry detection

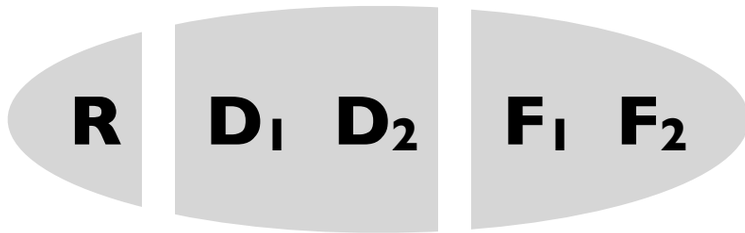
{ **R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$ }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D**$_1$⟩, ⟨**D**$_2$⟩}

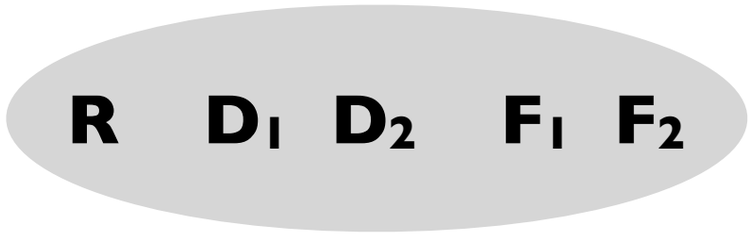
{} ⊆ File ⊆ {⟨**F**$_1$⟩, ⟨**F**$_2$⟩}

{} ⊆ contents ⊆ {**R**, **D**$_1$, **D**$_2$} × {**R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$}



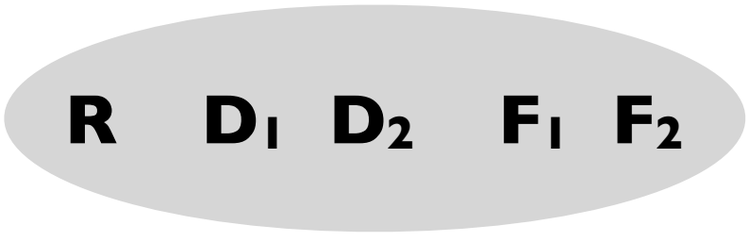
The coarsest partition of the universe such that each non-empty bound is expressible as a union of products of parts.

Finding the base partitioning



start with a single partition
and refine minimally for
each non-empty lower and
upper bound

Finding the base partitioning



$$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$$

start with a single partition
and refine minimally for
each non-empty lower and
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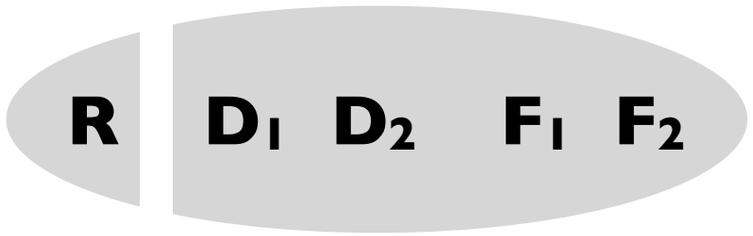
Finding the base partitioning



$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

start with a single partition
and refine minimally for
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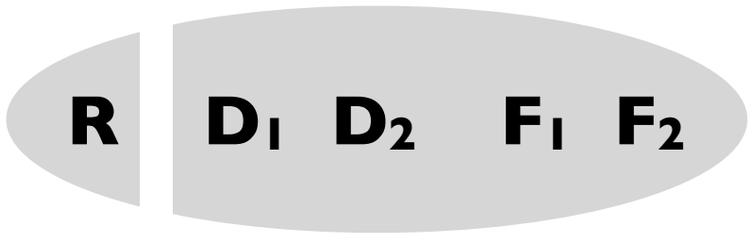
upper bound

Finding the base partitioning



$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

start with a single partition
and refine minimally for
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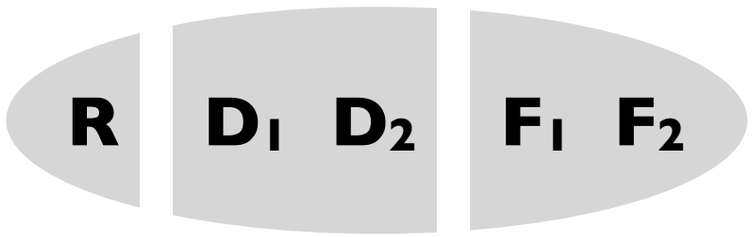
Finding the base partitioning



$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

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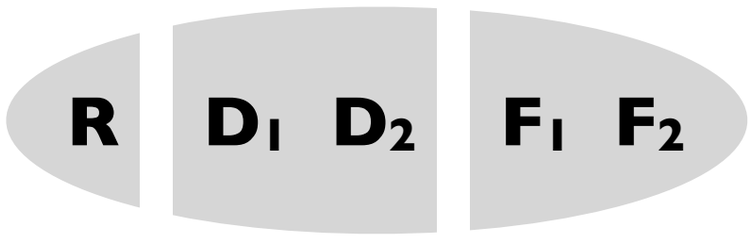

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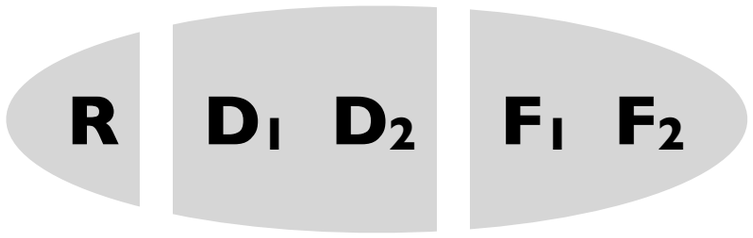
Finding the base partitioning



$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

$\{\} \subseteq \text{File} \subseteq \{\langle \mathbf{F_1} \rangle, \langle \mathbf{F_2} \rangle\}$

$\{\} \subseteq \text{contents} \subseteq \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}\} \times \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\}$

start with a single partition
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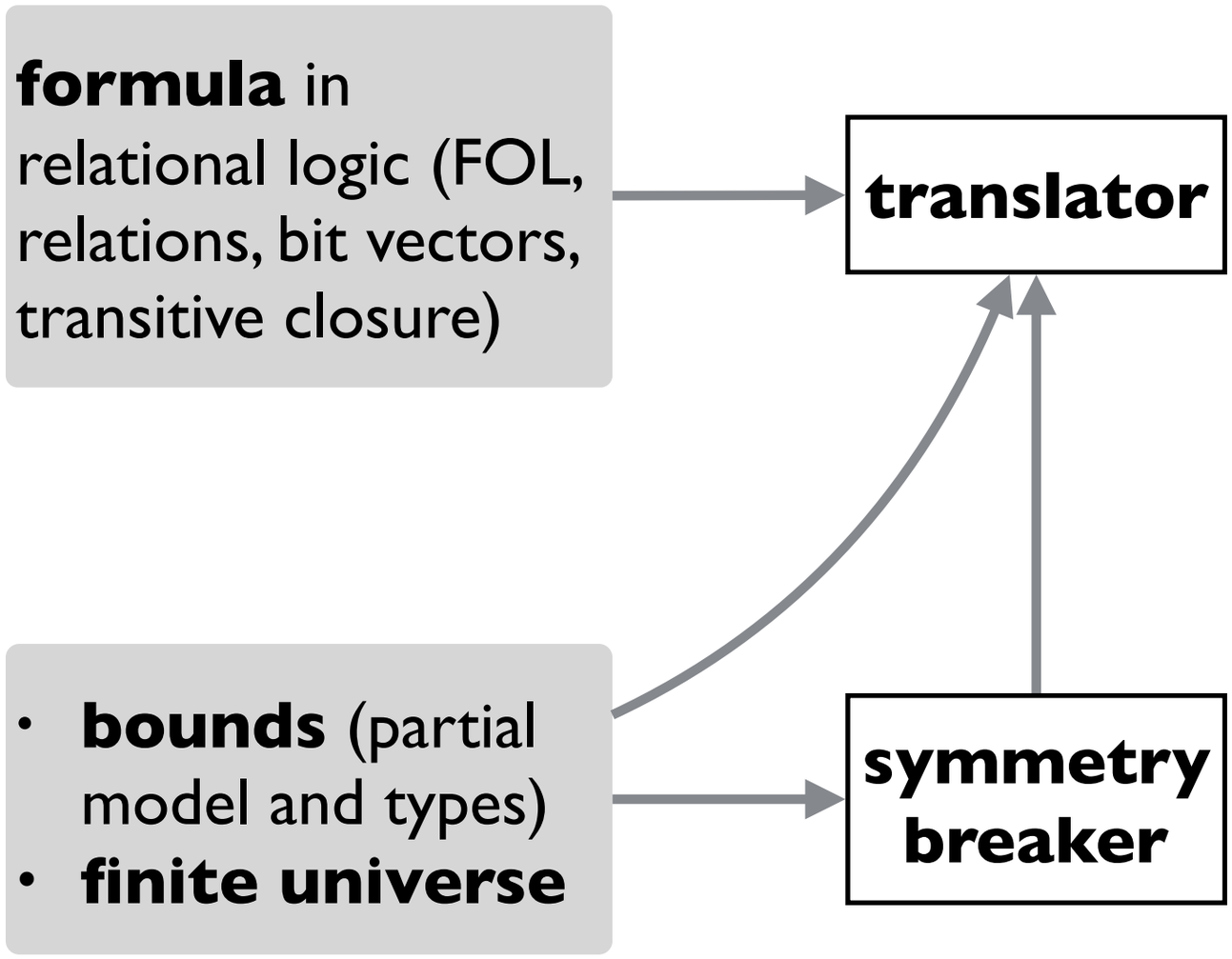
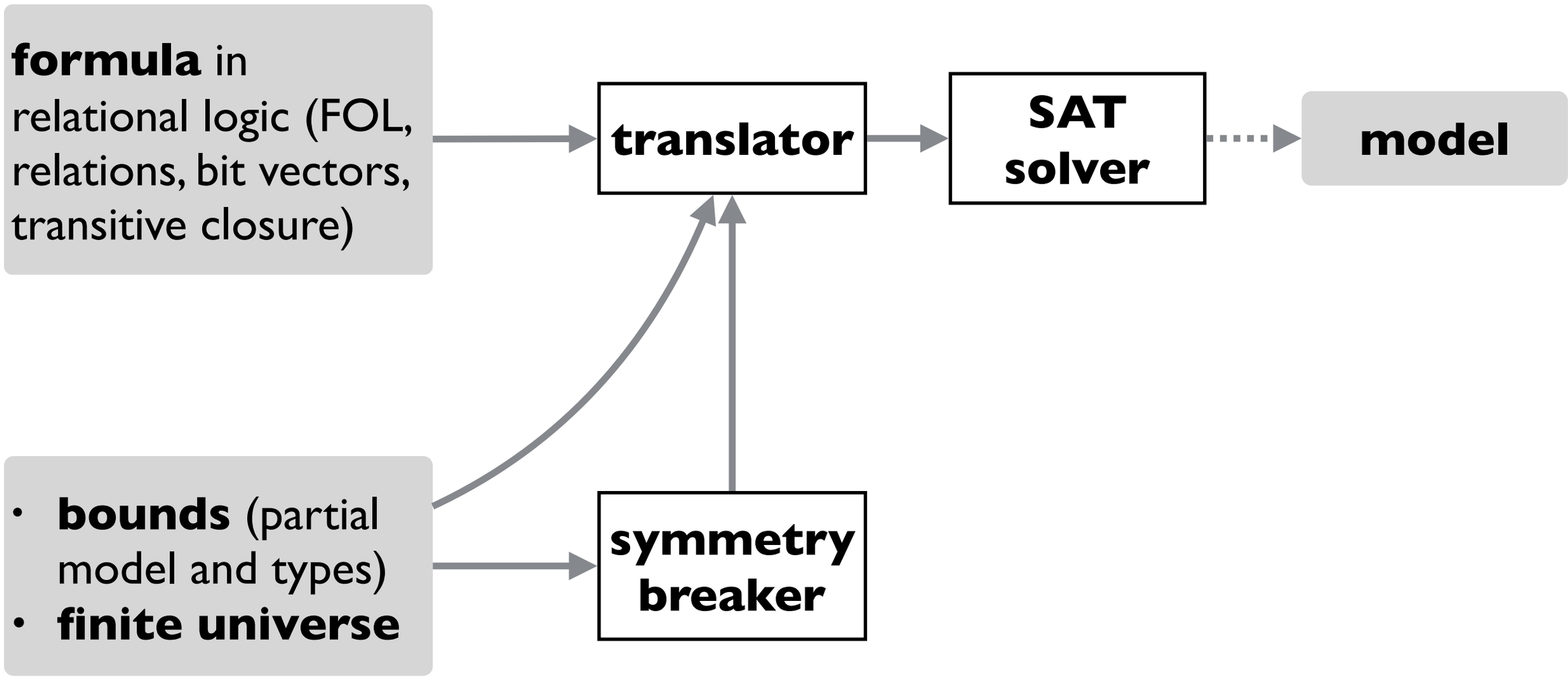
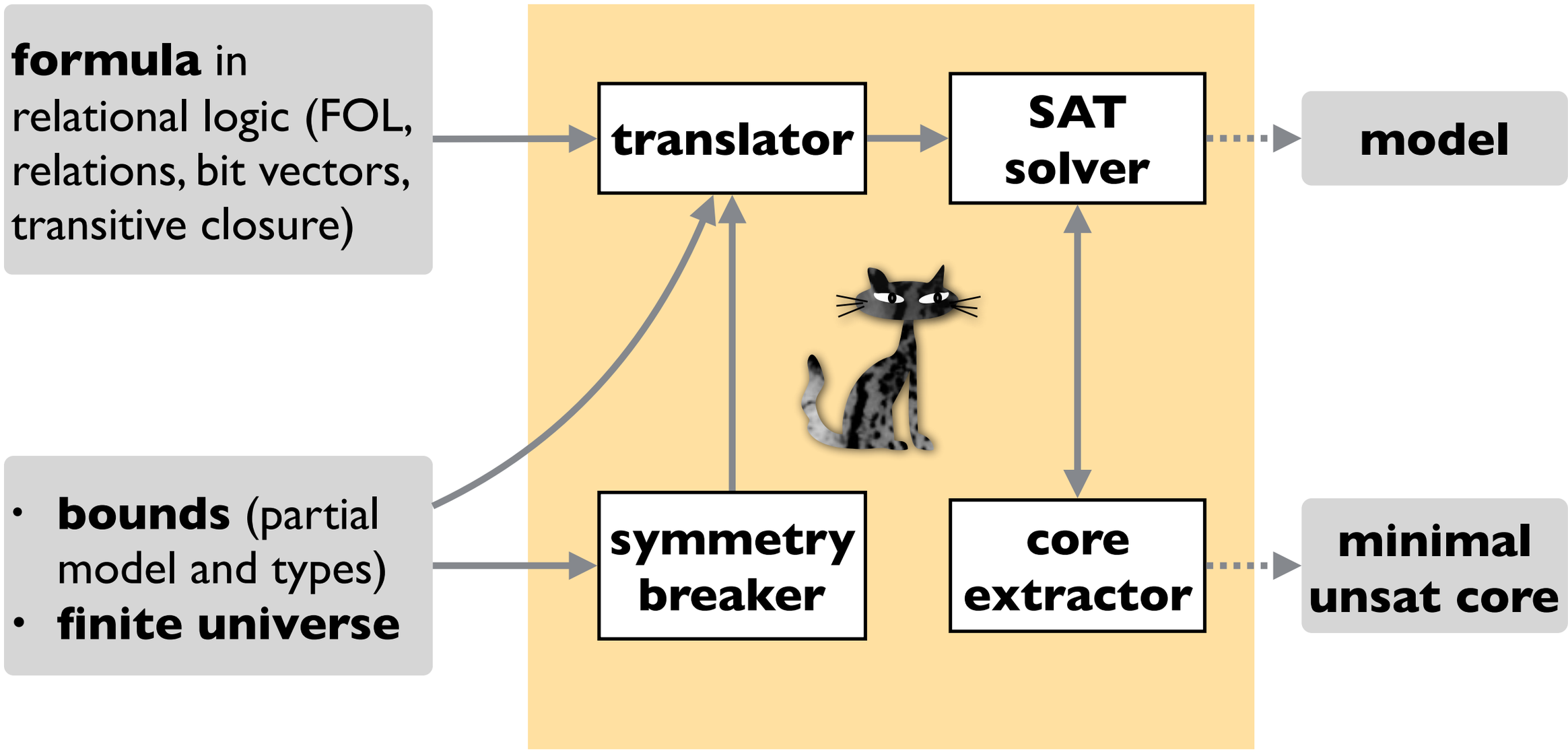
Overview of Kodkod



Overview of Kodkod



Overview of Kodkod



A bug in the tiny filesystem

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

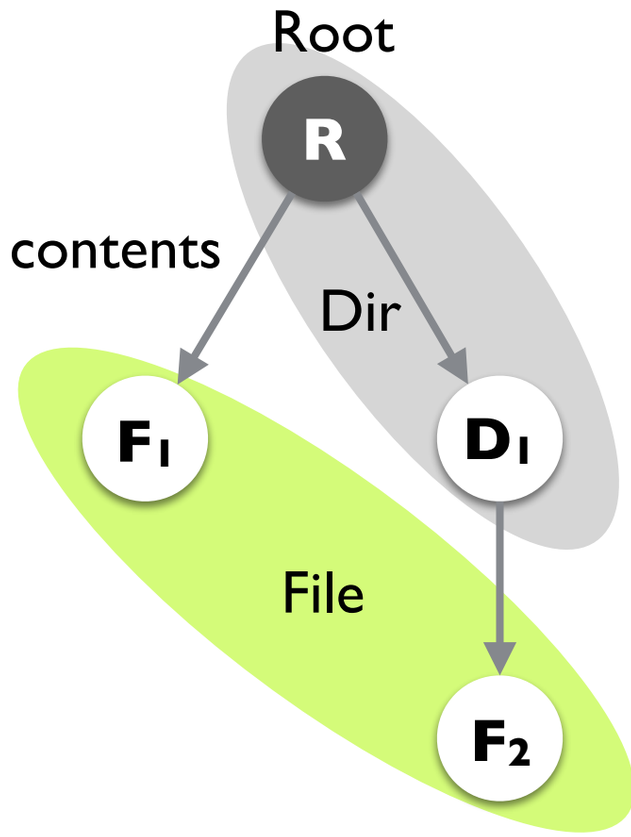
{ **R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$ }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D**$_1$⟩, ⟨**D**$_2$⟩}

{} ⊆ File ⊆ {⟨**F**$_1$⟩, ⟨**F**$_2$⟩}

{} ⊆ contents ⊆ {**R**, **D**$_1$, **D**$_2$} × {**R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$}



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(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

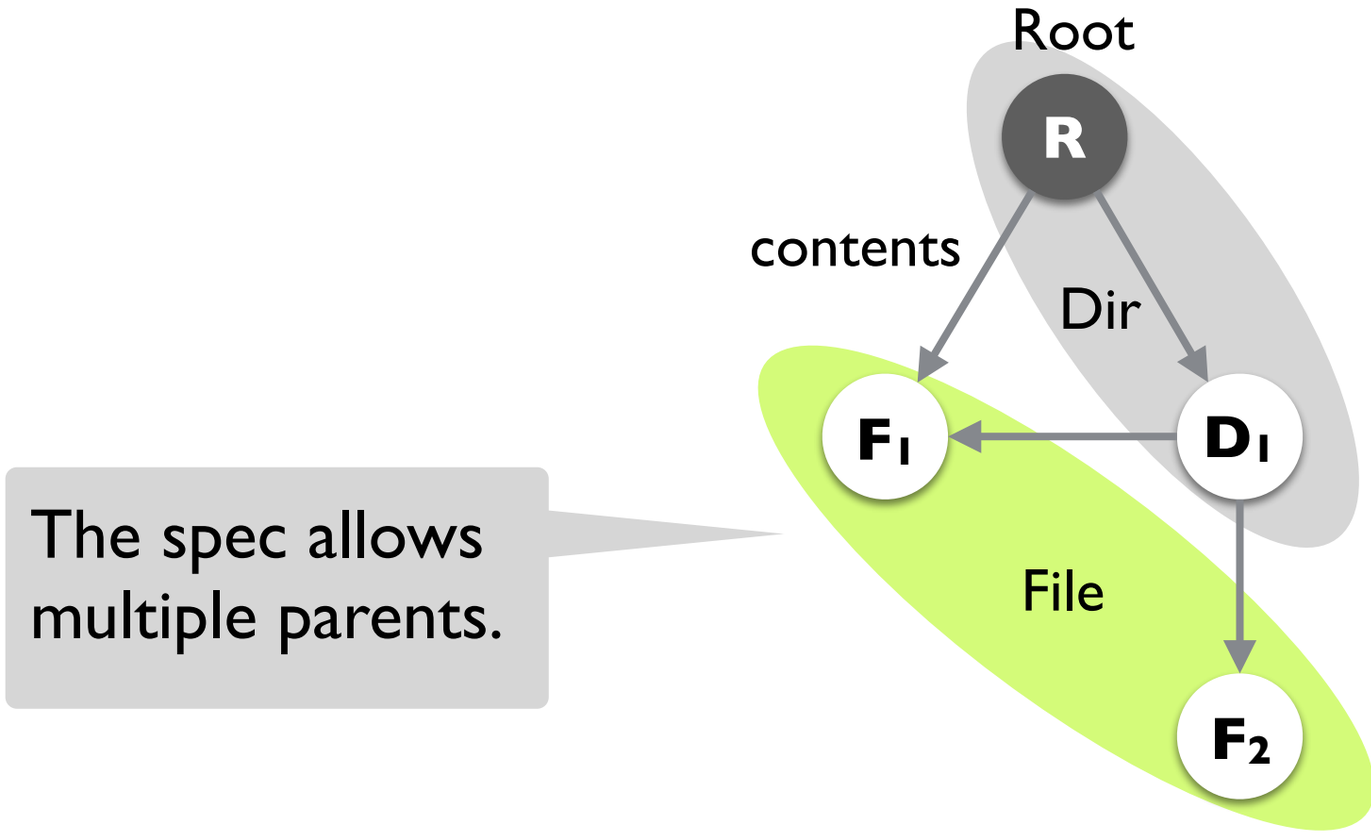
{ **R**, **D1**, **D2**, **F1**, **F2** }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D1**⟩, ⟨**D2**⟩}

{} ⊆ File ⊆ {⟨**F1**⟩, ⟨**F2**⟩}

{} ⊆ contents ⊆ {**R**, **D1**, **D2**} × {**R**, **D1**, **D2**, **F1**, **F2**}



Fixing the tiny filesystem

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

∀ f: File | one contents.f

∀ d: Dir | one contents.d

{ **R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$ }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D**$_1$⟩, ⟨**D**$_2$⟩}

{} ⊆ File ⊆ {⟨**F**$_1$⟩, ⟨**F**$_2$⟩}

{} ⊆ contents ⊆ {**R**, **D**$_1$, **D**$_2$} × {**R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$}

Fixing the tiny filesystem

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

∀ f: File | one contents.f

∀ d: Dir | one contents.d

Minimal unsatisfiable core: an unsatisfiable subset of a formula that becomes satisfiable if any of its members are removed.

{ **R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$ }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D**$_1$⟩, ⟨**D**$_2$⟩}

{} ⊆ File ⊆ {⟨**F**$_1$⟩, ⟨**F**$_2$⟩}

{} ⊆ contents ⊆ {**R**, **D**$_1$, **D**$_2$} × {**R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$}

Resolution-based core extraction

Root ⊆ Dir

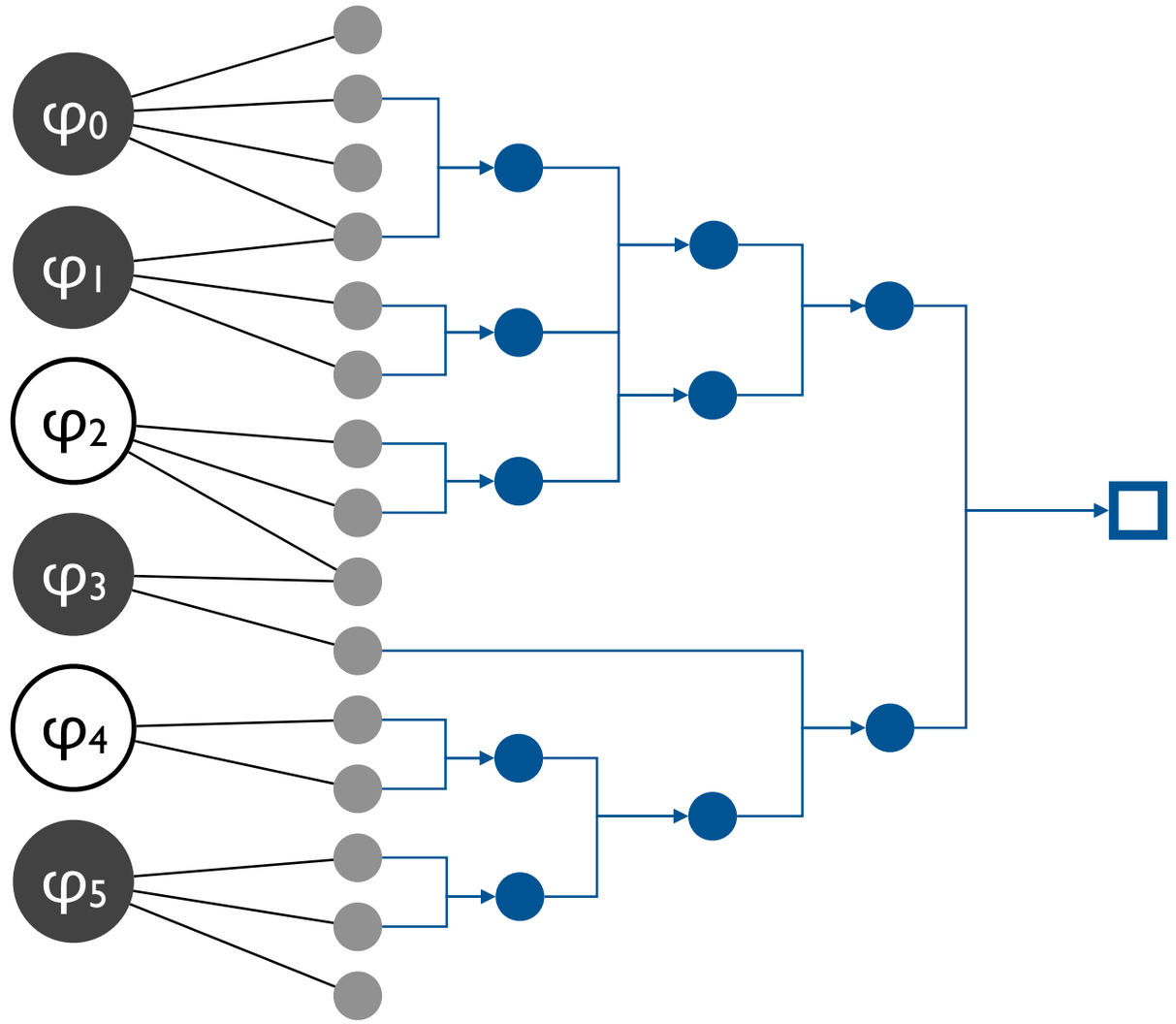
contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

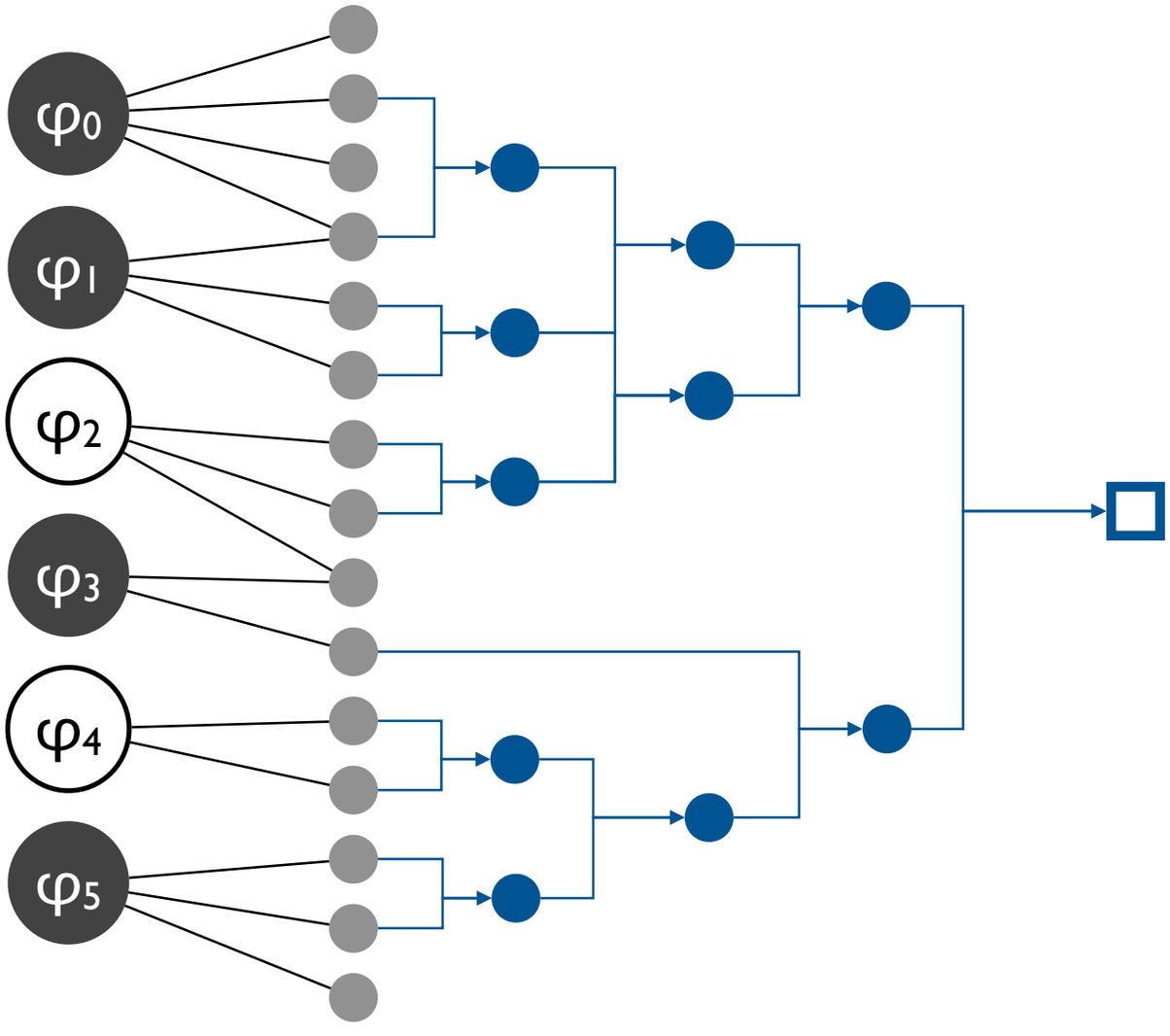
∀ d: Dir | ¬ (d ⊆ d.^contents)

∀ f: File | one contents.f

∀ d: Dir | one contents.d



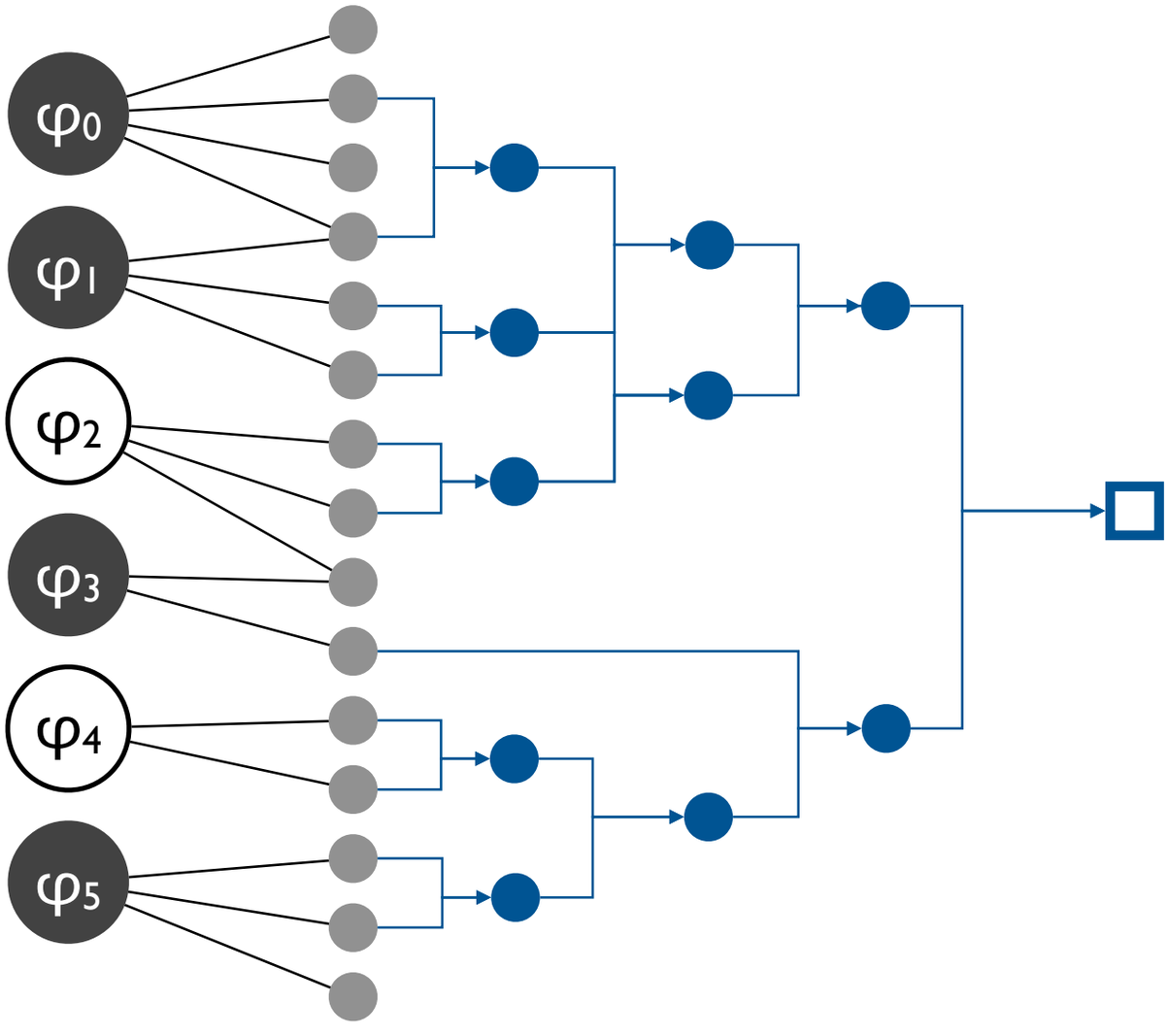
High-level minimal cores from low-level proofs



How to use the proof at the SAT level to find a minimal core at the specification level when

- SAT proof is not minimal
- minimal SAT core may map to a large specification core?

Recycling core extraction



Key idea: minimize core by removing constraints at the specification level but re-use valid resolvents from the previous step so that the solver doesn't have to re-derive them.

Summary

Today

- Finite model finding for first-order logic with quantifiers, relations, and transitive closure

Next lecture

- Reasoning about program correctness