Computer-Aided Reasoning for Software

CSE507

Finite Model Finding

courses.cs.washington.edu/courses/cse507/16sp/

Emina Torlak
emina@cs.washington.edu

Today

Last lecture

- The DPPL(T) framework for deciding quantifier-free SMT formulas

Today

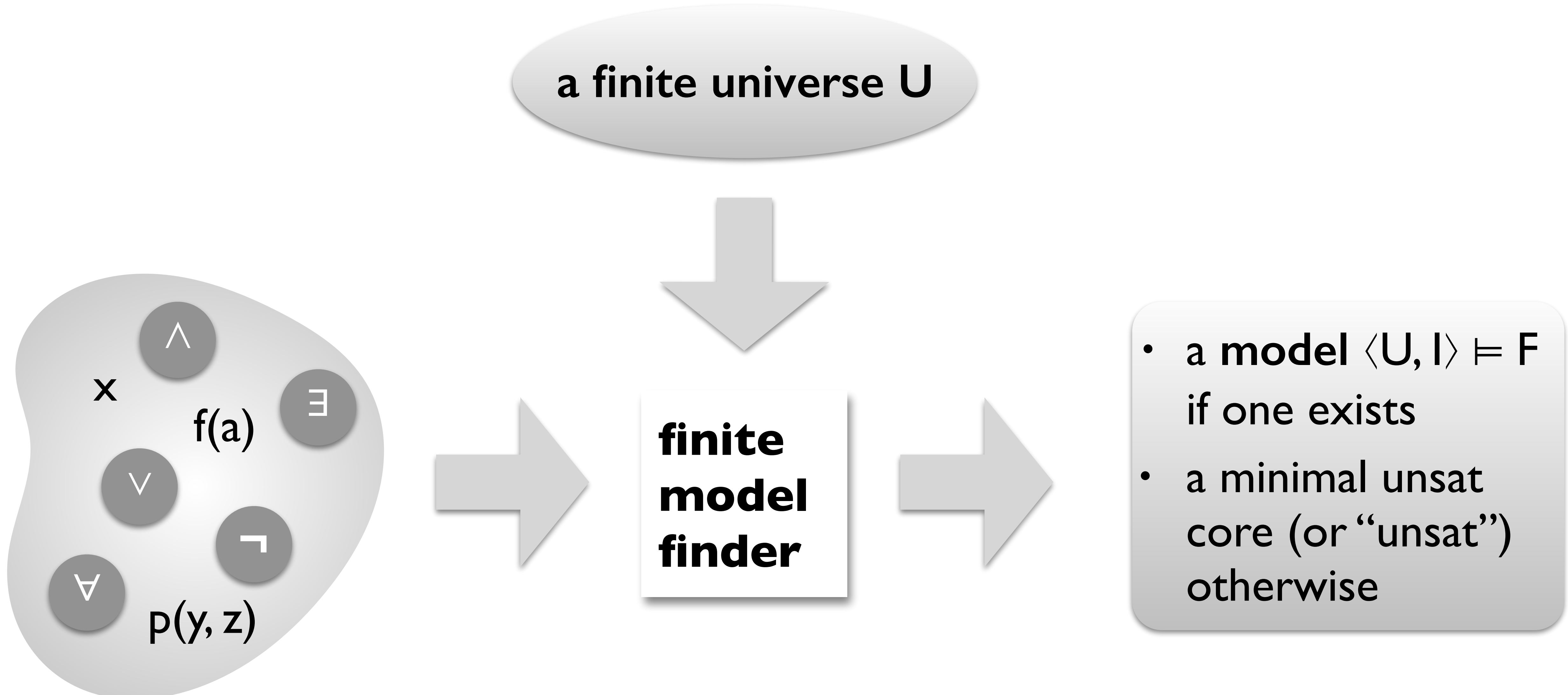
- Finite model finding for quantified FOL and beyond

Reminders

- HW2 is due in a week!
- No office hours on Friday.

topics

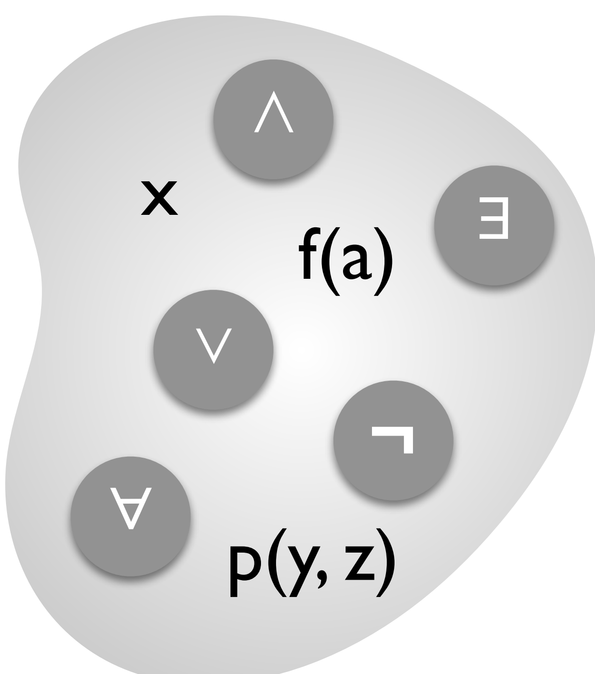
Finite model finding



a formula F in FOL

Finite model finding

a finite universe U



a formula F in FOL

finite model finder

- custom search (SEM, CVC4)
- reduction to SAT (Paradox, Kodkod)

- a **model** $\langle U, I\rangle \models F$ if one exists
- a minimal unsat core (or "unsat") otherwise

Some applications of finite model finding

Proving theorems in finite algebras (Finder, SEM, MACE)

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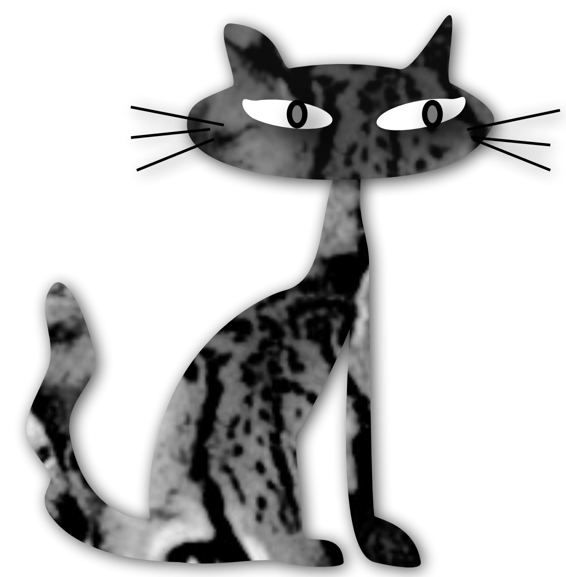
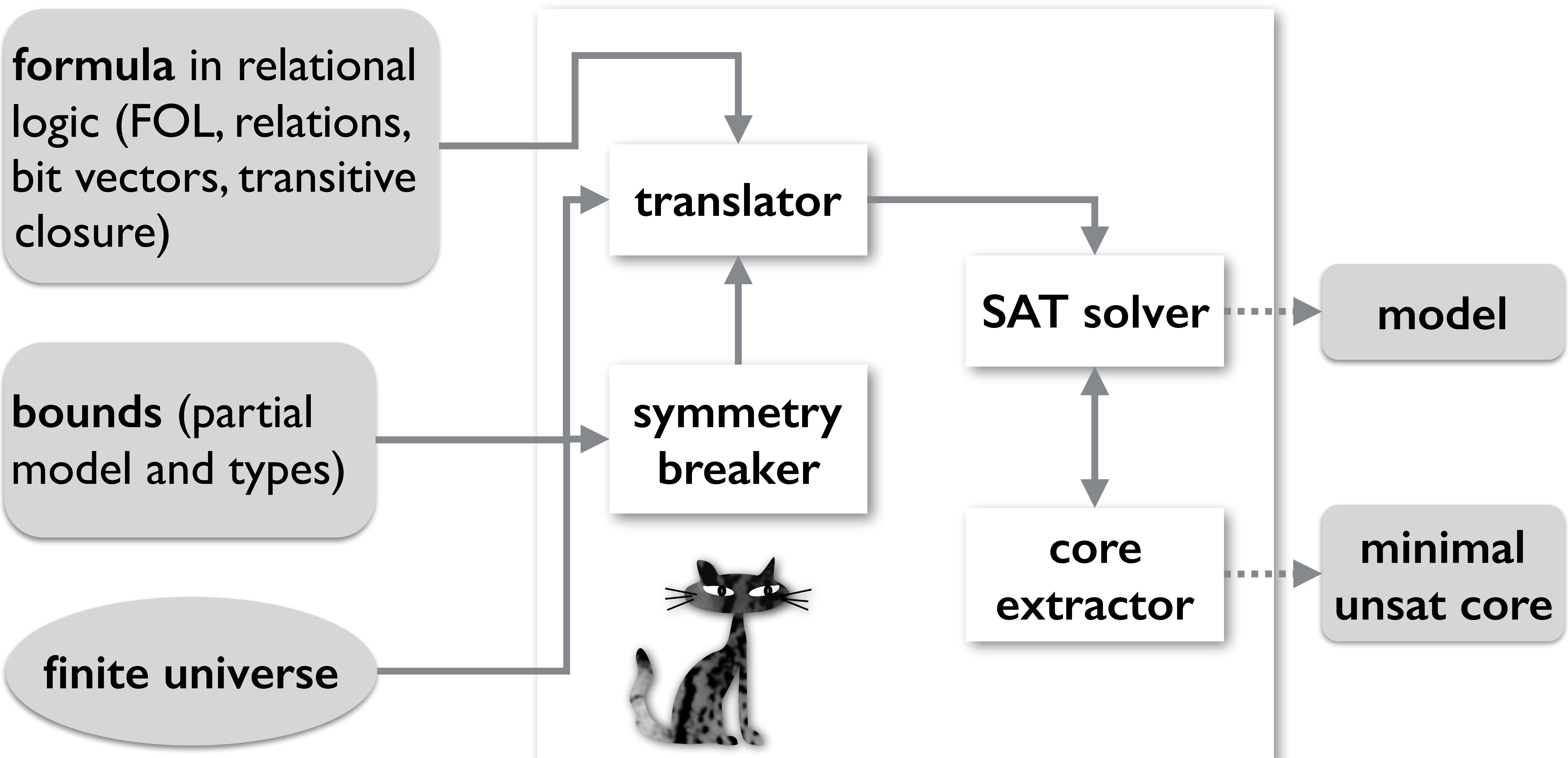


KODKOD

Overview of Kodkod



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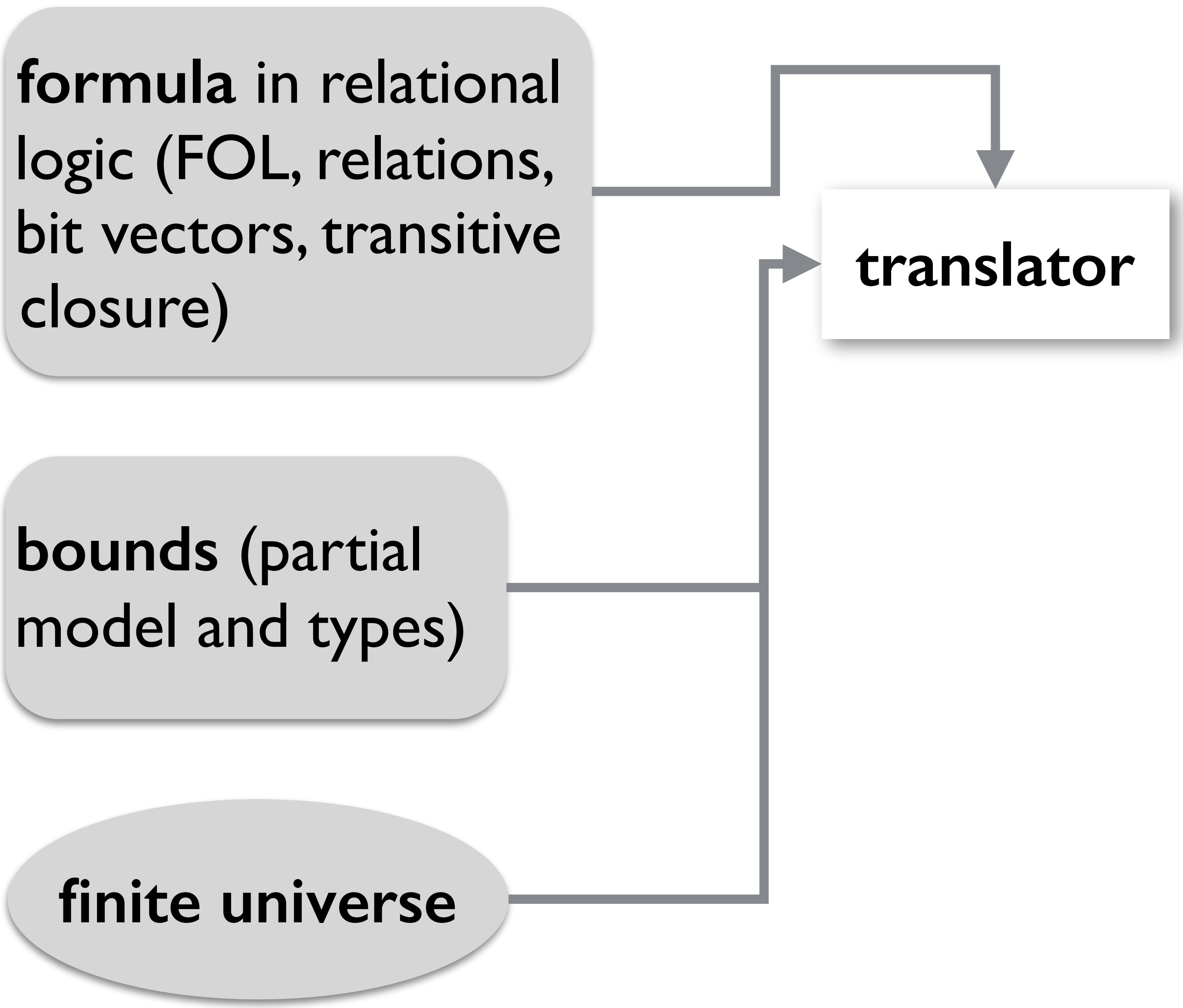

Relational logic by example

a minimalistic formal specification of a filesystem

Relational logic by example

Root ⊆ Dir

- The root of a filesystem hierarchy is a directory.

Relational logic by example

$Root \subseteq Dir$

$contents \subseteq Dir \times (File \cup Dir)$

- The root of a filesystem hierarchy is a directory.
- Directories may contain files or directories.

Relational logic by example

$Root \subseteq Dir$

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$(File \cup Dir) \subseteq Root.{*}contents$

- The root of a filesystem hierarchy is a directory.
- Directories may contain files or directories.
- All directories and files are reachable from the root.

Relational logic by example

$\text{Root} \subseteq \text{Dir}$

$\text{contents} \subseteq \text{Dir} \times (\text{File} \cup \text{Dir})$

$(\text{File} \cup \text{Dir}) \subseteq \text{Root}.{*}\text{contents}$

$\forall\, d: \text{Dir} \mid \neg\, (d \subseteq d.{\wedge}\text{contents})$

- The root of a filesystem hierarchy is a directory.
- Directories may contain files or directories.
- All directories and files are reachable from the root.
- The contents relation is acyclic.

Bounded relational logic by example

Root $\subseteq$ Dir

contents $\subseteq$ Dir × (File $\cup$ Dir)

(File $\cup$ Dir) $\subseteq$ Root.*contents

$\forall$ d: Dir | ¬ (d $\subseteq$ d.^contents)

{ **R, D_1, D_2, F_1, F_2** }

Finite universe of interpretation.

Bounded relational logic by example

$\text{Root} \subseteq \text{Dir}$

$\text{contents} \subseteq \text{Dir} \times (\text{File} \cup \text{Dir})$

$(\text{File} \cup \text{Dir}) \subseteq \text{Root}.{}^{*}\text{contents}$

$\forall\, d\colon \text{Dir} \mid \neg\, (d \subseteq d.{}^{\wedge}\text{contents})$

$\{\, \mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2} \,\}$

Finite universe of interpretation.

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

$\{\} \subseteq \text{File} \subseteq \{\langle \mathbf{F_1} \rangle, \langle \mathbf{F_2} \rangle\}$

$\{\} \subseteq \text{contents} \subseteq \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}\} \times \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\}$

Bounds for each relation:

- Tuples it *must* contain (partial model).
- Tuples it *may* contain (type).

Bounded relational logic by example

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$(\text{File} \cup \text{Dir}) \subseteq \text{Root.*contents}$

$\forall\ d: \text{Dir} \mid \neg\ (d \subseteq d.\text{^contents})$

$\{\ \mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\ \}$

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

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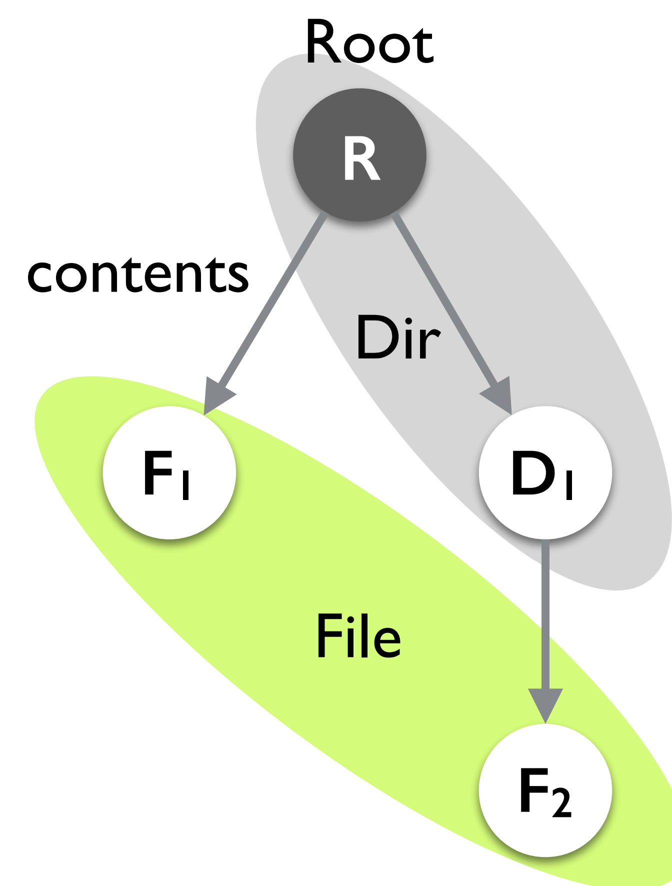

Translation by example

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Translation by example

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

{ **R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$ }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D**$_1$⟩, ⟨**D**$_2$⟩}

{} ⊆ File ⊆ {⟨**F**$_1$⟩, ⟨**F**$_2$⟩}

{} ⊆ contents ⊆ {**R**, **D**$_1$, **D**$_2$} × {**R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$}

Encode

- relational constants as boolean matrices
- relational expressions as matrix operations
- formulas as constraints over matrix entries

Relational constants as boolean matrices

Relational constants as boolean matrices

R	D_1	D_2	F_1	F_2
1	0	0	0	0

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

Relational constants as boolean matrices

R	D_1	D_2	F_1	F_2
1	0	0	0	0
d_0	d_1	d_2	0	0

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

Relational constants as boolean matrices

R	D_1	D_2	F_1	F_2	
1	0	0	0	0	$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$
d_0	d_1	d_2	0	0	$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$
0	0	0	f_0	f_1	$\{\} \subseteq \text{File} \subseteq \{\langle \mathbf{F_1} \rangle, \langle \mathbf{F_2} \rangle\}$

Relational constants as boolean matrices

R	D_1	D_2	F_1	F_2
1	0	0	0	0

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

R	D_1	D_2	F_1	F_2
d_0	d_1	d_2	0	0

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

R	D_1	D_2	F_1	F_2
0	0	0	f_0	f_1

$\{\} \subseteq \text{File} \subseteq \{\langle \mathbf{F_1} \rangle, \langle \mathbf{F_2} \rangle\}$

	R	D_1	D_2	F_1	F_2
R	c_0	c_1	c_2	c_3	c_4
D_1	c_5	c_6	c_7	c_8	c_9
D_2	c_{10}	c_{11}	c_{12}	c_{13}	c_{14}
F_1	0	0	0	0	0
F_2	0	0	0	0	0

$\{\} \subseteq \text{contents} \subseteq \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}\} \times \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\}$

Relational expressions as matrix operations

$$\overset{\text{File}}{\begin{bmatrix} 0 & 0 & 0 & f_0 & f_1 \end{bmatrix}} \vee \overset{\text{Dir}}{\begin{bmatrix} d_0 & d_1 & d_2 & 0 & 0 \end{bmatrix}} = \overset{\text{File} \cup \text{Dir}}{\begin{bmatrix} d_0 & d_1 & d_2 & f_0 & f_1 \end{bmatrix}}$$

$$\overset{\text{Dir}}{\begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ 0 \\ 0 \end{bmatrix}} \times \overset{\text{File} \cup \text{Dir}}{\begin{bmatrix} d_0 & d_1 & d_2 & f_0 & f_1 \end{bmatrix}} = \overset{\text{Dir} \times (\text{File} \cup \text{Dir})}{\begin{bmatrix} d_0 \wedge d_0 & d_0 \wedge d_1 & d_0 \wedge d_2 & d_0 \wedge f_0 & d_0 \wedge f_1 \\ d_1 \wedge d_0 & d_1 \wedge d_1 & d_1 \wedge d_2 & d_1 \wedge f_0 & d_1 \wedge f_1 \\ d_2 \wedge d_0 & d_2 \wedge d_1 & d_2 \wedge d_2 & d_2 \wedge f_0 & d_2 \wedge f_1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}$$

Formulas as constraints over matrix entries

contents

c_0	c_1	c_2	c_3	c_4
c_5	c_6	c_7	c_8	c_9
c_{10}	c_{11}	c_{12}	c_{13}	c_{14}
0	0	0	0	0
0	0	0	0	0

$\rightarrow$

Dir × (File ∪ Dir)

$d_0 \wedge d_0$	$d_0 \wedge d_1$	$d_0 \wedge d_2$	$d_0 \wedge f_0$	$d_0 \wedge f_1$
$d_1 \wedge d_0$	$d_1 \wedge d_1$	$d_1 \wedge d_2$	$d_1 \wedge f_0$	$d_1 \wedge f_1$
$d_2 \wedge d_0$	$d_2 \wedge d_1$	$d_2 \wedge d_2$	$d_2 \wedge f_0$	$d_2 \wedge f_1$
0	0	0	0	0
0	0	0	0	0

=

contents ⊆ Dir × (File ∪ Dir)

$$
\begin{aligned}
&(c_0 \longrightarrow d_0 \wedge d_0) \wedge \\
&(c_1 \longrightarrow d_0 \wedge d_1) \wedge \\
&(c_2 \longrightarrow d_0 \wedge d_2) \wedge \\
&(c_3 \longrightarrow d_0 \wedge f_0) \wedge \\
&(c_4 \longrightarrow d_0 \wedge f_1) \wedge \\
&(c_5 \longrightarrow d_1 \wedge d_0) \wedge \\
&\ldots \\
&(c_{14} \longrightarrow d_2 \wedge f_1)
\end{aligned}
$$

Dealing with sparseness and redundancy

Dir × (File ∪ Dir)

$d_0 \wedge d_0$	$d_0 \wedge d_1$	$d_0 \wedge d_2$	$d_0 \wedge f_0$	$d_0 \wedge f_1$
$d_1 \wedge d_0$	$d_1 \wedge d_1$	$d_1 \wedge d_2$	$d_1 \wedge f_0$	$d_1 \wedge f_1$
$d_2 \wedge d_0$	$d_2 \wedge d_1$	$d_2 \wedge d_2$	$d_2 \wedge f_0$	$d_2 \wedge f_1$
0	0	0	0	0
0	0	0	0	0

Dealing with sparseness and redundancy

Dir × (File ∪ Dir)

$d_0 \wedge d_0$	$d_0 \wedge d_1$	$d_0 \wedge d_2$	$d_0 \wedge f_0$	$d_0 \wedge f_1$
$d_1 \wedge d_0$	$d_1 \wedge d_1$	$d_1 \wedge d_2$	$d_1 \wedge f_0$	$d_1 \wedge f_1$
$d_2 \wedge d_0$	$d_2 \wedge d_1$	$d_2 \wedge d_2$	$d_2 \wedge f_0$	$d_2 \wedge f_1$
0	0	0	0	0
0	0	0	0	0

Empty regions in matrices
(exponential w.r.t. relation arity).

Dealing with sparseness and redundancy

Different circuits for the same formula.

Dir × (File ∪ Dir)

$d_0 \wedge d_0$	$d_0 \wedge d_1$	$d_0 \wedge d_2$	$d_0 \wedge f_0$	$d_0 \wedge f_1$
$d_1 \wedge d_0$	$d_1 \wedge d_1$	$d_1 \wedge d_2$	$d_1 \wedge f_0$	$d_1 \wedge f_1$
$d_2 \wedge d_0$	$d_2 \wedge d_1$	$d_2 \wedge d_2$	$d_2 \wedge f_0$	$d_2 \wedge f_1$
0	0	0	0	0
0	0	0	0	0

Empty regions in matrices (exponential w.r.t. relation arity).

Dealing with sparseness and redundancy

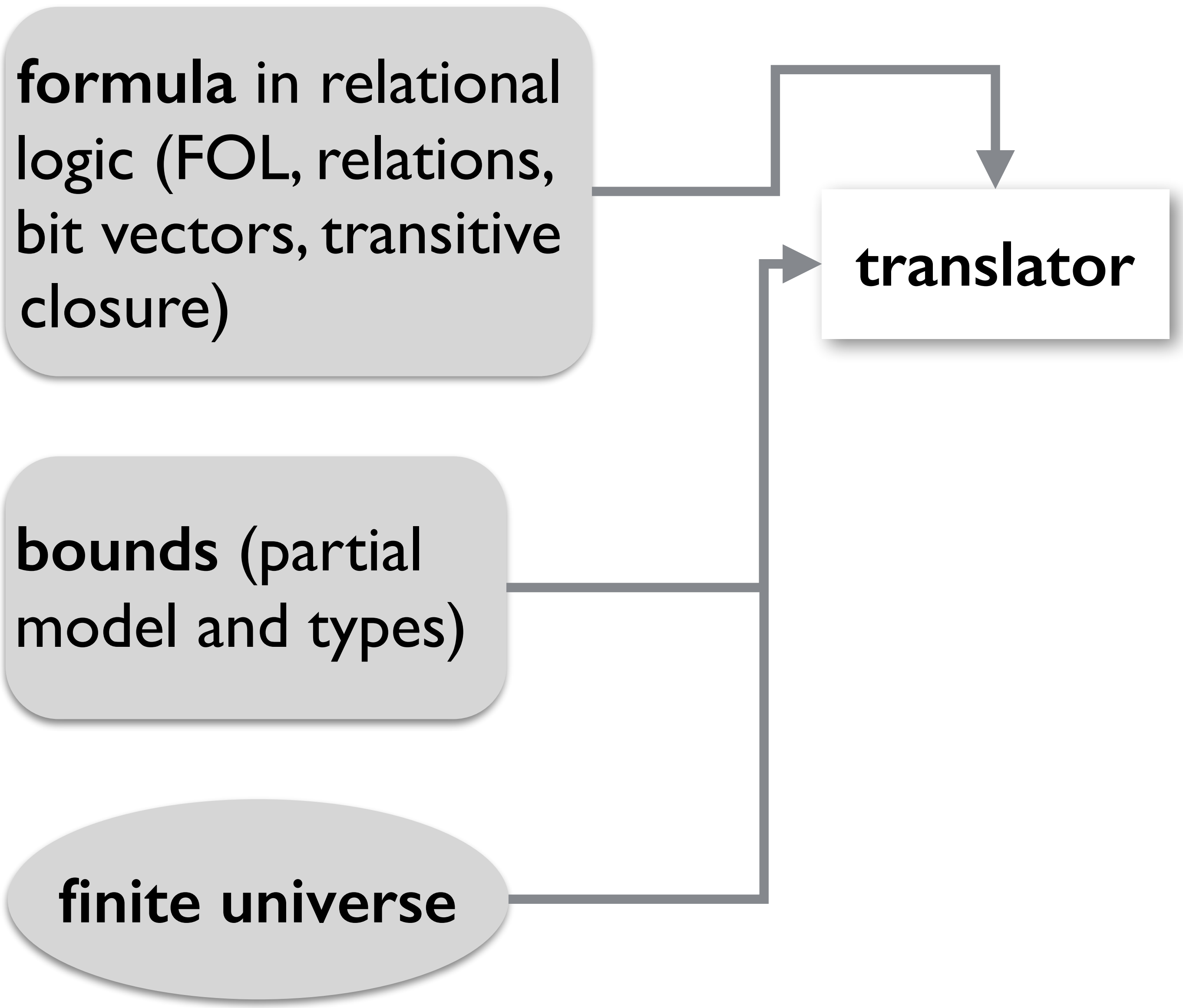
Dir × (File ∪ Dir)

$d_0 \wedge d_0$	$d_0 \wedge d_1$	$d_0 \wedge d_2$	$d_0 \wedge f_0$	$d_0 \wedge f_1$
$d_1 \wedge d_0$	$d_1 \wedge d_1$	$d_1 \wedge d_2$	$d_1 \wedge f_0$	$d_1 \wedge f_1$
$d_2 \wedge d_0$	$d_2 \wedge d_1$	$d_2 \wedge d_2$	$d_2 \wedge f_0$	$d_2 \wedge f_1$
0	0	0	0	0
0	0	0	0	0

Compact Boolean Circuits (CBCs).

Sparse matrices represented as interval trees.

Overview of Kodkod



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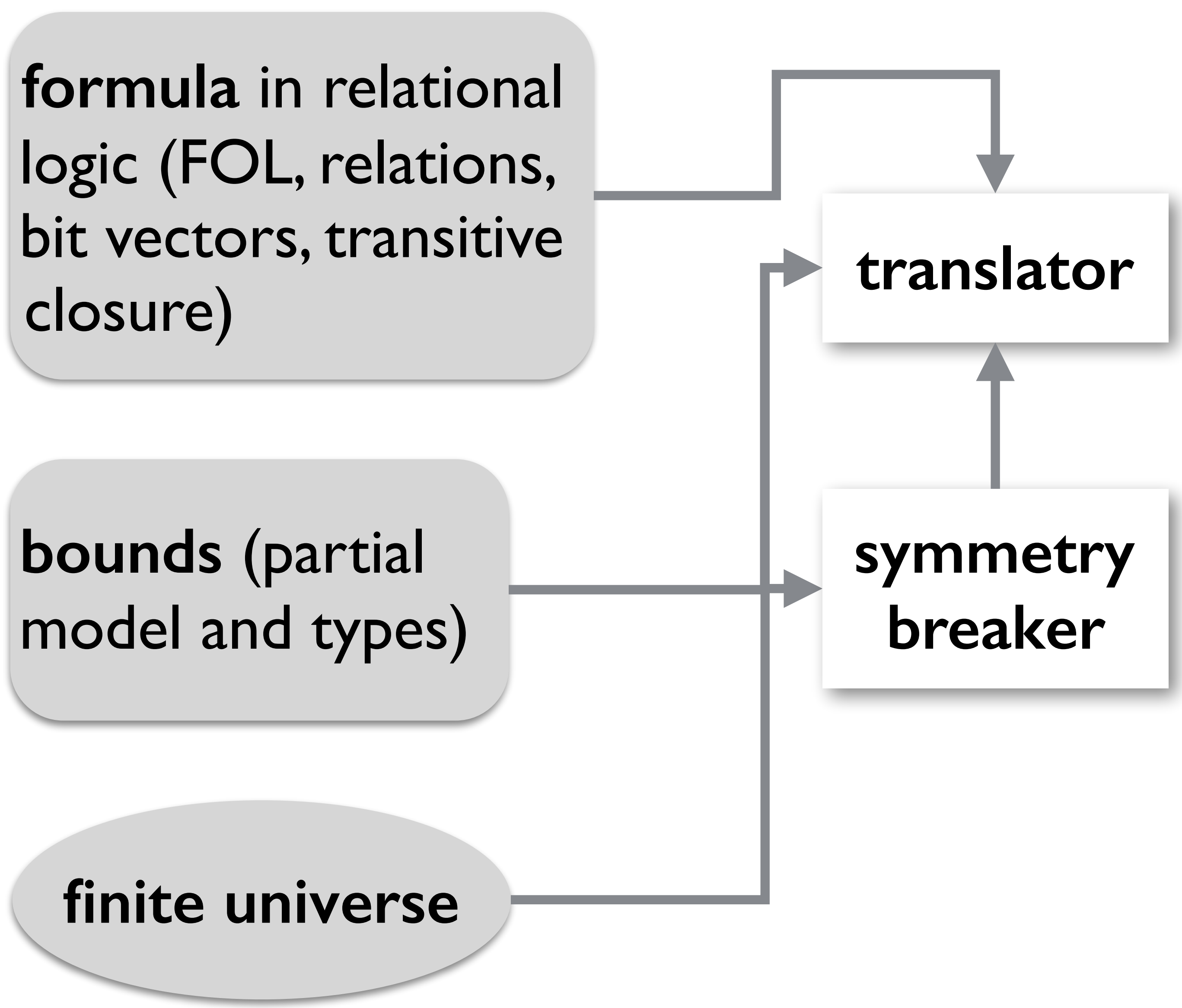

Symmetry by example

$\text{Root} \subseteq \text{Dir}$

$\text{contents} \subseteq \text{Dir} \times (\text{File} \cup \text{Dir})$

$(\text{File} \cup \text{Dir}) \subseteq \text{Root.*contents}$

$\forall\, d: \text{Dir} \mid \neg\, (d \subseteq d.\text{^contents})$

$\{\, \mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2} \,\}$

$\{\langle\mathbf{R}\rangle\} \subseteq \text{Root} \subseteq \{\langle\mathbf{R}\rangle\}$

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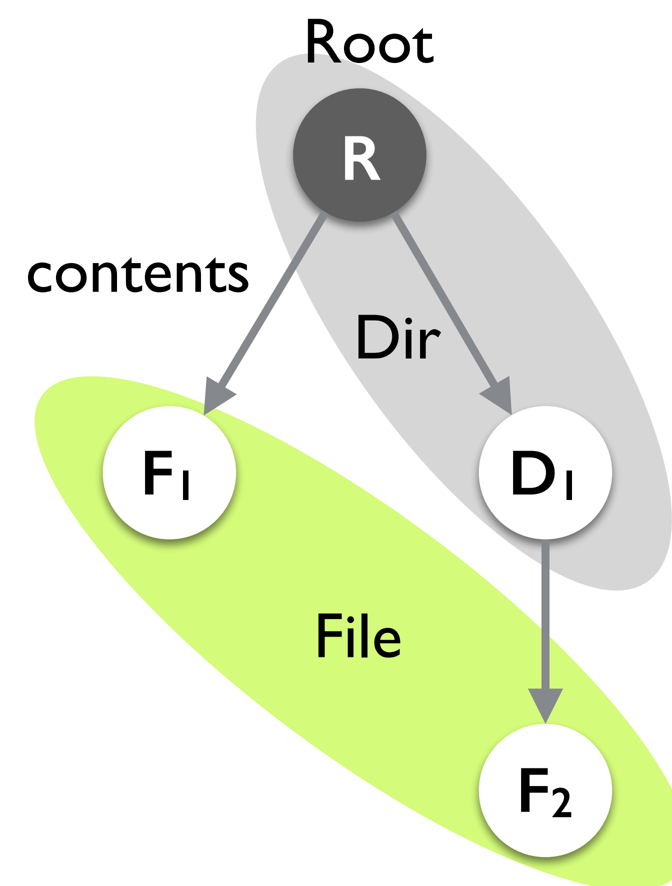


Symmetry by example

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

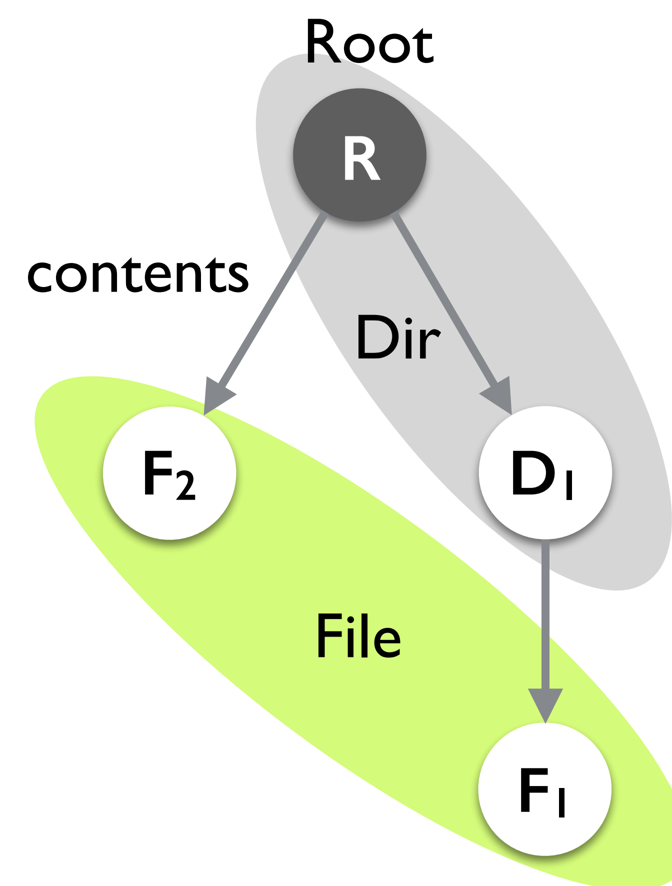
{ **R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$ }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D**$_1$⟩, ⟨**D**$_2$⟩}

{} ⊆ File ⊆ {⟨**F**$_1$⟩, ⟨**F**$_2$⟩}

{} ⊆ contents ⊆ {**R**, **D**$_1$, **D**$_2$} × {**R**, **D**$_1$, **D**$_2$, **F**$_1$, **F**$_2$}



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$\{ \mathbf{R}, \mathbf{D_1}\ \mathbf{D_2}\ \mathbf{F_1}, \mathbf{F_2} \}$

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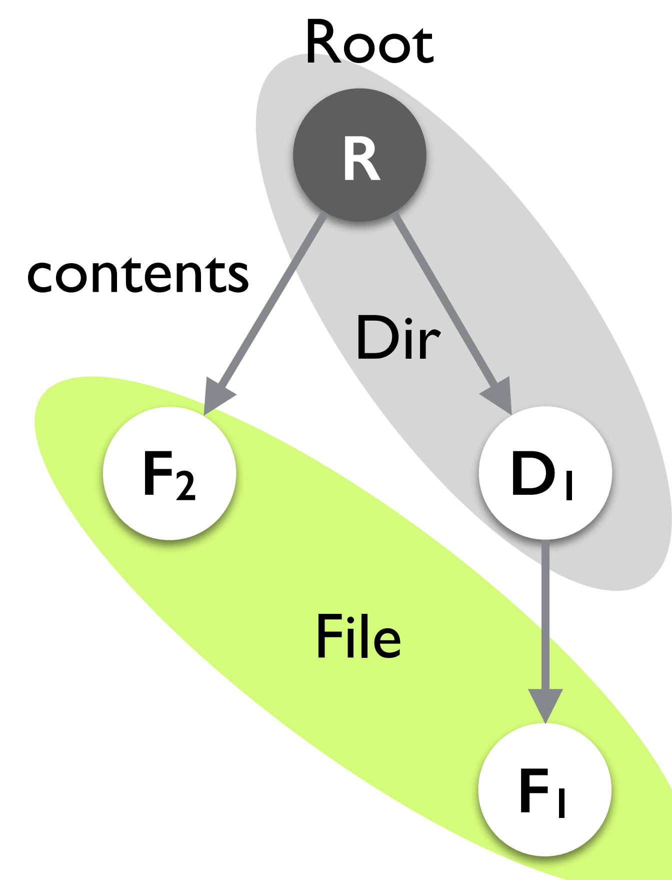

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{ **R**, **D$_1$**, **D$_2$**, **F$_1$**, **F$_2$** }

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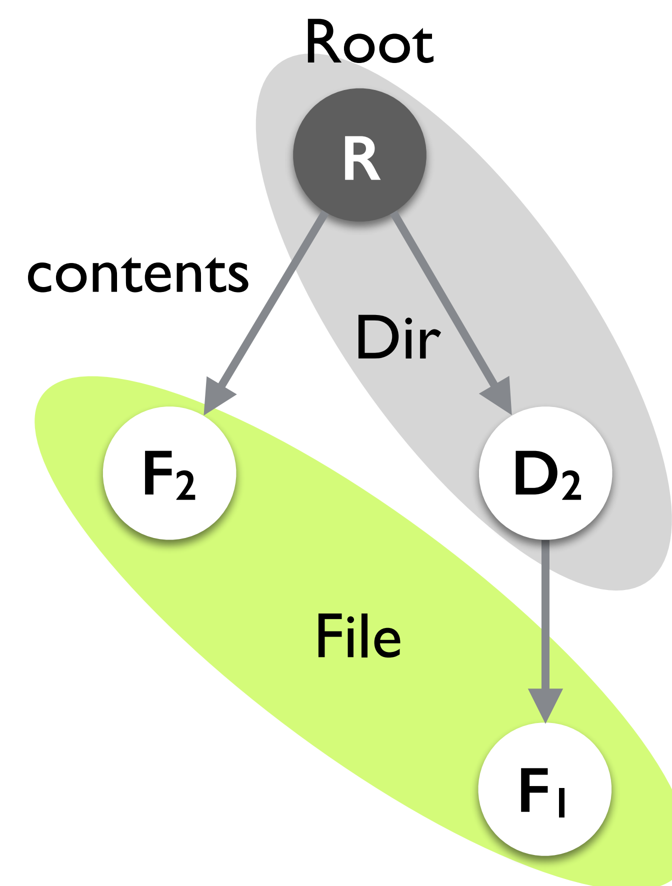
{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**D$_1$**⟩, ⟨**D$_2$**⟩}

{} ⊆ File ⊆ {⟨**F$_1$**⟩, ⟨**F$_2$**⟩}

{} ⊆ contents ⊆ {**R**, **D$_1$**, **D$_2$**} × {**R**, **D$_1$**, **D$_2$**, **F$_1$**, **F$_2$**}



Symmetries between models

$\text{Root} \subseteq \text{Dir}$

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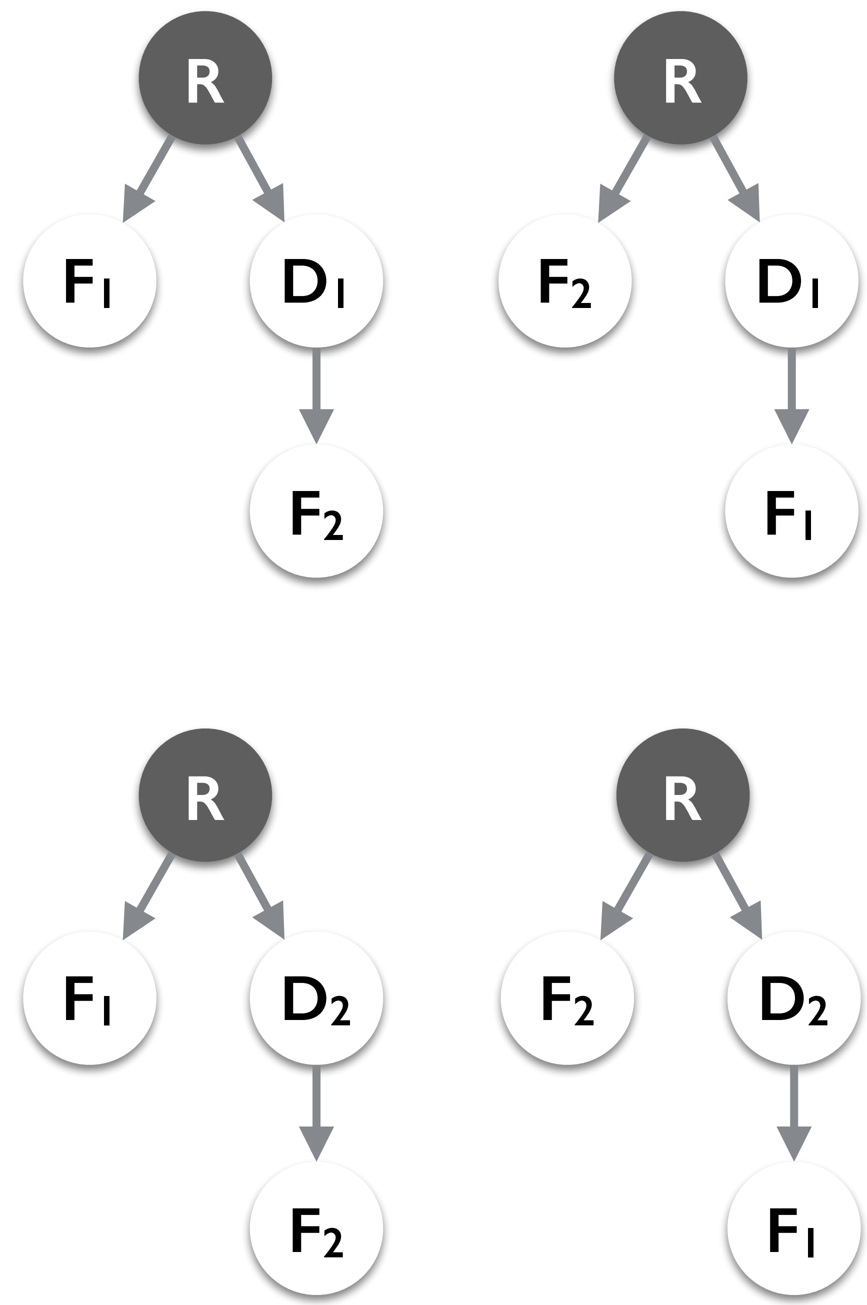
$(\text{File} \cup \text{Dir}) \subseteq \text{Root}.^{*}\text{contents}$

$\forall\, d: \text{Dir} \mid \neg\, (d \subseteq d.\hat{}\text{contents})$

$\{\, R, D_1, D_2, F_1, F_2 \,\}$



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$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

$\{\} \subseteq \text{File} \subseteq \{\langle \mathbf{F_1} \rangle, \langle \mathbf{F_2} \rangle\}$

$\{\} \subseteq \text{contents} \subseteq \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}\} \times \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\}$

Symmetries between non-models

$\text{Root} \subseteq \text{Dir}$

$\text{contents} \subseteq \text{Dir} \times (\text{File} \cup \text{Dir})$

$(\text{File} \cup \text{Dir}) \subseteq \text{Root.*contents}$

$\forall\ d\text{: Dir} \mid \neg\ (d \subseteq d\text{.\^{}contents})$

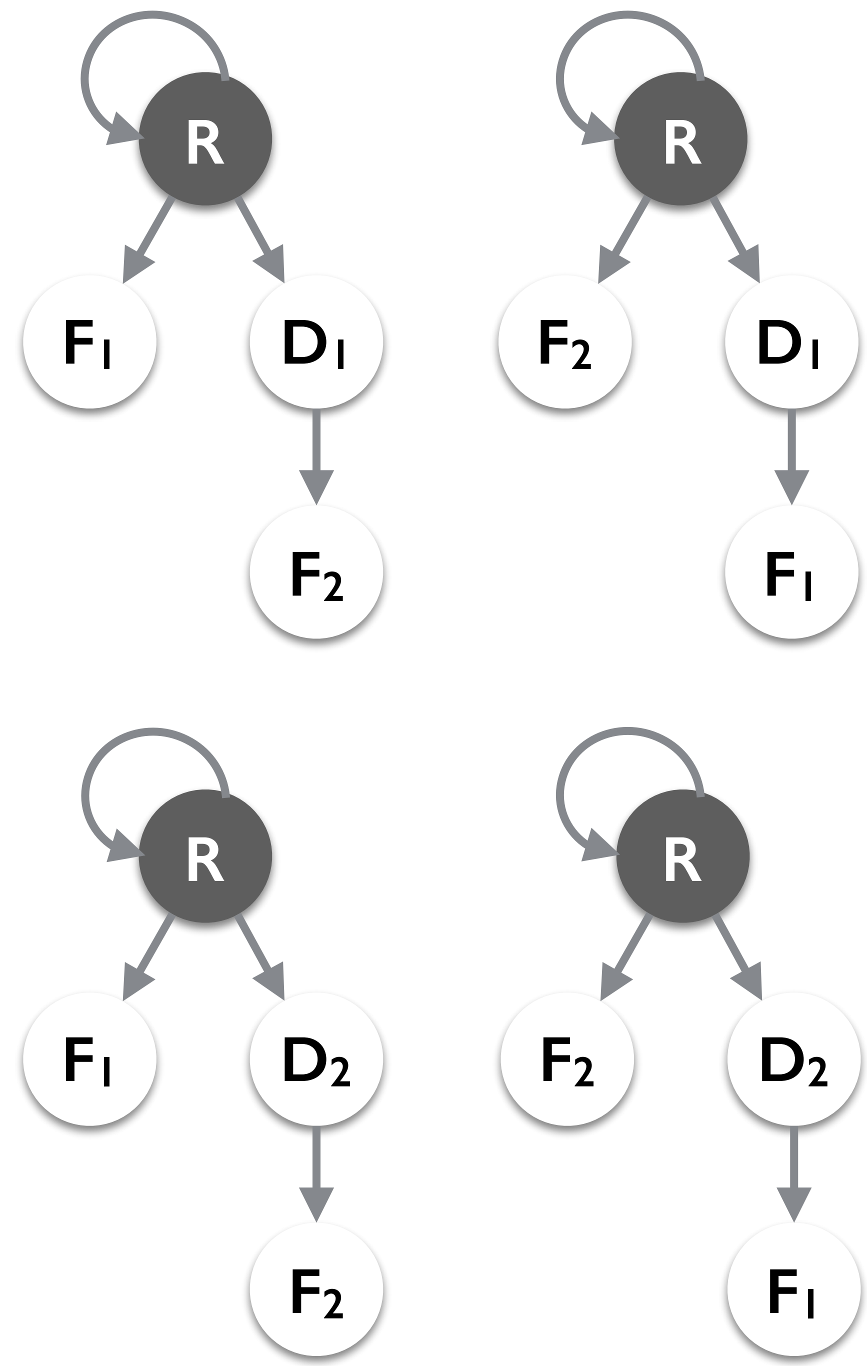
$\{\ R, D_1, D_2, F_1, F_2\ \}$

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

$\{\} \subseteq \text{File} \subseteq \{\langle \mathbf{F_1} \rangle, \langle \mathbf{F_2} \rangle\}$

$\{\} \subseteq \text{contents} \subseteq \{R, D_1, D_2\} \times \{R, D_1, D_2, F_1, F_2\}$



Symmetries induce equivalence classes

$\text{Root} \subseteq \text{Dir}$

$\text{contents} \subseteq \text{Dir} \times (\text{File} \cup \text{Dir})$

$(\text{File} \cup \text{Dir}) \subseteq \text{Root}.{}^{*}\text{contents}$

$\forall\, d\colon \text{Dir} \mid \neg\, (d \subseteq d.\hat{}\text{contents})$

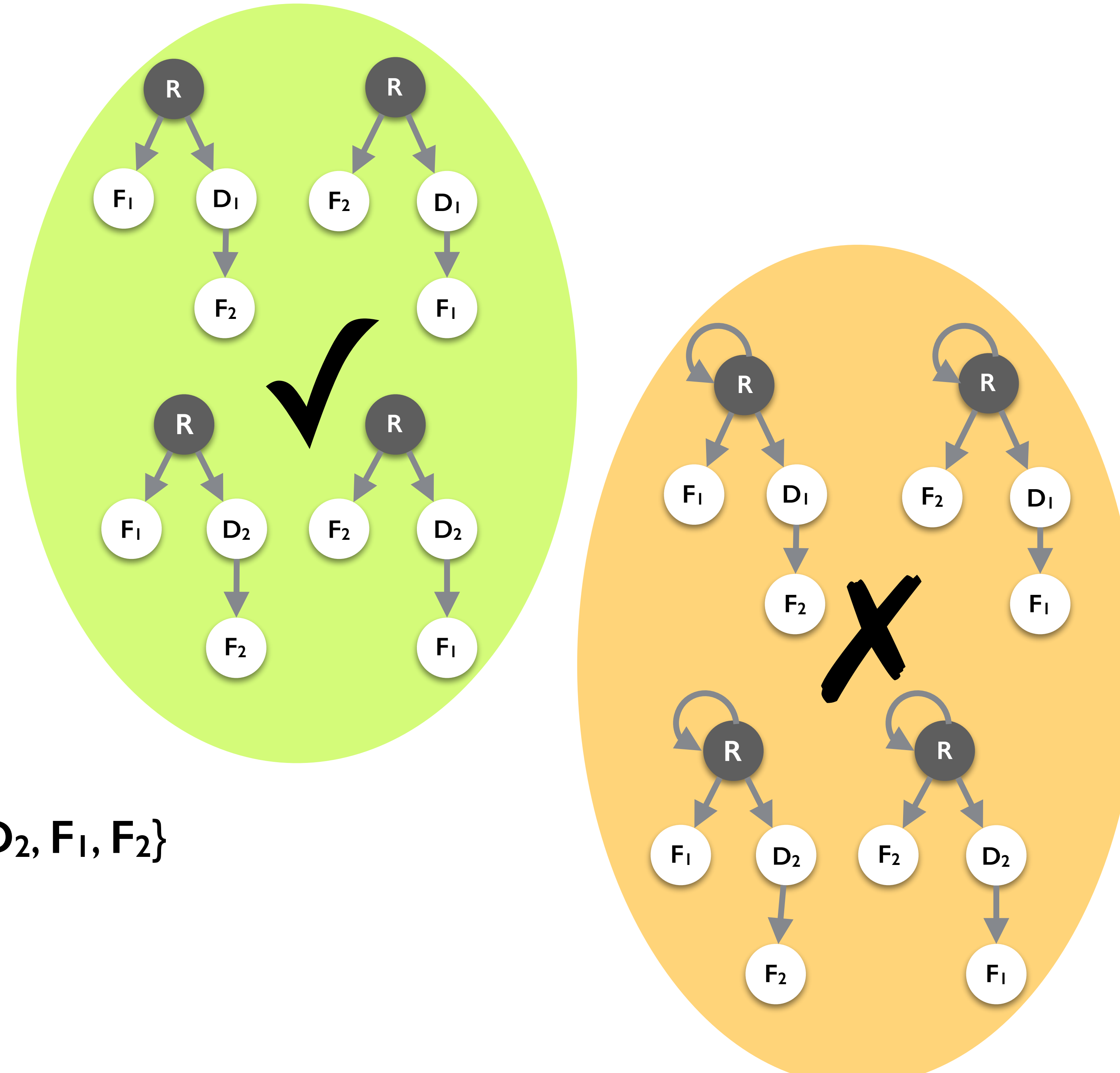
$\{\ \mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\ \}$

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

$\{\} \subseteq \text{File} \subseteq \{\langle \mathbf{F_1} \rangle, \langle \mathbf{F_2} \rangle\}$

$\{\} \subseteq \text{contents} \subseteq \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}\} \times \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\}$



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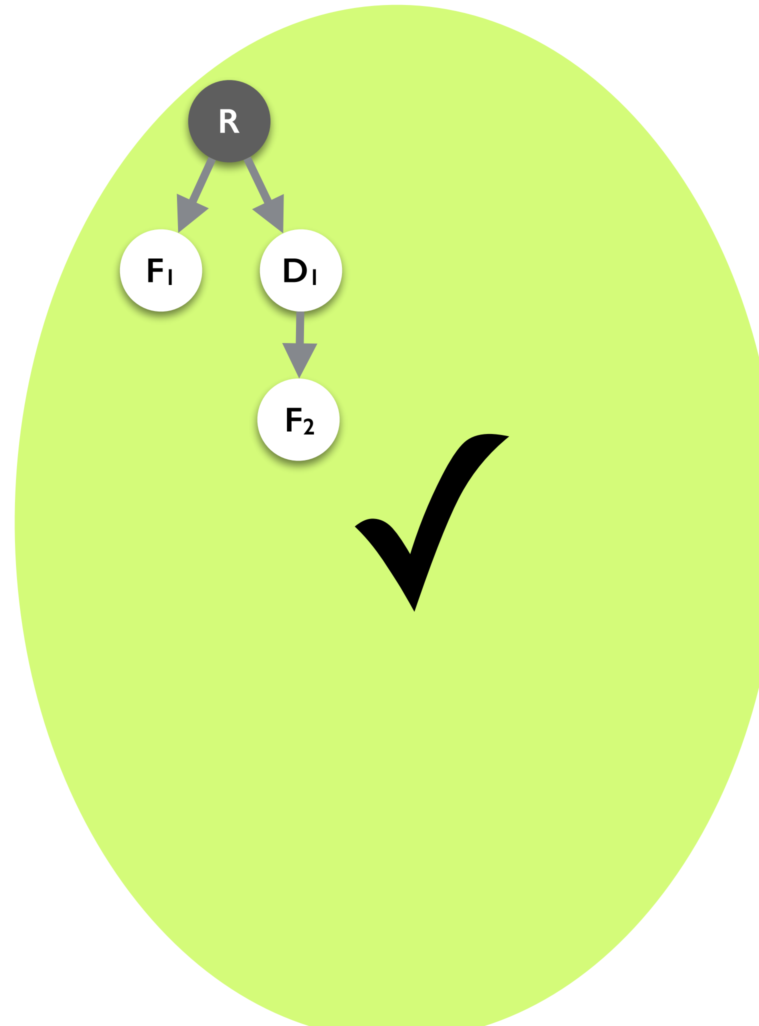
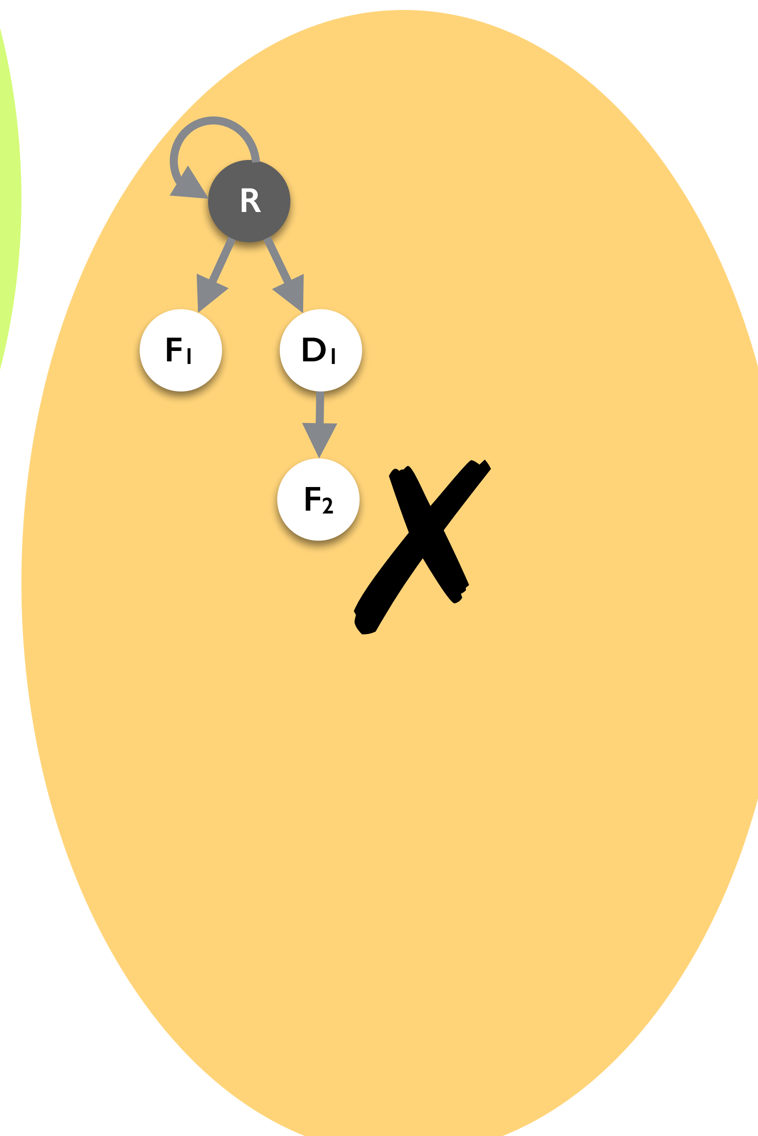


Sufficient to check one interpretation per equivalence class.



Symmetry detection

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

$\{ R, D_1, D_2, F_1, F_2 \}$

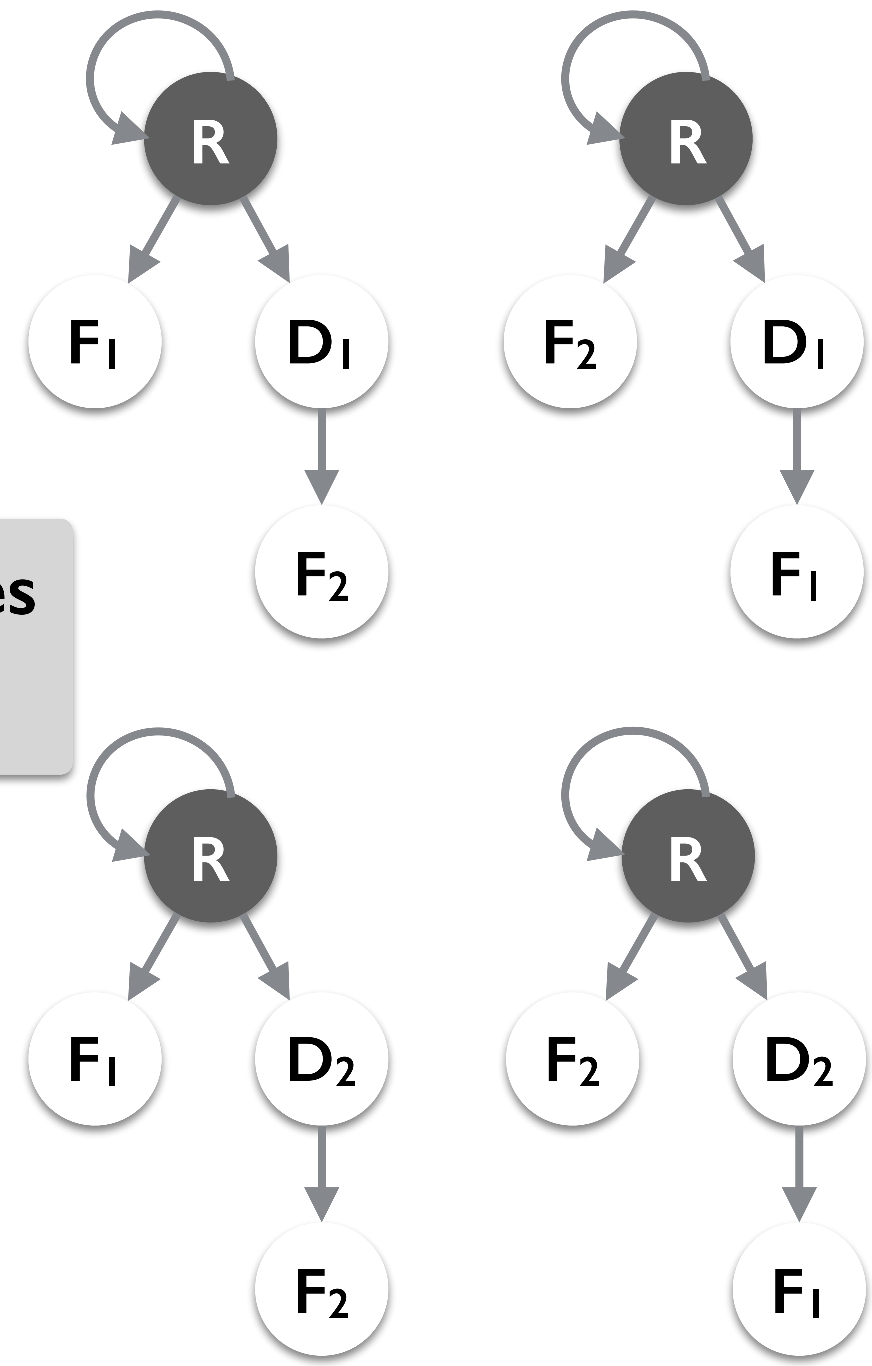
Interpretation symmetries
= bound symmetries

$\{\langle R \rangle\} \subseteq \text{Root} \subseteq \{\langle R \rangle\}$

$\{\} \subseteq \text{Dir} \subseteq \{\langle R \rangle, \langle D_1 \rangle, \langle D_2 \rangle\}$

$\{\} \subseteq \text{File} \subseteq \{\langle F_1 \rangle, \langle F_2 \rangle\}$

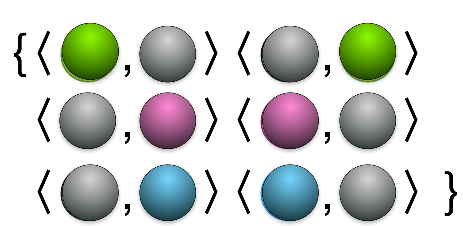
$\{\} \subseteq \text{contents} \subseteq \{R, D_1, D_2\} \times \{R, D_1, D_2, F_1, F_2\}$

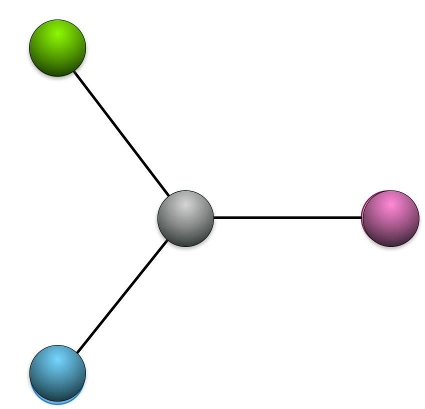


Detecting symmetries is hard …

Interpretation symmetries = bound symmetries

Graph automorphism detection





But only a few symmetries needed in practice

Greedy algorithm that partitions the universe into equivalence classes ✓

✗ Graph automorphism detection

Base partitioning: practical symmetry detection

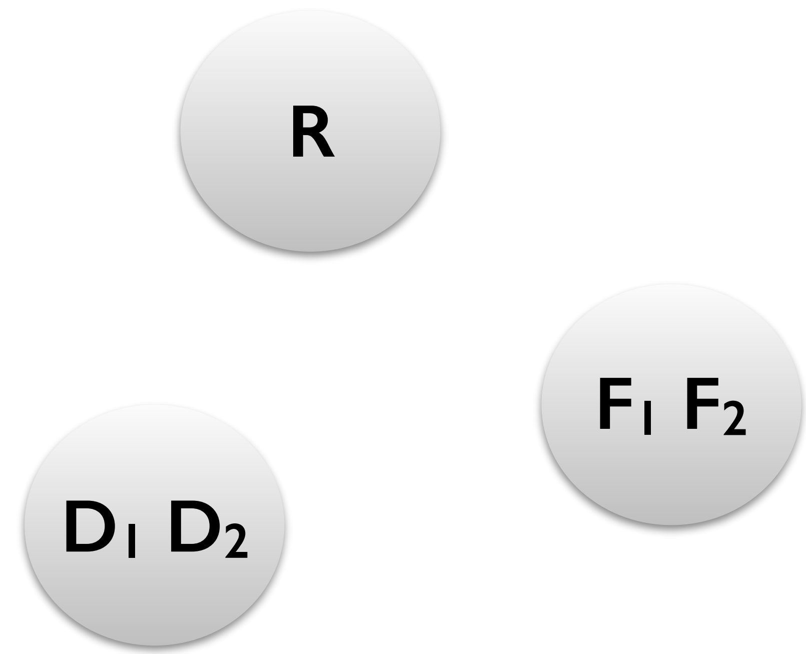
$\{ \mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2} \}$

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

$\{\} \subseteq \text{File} \subseteq \{\langle \mathbf{F_1} \rangle, \langle \mathbf{F_2} \rangle\}$

$\{\} \subseteq \text{contents} \subseteq \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}\} \times \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\}$



The coarsest partition of the universe such that each non-empty bound is expressible as a union of products of parts.

Finding the base partitioning

$R\ D_1\ D_2\ F_1\ F_2$

start with a single partition and refine minimally for each non-empty lower and upper bound

Finding base partitioning

$R\ D_1\ D_2\ F_1\ F_2$

Finding base partitioning

$R\ D_1\ D_2\ F_1\ F_2$

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

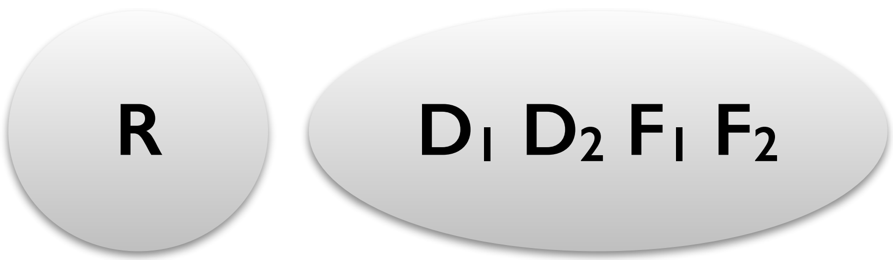
Finding base partitioning

R

D_1 D_2 F_1 F_2

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

Finding base partitioning



$\{\langle \mathbf{R} \rangle\} \subseteq \mathrm{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq \mathrm{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

Finding base partitioning

R

D_1 D_2

F_1 F_2

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

Finding base partitioning

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

$\{\} \subseteq \text{File} \subseteq \{\langle \mathbf{F_1} \rangle, \langle \mathbf{F_2} \rangle\}$

Finding base partitioning

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

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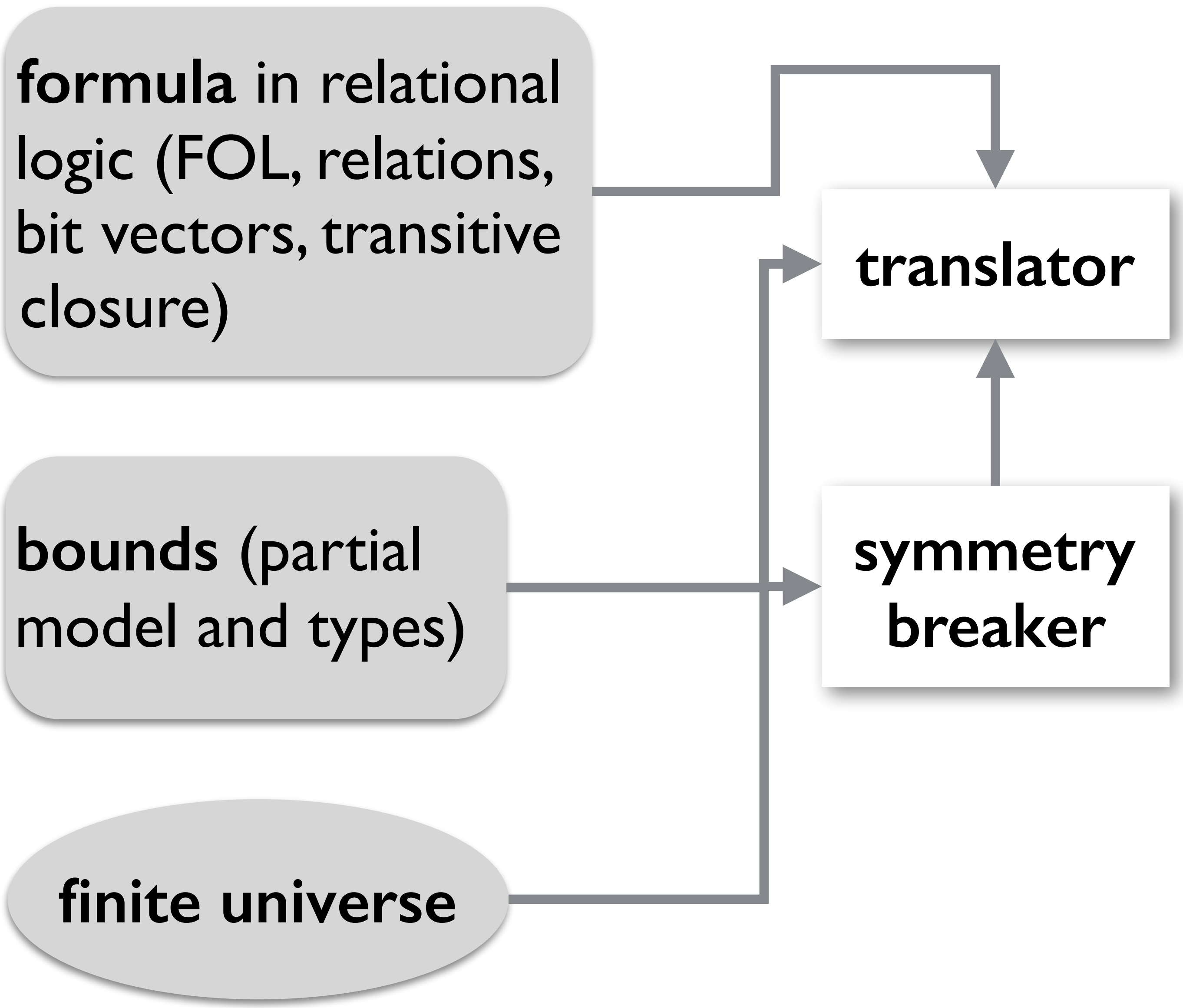
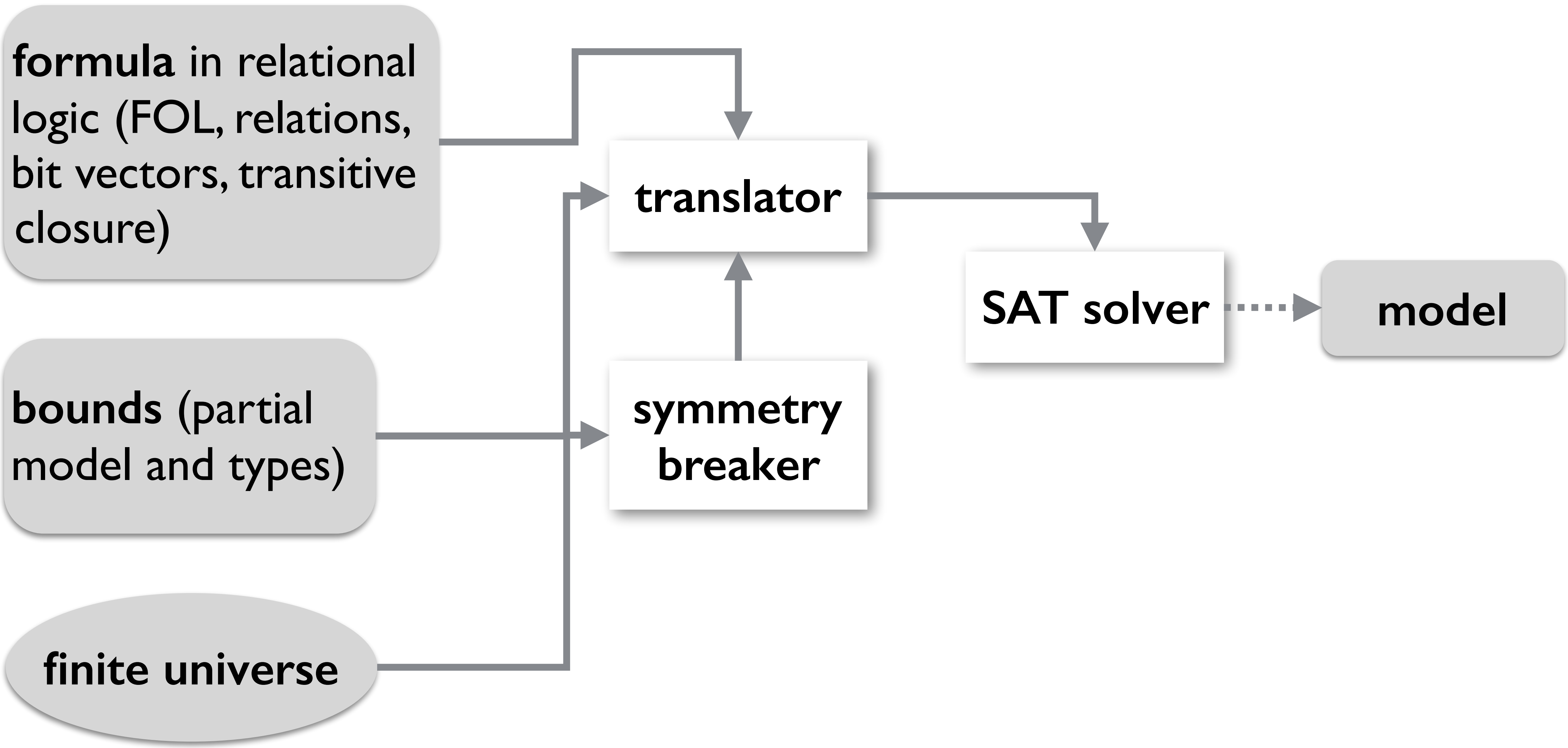
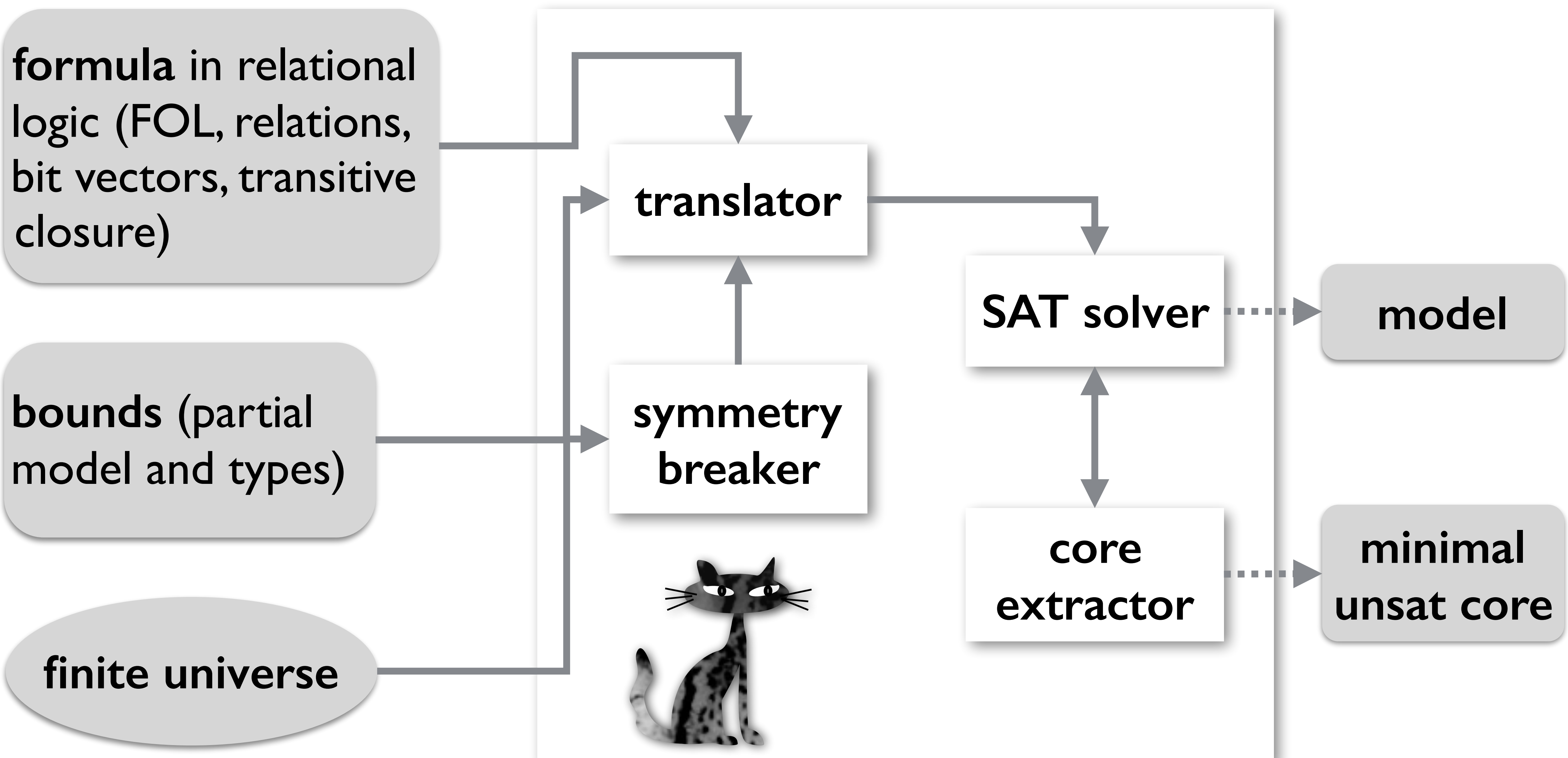
Overview of Kodkod



Overview of Kodkod



Overview of Kodkod



A bug in the tiny filesystem

Root ⊆ Dir

contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

∀ d: Dir | ¬ (d ⊆ d.^contents)

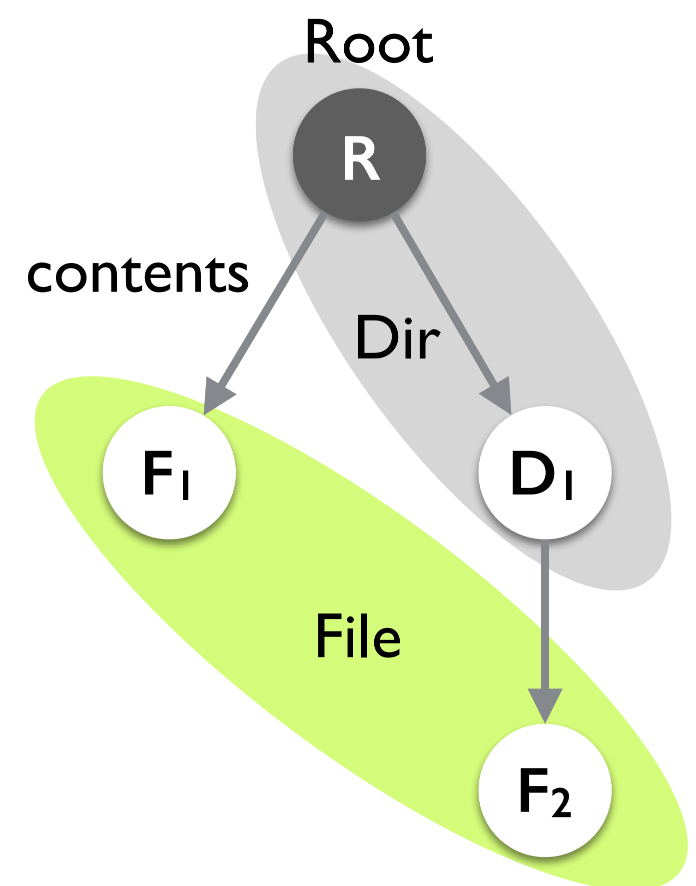
{ **R**, **$\mathbf{D_1}$**, **$\mathbf{D_2}$**, **$\mathbf{F_1}$**, **$\mathbf{F_2}$** }

{⟨**R**⟩} ⊆ Root ⊆ {⟨**R**⟩}

{} ⊆ Dir ⊆ {⟨**R**⟩, ⟨**$\mathbf{D_1}$**⟩, ⟨**$\mathbf{D_2}$**⟩}

{} ⊆ File ⊆ {⟨**$\mathbf{F_1}$**⟩, ⟨**$\mathbf{F_2}$**⟩}

{} ⊆ contents ⊆ {**R**, **$\mathbf{D_1}$**, **$\mathbf{D_2}$**} × {**R**, **$\mathbf{D_1}$**, **$\mathbf{D_2}$**, **$\mathbf{F_1}$**, **$\mathbf{F_2}$**}



A bug in the tiny filesystem

$\text{Root} \subseteq \text{Dir}$

$\text{contents} \subseteq \text{Dir} \times (\text{File} \cup \text{Dir})$

$(\text{File} \cup \text{Dir}) \subseteq \text{Root}.{}^{*}\text{contents}$

$\forall\, d{:}\, \text{Dir} \mid \neg\, (d \subseteq d.\hat{}\,\text{contents})$

$\{\, \mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2} \,\}$

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

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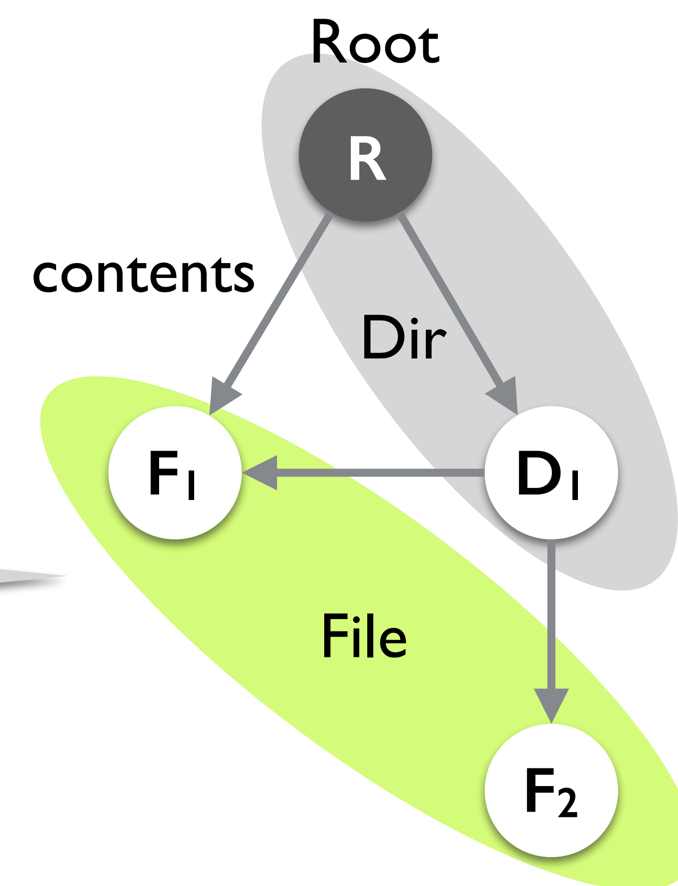
The spec allows multiple parents.



Fixing the tiny filesystem

$\text{Root} \subseteq \text{Dir}$

$\text{contents} \subseteq \text{Dir} \times (\text{File} \cup \text{Dir})$

$(\text{File} \cup \text{Dir}) \subseteq \text{Root.*contents}$

$\forall\ \text{d: Dir} \mid \neg\ (\text{d} \subseteq \text{d.\^{}contents})$

$\forall\ \text{f: File} \mid \text{one contents.f}$

$\forall\ \text{d: Dir} \mid \text{one contents.d}$

$\{\ \mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\ \}$

$\{\langle \mathbf{R} \rangle\} \subseteq \text{Root} \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq \text{Dir} \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

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Fixing the tiny filesystem

$Root \subseteq Dir$

$contents \subseteq Dir \times (File \cup Dir)$

$(File \cup Dir) \subseteq Root.{}^{*}contents$

$\forall\, d{:}\, Dir \mid \neg\, (d \subseteq d.{}^{\wedge}contents)$

$\forall\, f{:}\, File \mid one\ contents.f$

$\forall\, d{:}\, Dir \mid one\ contents.d$

Minimal unsatisfiable core: an unsatisfiable subset of a formula that becomes satisfiable if any of its members are removed.

$\{\ \mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\ \}$

$\{\langle \mathbf{R} \rangle\} \subseteq Root \subseteq \{\langle \mathbf{R} \rangle\}$

$\{\} \subseteq Dir \subseteq \{\langle \mathbf{R} \rangle, \langle \mathbf{D_1} \rangle, \langle \mathbf{D_2} \rangle\}$

$\{\} \subseteq File \subseteq \{\langle \mathbf{F_1} \rangle, \langle \mathbf{F_2} \rangle\}$

$\{\} \subseteq contents \subseteq \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}\} \times \{\mathbf{R}, \mathbf{D_1}, \mathbf{D_2}, \mathbf{F_1}, \mathbf{F_2}\}$

Resolution-based core extraction

Root ⊆ Dir

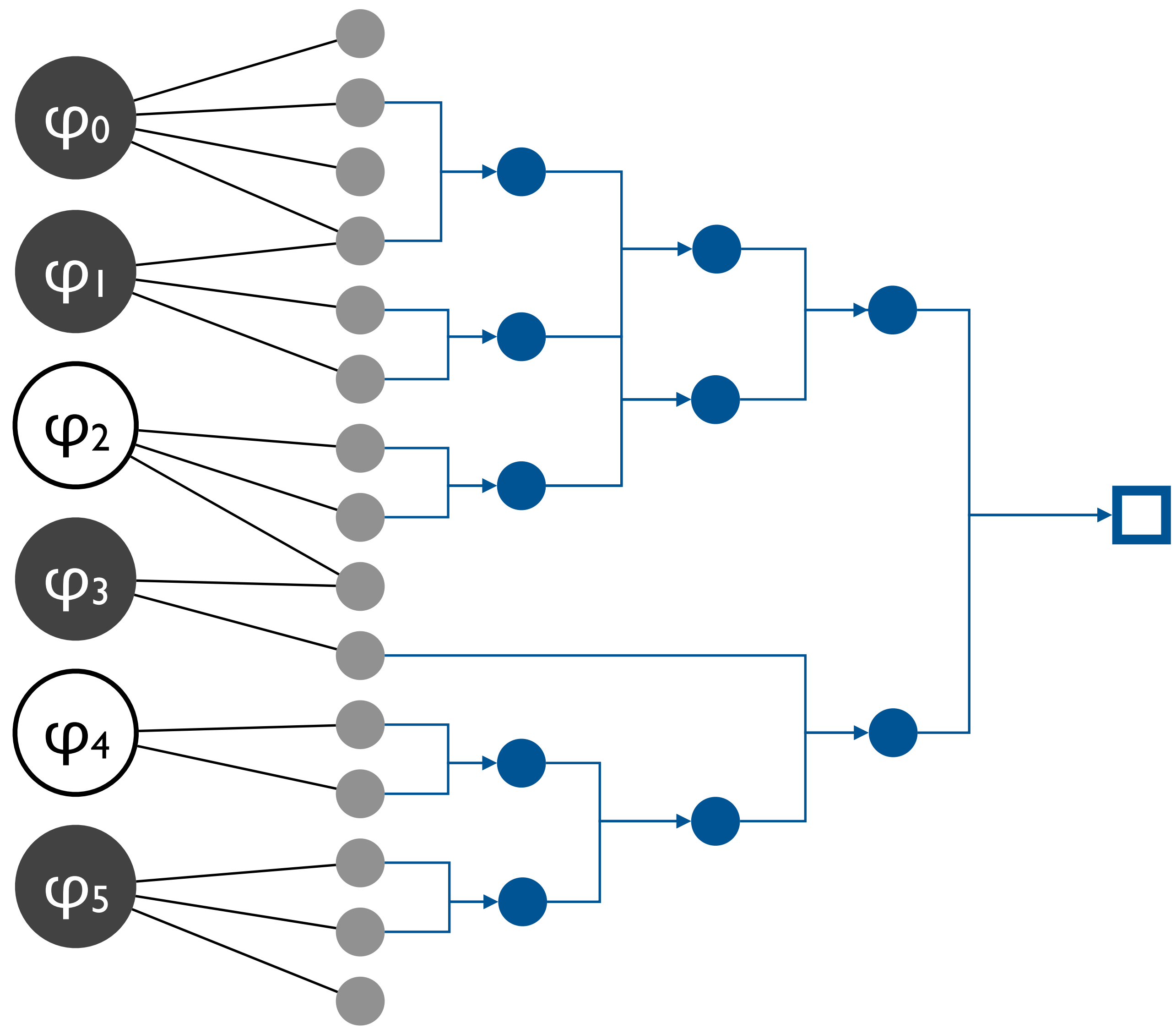
contents ⊆ Dir × (File ∪ Dir)

(File ∪ Dir) ⊆ Root.*contents

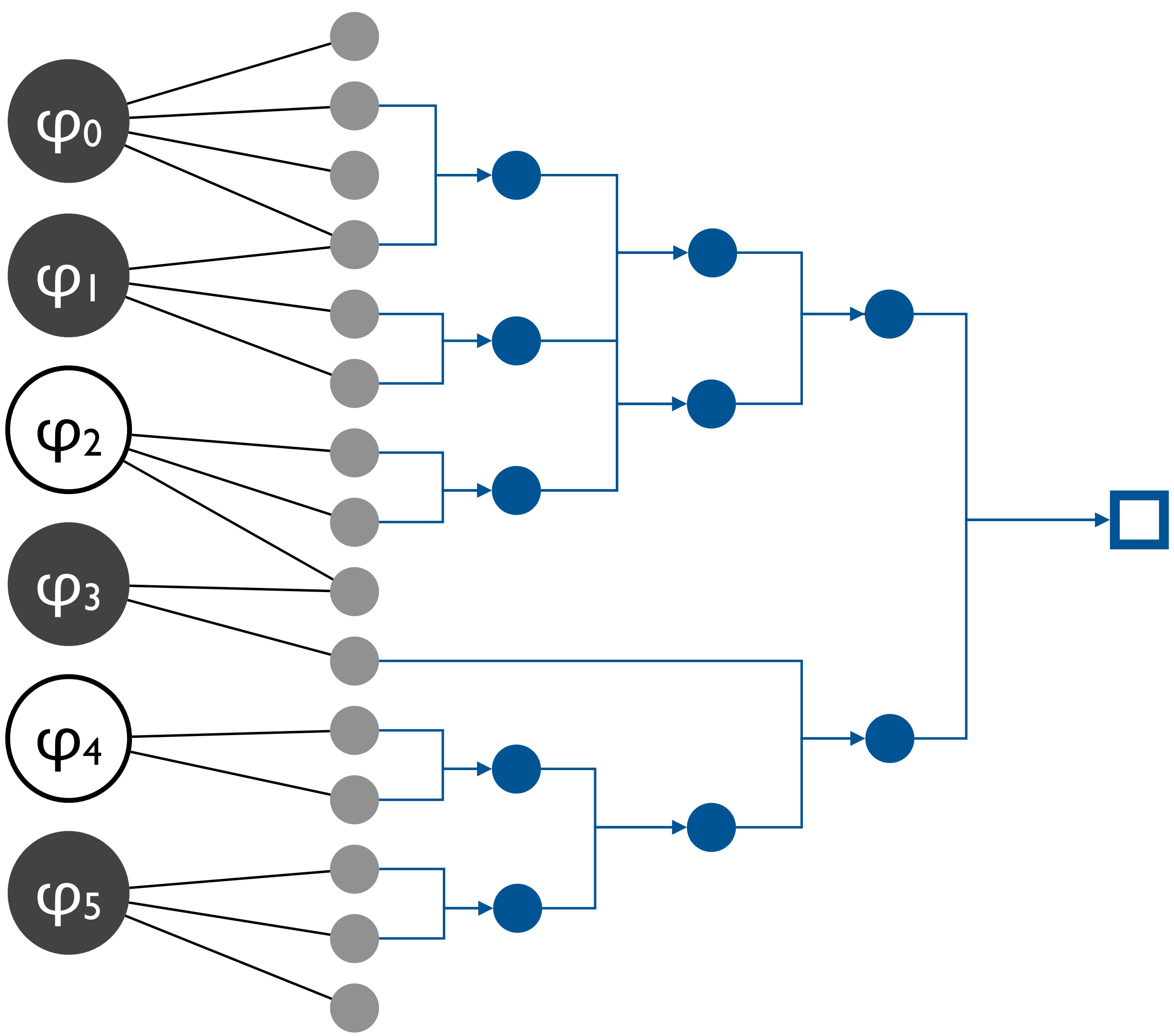
∀ d: Dir | ¬ (d ⊆ d.^contents)

∀ f: File | one contents.f

∀ d: Dir | one contents.d



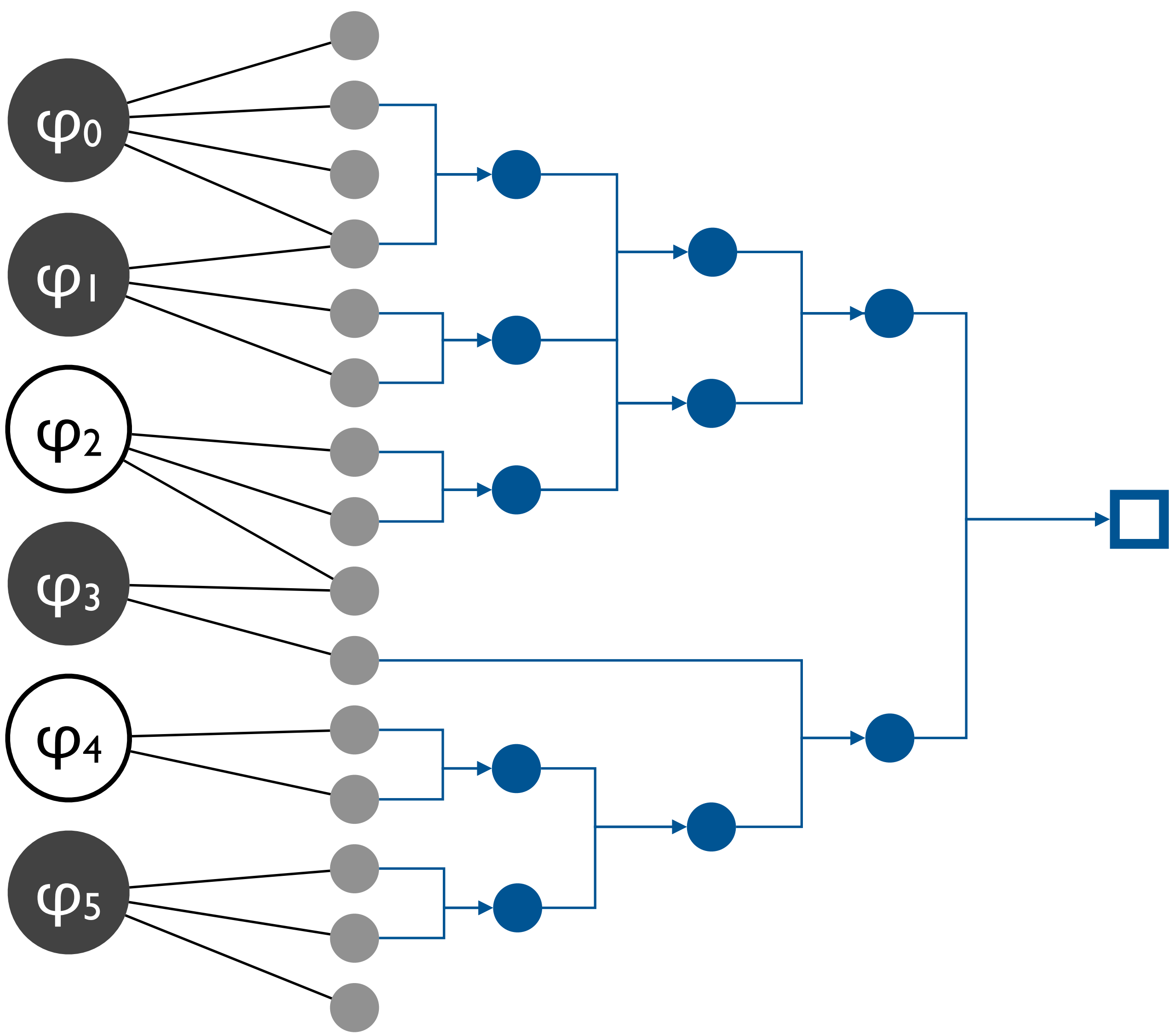
High-level minimal cores from low-level proofs



How to use the proof at the SAT level to find a minimal core at the specification level when

- SAT proof is not minimal
- minimal SAT core may map to a large specification core?

Recycling core extraction



Key idea: minimize core by removing constraints at the specification level but re-use valid resolvents from the previous step so that the solver doesn't have to re-derive them.

Summary

Today

- Finite model finding for first-order logic with quantifiers, relations, and transitive closure

Next lecture

- Reasoning about program correctness