Computer-Aided Reasoning for Software

CSE507

Reasoning about Programs

courses.cs.washington.edu/courses/cse507/16sp/

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Overview

topics

Last lecture

- Finite model finding for first-order logic with quantifiers, relations, and transitive closure

Today

- Reasoning about (partial) correctness of programs
  - Hoare Logic
  - Verification Condition Generation

A look ahead (L9–L14)

Classic verification (L9, L10)

- Checking that all (terminating) executions satisfy an FOL property on all inputs

Bounded verification (L11)

- Scope-complete checking of FOL properties

Symbolic execution (L12)

- Systematic checking of FOL properties

Model checking (L13, L14)

- Exhaustive checking of temporal properties of abstracted programs

Active research topic for 45 years

Classic ideas every computer scientist should know

Understanding the ideas can help you become a better programmer

A bit of history

1967: *Assigning Meaning to Programs* (Floyd)

- 1978 Turing Award

1969: *An Axiomatic Basis for Computer Programming* (Hoare)

- 1980 Turing Award

1975: *Guarded Commands, Nondeterminacy and Formal Derivation of Programs* (Dijkstra)

- 1972 Turing Award

A tiny Imperative Programming Language (IMP)

Expression E

- Z | V | $E_1 + E_2$ | $E_1 * E_2$

Conditional C

- true | false | $E_1 = E_2$ | $E_1 \leq E_2$

Statement S

- **skip** (Skip)
- V := E (Assignment)
- S_1; S_2 (Composition)
- **if** C **then** S_1 **else** S_2 (If)
- **while** C **do** S (While)

A minimalist programming language for demonstrating key features of Hoare logic.

Specifying correctness in Hoare logic

{P} S {Q}

Specifying correctness in Hoare logic

Hoare triple

{P} S {Q}

- S is a program statement (in IMP).
- P and Q are FOL formulas over program variables.
- P is called a *precondition* and Q is a *postcondition*.

safety

Partial correctness (Hoare triple semantics)

- If S executes from a state satisfying P, and if its execution terminates, then the resulting state satisfies Q.

liveness

[P] S [Q]

Total correctness

- If S executes from a state satisfying P, then its execution terminates and the resulting state satisfies Q.

Examples of Hoare triples

{false} S {Q}

- Valid for all S and Q.

{P} while (true) do skip {Q}

- Valid for all P and Q.

{true} S {Q}

- If S terminates, the resulting state satisfies Q.

{P} S {true}

- Valid for all P and S.

Proving partial correctness in Hoare logic

Expression E

- Z | V | E_1 + E_2 | E_1 * E_2

Conditional C

- true | false | $E_1 = E_2$ | $E_1 \leq E_2$

Statement S

- **skip** (Skip)
- V := E (Assignment)
- S_1; S_2 (Composition)
- **if** C **then** S_1 **else** S_2 (If)
- **while** C **do** S (While)

One inference rule for every statement in the language:

$$\frac{\vdash \{P_1\}S_1\{Q_1\} \;\ldots\; \vdash \{P_n\}S_n\{Q_n\}}{\vdash \{P\}S\{Q\}}$$

If the Hoare triples $\{P_1\} S_1\{Q_1\}$ … $\{P_n\}S_n\{Q_n\}$ are provable, then so is $\{P\}S\{Q\}$.

Inference rules for Hoare logic

$$\frac{}{\vdash \{P\}\ \textbf{skip}\ \{P\}}$$

$$\frac{\vdash \{P\}\ S_1\ \{R\} \qquad \vdash \{R\}\ S_2\ \{Q\}}{\vdash \{P\}\ S_1; S_2\ \{Q\}}$$

$$\frac{}{\vdash \{Q[E/x]\}\ x := E\ \{Q\}}$$

$$\frac{\vdash \{P \land C\}\ S_1\ \{Q\} \quad \vdash \{P \land \neg C\}\ S_2\ \{Q\}}{\vdash \{P\}\ \textbf{if}\ C\ \textbf{then}\ S_1\ \textbf{else}\ S_2\ \{Q\}}$$

$$\frac{\vdash \{P_1\}\ S\ \{Q_1\} \qquad P \Rightarrow P_1 \quad Q_1 \Rightarrow Q}{\vdash \{P\}\ S\ \{Q\}}$$

$$\frac{\vdash \{P \land C\}\ S\ \{P\}}{\vdash \{P\}\ \textbf{while}\ C\ \textbf{do}\ S\ \{P \land \neg C\}}$$

loop invariant

Example: proof outline

$\{x \leq n\}$
while $(x < n)$ **do**
  $\{x \leq n \wedge x < n\}$
  $\{x+1 \leq n\}$ // consequence
  $x := x + 1$
  $\{x \leq n\}$ // assignment
$\{x \leq n \wedge x \geq n\}$ // while
$\{x \geq n\}$ // consequence

Example: proof outline with auxiliary variables

$\{x = X \wedge y = Y\}$

$\{y = Y \wedge x = X\}$

t := x

$\{y = Y \wedge t = X\}$ // assignment

x := y

$\{x = Y \wedge t = X\}$ // assignment

y := t

$\{x = Y \wedge y = X\}$ // assignment

Soundness and relative completeness

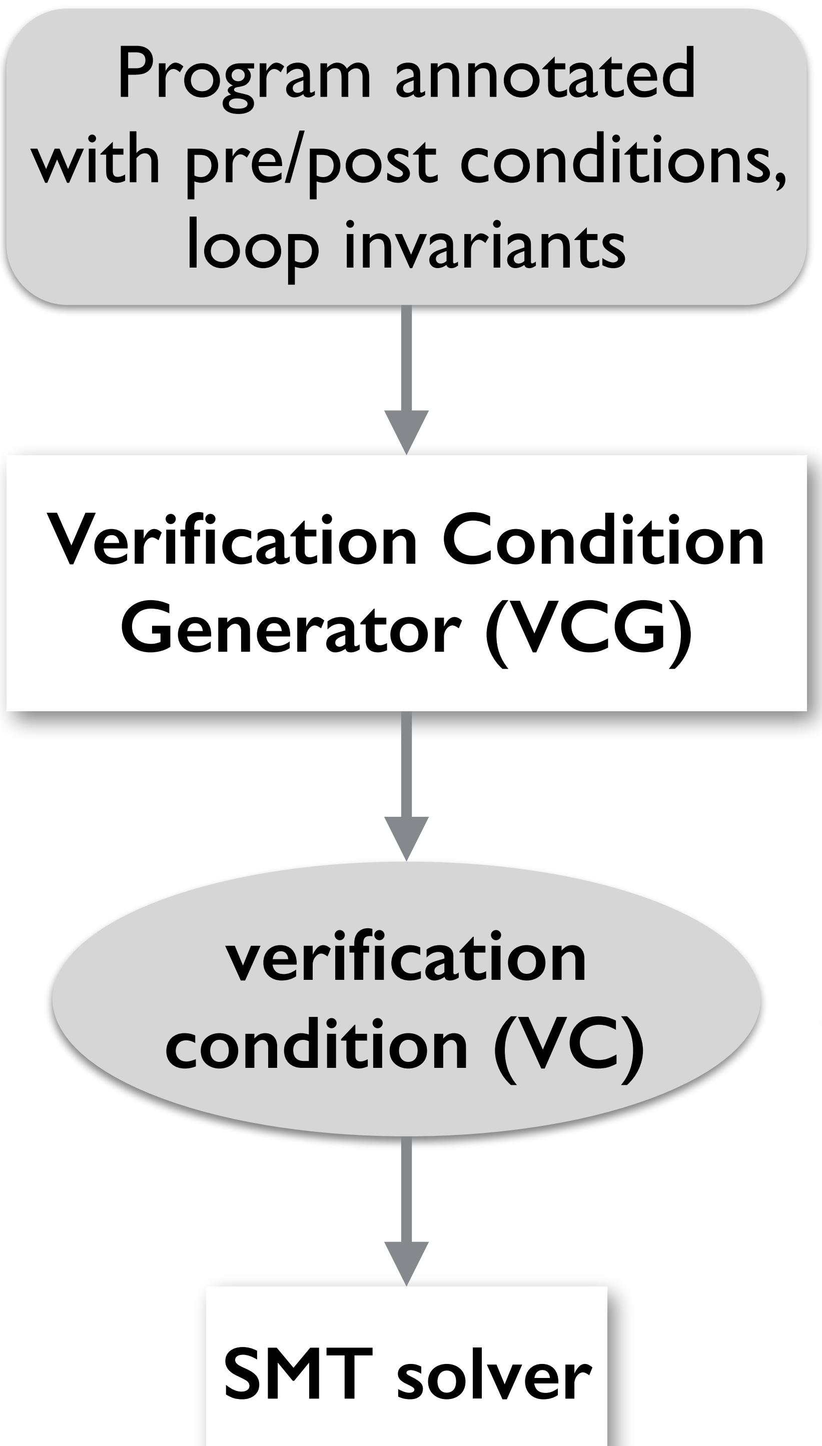
Proof rules for Hoare logic are sound

If $\vdash \{P\}\ S\ \{Q\}$ then $\models \{P\}\ S\ \{Q\}$

Proof rules for Hoare logic are relatively complete

If $\models \{P\}\ S\ \{Q\}$ then $\vdash \{P\}\ S\ \{Q\}$, assuming an oracle for deciding implications

Automating Hoare logic with VC generation



Forwards computation:

- Starting from the precondition, generate formulas to prove the postcondition.
- Based on computing ***strongest postconditions (sp).***

Backwards computation:

- Starting from the postcondition, generate formulas to prove the precondition.
- Based on computing ***weakest liberal preconditions (wp).***

VC generation with WP and SP

wp(S, Q)

- The weakest predicate that guarantees Q will hold after executing S from a state satisfying that predicate.

sp(S, P)

- The strongest predicate that holds after S is executed from a state satisfying P.

{P} S {Q} is valid iff

- $P \Rightarrow wp(S, Q)$ or
- $sp(S, P) \Rightarrow Q$

Computing wp(S, Q)

wp(S, Q):

- wp(**skip**, Q) = Q
- wp(x := E, Q) = Q[E / x]
- wp(S_1; S_2, Q) = wp(S_1, wp(S_2, Q))
- wp(**if** C **then** S_1 **else** S_2, Q) = C → wp(S_1, Q) ∧ ¬C → wp(S_2, Q)
- wp(**while** C **do** S, Q) = ✗

Approximate wp(S, Q) with awp(S, Q).

terms of the postcondition.

Computing awp(S, Q)

awp(S, Q):

- awp(**skip**, Q) = Q
- awp(x := E, Q) = Q[E / x]
- awp(S_1; S_2, Q) = awp(S_1, awp(S_2, Q))
- awp(**if** C **then** S_1 **else** S_2, Q) = C → awp(S_1, Q) ∧ ¬C → awp(S_2, Q)
- awp(**while** C **do** {I} S, Q) = I

For each statement S, also define VC(S,Q) that encodes additional conditions that must be checked.

Computing VC(S, Q)

VC(S, Q):

- VC(**skip**, Q) = true
- VC(x := E, Q) = true
- VC(S_1; S_2, Q) = VC(S_2, Q) $\wedge$ VC(S_1, awp(S_2, Q))
- VC(**if** C **then** S_1 **else** S_2, Q) = VC(S_1, Q) $\wedge$ VC(S_2, Q)
- VC(**while** C **do** {I} S, Q) = (I$\wedge$C $\Rightarrow$ awp(S,I)) $\wedge$ VC(S,I) $\wedge$ (I$\wedge\neg$C $\Rightarrow$ Q)

I is an invariant.

I is strong enough.

Verifying a Hoare triple

Theorem: {P} S {Q} is valid if

$$VC(S, Q) \land (P \rightarrow awp(S, Q))$$

The other direction doesn't hold because loop invariants may not be strong enough or they may be incorrect.

Might get false alarms.

Summary

Today

- Reasoning about partial correctness of programs
  - Hoare Logic
  - VCG, WP, SP

Next lecture

- Guest lecture by Rustan Leino!
- Verification with Dafny, Boogie, and Z3.