Computer-Aided Reasoning for Software

CSE507

The DPLL(T) Framework

courses.cs.washington.edu/courses/cse507/18sp/

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Today

Last lecture

- Deciding conjunctions of $(T_1 \cup T_2)$-constraints with Nelson-Oppen

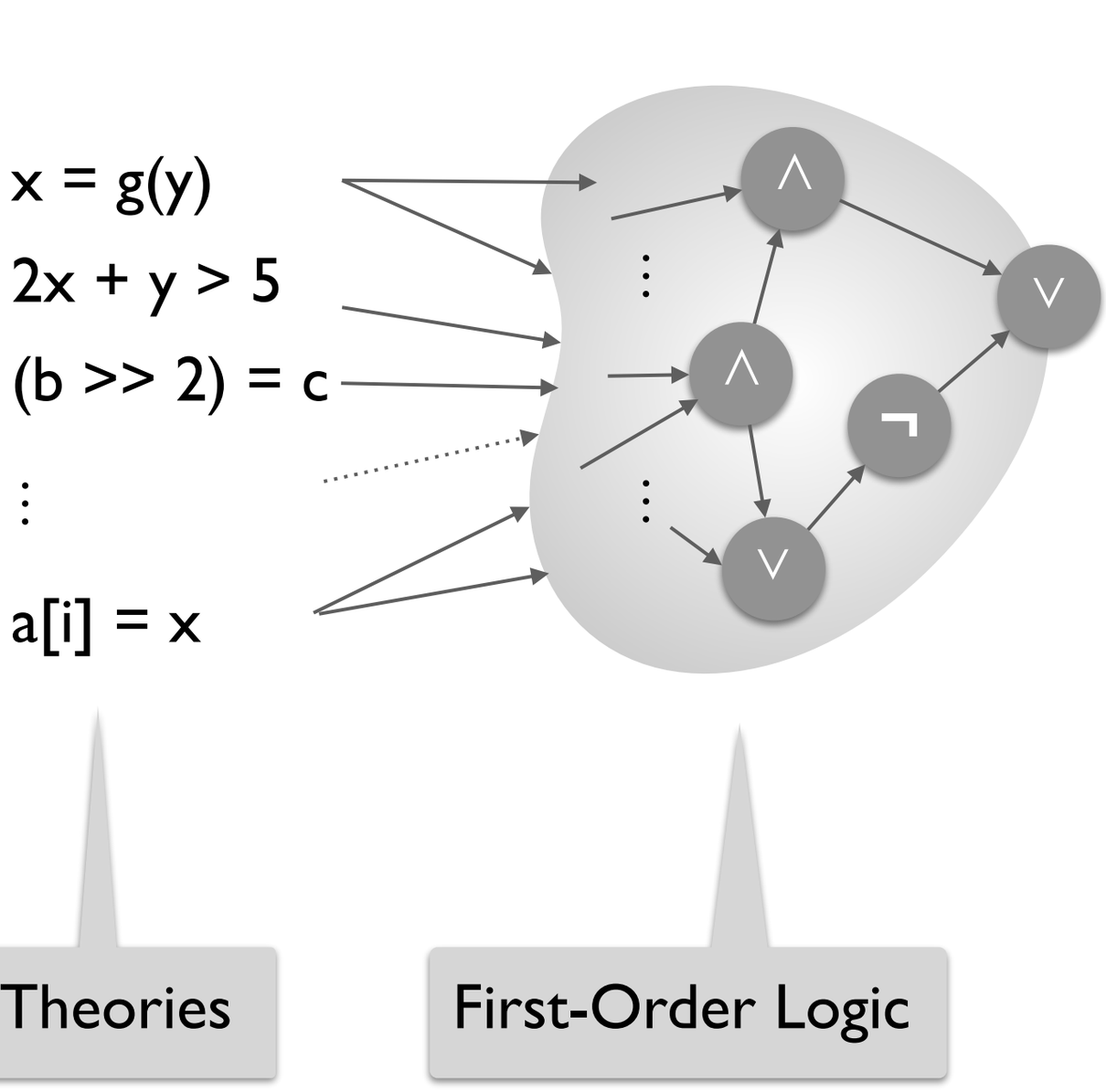
Today

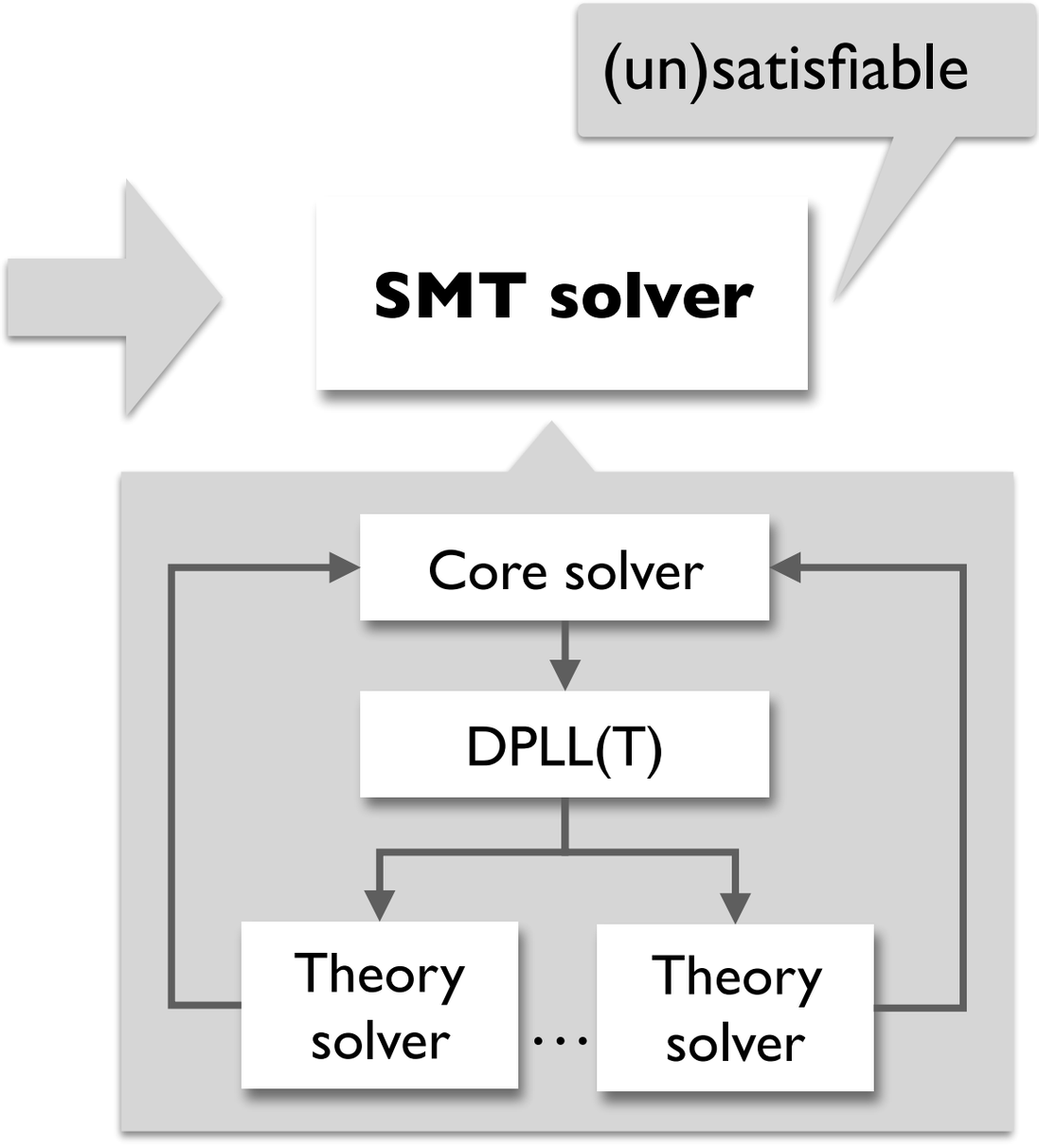
- Deciding arbitrary boolean combinations of theory constraints

Reminders

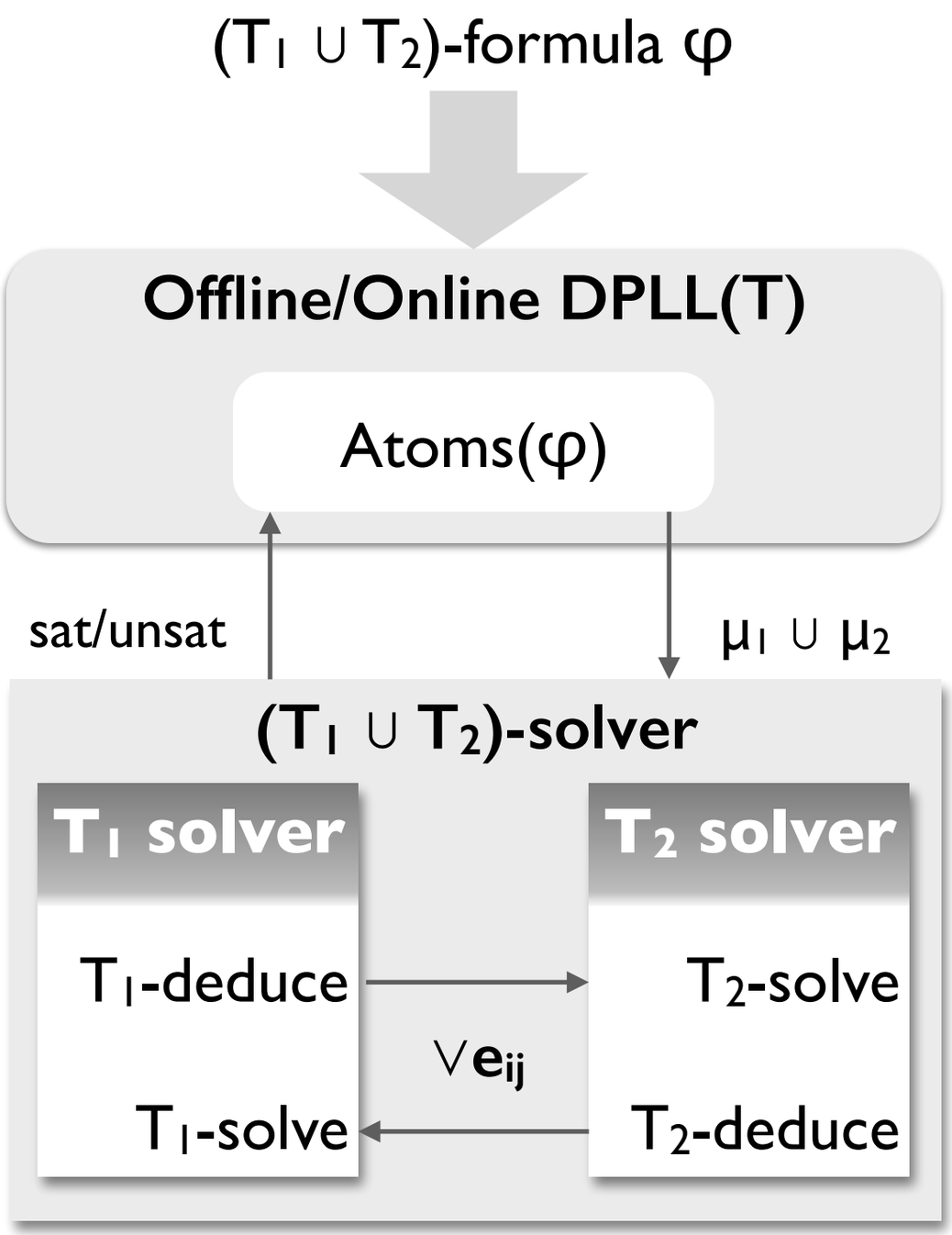
- HW1 due on Friday.
- HW2 out; start early.

Satisfiability Modulo Theories (SMT)

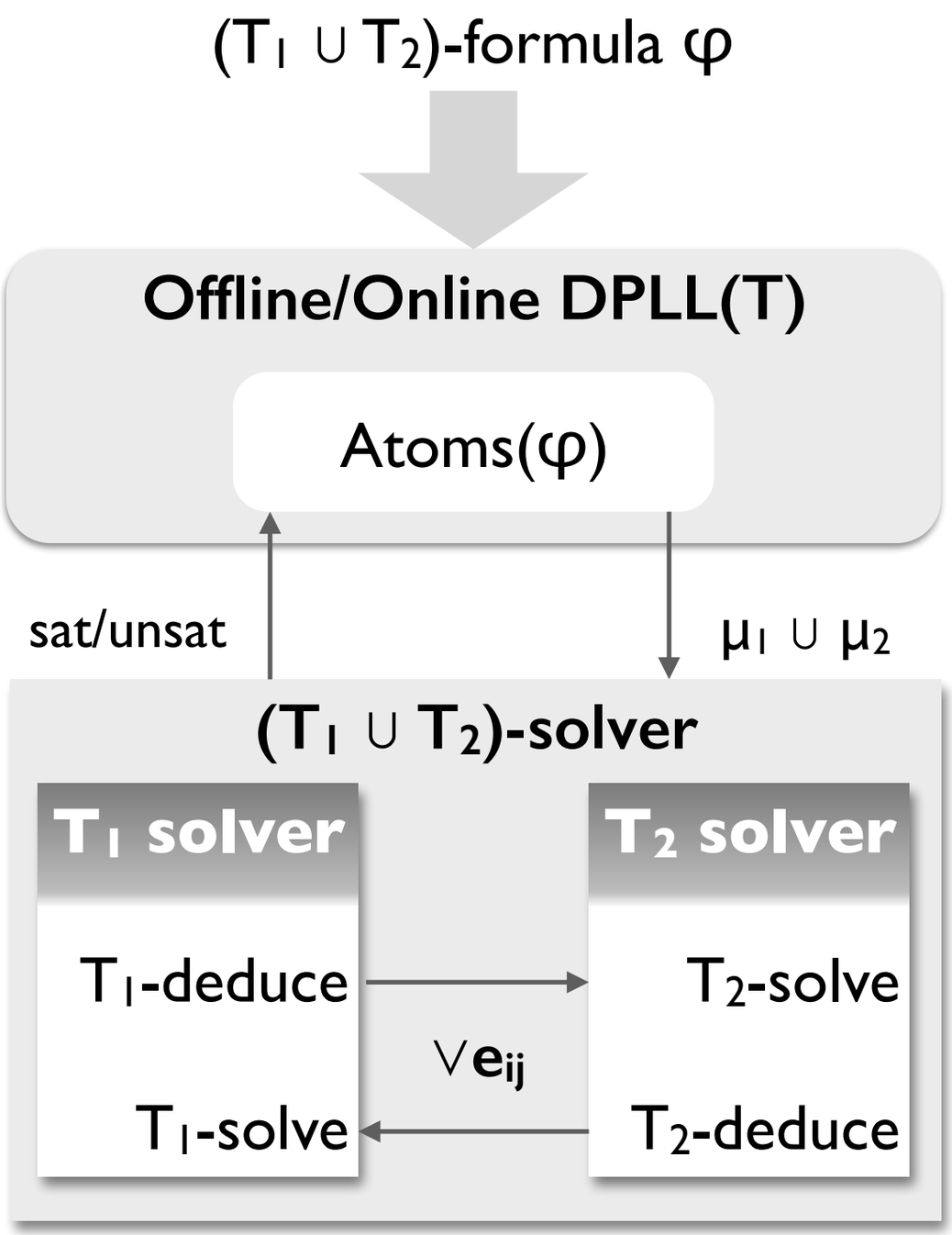


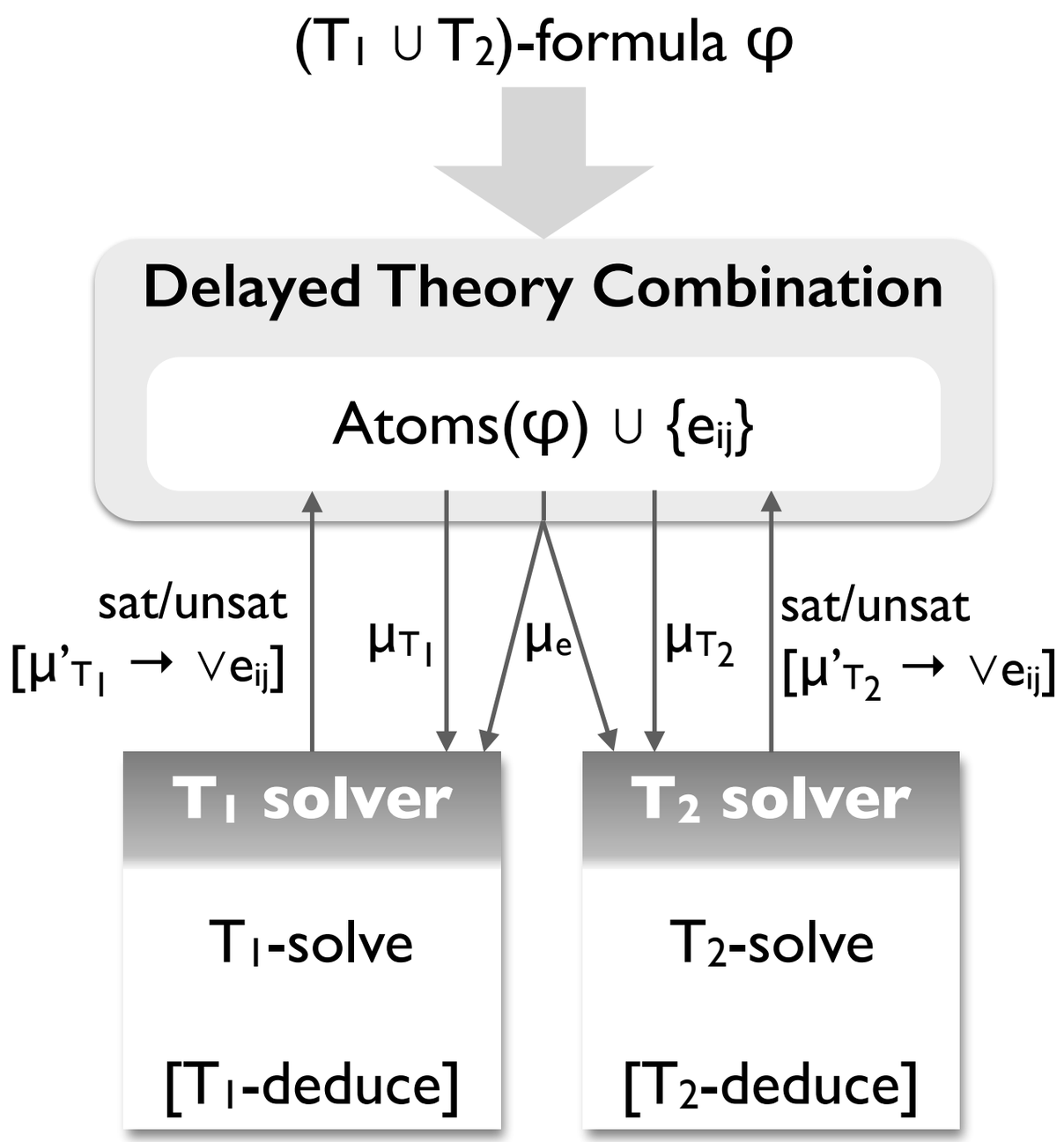


The DPLL(T) Framework



The DPLL(T) Framework





Offline DPLL(T)

Offline-DPLL$_T$(T-formula φ)
  $\varphi_P \leftarrow$ **T2B**(φ)
  while (TRUE) **do**
    μ_P, res $\leftarrow$ **CDCL**(φ_P)
    if res = UNSAT **then return** UNSAT
    else
      T-res $\leftarrow$ T-solve(**B2T**(μ_P))
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Assume φ is in CNF.

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T2B computes the *boolean abstraction* (aka *boolean skeleton*) of φ by replacing every atom in φ with a fresh boolean variable.

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B2T computes the *boolean refinement* of the current propositional assignment μ^P.

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B2T computes the *boolean refinement* of the current propositional assignment μ^P.

Theory conflict clause.

Boolean abstraction (T2B) and refinement (B2T)

T2B(φ)

- T2B(a_i) = b_i, if a_i is a theory atom and b_i is a fresh boolean variable
- T2B(b_j) = b_j, if b_j is a boolean variable
- T2B($\varphi_1 \wedge \varphi_2$) = T2B($\varphi_1$) $\wedge$ T2B(φ_2)
- T2B($\varphi_1 \vee \varphi_2$) = T2B(φ_1) $\vee$ T2B(φ_2)
- T2B($\neg\varphi_1$) = $\neg$T2B(φ_1)

B2T(φ^P) = T2B^{-1}(φ^P)

Boolean abstraction (T2B) and refinement (B2T)

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- $T2B(\neg\varphi_1) = \neg T2B(\varphi_1)$

φ: $(x = 1) \wedge ((x = 2) \vee (x = 3))$

$B2T(\varphi^P) = T2B^{-1}(\varphi^P)$

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φ: $(x = 1) \wedge ((x = 2) \vee (x = 3))$

$T2B(\varphi)$: $b_1 \wedge (b_2 \vee b_3)$

$B2T(\varphi^P) = T2B^{-1}(\varphi^P)$

Boolean abstraction (T2B) and refinement (B2T)

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$T2B(\varphi)$: $b_1 \wedge (b_2 \vee b_3)$

$B2T(b_1 \wedge b_3)$: $(x = 1) \wedge (x = 3)$

$B2T(\varphi^P) = T2B^{-1}(\varphi^P)$

Soundness and termination of offline DPLL(T)

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The algorithm is sound because φ^P overapproximates the satisfiability of φ at the beginning of each loop iteration (so it never returns UNSAT for SAT formulas), and it only returns SAT when μ^P both satisfies φ^P and is consistent with the theory axioms.

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The algorithm terminates because there are finitely many satisfying assignments to the boolean abstraction, and we get a different one every time.

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The *theory conflict clause* $\neg\mu^P$ is too weak; it blocks one assignment at a time. What is a better clause?

- $\varphi \leftarrow (x = 1) \wedge ((x = 2) \vee (x = 3))$
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$\neg$T2B(MINIMALUNSATCORE(B2T(μ^P)))

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      else
        t $\leftarrow$ T2B(MINUNSATCORE(B2T(μ^P)))
        $\varphi^P \leftarrow \varphi^P \wedge \neg t$

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  - $\varphi^P \leftarrow b_1 \wedge (b_2 \vee b_3) \wedge (\neg b_1 \vee \neg b_2)$
- $\mu^P \leftarrow b_1 \wedge \neg b_2 \wedge b_3$
  - $\varphi^P \leftarrow b_1 \wedge (b_2 \vee b_3) \wedge (\neg b_1 \vee \neg b_2) \wedge (\neg b_1 \vee \neg b_3)$
- UNSAT

Offline DPLL(T): improvements

Offline-DPLL$_T$(T-formula φ)
  $\varphi^P \leftarrow$ **T2B**(φ)
  while (TRUE) **do**
    μ^P, res $\leftarrow$ **CDCL**(φ^P)
    if res = UNSAT **then return** UNSAT
    else
      T-res $\leftarrow$ T-solve(**B2T**(μ^P))
      if T-res = SAT **then return** SAT
      else
        t $\leftarrow$ T2B(MINUNSATCORE(B2T(μ^P)))
        $\varphi^P \leftarrow \varphi^P \wedge \neg t$

- $\varphi \leftarrow (x = 1) \wedge ((x = 2) \vee (x = 3))$
- $\varphi^P \leftarrow b_1 \wedge (b_2 \vee b_3)$
- $\mu^P \leftarrow b_1 \wedge b_2 \wedge b_3$
  - $\varphi^P \leftarrow b_1 \wedge (b_2 \vee b_3) \wedge (\neg b_1 \vee \neg b_2)$
- $\mu^P \leftarrow b_1 \wedge \neg b_2 \wedge b_3$
  - $\varphi^P \leftarrow b_1 \wedge (b_2 \vee b_3) \wedge (\neg b_1 \vee \neg b_2) \wedge (\neg b_1 \vee \neg b_3)$
- UNSAT

Better but still need a *full assignment* to the boolean abstraction in order to generate a conflict clause.

Online DPLL(T) address this issue.

Online DPLL(T)

```
Online-DPLL_T(T-formula φ, T-assignment μ)
  if T-PREPROCESS(φ, μ) = CONFLICT then
    return UNSAT
  φ^P , μ^P ← T2B(φ), T2B(μ)
  while (TRUE) do
    T-DECIDE(φ^P, μ^P)
    while (TRUE) do
      res ← T-DEDUCE(φ^P, μ^P)
      if res = SAT then return SAT
      else if res = CONFLICT
        blevel ← T-ANALZECONFLICT(φ^P, μ^P)
        if (blevel < 0) then return UNSAT
        else T-BACKTRACK(blevel, φ^P, μ^P)
      else break
```

Online DPLL(T)

```
Online-DPLL_T(T-formula φ, T-assignment μ)
  if T-PREPROCESS(φ, μ) = CONFLICT then
    return UNSAT
  φ^P , μ^P ← T2B(φ), T2B(μ)
  while (TRUE) do
    T-DECIDE(φ^P, μ^P)
    while (TRUE) do
      res ← T-DEDUCE(φ^P, μ^P)
      if res = SAT then return SAT
      else if res = CONFLICT
        blevel ← T-ANALZECONFLICT(φ^P, μ^P)
        if (blevel < 0) then return UNSAT
        else T-BACKTRACK(blevel, φ^P, μ^P)
      else break
```

Everything passed by reference.

All procedures have access to T2B and B2T.

Online DPLL(T): T-PREPROCESS

```
Online-DPLL_T(T-formula φ, T-assignment μ)
  if T-PREPROCESS(φ, μ) = CONFLICT then
    return UNSAT
  φ^P , μ^P ← T2B(φ), T2B(μ)
  while (TRUE) do
    T-DECIDE(φ^P, μ^P)
    while (TRUE) do
      res ← T-DEDUCE(φ^P, μ^P)
      if res = SAT then return SAT
      else if res = CONFLICT
        blevel ← T-ANALZECONFLICT(φ^P, μ^P)
        if (blevel < 0) then return UNSAT
        else T-BACKTRACK(blevel, φ^P, μ^P)
      else break
```

Simplifies φ and updates μ, if needed, so that equisatisfiability is preserved.

Common simplifications:

- Drop dual operators
- Exploit associativity
- Sort arguments
- Exploit theory-specific properties

Online DPLL(T): T-PREPROCESS

```
Online-DPLL_T(T-formula φ, T-assignment μ)
  if T-PREPROCESS(φ, μ) = CONFLICT then
    return UNSAT
  φ^P , μ^P ← T2B(φ), T2B(μ)
  while (TRUE) do
    T-DECIDE(φ^P, μ^P)
  while (TRUE) do
    res ← T-DEDUCE(φ^P, μ^P)
    if res = SAT then return SAT
    else if res = CONFLICT
      blevel ← T-ANALZECONFLICT(φ^P, μ^P)
      if (blevel < 0) then return UNSAT
      else T-BACKTRACK(blevel, φ^P, μ^P)
    else break
```

Simplifies φ and updates μ, if needed, so that equisatisfiability is preserved.

Common simplifications:

- Drop dual operators
- Exploit associativity
- Sort arguments
- Exploit theory-specific properties

Online DPLL(T): T-DECIDE

```
Online-DPLL_T(T-formula φ, T-assignment μ)
  if T-PREPROCESS(φ, μ) = CONFLICT then
    return UNSAT
  φ^P , μ^P ← T2B(φ), T2B(μ)
  while (TRUE) do
    T-DECIDE(φ^P, μ^P)
    while (TRUE) do
      res ← T-DEDUCE(φ^P, μ^P)
      if res = SAT then return SAT
      else if res = CONFLICT
        blevel ← T-ANALZECONFLICT(φ^P, μ^P)
        if (blevel < 0) then return UNSAT
        else T-BACKTRACK(blevel, φ^P, μ^P)
      else break
```

Analogous to DECIDE in CDCL:

- Selects an unassigned l^P literal and adds it to μ^P.
- May consider the semantics of literals in T.

Online DPLL(T): T-DECIDE

```
Online-DPLL_T(T-formula φ, T-assignment μ)
  if T-PREPROCESS(φ, μ) = CONFLICT then
    return UNSAT
  φ^P , μ^P ← T2B(φ), T2B(μ)
  while (TRUE) do
    T-DECIDE(φ^P, μ^P)
    while (TRUE) do
      res ← T-DEDUCE(φ^P, μ^P)
      if res = SAT then return SAT
      else if res = CONFLICT
        blevel ← T-ANALZECONFLICT(φ^P, μ^P)
        if (blevel < 0) then return UNSAT
        else T-BACKTRACK(blevel, φ^P, μ^P)
      else break
```

Analogous to DECIDE in CDCL:

- Selects an unassigned l^P literal and adds it to μ^P.
- May consider the semantics of literals in T.

Online DPLL(T): T-DEDUCE

```
Online-DPLL_T(T-formula φ, T-assignment μ)
  if T-PREPROCESS(φ, μ) = CONFLICT then
    return UNSAT
  φ^P , μ^P ← T2B(φ), T2B(μ)
  while (TRUE) do
    T-DECIDE(φ^P, μ^P)
    while (TRUE) do
      res ← T-DEDUCE(φ^P, μ^P)
      if res = SAT then return SAT
      else if res = CONFLICT
        blevel ← T-ANALZECONFLICT(φ^P, μ^P)
        if (blevel < 0) then return UNSAT
        else T-BACKTRACK(blevel, φ^P, μ^P)
      else break
```

Applies BCP to φ^P and μ^P until

- μ^P propositionally violates φ^P: returns CONFLICT.
- μ^P propositionally satisfies φ^P: invokes T-solver on B2T(μ^P) and returns SAT if T-solver does. Otherwise returns CONFLICT.
- no more literals can be deduced: invokes T-solver on partial assignment B2T(μ^P) and returns CONFLICT is T-solver returns UNSAT. This is *early pruning*. May also do *theory propagation*.

Online DPLL(T): T-DEDUCE

```
Online-DPLL_T(T-formula φ, T-assignment μ)
  if T-PREPROCESS(φ, μ) = CONFLICT then
    return UNSAT
  φ^P , μ^P ← T2B(φ), T2B(μ)
  while (TRUE) do
    T-DECIDE(φ^P, μ^P)
    while (TRUE) do
      res ← T-DEDUCE(φ^P, μ^P)
      if res = SAT then return SAT
      else if res = CONFLICT
        blevel ← T-ANALZECONFLICT(φ^P, μ^P)
        if (blevel < 0) then return UNSAT
        else T-BACKTRACK(blevel, φ^P, μ^P)
      else break
```

Applies BCP to φ^P and μ^P until

- μ^P propositionally violates φ^P: returns CONFLICT.
- μ^P propositionally satisfies φ^P: invokes T-solver on B2T(μ^P) and returns SAT if T-solver does. Otherwise returns CONFLICT.
- no more literals can be deduced: invokes T-solver on partial assignment B2T(μ^P) and returns CONFLICT is T-solver returns UNSAT. This is *early pruning*. May also do *theory propagation*.

Online DPLL(T): T-DEDUCE

```
Online-DPLL_T(T-formula φ, T-assignment μ)
  if T-PREPROCESS(φ, μ) = CONFLICT then
    return UNSAT
  φ^P , μ^P ← T2B(φ), T2B(μ)
  while (TRUE) do
    T-DECIDE(φ^P, μ^P)
    while (TRUE) do
      res ← T-DEDUCE(φ^P, μ^P)
      if res = SAT then return SAT
      else if res = CONFLICT
        blevel ← T-ANALZECONFLICT(φ^P, μ^P)
        if (blevel < 0) then return UNSAT
        else T-BACKTRACK(blevel, φ^P, μ^P)
      else break
```

Applies BCP to φ^P and μ^P until

- μ^P propositionally violates φ^P: returns CONFLICT.
- μ^P propositionally satisfies φ^P: invokes T-solver on B2T(μ^P) and returns SAT if T-solver does. Otherwise returns CONFLICT.
- no more literals can be deduced: invokes T-solver on partial assignment B2T(μ^P) and returns CONFLICT is T-solver returns UNSAT. This is *early pruning*. May also do *theory propagation*.

Online DPLL(T): T-DEDUCE

```
Online-DPLL_T(T-formula φ, T-assignment μ)
  if T-PREPROCESS(φ, μ) = CONFLICT then
    return UNSAT
  φ^P , μ^P ← T2B(φ), T2B(μ)
  while (TRUE) do
    T-DECIDE(φ^P, μ^P)
    while (TRUE) do
      res ← T-DEDUCE(φ^P, μ^P)
      if res = SAT then return SAT
      else if res = CONFLICT
        blevel ← T-ANALZECONFLICT(φ^P, μ^P)
        if (blevel < 0) then return UNSAT
        else T-BACKTRACK(blevel, φ^P, μ^P)
      else break
```

Applies BCP to φ^P and μ^P until

- μ^P propositionally violates φ^P: returns CONFLICT.
- μ^P propositionally satisfies φ^P: invokes T-solver on B2T(μ^P) and returns SAT if T-solver does. Otherwise returns CONFLICT.
- no more literals can be deduced: invokes T-solver on partial assignment B2T(μ^P) and returns CONFLICT is T-solver returns UNSAT. This is *early pruning*. May also do *theory propagation*.

Online DPLL(T): T-AnalyzeConflict

```
Online-DPLL_T(T-formula φ, T-assignment μ)
  if T-Preprocess(φ, μ) = CONFLICT then
    return UNSAT
  φ^P , μ^P ← T2B(φ), T2B(μ)
  while (TRUE) do
    T-Decide(φ^P, μ^P)
    while (TRUE) do
      res ← T-Deduce(φ^P, μ^P)
      if res = SAT then return SAT
      else if res = CONFLICT
        blevel ← T-AnalzeConflict(φ^P, μ^P)
        if (blevel < 0) then return UNSAT
        else T-Backtrack(blevel, φ^P, μ^P)
      else break
```

Extends AnalyzeConflict from CDCL:

- if the conflict is caused by a boolean (BCP) failure, returns the same blevel and conflict clause as AnalyzeConflict
- if the conflict is caused by a theory failure, returns a mixed boolean+theory conflict clause

Online DPLL(T): T-AnalyzeConflict

```
Online-DPLL_T(T-formula φ, T-assignment μ)
  if T-Preprocess(φ, μ) = CONFLICT then
    return UNSAT
  φ^P , μ^P ← T2B(φ), T2B(μ)
  while (TRUE) do
    T-Decide(φ^P, μ^P)
    while (TRUE) do
      res ← T-Deduce(φ^P, μ^P)
      if res = SAT then return SAT
      else if res = CONFLICT
        blevel ← T-AnalzeConflict(φ^P, μ^P)
        if (blevel < 0) then return UNSAT
        else T-Backtrack(blevel, φ^P, μ^P)
      else break
```

Extends AnalyzeConflict from CDCL:

- if the conflict is caused by a boolean (BCP) failure, returns the same blevel and conflict clause as AnalyzeConflict
- if the conflict is caused by a theory failure, returns a mixed boolean+theory conflict clause

Online DPLL(T): T-AnalyzeConflict

```
Online-DPLL_T(T-formula φ, T-assignment μ)
  if T-Preprocess(φ, μ) = CONFLICT then
    return UNSAT
  φ^P , μ^P ← T2B(φ), T2B(μ)
  while (TRUE) do
    T-Decide(φ^P, μ^P)
    while (TRUE) do
      res ← T-Deduce(φ^P, μ^P)
      if res = SAT then return SAT
      else if res = CONFLICT
        blevel ← T-AnalzeConflict(φ^P, μ^P)
        if (blevel < 0) then return UNSAT
        else T-Backtrack(blevel, φ^P, μ^P)
      else break
```

Extends AnalyzeConflict from CDCL:

- if the conflict is caused by a boolean (BCP) failure, returns the same blevel and conflict clause as AnalyzeConflict
- if the conflict is caused by a theory failure, returns a mixed boolean+theory conflict clause

Online DPLL(T): T-BACKTRACK

```
Online-DPLL_T(T-formula φ, T-assignment μ)
  if T-PREPROCESS(φ, μ) = CONFLICT then
    return UNSAT
  φ^P , μ^P ← T2B(φ), T2B(μ)
  while (TRUE) do
    T-DECIDE(φ^P, μ^P)
    while (TRUE) do
      res ← T-DEDUCE(φ^P, μ^P)
      if res = SAT then return SAT
      else if res = CONFLICT
        blevel ← T-ANALZECONFLICT(φ^P, μ^P)
        if (blevel < 0) then return UNSAT
        else T-BACKTRACK(blevel, φ^P, μ^P)
      else break
```

Analogous to BACKTRACK in CDCL:

- adds learned clause to φ^P (*T-learning*).
- backtracks to blevel by undoing all the assignments > blevel (*T-backjumping*).

Online DPLL(T): T-BACKTRACK

```
Online-DPLL_T(T-formula φ, T-assignment μ)
  if T-PREPROCESS(φ, μ) = CONFLICT then
    return UNSAT
  φ^P , μ^P ← T2B(φ), T2B(μ)
  while (TRUE) do
    T-DECIDE(φ^P, μ^P)
    while (TRUE) do
      res ← T-DEDUCE(φ^P, μ^P)
      if res = SAT then return SAT
      else if res = CONFLICT
        blevel ← T-ANALZECONFLICT(φ^P, μ^P)
        if (blevel < 0) then return UNSAT
        else T-BACKTRACK(blevel, φ^P, μ^P)
      else break
```

Analogous to BACKTRACK in CDCL:

- adds learned clause to φ^P (*T-learning*).
- backtracks to blevel by undoing all the assignments > blevel (*T-backjumping*).

Online DPLL(T) example

T_R-formula φ and φ^P:

1. $\neg(2x_2 - x_3 > 2) \vee A_1$, $\neg b_1 \vee A_1$
2. $\neg A_2 \vee (x_1 - x_5 \leq 1)$, $\neg A_2 \vee b_2$
3. $(3x_1 - 2x_2 \leq 3) \vee A_2$, $b_3 \vee A_2$
4. $\neg(2x_3 + x_4 \geq 5) \vee \neg(3x_1 - x_3 \leq 6) \vee \neg A_1$, $\neg b_4 \vee \neg b_5 \vee \neg A_1$
5. $A_1 \vee (3x_1 - 2x_2 \leq 3)$, $A_1 \vee b_3$
6. $(x_2 - x_4 \leq 6) \vee (x_5 = 5 - 3x_4) \vee \neg A_1$, $b_6 \vee b_7 \vee \neg A_1$
7. $A_1 \vee (x_3 = 3x_5 + 4) \vee A_2$, $A_1 \vee b_8 \vee A_2$

Online DPLL(T) example: T-Decide

T_R-formula φ and φ^P:

1. $\neg(2x_2 - x_3 > 2) \vee A_1, \neg b_1 \vee A_1$
2. $\neg A_2 \vee (x_1 - x_5 \leq 1), \neg A_2 \vee b_2$
3. $(3x_1 - 2x_2 \leq 3) \vee A_2, \; b_3 \vee A_2$
4. $\neg(2x_3 + x_4 \geq 5) \vee \neg(3x_1 - x_3 \leq 6) \vee \neg A_1, \; \neg b_4 \vee \neg b_5 \vee \neg A_1$
5. $A_1 \vee (3x_1 - 2x_2 \leq 3), \; A_1 \vee b_3$
6. $(x_2 - x_4 \leq 6) \vee (x_5 = 5 - 3x_4) \vee \neg A_1, \; b_6 \vee b_7 \vee \neg A_1$
7. $A_1 \vee (x_3 = 3x_5 + 4) \vee A_2, \; A_1 \vee b_8 \vee A_2$

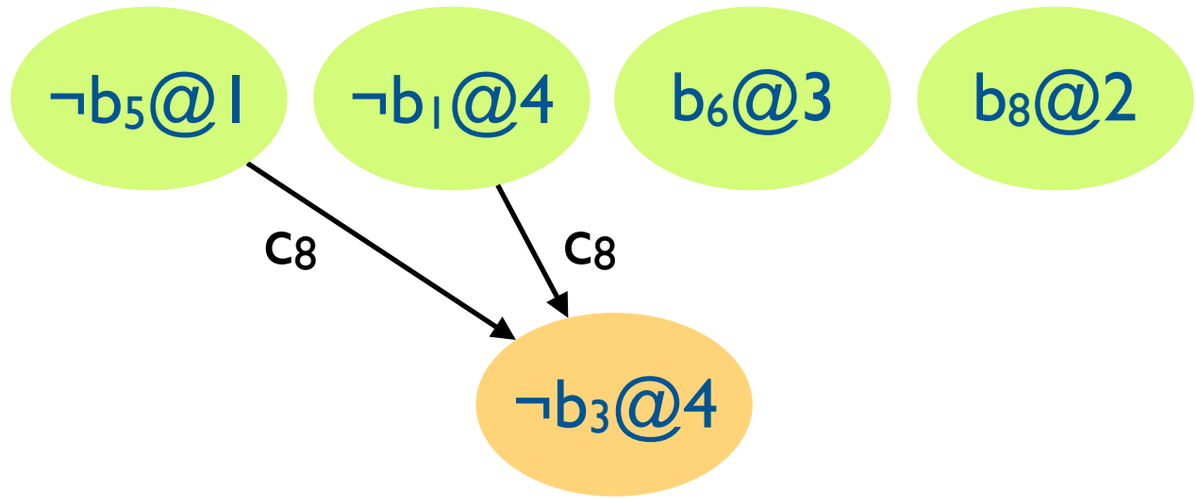
$\neg b_5@1$ $\neg b_1@4$ $b_6@3$ $b_8@2$

T-DECIDE makes 4 decisions.

Online DPLL(T) example: T-Deduce

T_R-formula φ and φ^P:

1. $\neg(2x_2 - x_3 > 2) \vee A_1, \neg b_1 \vee A_1$
2. $\neg A_2 \vee (x_1 - x_5 \leq 1), \neg A_2 \vee b_2$
3. $(3x_1 - 2x_2 \leq 3) \vee A_2, \; b_3 \vee A_2$
4. $\neg(2x_3 + x_4 \geq 5) \vee \neg(3x_1 - x_3 \leq 6) \vee \neg A_1, \; \neg b_4 \vee \neg b_5 \vee \neg A_1$
5. $A_1 \vee (3x_1 - 2x_2 \leq 3), \; A_1 \vee b_3$
6. $(x_2 - x_4 \leq 6) \vee (x_5 = 5 - 3x_4) \vee \neg A_1, \; b_6 \vee b_7 \vee \neg A_1$
7. $A_1 \vee (x_3 = 3x_5 + 4) \vee A_2, \; A_1 \vee b_8 \vee A_2$
8. $b_5 \vee b_1 \vee \neg b_3$

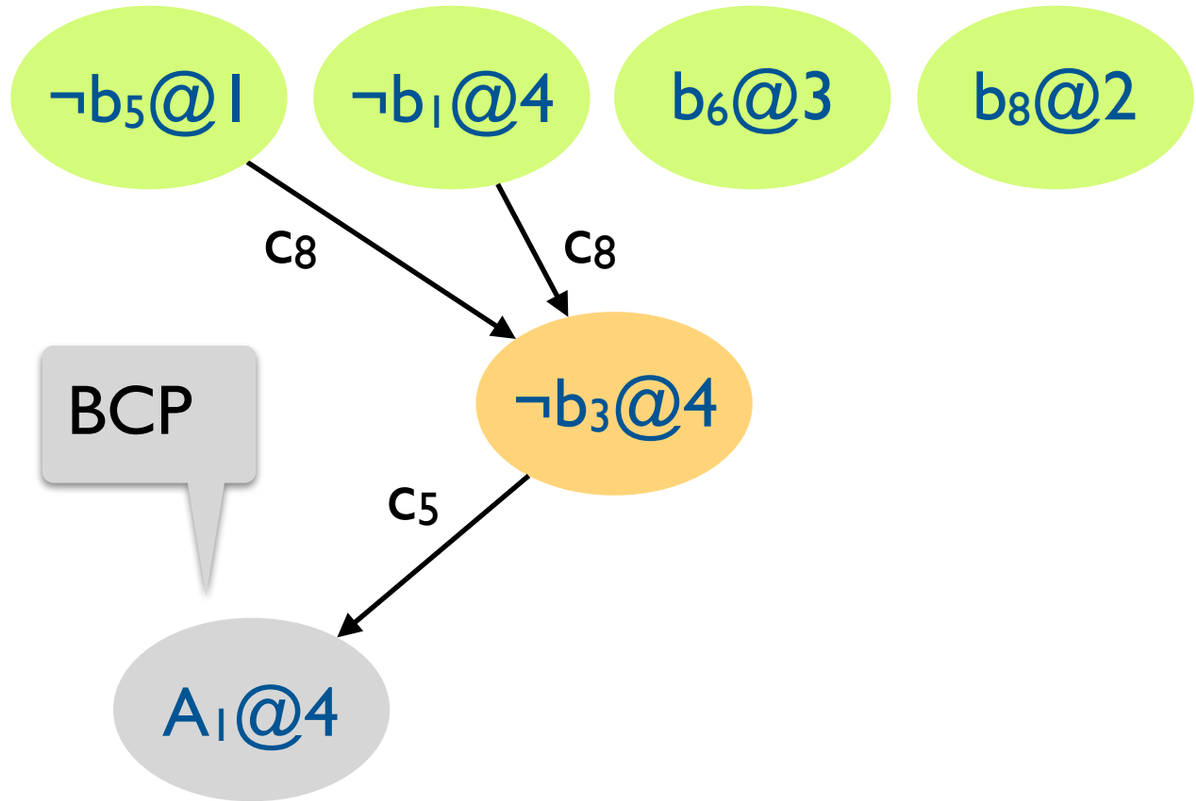


Early pruning & T-propagation.

Online DPLL(T) example: T-Deduce

T_R-formula φ and φ^P:

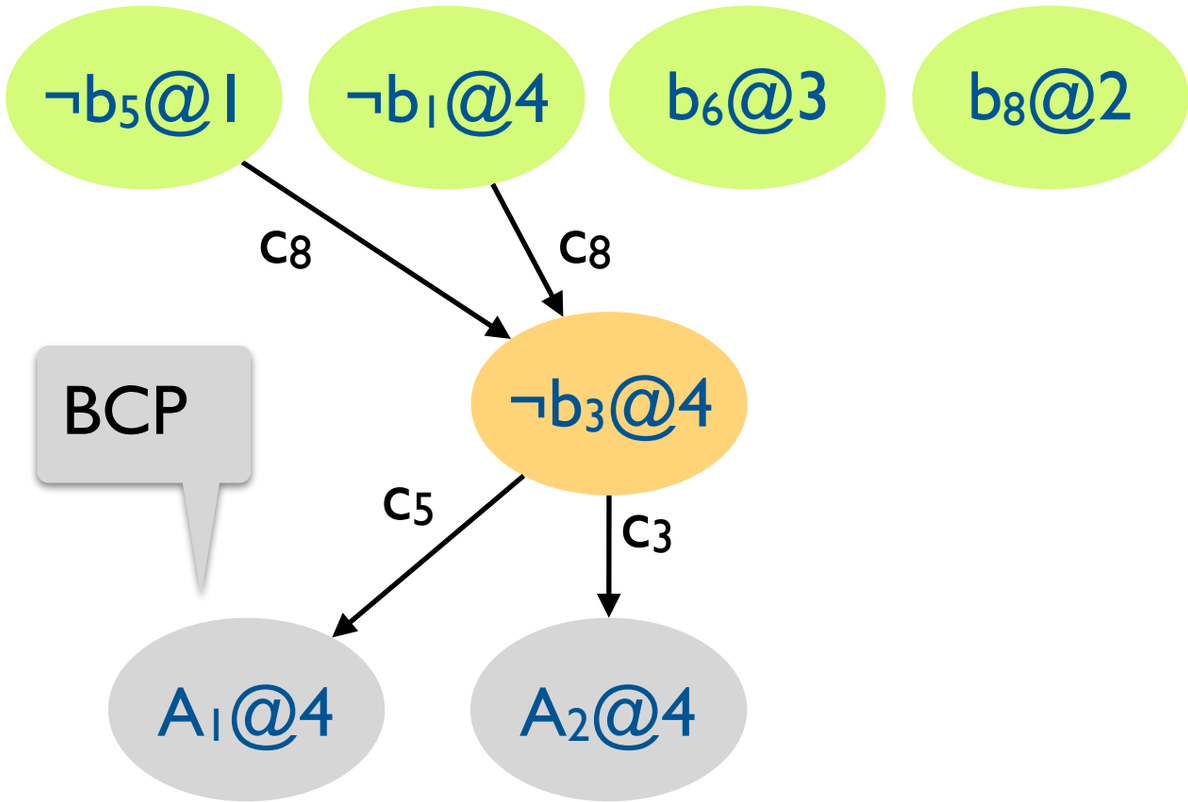
1. $\neg(2x_2 - x_3 > 2) \vee A_1$, $\neg b_1 \vee A_1$
2. $\neg A_2 \vee (x_1 - x_5 \leq 1)$, $\neg A_2 \vee b_2$
3. $(3x_1 - 2x_2 \leq 3) \vee A_2$, $b_3 \vee A_2$
4. $\neg(2x_3 + x_4 \geq 5) \vee \neg(3x_1 - x_3 \leq 6) \vee \neg A_1$, $\neg b_4 \vee \neg b_5 \vee \neg A_1$
5. $A_1 \vee (3x_1 - 2x_2 \leq 3)$, $A_1 \vee b_3$
6. $(x_2 - x_4 \leq 6) \vee (x_5 = 5 - 3x_4) \vee \neg A_1$, $b_6 \vee b_7 \vee \neg A_1$
7. $A_1 \vee (x_3 = 3x_5 + 4) \vee A_2$, $A_1 \vee b_8 \vee A_2$
8. $b_5 \vee b_1 \vee \neg b_3$



Online DPLL(T) example: T-Deduce

T_R-formula φ and φ^P:

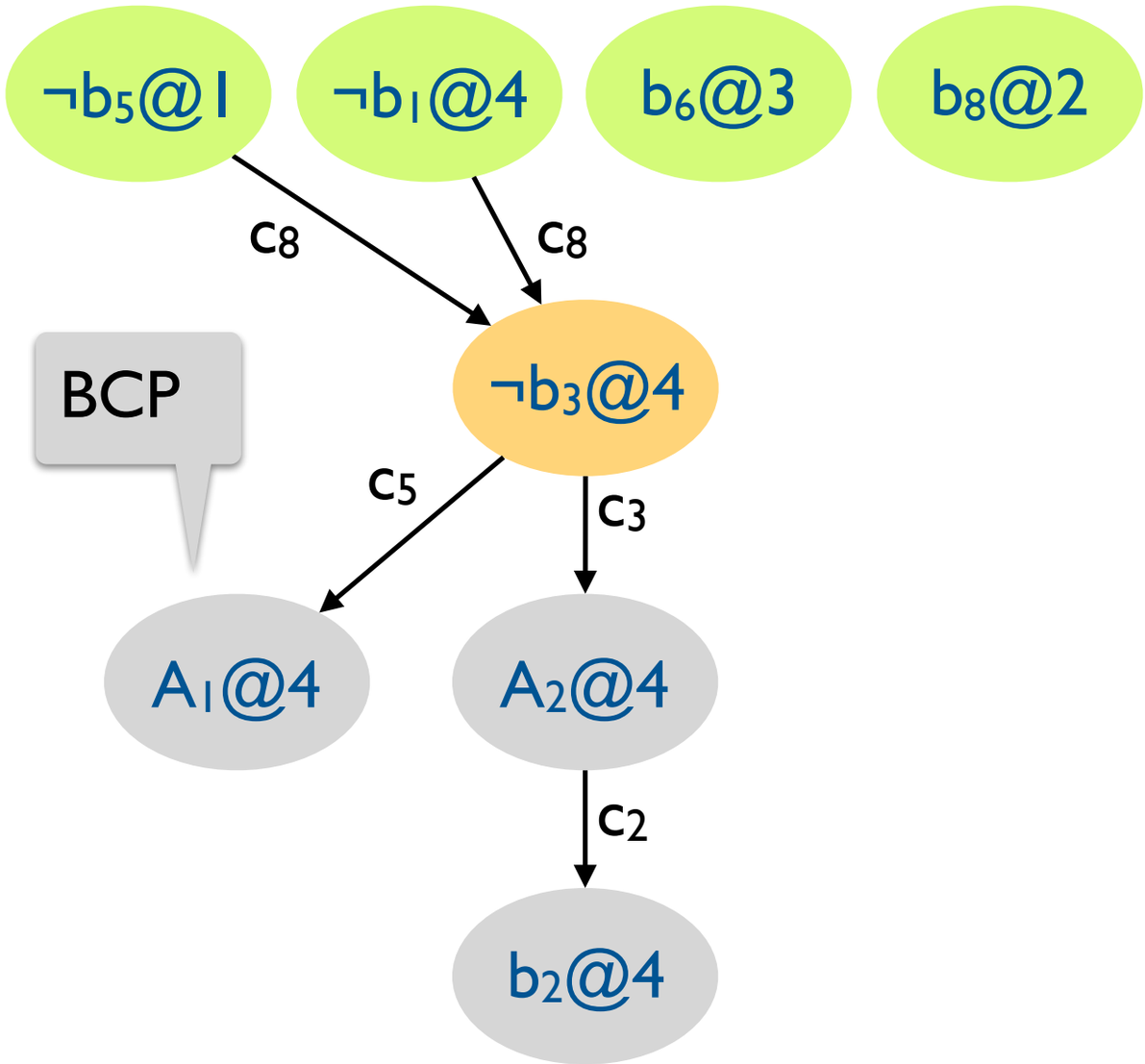
1. $\neg(2x_2 - x_3 > 2) \vee A_1$, $\neg b_1 \vee A_1$
2. $\neg A_2 \vee (x_1 - x_5 \leq 1)$, $\neg A_2 \vee b_2$
3. $(3x_1 - 2x_2 \leq 3) \vee A_2$, $b_3 \vee A_2$
4. $\neg(2x_3 + x_4 \geq 5) \vee \neg(3x_1 - x_3 \leq 6) \vee \neg A_1$, $\neg b_4 \vee \neg b_5 \vee \neg A_1$
5. $A_1 \vee (3x_1 - 2x_2 \leq 3)$, $A_1 \vee b_3$
6. $(x_2 - x_4 \leq 6) \vee (x_5 = 5 - 3x_4) \vee \neg A_1$, $b_6 \vee b_7 \vee \neg A_1$
7. $A_1 \vee (x_3 = 3x_5 + 4) \vee A_2$, $A_1 \vee b_8 \vee A_2$
8. $b_5 \vee b_1 \vee \neg b_3$



Online DPLL(T) example: T-Deduce

T_R-formula φ and φ^P:

1. $\neg(2x_2 - x_3 > 2) \vee A_1$, $\neg b_1 \vee A_1$
2. $\neg A_2 \vee (x_1 - x_5 \leq 1)$, $\neg A_2 \vee b_2$
3. $(3x_1 - 2x_2 \leq 3) \vee A_2$, $b_3 \vee A_2$
4. $\neg(2x_3 + x_4 \geq 5) \vee \neg(3x_1 - x_3 \leq 6) \vee \neg A_1$, $\neg b_4 \vee \neg b_5 \vee \neg A_1$
5. $A_1 \vee (3x_1 - 2x_2 \leq 3)$, $A_1 \vee b_3$
6. $(x_2 - x_4 \leq 6) \vee (x_5 = 5 - 3x_4) \vee \neg A_1$, $b_6 \vee b_7 \vee \neg A_1$
7. $A_1 \vee (x_3 = 3x_5 + 4) \vee A_2$, $A_1 \vee b_8 \vee A_2$
8. $b_5 \vee b_1 \vee \neg b_3$



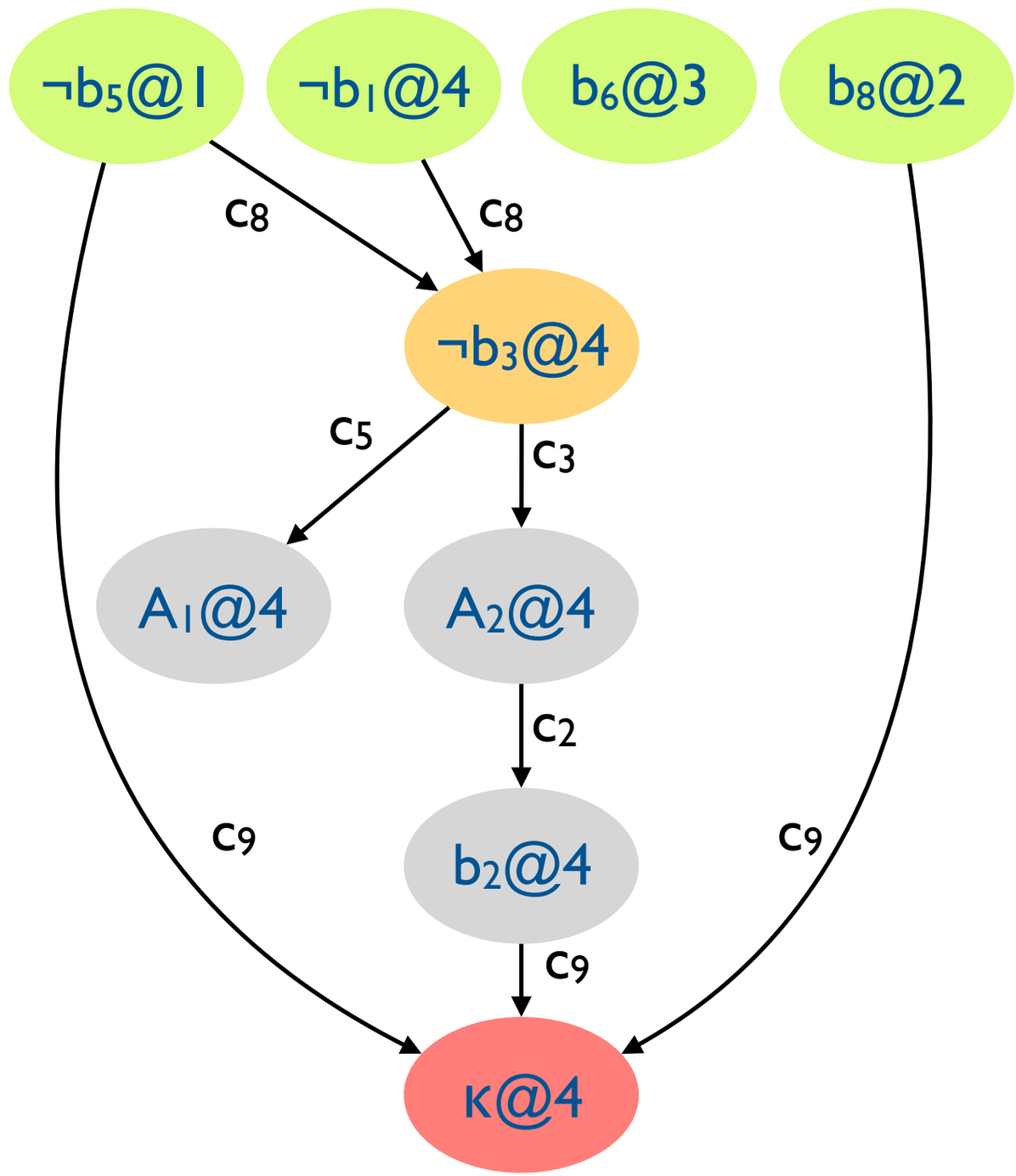
Online DPLL(T) example: T-Deduce

Current partial assignment is T-UNSAT.

T_R-formula φ and φ^P:

1. $\neg(2x_2 - x_3 > 2) \vee A_1$, $\neg b_1 \vee A_1$
2. $\neg A_2 \vee (x_1 - x_5 \leq 1)$, $\neg A_2 \vee b_2$
3. $(3x_1 - 2x_2 \leq 3) \vee A_2$, $b_3 \vee A_2$
4. $\neg(2x_3 + x_4 \geq 5) \vee \neg(3x_1 - x_3 \leq 6) \vee \neg A_1$, $\neg b_4 \vee \neg b_5 \vee \neg A_1$
5. $A_1 \vee (3x_1 - 2x_2 \leq 3)$, $A_1 \vee b_3$
6. $(x_2 - x_4 \leq 6) \vee (x_5 = 5 - 3x_4) \vee \neg A_1$, $b_6 \vee b_7 \vee \neg A_1$
7. $A_1 \vee (x_3 = 3x_5 + 4) \vee A_2$, $A_1 \vee b_8 \vee A_2$
8. $b_5 \vee b_1 \vee \neg b_3$
9. $b_5 \vee \neg b_8 \vee \neg b_2$

Theory conflict.

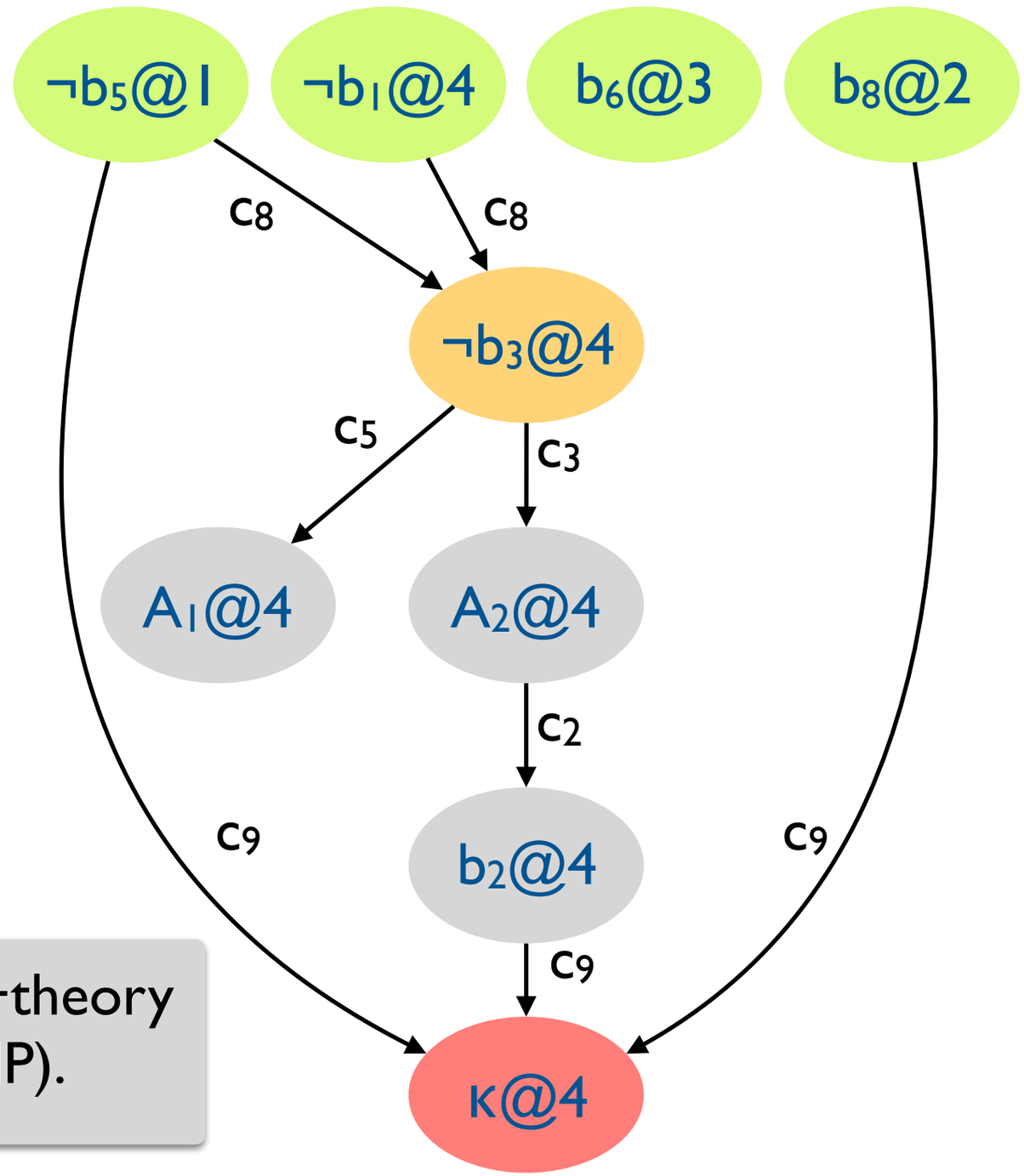


Online DPLL(T) example: T-AnalyzeConflict

T_R-formula φ and φ^P:

1. $\neg(2x_2 - x_3 > 2) \vee A_1$, $\neg b_1 \vee A_1$
2. $\neg A_2 \vee (x_1 - x_5 \leq 1)$, $\neg A_2 \vee b_2$
3. $(3x_1 - 2x_2 \leq 3) \vee A_2$, $b_3 \vee A_2$
4. $\neg(2x_3 + x_4 \geq 5) \vee \neg(3x_1 - x_3 \leq 6) \vee \neg A_1$, $\neg b_4 \vee \neg b_5 \vee \neg A_1$
5. $A_1 \vee (3x_1 - 2x_2 \leq 3)$, $A_1 \vee b_3$
6. $(x_2 - x_4 \leq 6) \vee (x_5 = 5 - 3x_4) \vee \neg A_1$, $b_6 \vee b_7 \vee \neg A_1$
7. $A_1 \vee (x_3 = 3x_5 + 4) \vee A_2$, $A_1 \vee b_8 \vee A_2$
8. $b_5 \vee b_1 \vee \neg b_3$
9. $b_5 \vee \neg b_8 \vee \neg b_2$

Mixed boolean+theory conflict (first UIP).

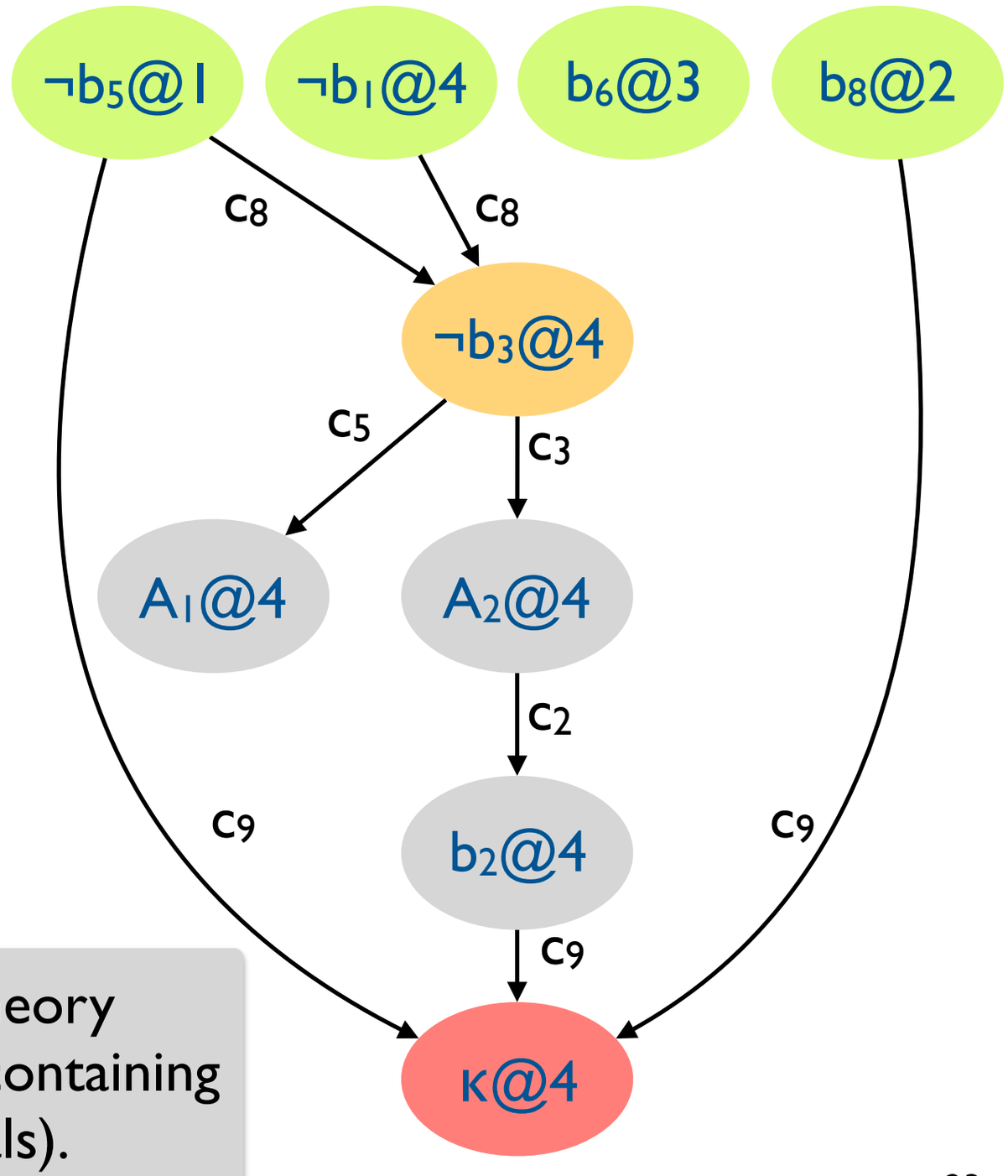


Online DPLL(T) example: T-AnalyzeConflict

T_R-formula φ and φ^P:

1. $\neg(2x_2 - x_3 > 2) \vee A_1$, $\neg b_1 \vee A_1$
2. $\neg A_2 \vee (x_1 - x_5 \leq 1)$, $\neg A_2 \vee b_2$
3. $(3x_1 - 2x_2 \leq 3) \vee A_2$, $b_3 \vee A_2$
4. $\neg(2x_3 + x_4 \geq 5) \vee \neg(3x_1 - x_3 \leq 6) \vee \neg A_1$, $\neg b_4 \vee \neg b_5 \vee \neg A_1$
5. $A_1 \vee (3x_1 - 2x_2 \leq 3)$, $A_1 \vee b_3$
6. $(x_2 - x_4 \leq 6) \vee (x_5 = 5 - 3x_4) \vee \neg A_1$, $b_6 \vee b_7 \vee \neg A_1$
7. $A_1 \vee (x_3 = 3x_5 + 4) \vee A_2$, $A_1 \vee b_8 \vee A_2$
8. $b_5 \vee b_1 \vee \neg b_3$
9. $b_5 \vee \neg b_8 \vee \neg b_2$
10. $b_5 \vee \neg b_8 \vee b_1$

Mixed boolean+theory conflict (last UIP, containing just decision literals).



Online DPLL(T) example: T-Backtrack

T_R-formula φ and φ^P:

1. $\neg(2x_2 - x_3 > 2) \vee A_1$, $\neg b_1 \vee A_1$
2. $\neg A_2 \vee (x_1 - x_5 \leq 1)$, $\neg A_2 \vee b_2$
3. $(3x_1 - 2x_2 \leq 3) \vee A_2$, $b_3 \vee A_2$
4. $\neg(2x_3 + x_4 \geq 5) \vee \neg(3x_1 - x_3 \leq 6) \vee \neg A_1$, $\neg b_4 \vee \neg b_5 \vee \neg A_1$
5. $A_1 \vee (3x_1 - 2x_2 \leq 3)$, $A_1 \vee b_3$
6. $(x_2 - x_4 \leq 6) \vee (x_5 = 5 - 3x_4) \vee \neg A_1$, $b_6 \vee b_7 \vee \neg A_1$
7. $A_1 \vee (x_3 = 3x_5 + 4) \vee A_2$, $A_1 \vee b_8 \vee A_2$
8. $b_5 \vee b_1 \vee \neg b_3$
9. $b_5 \vee \neg b_8 \vee \neg b_2$
10. $b_5 \vee \neg b_8 \vee b_1$

$\neg b_5$@1

b_8@2

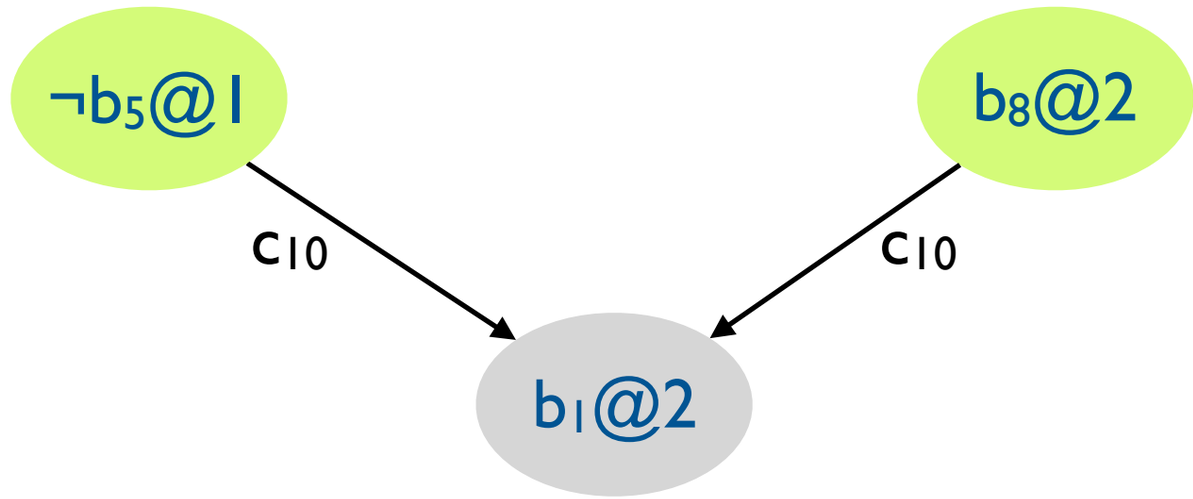
Backtrack to the second highest decision level (2).

Online DPLL(T) example: T-Backtrack

T_R-formula φ and φ^P:

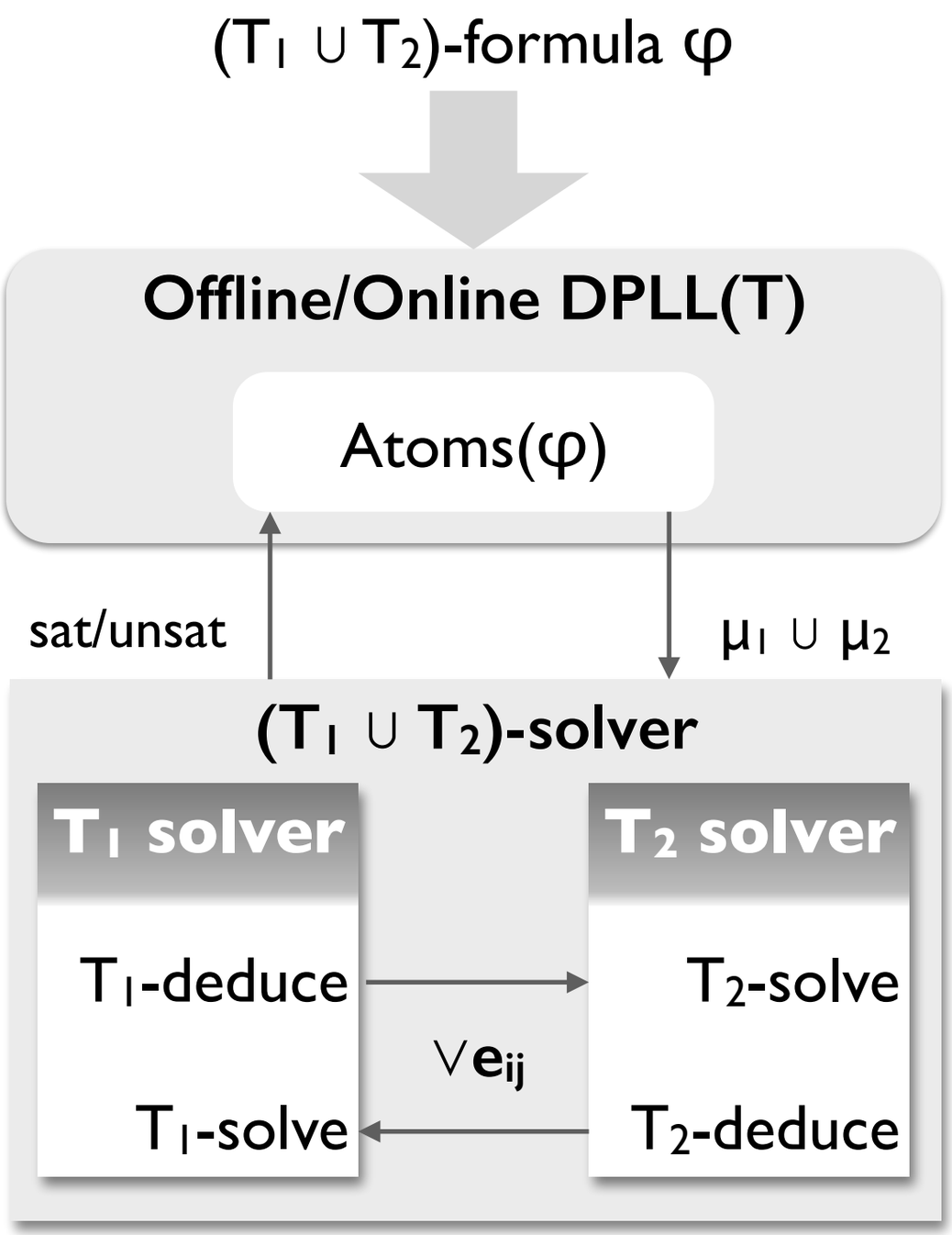
1. $\neg(2x_2 - x_3 > 2) \vee A_1$, $\neg b_1 \vee A_1$
2. $\neg A_2 \vee (x_1 - x_5 \leq 1)$, $\neg A_2 \vee b_2$
3. $(3x_1 - 2x_2 \leq 3) \vee A_2$, $b_3 \vee A_2$
4. $\neg(2x_3 + x_4 \geq 5) \vee \neg(3x_1 - x_3 \leq 6) \vee \neg A_1$, $\neg b_4 \vee \neg b_5 \vee \neg A_1$
5. $A_1 \vee (3x_1 - 2x_2 \leq 3)$, $A_1 \vee b_3$
6. $(x_2 - x_4 \leq 6) \vee (x_5 = 5 - 3x_4) \vee \neg A_1$, $b_6 \vee b_7 \vee \neg A_1$
7. $A_1 \vee (x_3 = 3x_5 + 4) \vee A_2$, $A_1 \vee b_8 \vee A_2$
8. $b_5 \vee b_1 \vee \neg b_3$
9. $b_5 \vee \neg b_8 \vee \neg b_2$
10. $b_5 \vee \neg b_8 \vee b_1$

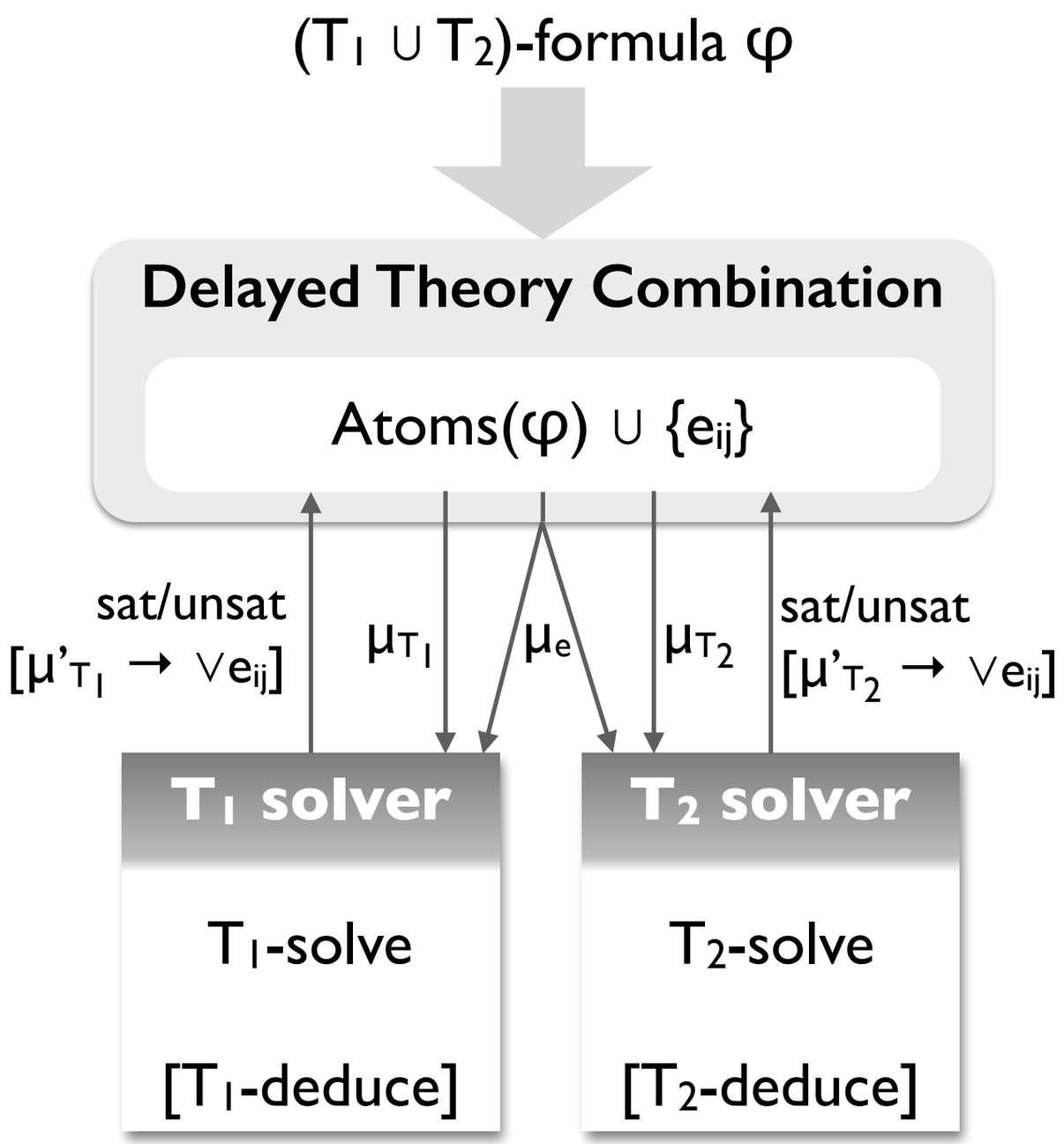
Backtrack to the second highest decision level (2).



b_1 immediately implied by BCP.

The DPLL(T) Framework





R. Bruttomesso et al., DTC vs NO, 2009

Delayed Theory Combination (DTC)

Online-DPLL$_T$(T-formula φ, T-assignment μ)
  if **T-PREPROCESS**(φ, μ) = CONFLICT **then**
    return UNSAT
  φ^P , μ^P ← **T2B**(φ), **T2B**(μ)
  while (TRUE) **do**
    T-DECIDE(φ^P, μ^P)
    while (TRUE) **do**
      res ← **T-DEDUCE**(φ^P, μ^P)
      if res = SAT **then return** SAT
      else if res = CONFLICT
        blevel ← **T-ANALZECONFLICT**(φ^P, μ^P)
        if (blevel < 0) **then return** UNSAT
        else **T-BACKTRACK**(blevel, φ^P, μ^P)
      else break

To get DTC, modify Online-DPLL$_T$ so that

- Truth values assigned to both atoms in φ and the *interface equalities* e_{ij} not in φ.
- T-DECIDE branches on interface equalities e_{ij} after μ propositionally satisfies φ.
- T-DEDUCE passes $\mu_i \cup \mu_e$ to each T_i-solver and returns SAT if both return SAT. Otherwise returns CONFLICT.
- T-ANALYZECONFLICT and T-BACKTRACK use the conflict clause (possibly containing interface equalities) from one of the T_i-solvers.
- Early pruning and T-PROPAGATION are performed.

Summary

Today

- The DPLL(T) framework for deciding SMT formulas

Next lecture

- Finite model finding: reasoning about quantified formulas over finite domains
- Last lecture on **computer-aided reasoning**
- It's all **for software** afterwards!