Practical Applications of SAT

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Today

Past 2 lectures

• The theory and mechanics of SAT solving

Today

• Practical applications of SAT
• Variants of the SAT problem
• Motivating the next lecture on SMT
A brief history of SAT solving and applications

Based on a slide from Vijay Ganesh
Bounded Model Checking.
First presented at FMCAD’98. In an unusual move, the Chairs included an extra talk on BMC. A 1999 paper describes its application at Motorola to verify a PowerPC processor.
A brief history of SAT solving and applications

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MiniSAT, ’03

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Concolic Testing, Program Analysis, Mercedes Product Configuration
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Concolic Testing, Program Analysis, Mercedes Product Configuration

Synthesis, Type Systems, Bio, Configuration Management, SMT

Based on a slide from Vijay Ganesh
Bounded Model Checking (BMC) & Configuration Management
Bounded Model Checking (in general)

Given a system and a property, BMC checks if the property is satisfied by all executions of the system with $\leq k$ steps, on all inputs of size $\leq n$. 
Bounded Model Checking (in general)

Given a system and a property, BMC checks if the property is satisfied by all executions of the system with \( \leq k \) steps, on all inputs of size \( \leq n \).

We will focus on safety properties (i.e., making sure a bad state, such as an assertion violation, is not reached).
Bounded Model Checking (in general)

BMC: checks all executions of size $\leq k$

low confidence  high confidence
low human labor  high human labor
Bounded Model Checking (in general)

Testing: checks a few executions of arbitrary size

BMC: checks all executions of size $\leq k$

low confidence  high confidence

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Bounded Model Checking (in general)

Testing: checks a few executions of arbitrary size

BMC: checks all executions of size $\leq k$

Verification: checks all executions of every size

low confidence \quad low human labor

high confidence \quad high human labor
Bounded Model Checking (in general)

Testing: checks a few executions of arbitrary size

BMC: checks all executions of size $\leq k$

Verification: checks all executions of every size

The small scope hypothesis: most bugs can be triggered with small inputs and executions.

low confidence → low human labor

The small scope hypothesis: most bugs can be triggered with small inputs and executions.

high confidence → high human labor
BMC by example
BMC by example

```c
int daysToYear(int days) {
    int year = 1980;
    while (days > 365) {
        if (isLeapYear(year)) {
            if (days > 366) {
                days -= 366;
                year += 1;
            }
        } else {
            days -= 365;
            year += 1;
        }
    }
    return year;
}
```

**The Zune Bug:** on December 31, 2008, all first generation Zune players from Microsoft became unresponsive because of this code. What’s wrong?
BMC by example

```c
int daysToYear(int days) {
    int year = 1980;
    while (days > 365) {
        if (isLeapYear(year)) {
            if (days > 366) {
                days -= 366;
                year += 1;
            }
        } else {
            days -= 365;
            year += 1;
        }
    }
    return year;
}
```

The Zune Bug: on December 31, 2008, all first generation Zune players from Microsoft became unresponsive because of this code. What’s wrong?

Infinite loop triggered on the last day of every leap year.
BMC by example

The Zune Bug: on December 31, 2008, all first generation Zune players from Microsoft became unresponsive because of this code. What’s wrong?

Infinite loop triggered on the last day of every leap year.

A desired safety property: the value of the days variable decreases in every loop iteration.
int daysToYear(int days) {
    int year = 1980;
    while (days > 365) {
        int oldDays = days;
        if (isLeapYear(year)) {
            if (days > 366) {
                days -= 366;
                year += 1;
            } else {
                days -= 365;
                year += 1;
            }
        } else {
            days -= 365;
            year += 1;
        }
    }
    assert days < oldDays;
    return year;
}
BMC step 1 of 4: finitize loops

int daysToYear(int days) {
    int year = 1980;
    if (days > 365) {
        int oldDays = days;
        if (isLeapYear(year)) {
            if (days > 366) {
                days -= 366;
                year += 1;
            }
        } else {
            days -= 365;
            year += 1;
        }
    } else {
        days -= 365;
        year += 1;
    }
    assert days < oldDays;
    assert days <= 365;
}
return year;

• Unwind all loops k times (e.g., k=1), and add an unwinding assertion at the end.
int daysToYear(int days) {
    int year = 1980;
    if (days > 365) {
        int oldDays = days;
        if (isLeapYear(year)) {
            if (days > 366) {
                days -= 366;
                year += 1;
            }
        } else {
            days -= 365;
            year += 1;
        }
    } else {
        days -= 365;
        year += 1;
    }
    assert days < oldDays;
    assert days <= 365;
}

return year;

• Unwind all loops k times (e.g., k=1), and add an **unwinding assertion** at the end.
• If a CEX violates a program assertion, we have found a buggy behavior of length ≤k.
BMC step 1 of 4: finitize loops

```c
int daysToYear(int days) {
    int year = 1980;
    if (days > 365) {
        int oldDays = days;
        if (isLeapYear(year)) {
            if (days > 366) {
                days -= 366;
                year += 1;
            }
        } else {
            days -= 365;
            year += 1;
        }
        assert days < oldDays;
        assert days <= 365;
    }
    return year;
}
```

- Unwind all loops \( k \) times (e.g., \( k=1 \)), and add an **unwinding assertion** at the end.
- If a CEX violates a program assertion, we have found a buggy behavior of length \( \leq k \).
- If a CEX violates an unwinding assertion, the program has no buggy behavior of length \( \leq k \), but it may have a longer one.
BMC step 1 of 4: finitize loops

```c
int daysToYear(int days) {
    int year = 1980;
    if (days > 365) {
        int oldDays = days;
        if (isLeapYear(year)) {
            if (days > 366) {
                days -= 366;
                year += 1;
            }
        } else {
            days -= 365;
            year += 1;
        }
        assert days < oldDays;
        assert days <= 365;
    }
    return year;
}
```

- Unwind all loops \( k \) times (e.g., \( k=1 \)), and add an **unwinding assertion** at the end.
- If a CEX violates a program assertion, we have found a buggy behavior of length \( \leq k \).
- If a CEX violates an unwinding assertion, the program has no buggy behavior of length \( \leq k \), but it may have a longer one.
- If there is no CEX, the program is correct for all \( k \)!
int daysToYear(int days) {
    int year = 1980;
    if (days > 365) {
        int oldDays = days;
        if (isLeapYear(year)) {
            if (days > 366) {
                days -= 366;
                year += 1;
            }
        } else {
            days -= 365;
            year += 1;
        }
        assert days < oldDays;
    }
    assert days <= 365;
    return year;
}
```c
int daysToYear(int days) {
    int year = 1980;
    if (days > 365) {
        int oldDays = days;
        if (isLeapYear(year)) {
            if (days > 366) {
                days -= 366;
                year += 1;
            }
        } else {
            days -= 365;
            year += 1;
        }
        assert days < oldDays;
        assert days <= 365;
    }
    return year;
}
```
BMC step 2 of 4: eliminate side effects

```c
int days;
int year = 1980;
if (days > 365) {
    int oldDays = days;
    if (isLeapYear(year)) {
        if (days > 366) {
            days = days - 366;
            year = year + 1;
        }
    } else {
        days = days - 365;
        year = year + 1;
    }
    assert days < oldDays;
    assert days <= 365;
}
return year;
```

Convert to **Static Single Assignment** (SSA) form:
BMC step 2 of 4: eliminate side effects

int days₀;
int year₀ = 1980;
if (days₀ > 365) {
    int oldDays₀ = days₀;
    if (isLeapYear(year₀)) {
        if (days₀ > 366) {
            days₁ = days₀ - 366;
            year₁ = year₀ + 1;
        }
    } else {
        days₃ = days₀ - 365;
        year₃ = year₀ + 1;
    }
    assert days₄ < oldDays₀;
    assert days₄ <= 365;
}
return year₅;
BMC step 2 of 4: eliminate side effects

int days0;
int year0 = 1980;
boolean g0 = (days0 > 365);
int oldDays0 = days0;
boolean g1 = isLeapYear(year0);
boolean g2 = days0 > 366;
days1 = days0 - 366;
year1 = year0 + 1;
days2 = φ(g1 && g2, days1, days0);
year2 = φ(g1 && g2, year1, year0);
days3 = days0 - 365;
year3 = year0 + 1;
days4 = φ(g1, days2, days3);
year4 = φ(g1, year2, year3);
assert days4 < oldDays0;
assert days4 <= 365;
year5 = φ(g0, year4, year0);
return year5;

Convert to **Static Single Assignment** (SSA) form:

- Replace each assignment to a variable \( v \) with a definition of a fresh variable \( v_i \).
- Change uses of variables so that they refer to the correct definition (version).
- Make conditional dependences explicit with gated \( \phi \) nodes.
int days0;
int year0 = 1980;
boolean g0 = (days0 > 365);
int oldDays0 = days0;
boolean g1 = isLeapYear(year0);
boolean g2 = days0 > 366;
days1 = days0 - 366;
year1 = year0 + 1;
days2 = φ(g1 && g2, days1, days0);
year2 = φ(g1 && g2, year1, year0);
days3 = days0 - 365;
year3 = year0 + 1;
days4 = φ(g1, days2, days3);
year4 = φ(g1, year2, year3);
assert days4 < oldDays0;
assert days4 <= 365;
year5 = φ(g0, year4, year0);
return year5;
int days₀;
int year₀ = 1980;
boolean g₀ = (days₀ > 365);
int oldDays₀ = days₀;
boolean g₁ = isLeapYear(year₀);
boolean g₂ = days₀ > 366;
days₁ = days₀ - 366;
year₁ = year₀ + 1;
days₂ = \phi(g₁ && g₂, days₁, days₀);
year₂ = \phi(g₁ && g₂, year₁, year₀);
days₃ = days₀ - 365;
year₃ = year₀ + 1;
days₄ = \phi(g₁, days₂, days₃);
year₄ = \phi(g₁, year₂, year₃);
assert days₄ < oldDays₀;
assert days₄ <= 365;
year₅ = \phi(g₀, year₄, year₀);
return year₅;
int year₀ = 1980;
boolean g₀ = (days₀ > 365);
int oldDays₀ = days₀;
boolean g₁ = isLeapYear(year₀);
boolean g₂ = days₀ > 366;
days₁ = days₀ - 366;
year₁ = year₀ + 1;
days₂ = φ(g₁ && g₂, days₁, days₀);
year₂ = φ(g₁ && g₂, year₁, year₀);
days₃ = days₀ - 365;
year₃ = year₀ + 1;
days₄ = φ(g₁, days₂, days₃);
year₄ = φ(g₁, year₂, year₃);
assert days₄ < oldDays₀;
assert days₄ <= 365;

We can now read off the equations that encode the program semantics …
BMC step 3 of 4: convert into equations

\begin{align*}
\text{year}_0 &= 1980; \\
\text{g}_0 &= (\text{days}_0 > 365); \\
\text{oldDays}_0 &= \text{days}_0; \\
\text{g}_1 &= \text{isLeapYear}(\text{year}_0); \\
\text{g}_2 &= \text{days}_0 > 366; \\
\text{days}_1 &= \text{days}_0 - 366; \\
\text{year}_1 &= \text{year}_0 + 1; \\
\text{days}_2 &= \varphi(\text{g}_1 \&\& \text{g}_2, \text{days}_1, \text{days}_0); \\
\text{year}_2 &= \varphi(\text{g}_1 \&\& \text{g}_2, \text{year}_1, \text{year}_0); \\
\text{days}_3 &= \text{days}_0 - 365; \\
\text{year}_3 &= \text{year}_0 + 1; \\
\text{days}_4 &= \varphi(\text{g}_1, \text{days}_2, \text{days}_3); \\
\text{year}_4 &= \varphi(\text{g}_1, \text{year}_2, \text{year}_3); \\
\text{assert} & \quad \text{days}_4 < \text{oldDays}_0; \\
\text{assert} & \quad \text{days}_4 \leq 365;
\end{align*}

We can now read off the equations that encode the program semantics …
We can now read off the equations that encode the program semantics ...

\[
\begin{align*}
\text{year}_0 &= 1980 \land \\
\text{g}_0 &= (\text{days}_0 > 365) \land \\
\text{oldDays}_0 &= \text{days}_0 \land \\
\text{g}_1 &= \text{isLeapYear(\text{year}_0)} \land \\
\text{g}_2 &= \text{days}_0 > 366 \land \\
\text{days}_1 &= \text{days}_0 - 366 \land \\
\text{year}_1 &= \text{year}_0 + 1 \land \\
\text{days}_2 &= \varphi(\text{g}_1 \land \text{g}_2, \text{days}_1, \text{days}_0) \land \\
\text{year}_2 &= \varphi(\text{g}_1 \land \text{g}_2, \text{year}_1, \text{year}_0) \land \\
\text{days}_3 &= \text{days}_0 - 365 \land \\
\text{year}_3 &= \text{year}_0 + 1 \land \\
\text{days}_4 &= \varphi(\text{g}_1, \text{days}_2, \text{days}_3) \land \\
\text{year}_4 &= \varphi(\text{g}_1, \text{year}_2, \text{year}_3) \land \\
\text{assert} \text{ days}_4 < \text{oldDays}_0; \\
\text{assert} \text{ days}_4 \leq 365;
\end{align*}
\]
BMC step 3 of 4: convert into equations

We can now read off the equations that encode the program semantics ...

\[
\begin{align*}
\text{year}_0 &= 1980 \land \\
\text{g}_0 &= (\text{days}_0 > 365) \land \\
\text{oldDays}_0 &= \text{days}_0 \land \\
\text{g}_1 &= \text{isLeapYear} (\text{year}_0) \land \\
\text{g}_2 &= \text{days}_0 > 366 \land \\
\text{days}_1 &= \text{days}_0 - 366 \land \\
\text{year}_1 &= \text{year}_0 + 1 \land \\
\text{days}_2 &= \text{ite} (\text{g}_1 \land \text{g}_2, \text{days}_1, \text{days}_0) \land \\
\text{year}_2 &= \text{ite} (\text{g}_1 \land \text{g}_2, \text{year}_1, \text{year}_0) \land \\
\text{days}_3 &= \text{days}_0 - 365 \land \\
\text{year}_3 &= \text{year}_0 + 1 \land \\
\text{days}_4 &= \text{ite} (\text{g}_1, \text{days}_2, \text{days}_3) \land \\
\text{year}_4 &= \text{ite} (\text{g}_1, \text{year}_2, \text{year}_3) \land \\
\text{assert} \text{ days}_4 &< \text{oldDays}_0; \\
\text{assert} \text{ days}_4 &\leq 365;
\end{align*}
\]
BMC step 3 of 4: convert into equations

\[\begin{align*}
\text{year}_0 &= 1980 \land \\
\text{g}_0 &= (\text{days}_0 > 365) \land \\
\text{oldDays}_0 &= \text{days}_0 \land \\
\text{g}_1 &= \text{isLeapYear}(\text{year}_0) \land \\
\text{g}_2 &= \text{days}_0 > 366 \land \\
\text{days}_1 &= \text{days}_0 - 366 \land \\
\text{year}_1 &= \text{year}_0 + 1 \land \\
\text{days}_2 &= \text{ite}(\text{g}_1 \land \text{g}_2, \text{days}_1, \text{days}_0) \land \\
\text{year}_2 &= \text{ite}(\text{g}_1 \land \text{g}_2, \text{year}_1, \text{year}_0) \land \\
\text{days}_3 &= \text{days}_0 - 365 \land \\
\text{year}_3 &= \text{year}_0 + 1 \land \\
\text{days}_4 &= \text{ite}(\text{g}_1, \text{days}_2, \text{days}_3) \land \\
\text{year}_4 &= \text{ite}(\text{g}_1, \text{year}_2, \text{year}_3) \land \\
(\neg(\text{days}_4 < \text{oldDays}_0) \lor \\
\neg(\text{days}_4 \leq 365))
\end{align*}\]
BMC step 3 of 4: convert into equations

We can now read off the equations that encode the program semantics, and the assertions to be checked.

A solution to this formula is a sound counterexample: an interpretation for all logical variables that satisfies the program semantics (for up to k unwindings) but violates at least one of the assertions.

\begin{align*}
  \text{year}_0 &= 1980 \land \\
  \text{g}_0 &= (\text{days}_0 > 365) \land \\
  \text{oldDays}_0 &= \text{days}_0 \land \\
  \text{g}_1 &= \text{isLeapYear}(\text{year}_0) \land \\
  \text{g}_2 &= \text{days}_0 > 366 \land \\
  \text{days}_1 &= \text{days}_0 - 366 \land \\
  \text{year}_1 &= \text{year}_0 + 1 \land \\
  \text{days}_2 &= \text{ite}(\text{g}_1 \land \text{g}_2, \text{days}_1, \text{days}_0) \land \\
  \text{year}_2 &= \text{ite}(\text{g}_1 \land \text{g}_2, \text{year}_1, \text{year}_0) \land \\
  \text{days}_3 &= \text{days}_0 - 365 \land \\
  \text{year}_3 &= \text{year}_0 + 1 \land \\
  \text{days}_4 &= \text{ite}(\text{g}_1, \text{days}_2, \text{days}_3) \land \\
  \text{year}_4 &= \text{ite}(\text{g}_1, \text{year}_2, \text{year}_3) \land \\
  (\neg(\text{days}_4 < \text{oldDays}_0) \lor \\
  (\neg(\text{days}_4 <= 365))
\end{align*}
BMC step 4 of 4: convert into CNF

\[ \text{year}_1 = \text{year}_0 + 1 \]
BMC step 4 of 4: convert into CNF

Represent numbers as arrays of bits.

\[ \text{year}_1 = \text{year}_0 + 1 \]

\[ \text{year}_0 = 000 \ldots 000 \]

\[ \begin{array}{cccc}
31 & 30 & 29 & 2 \ 1 \ 0
\end{array} \]
Represent numbers as arrays of bits. Use one boolean variable per bit for each number.

BMC step 4 of 4: convert into CNF

\[ \text{year}_1 = \text{year}_0 + 1 \]

\[ \text{year}_0 = \underbrace{000 \ldots 000}_{31 \text{ bits}} \]
BMC step 4 of 4: convert into CNF

Represent numbers as arrays of bits. Use one boolean variable per bit for each number.

\[ \text{year}_1 = \text{year}_0 + 1 \]

\[ \text{year}_0 = 000 \ldots 000 \]

Construct an adder circuit for \( \text{year}_0 + 1 \).
BMC step 4 of 4: convert into CNF

Represent numbers as arrays of bits. Use one boolean variable per bit for each number.

\[ \text{year}_1 = \text{year}_0 + 1 \]

\[ \text{year}_0 = 000 \ldots 000 \]

Construct an adder circuit for \( \text{year}_0 + 1 \).

Introduce new clauses to constrain bits in \( \text{year}_1 \) to match bits in the sum.
BMC counterexample for $k=1$

```c
int daysToYear(int days) {
    int year = 1980;
    while (days > 365) {
        if (isLeapYear(year)) {
            if (days > 366) {
                days -= 366;
                year += 1;
            }
        } else {
            days -= 365;
            year += 1;
        }
    }
    return year;
}
```
Bounded Model Checking (BMC) & Configuration Management
Configuration Management

Given a configuration, consisting of a set of components, their dependencies, and conflicts:

- Decide if a new component can be added to the configuration.
- Add the component while optimizing some linear function.
- If the component cannot be added, find a way to add it by removing as few conflicting components from the current configuration as possible.
Configuration Management

Given a configuration, consisting of a set of components, their dependencies, and conflicts:

• Decide if a new component can be added to the configuration.

• Add the component while optimizing some linear function.

• If the component cannot be added, find a way to add it by removing as few conflicting components from the current configuration as possible.
Configuration Management

Given a configuration, consisting of a set of components, their dependencies, and conflicts:

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Configuration Management

Given a configuration, consisting of a set of components, their dependencies, and conflicts:

• Decide if a new component can be added to the configuration.
• Add the component while optimizing some linear function.
• If the component cannot be added, find a way to add it by removing as few conflicting components from the current configuration as possible.

SAT
Pseudo-Boolean Constraints
Partial (Weighted) MaxSAT
Deciding if a component can be installed
Deciding if a component can be installed

z already installed.
Deciding if a component can be installed

a depends on b, c, z.

z already installed.
Deciding if a component can be installed

- **a** depends on **b, c, z.**
- **z** already installed.
- **c** needs **f or g.**
Deciding if a component can be installed

Conflict: d and e cannot both be installed.

- a depends on b, c, z.
- z already installed.
- c needs f or g.
Deciding if a component can be installed

To install a, CNF constraints are:

- a depends on b, c, z.
- z already installed.
- c needs f or g.

Conflict: d and e cannot both be installed.
Deciding if a component can be installed

To install a, CNF constraints are:

\((\neg a \lor b) \land (\neg a \lor c) \land (\neg a \lor z) \land z\text{ already installed.}\)

Conflict: d and e cannot both be installed.
Deciding if a component can be installed

To install a, CNF constraints are:

\[(\neg a \lor b) \land (\neg a \lor c) \land (\neg a \lor z) \land (\neg b \lor d) \land \]

Conflict: d and e cannot both be installed.
Deciding if a component can be installed

To install a, CNF constraints are:

\[ \neg a \lor b \land \neg a \lor c \land \neg a \lor z \land \neg b \lor d \land \neg c \lor d \lor e \land \neg c \lor f \lor g \land z \text{ already installed.} \]

Conflict: d and e cannot both be installed.
Deciding if a component can be installed

To install a, CNF constraints are:

\[(\neg a \lor b) \land (\neg a \lor c) \land (\neg a \lor z) \land (\neg b \lor d) \land (\neg c \lor d \lor e) \land (\neg c \lor f \lor g) \land (\neg d \lor \neg e) \land \]

Conflict: d and e cannot both be installed.
Deciding if a component can be installed

a depends on b, c, z.

z already installed.

c needs f or g.

Conflict: d and e cannot both be installed.

To install a, CNF constraints are:

\((\neg a \lor b) \land (\neg a \lor c) \land (\neg a \lor z) \land (\neg b \lor d) \land (\neg c \lor d \lor e) \land (\neg c \lor f \lor g) \land (\neg d \lor \neg e) \land (\neg y \lor z) \land z \text{ already installed.}\)
Deciding if a component can be installed

To install a, CNF constraints are:

\[(\neg a \lor b) \land (\neg a \lor c) \land (\neg a \lor z) \land (\neg b \lor d) \land (\neg c \lor d \lor e) \land (\neg c \lor f \lor g) \land (\neg d \lor \neg e) \land (\neg y \lor z) \land a \land z\]

Conflict: d and e cannot both be installed.
Optimal installation

Assume f and g are 5MB and 2MB each, and all other components are 1MB. How to install a, while minimizing total size?

\[ \neg a \lor b \land (\neg a \lor c) \land (\neg a \lor z) \land \\
(\neg b \lor d) \land \\
(\neg c \lor d \lor e) \land (\neg c \lor f \lor g) \land \\
(\neg d \lor \neg e) \land \\
(\neg y \lor z) \land \\
a \land z \]
Assume \( f \) and \( g \) are 5MB and 2MB each, and all other components are 1MB. How to install \( a \), while minimizing total size?

Pseudo-boolean solvers accept a linear function to minimize, in addition to a (weighted) CNF.

\[
\min c_1 x_1 + \ldots + c_n x_n \\
a_1 x_1 + \ldots + a_n x_n \geq b_1 \wedge \ldots \wedge \\
a_{k_1} x_1 + \ldots + a_{k_n} x_n \geq b_k
\]

\[
(\neg a \lor b) \wedge (\neg a \lor c) \wedge (\neg a \lor z) \wedge \\
(\neg b \lor d) \wedge \\
(\neg c \lor d \lor e) \wedge (\neg c \lor f \lor g) \wedge \\
(\neg d \lor \neg e) \wedge \\
(\neg y \lor z) \wedge \\
a \wedge z
\]
Assume $f$ and $g$ are 5MB and 2MB each, and all other components are 1MB. How to install $a$, while minimizing total size?

Pseudo-boolean solvers accept a linear function to minimize, in addition to a (weighted) CNF.

$$\text{min } a + b + c + d + e + 5f + 2g + y + 0z$$

$$(-a + b \geq 0) \land (-a + c \geq 0) \land (-a + z \geq 0) \land$$
$$(-b + d \geq 0) \land$$
$$(-c + d + e \geq 0) \land (-c + f + g \geq 0) \land$$
$$(-d + -e \geq -1) \land$$
$$(-y + z \geq 0) \land$$
$$(a \geq 1) \land (z \geq 1)$$
Installation in the presence of conflicts
Installation in the presence of conflicts

a cannot be installed because it requires b, which requires d, which conflicts with e.
Installation in the presence of conflicts

Partial MaxSAT solver takes as input a set of **hard** clauses and a set of **soft** clauses, and it produces an assignment that satisfies all hard clauses and the greatest number of soft clauses.

a cannot be installed because it requires b, which requires d, which conflicts with e.
Installation in the presence of conflicts

Partial MaxSAT solver takes as input a set of **hard** clauses and a set of **soft** clauses, and it produces an assignment that satisfies all hard clauses and the greatest number of soft clauses.

To install a, while minimizing the number of removed components, Partial MaxSAT constraints are:

**hard:** \[(\neg a \lor b) \land (\neg a \lor c) \land (\neg a \lor z) \land (\neg b \lor d) \land (\neg c \lor d \lor e) \land (\neg c \lor f \lor g) \land (\neg d \lor \neg e) \land (\neg y \lor z) \land a\]

**soft:** \[e \land z\]
Summary

Today

• SAT solvers have been used successfully in many applications & domains
• But reducing problems to SAT is a lot like programming in assembly …
• We need higher-level logics!

Next lecture

• On to richer logics: introduction to Satisfiability Modulo Theories (SMT)