Computer-Aided Reasoning for Software

CSE507

Practical Applications of SAT

courses.cs.washington.edu/courses/cse507/14au/

Emina Torlak
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Today

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Past 3 lectures

- The theory and mechanics of SAT solving

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- Practical applications of SAT
- Variants of the SAT problem
- Motivating the next lecture on SMT

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Past 3 lectures

- The theory and mechanics of SAT solving

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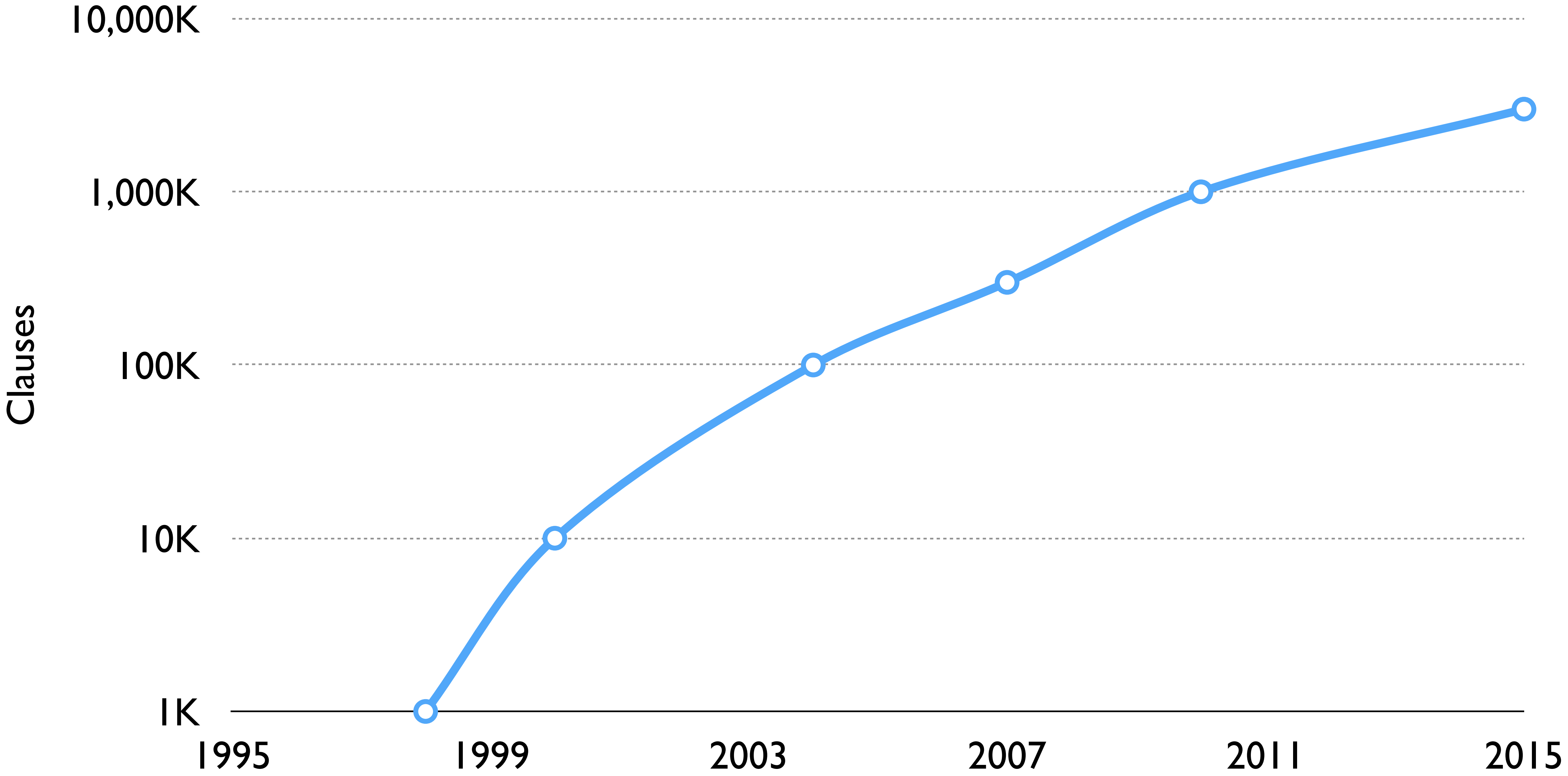
- Practical applications of SAT
- Variants of the SAT problem
- Motivating the next lecture on SMT

But first …

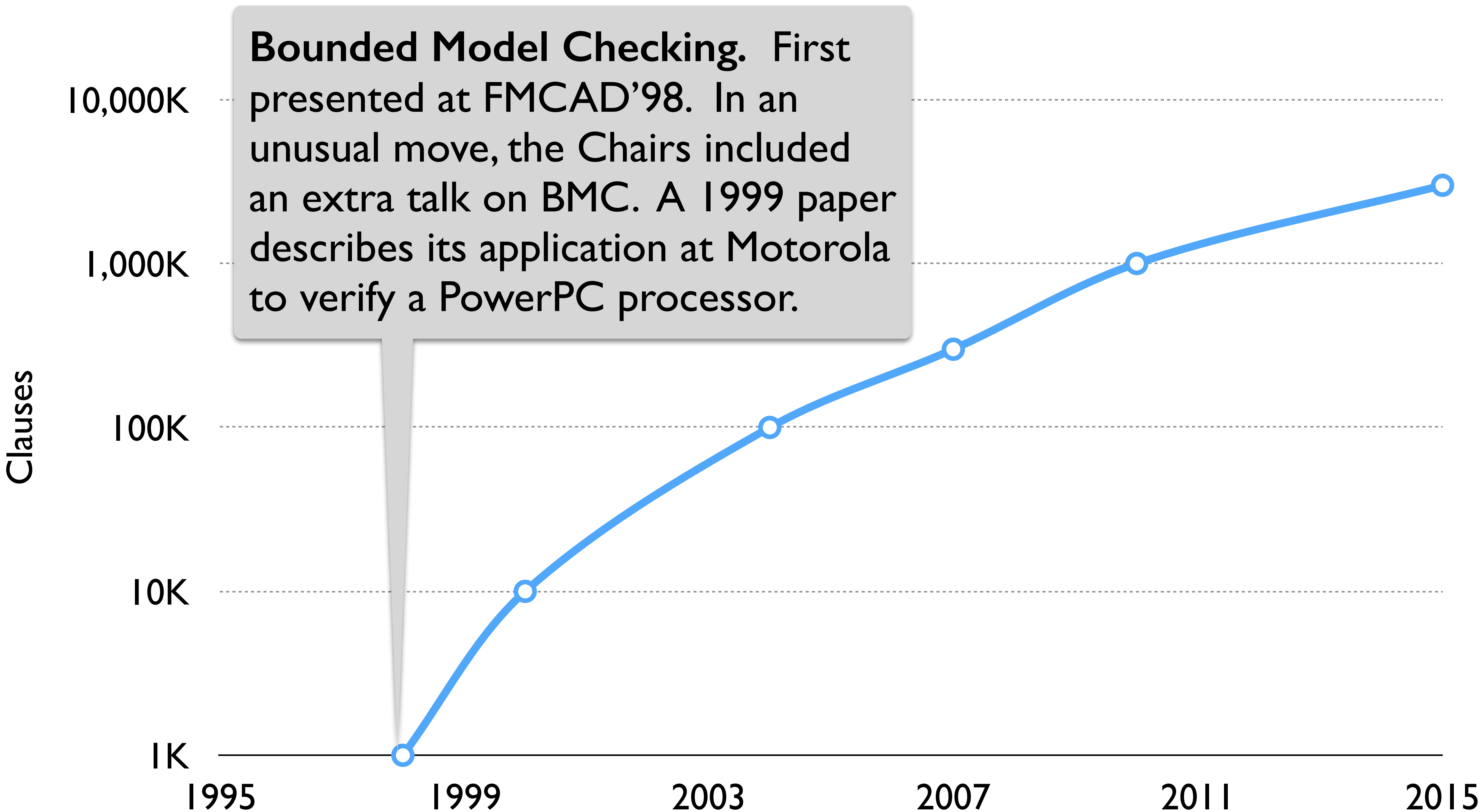
- A brief Q&A session for Homework 1
- Live SAT solving (or, partial assignment of students to project teams)

A brief history of SAT solving and applications



Based on a slide from Vijay Ganesh

A brief history of SAT solving and applications



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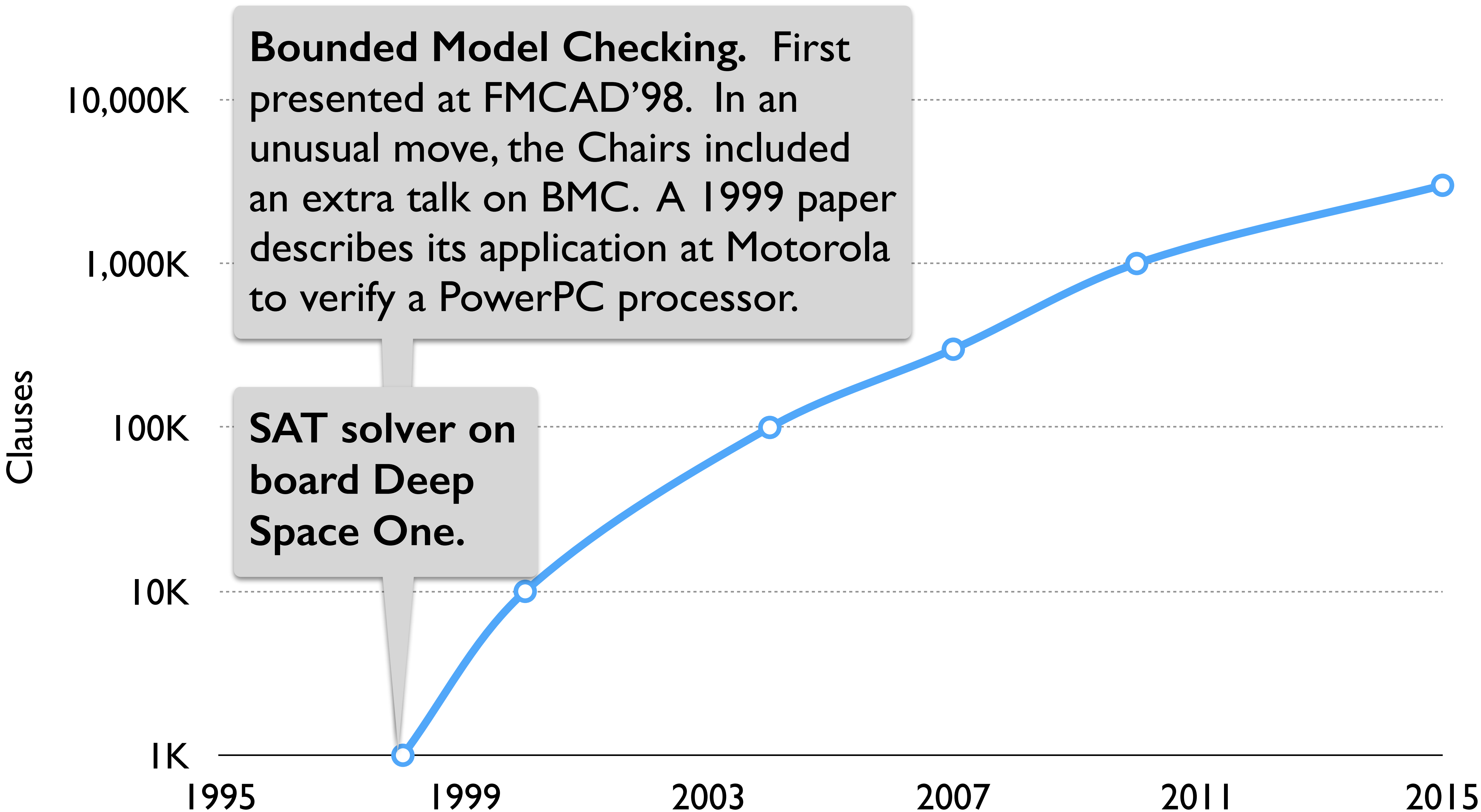

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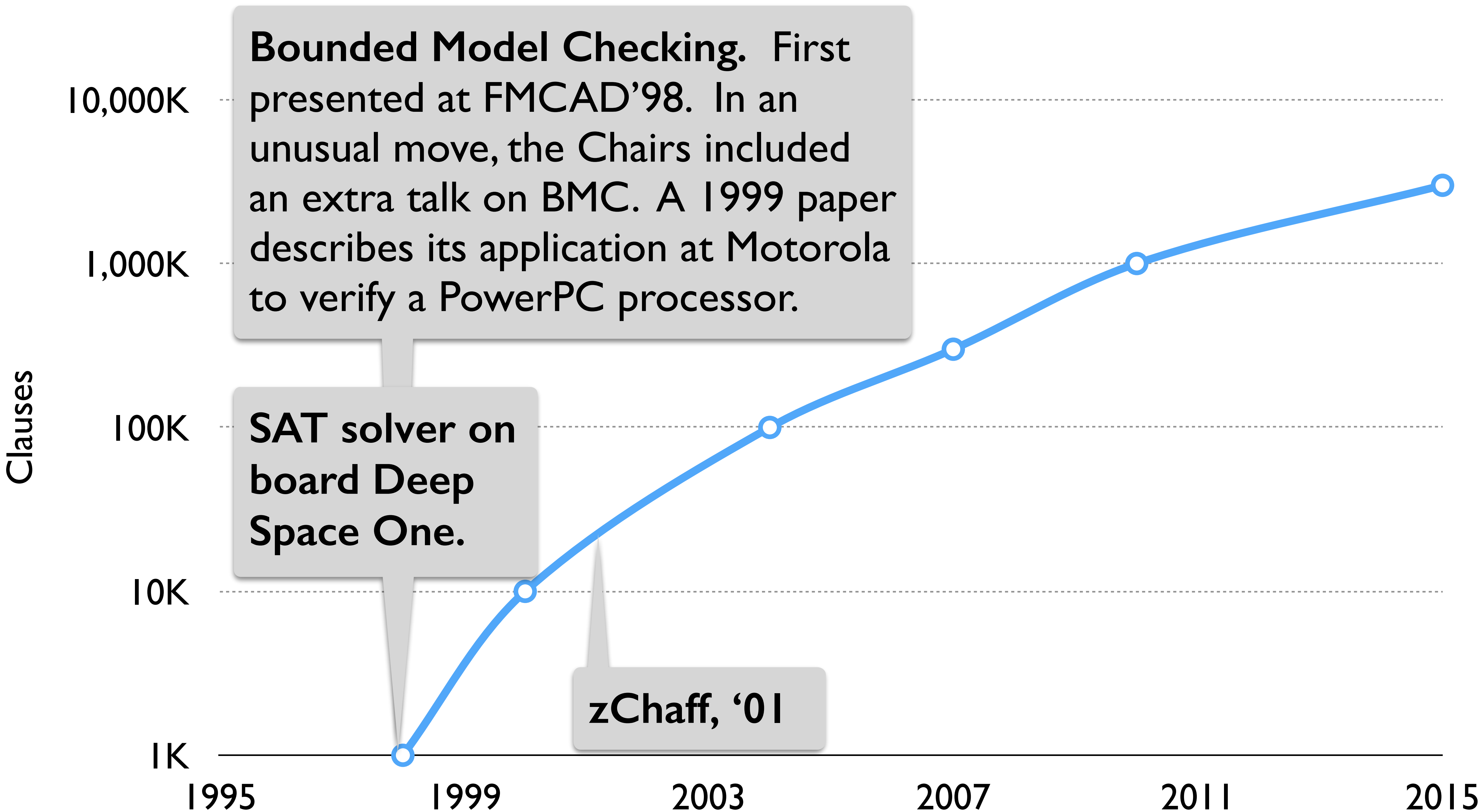

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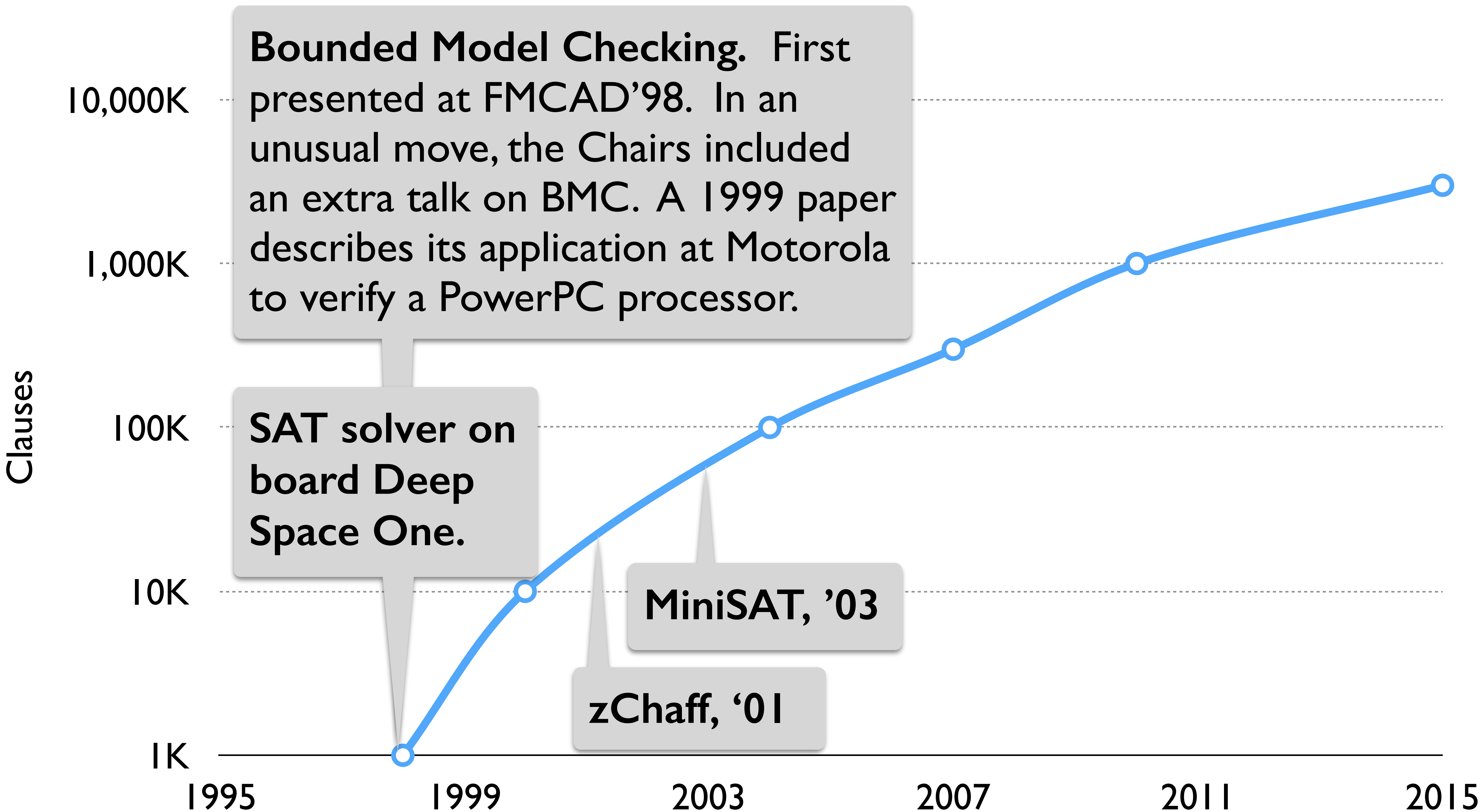

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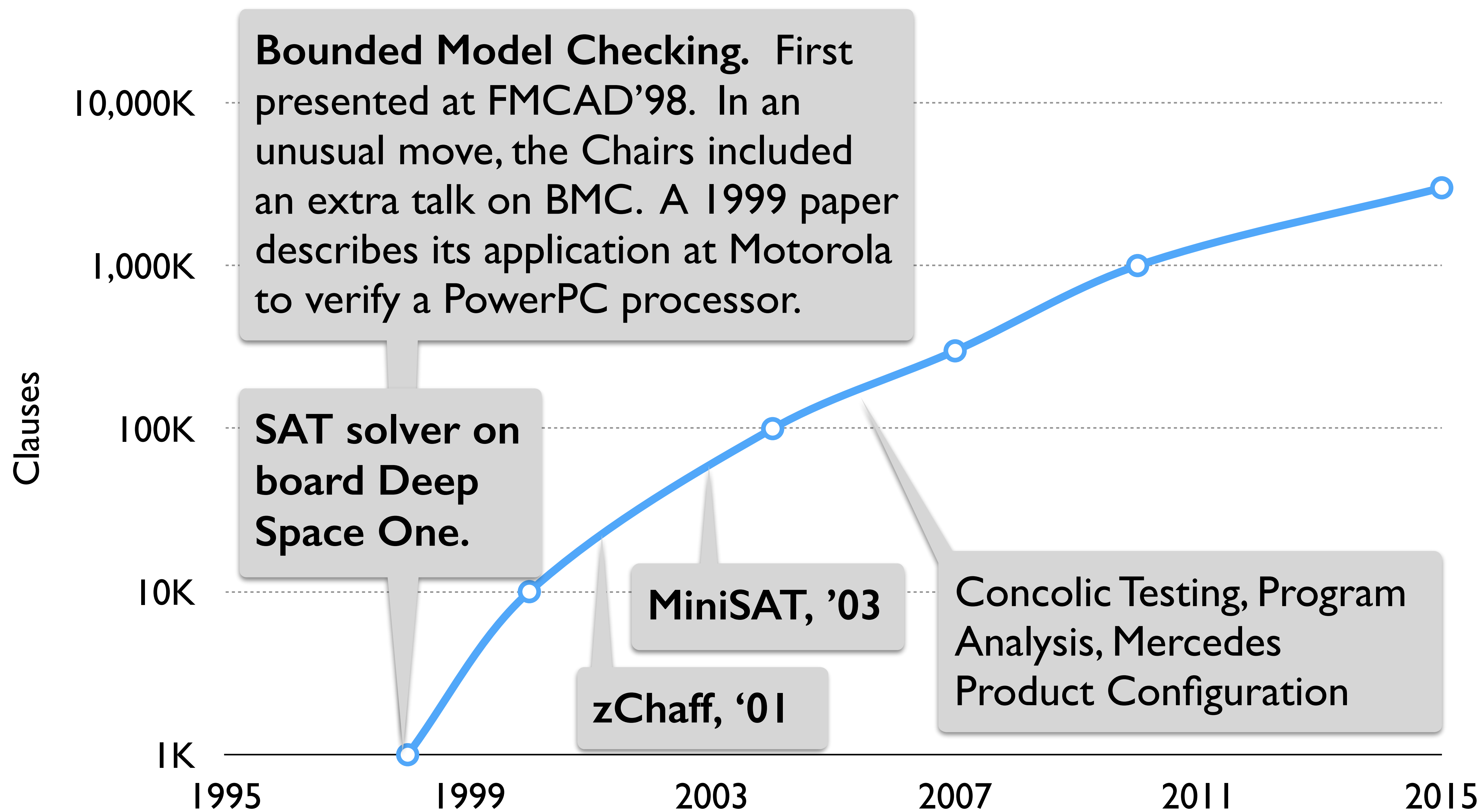

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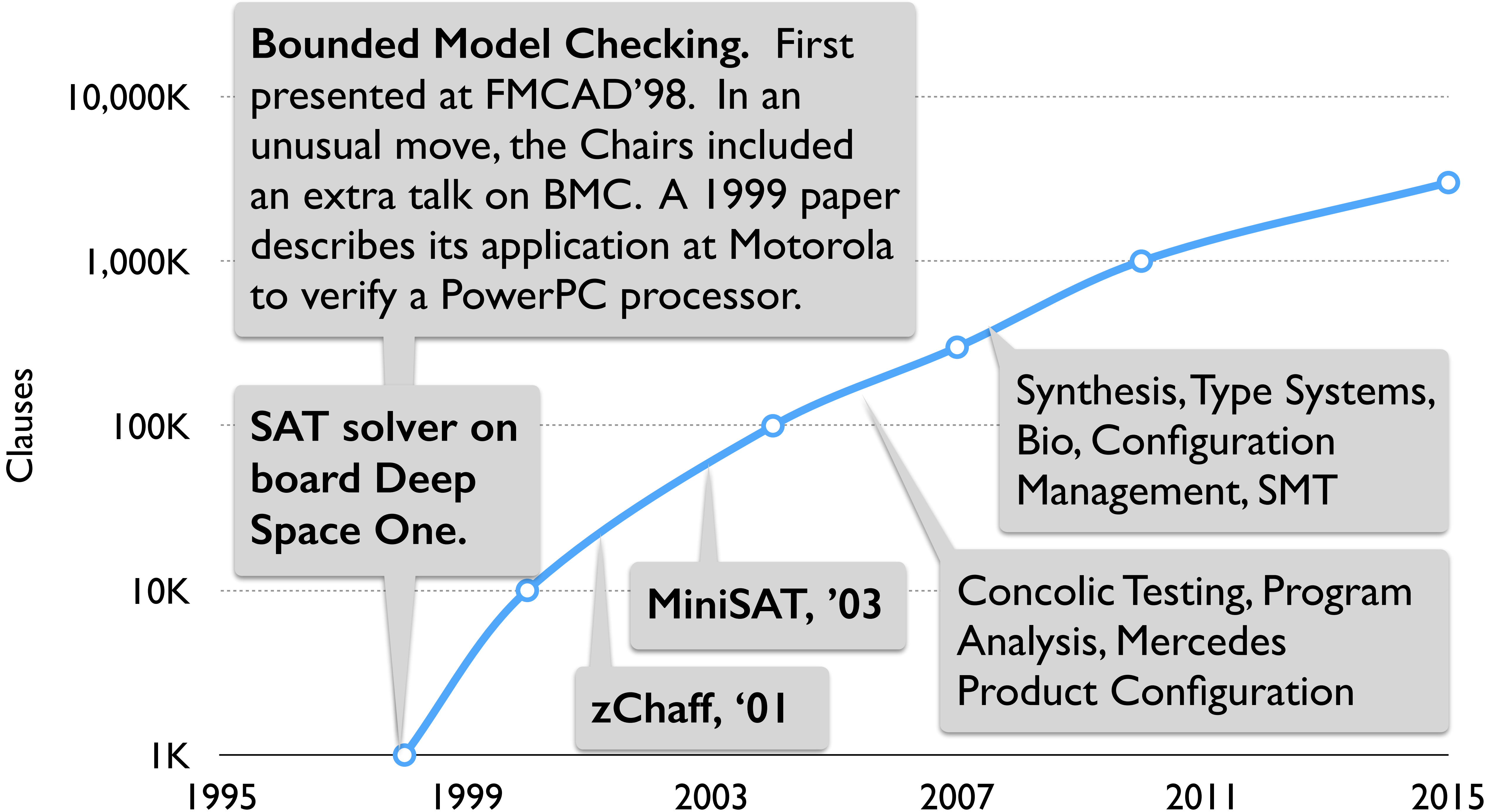

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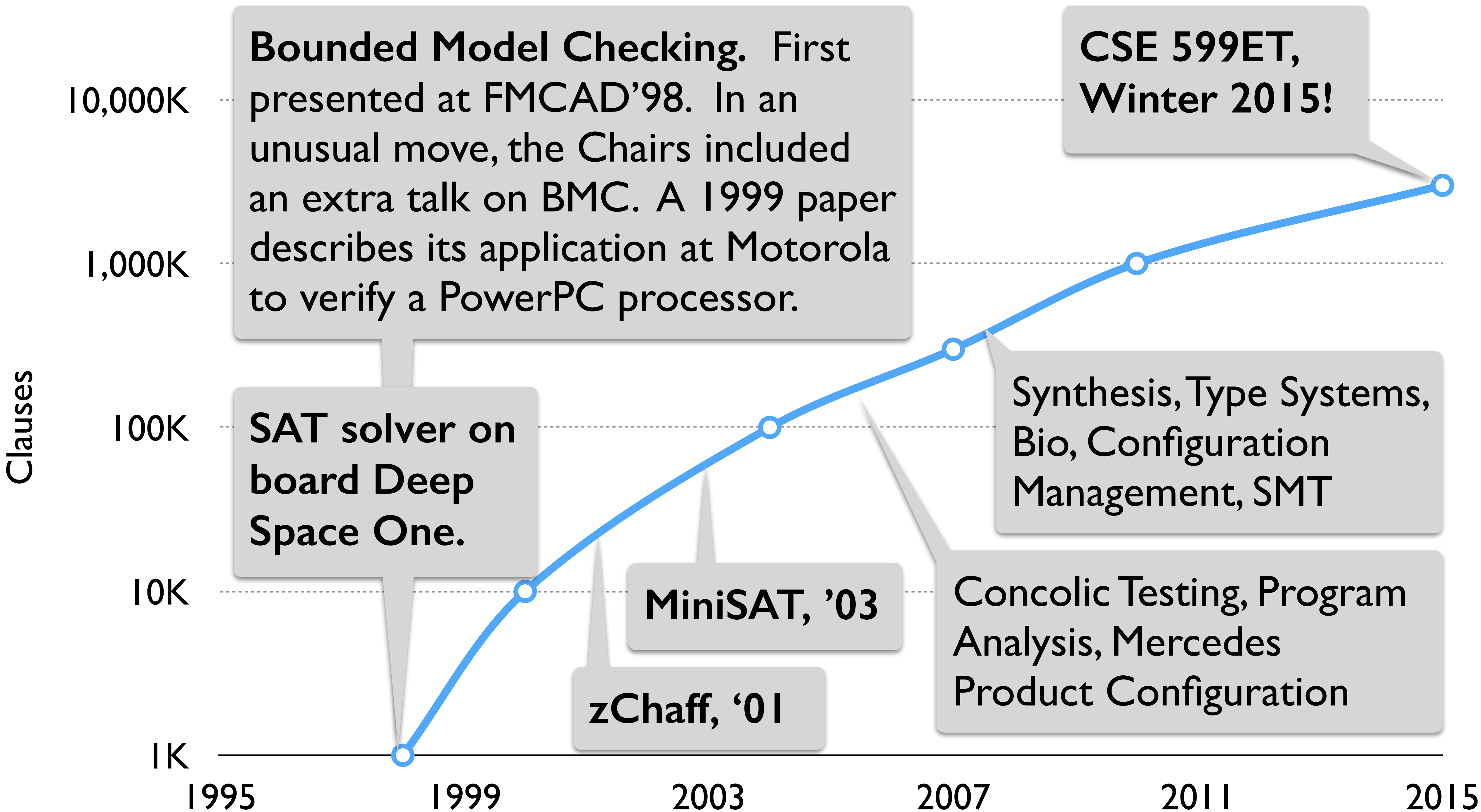
A brief history of SAT solving and applications



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today

Bounded Model Checking (BMC) & Configuration Management

Bounded Model Checking (in general)

Given a system and a property, BMC checks if the property is satisfied by all executions of the system with ≤k steps, on all inputs of size ≤n.

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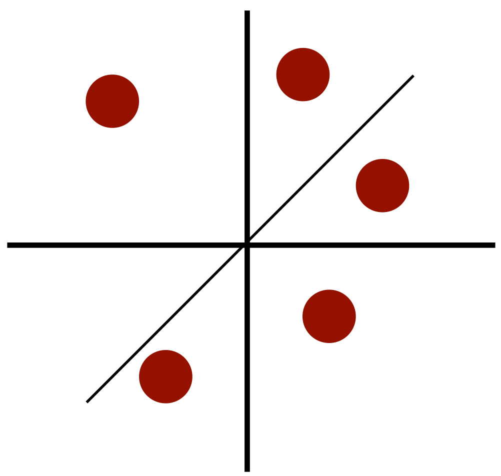
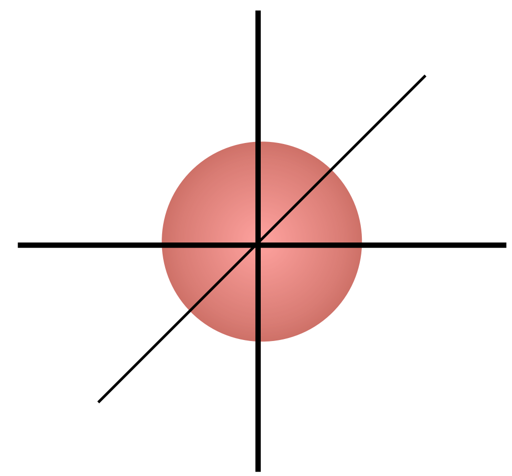
Given a system and a property, BMC checks if the property is satisfied by all executions of the system with ≤k steps, on all inputs of size ≤n.

We will focus on **safety properties** (i.e., making sure a bad state, such as an assertion violation, is not reached).

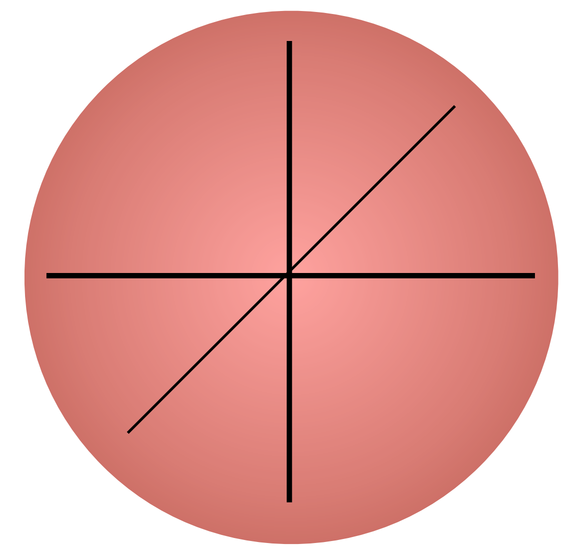
Bounded Model Checking (in general)



Testing: checks a few executions of arbitrary size



BMC: checks all executions of size $\leq k$



Verification: checks all executions of every size

low confidence | high confidence

low human labor | high human labor

BMC by example

```
int daysToYear(int days) {
  int year = 1980;
  while (days > 365) {
    if (isLeapYear(year)) {
      if (days > 366) {
        days -= 366;
        year += 1;
      }
    } else {
      days -= 365;
      year += 1;
    }
  }
  return year;
}
```

BMC by example

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    } else {
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      year += 1;
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}
```

The Zune Bug: on December 31, 2008, all first generation Zune players from Microsoft became unresponsive because of this code. What's wrong?

BMC by example

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      year += 1;
    }
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}
```

Infinite loop triggered on the last day of every leap year.

BMC by example

```
int daysToYear(int days) {
  int year = 1980;
  while (days > 365) {
    int oldDays = days;
    if (isLeapYear(year)) {
      if (days > 366) {
        days -= 366;
        year += 1;
      }
    } else {
      days -= 365;
      year += 1;
    }
    assert days < oldDays;
  }
  return year;
}
```

A desired safety property: the value of the days variable decreases in every loop iteration.

BMC step 1 of 4: finitize loops & inline calls

```
int daysToYear(int days) {
  int year = 1980;
  while (days > 365) {
    int oldDays = days;
    if (isLeapYear(year)) {
      if (days > 366) {
        days -= 366;
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    } else {
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BMC step 1 of 4: finitize loops & inline calls

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      days -= 365;
      year += 1;
    }
    assert days < oldDays;
    assert days <= 365;
  }
  return year;
}
```

- Unwind all loops k times (e.g., k=1), and add an **unwinding assertion** after each.

BMC step 1 of 4: finitize loops & inline calls

```
int daysToYear(int days) {
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      year += 1;
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    assert days <= 365;
  }
  return year;
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- Unwind all loops k times (e.g., k=1), and add an **unwinding assertion** after each.
- If a CEX violates a program assertion, we have found a buggy behavior of length ≤k.

BMC step 1 of 4: finitize loops & inline calls

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- Unwind all loops k times (e.g., k=1), and add an **unwinding assertion** after each.
- If a CEX violates a program assertion, we have found a buggy behavior of length ≤k.
- If a CEX violates an unwinding assertion, the program has no buggy behavior of length ≤k, but it may have a longer one.

BMC step 1 of 4: finitize loops & inline calls

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int daysToYear(int days) {
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    }
    assert days < oldDays;
    assert days <= 365;
  }
  return year;
}
```

- Unwind all loops k times (e.g., k=1), and add an **unwinding assertion** after each.
- If a CEX violates a program assertion, we have found a buggy behavior of length ≤k.
- If a CEX violates an unwinding assertion, the program has no buggy behavior of length ≤k, but it may have a longer one.
- If there is no CEX, the program is correct for all k!

BMC step 1 of 4: finitize loops & inline calls

```
int daysToYear(int days) {
  int year = 1980;
  if (days > 365) {
    int oldDays = days;
    if (isLeapYear(year)) {
      if (days > 366) {
        days -= 366;
        year += 1;
      }
    } else {
      days -= 365;
      year += 1;
    }
    assert days < oldDays;
    assert days <= 365;
  }
  return year;
}
```

Assume call to isLeapYear is inlined (replaced with the procedure body). We'll keep it for readability.

BMC step 2 of 4: eliminate side effects

```
int daysToYear(int days) {
  int year = 1980;
  if (days > 365) {
    int oldDays = days;
    if (isLeapYear(year)) {
      if (days > 366) {
        days -= 366;
        year += 1;
      }
    } else {
      days -= 365;
      year += 1;
    }
    assert days < oldDays;
    assert days <= 365;
  }
  return year;
}
```

BMC step 2 of 4: eliminate side effects

```
int days;
int year = 1980;
if (days > 365) {
  int oldDays = days;
  if (isLeapYear(year)) {
    if (days > 366) {
      days = days - 366;
      year = year + 1;
    }
  } else {
    days = days - 365;
    year = year + 1;
  }
  assert days < oldDays;
  assert days <= 365;
}
return year;
```

Convert to **Static Single Assignment** (SSA) form:

-
-
-

BMC step 2 of 4: eliminate side effects

```
int days0;
int year0 = 1980;
if (days0 > 365) {
  int oldDays0 = days0;
  if (isLeapYear(year0)) {
    if (days0 > 366) {
      days1 = days0 - 366;
      year1 = year0 + 1;
    }
  } else {
    days3 = days0 - 365;
    year3 = year0 + 1;
  }
  assert days4 < oldDays0;
  assert days4 <= 365;
}
return year4;
```

Convert to **Static Single Assignment** (SSA) form:

- Replace each assignment to a variable v with a definition of a fresh variable v_i.
- Change uses of variables so that they refer to the correct definition (version).
-

BMC step 2 of 4: eliminate side effects

```
int days0;
int year0 = 1980;
boolean g0 = (days0 > 365);
int oldDays0 = days0;
boolean g1 = isLeapYear(year0);
boolean g2 = days0 > 366;
days1 = days0 - 366;
year1 = year0 + 1;
days2 = φ(g1 && g2, days1, days0)
year2 = φ(g1 && g2, year1, year0)
days3 = days0 - 365;
year3 = year0 + 1;
days4 = φ(g1, days2, days3)
year4 = φ(g1, year2, year3)
assert days4 < oldDays0;
assert days4 <= 365;
return year4;
```

Convert to **Static Single Assignment** (SSA) form:

- Replace each assignment to a variable v with a definition of a fresh variable v_i.
- Change uses of variables so that they refer to the correct definition (version).
- Make conditional dependences explicit with gated φ nodes.

BMC step 2 of 4: eliminate side effects

```
int days0;
int year0 = 1980;
if (days0 > 365) {
  int oldDays0 = days0;
  if (isLeapYear(year0)) {
    if (days0 > 366) {
      days1 = days0 - 366;
      year1 = year0 + 1;
    }
  } else {
    days3 = days0 - 365;
    year3 = year0 + 1;
  }
  assert days4 < oldDays0;
  assert days4 <= 365;
}
return year4;
```

```
int days0;
int year0 = 1980;
boolean g0 = (days0 > 365);
int oldDays0 = days0;
boolean g1 = isLeapYear(year0);
boolean g2 = days0 > 366;
days1 = days0 - 366;
year1 = year0 + 1;
days2 = φ(g1 && g2, days1, days0)
year2 = φ(g1 && g2, year1, year0)
days3 = days0 - 365;
year3 = year0 + 1;
days4 = φ(g1, days2, days3)
year4 = φ(g1, year2, year3)
assert days4 < oldDays0;
assert days4 <= 365;
return year4;
```

BMC step 3 of 4: convert into equations

```
int days0;
int year0 = 1980;
boolean g0 = (days0 > 365);
int oldDays0 = days0;
boolean g1 = isLeapYear(year0);
boolean g2 = days0 > 366;
days1 = days0 - 366;
year1 = year0 + 1;
days2 = φ(g1 && g2, days1, days0)
year2 = φ(g1 && g2, year1, year0)
days3 = days0 - 365;
year3 = year0 + 1;
days4 = φ(g1, days2, days3)
year4 = φ(g1, year2, year3)
assert days4 < oldDays0;
assert days4 <= 365;
return year4;
```

BMC step 3 of 4: convert into equations

$$
\begin{aligned}
& year_0 = 1980 \wedge \\
& g_0 = (days_0 > 365) \wedge \\
& oldDays_0 = days_0 \wedge \\
& g_1 = \mathit{isLeapYear}(year_0) \wedge \\
& g_2 = days_0 > 366 \wedge \\
& days_1 = days_0 - 366 \wedge \\
& year_1 = year_0 + 1 \wedge \\
& days_2 = ite(g_1 \wedge g_2, days_1, days_0) \wedge \\
& year_2 = ite(g_1 \wedge g_2, year_1, year_0) \wedge \\
& days_3 = days_0 - 365 \wedge \\
& year_3 = year_0 + 1 \wedge \\
& days_4 = \varphi(g_1, days_2, days_3) \wedge \\
& year_4 = \varphi(g_1, year_2, year_3) \wedge \\
& (\neg(days_4 < oldDays_0) \vee \\
& \;\neg(days_4 <= 365))
\end{aligned}
$$

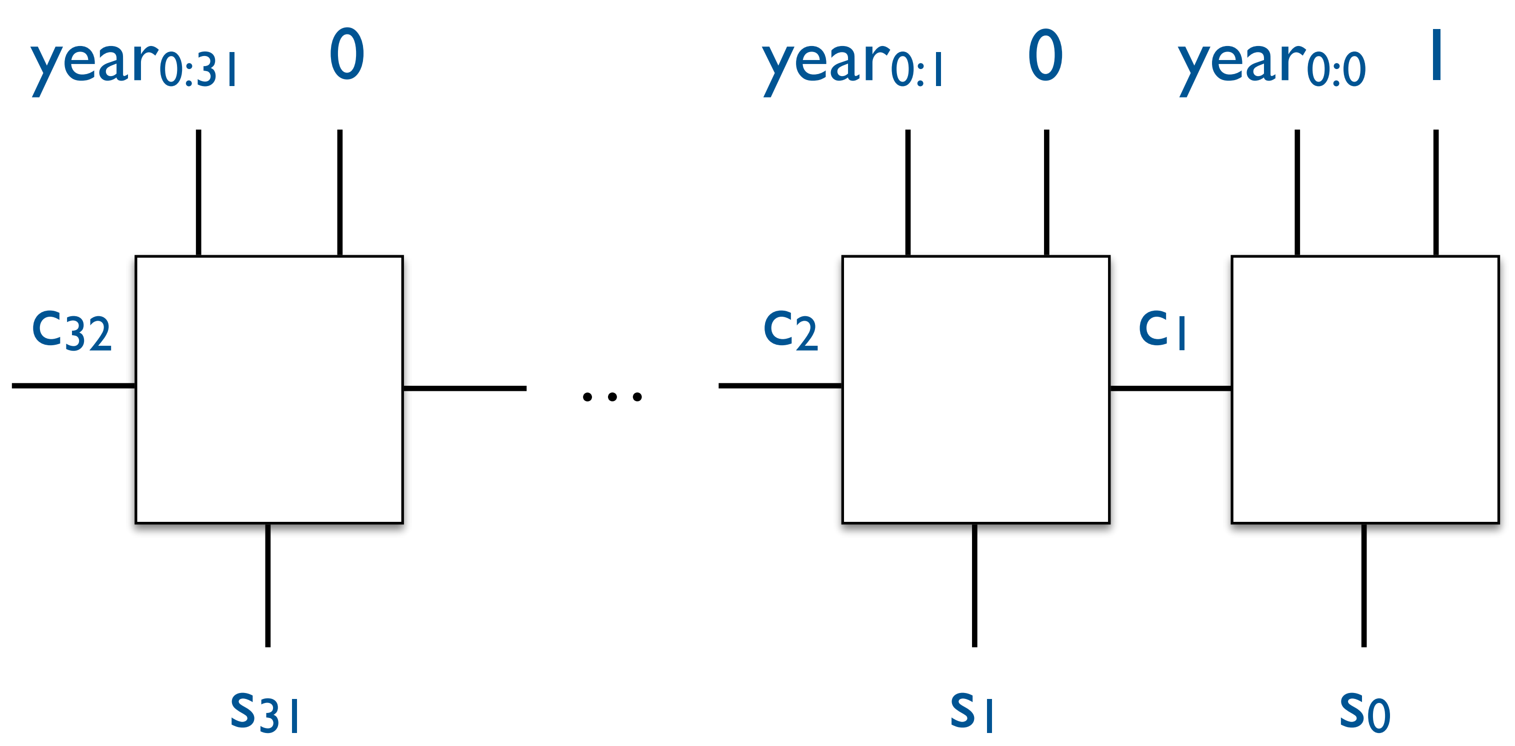
A solution to these equations is a sound **counterexample**: an interpretation for all logical variables that satisfies the program semantics (for up to k unwindings) but violates at least one of the assertions.

BMC step 4 of 4: convert into CNF

$year_1 = year_0 + 1$

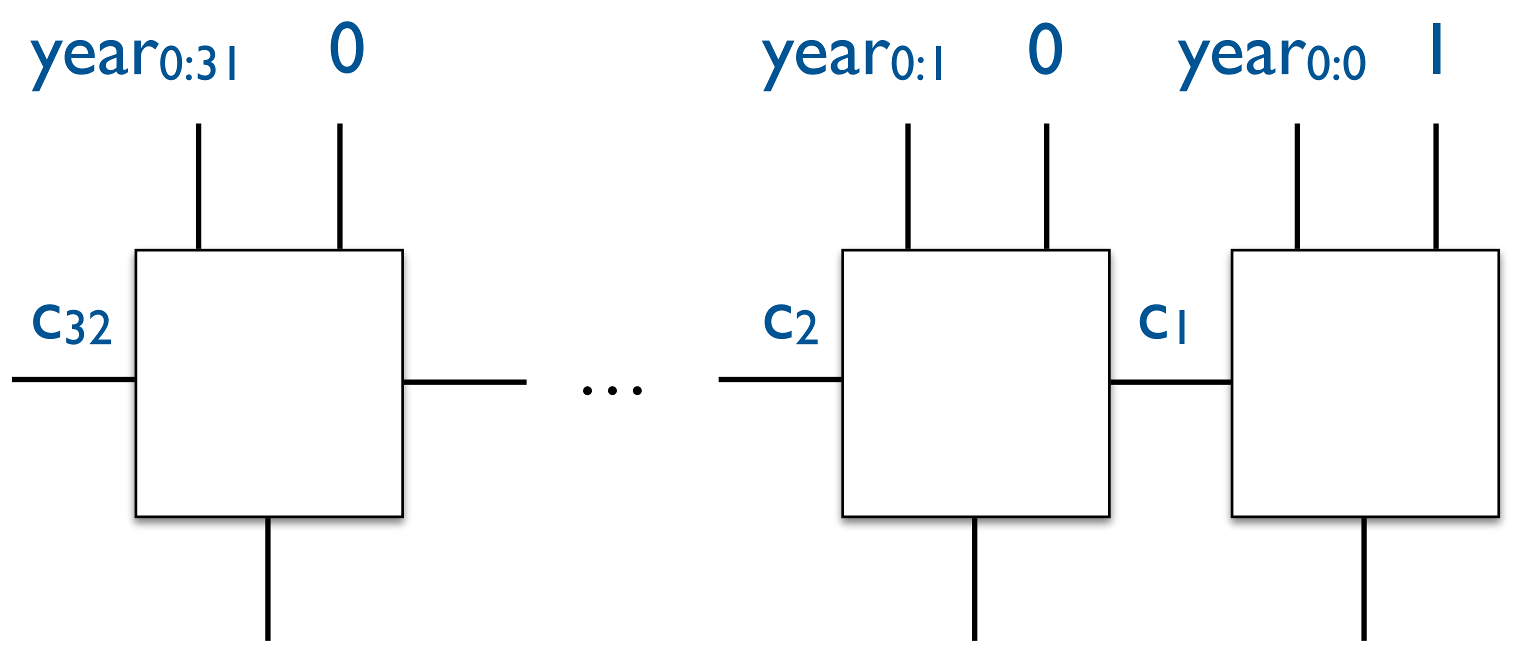
BMC step 4 of 4: convert into CNF

```
year₁ = year₀ + 1
```



BMC step 4 of 4: convert into CNF

```
year₁ = year₀ + 1
```



$$year_{1:31} \Longleftrightarrow s_{31} \wedge \ldots \wedge year_{1:1} \Longleftrightarrow s_1 \wedge s_0 \Longleftrightarrow year_{1:0}$$

BMC counterexample for k=1

```
int daysToYear(int days) {
  int year = 1980;
  while (days > 365) {
    int oldDays = days;
    if (isLeapYear(year)) {
      if (days > 366) {
        days -= 366;
        year += 1;
      }
    } else {
      days -= 365;
      year += 1;
    }
    assert days < oldDays;
  }
  return year;
}
```

days = 366

today

Bounded Model Checking (BMC) & Configuration Management

Configuration Management

Given a configuration, consisting of a set of components, their dependencies, and conflicts:

- Decide if a new component can be added to the configuration.
- Add the component while optimizing some linear function.
- If the component cannot be added, find way to add it by removing as few conflicting components from the current configuration as possible.

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SAT

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SAT

Pseudo-Boolean Constraints

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SAT

Pseudo-Boolean Constraints

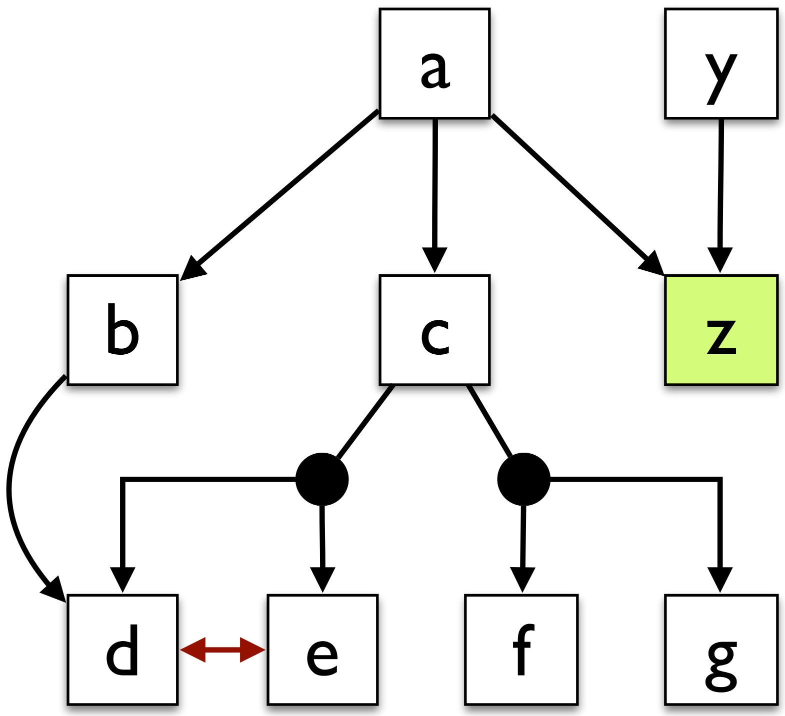
Partial (Weighted) MaxSAT

maven

eclipse

suse

Deciding if a component can be installed



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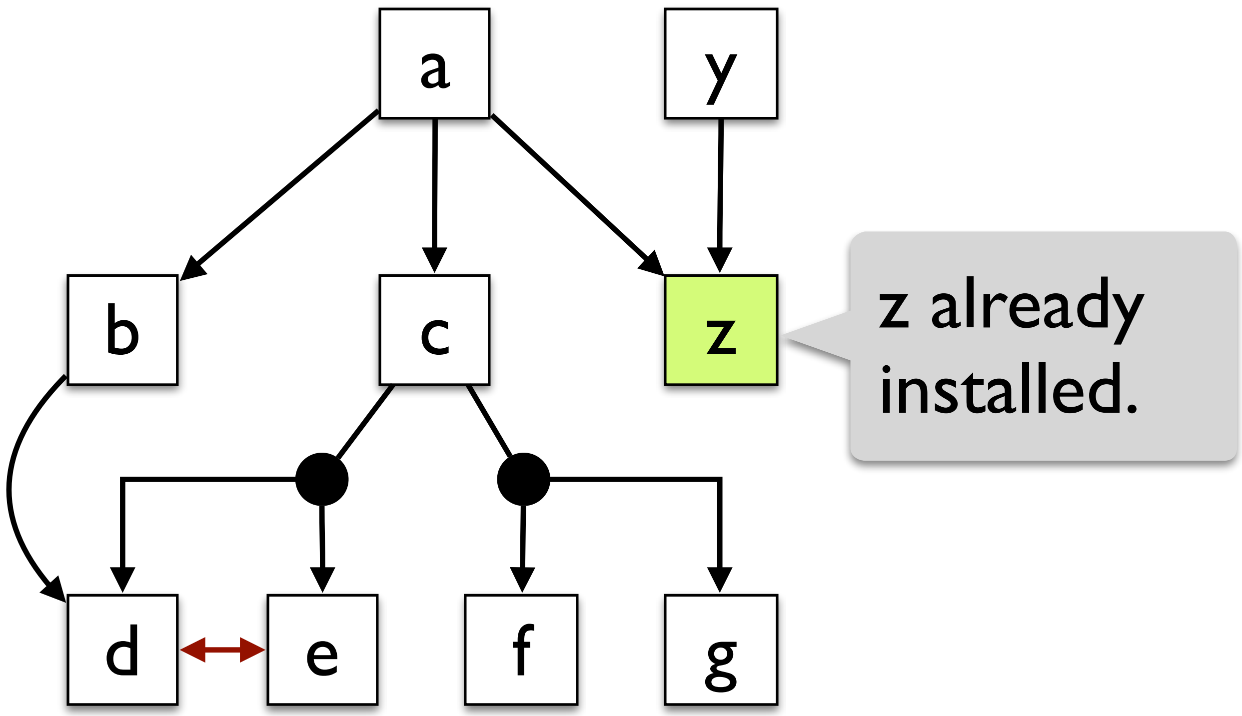

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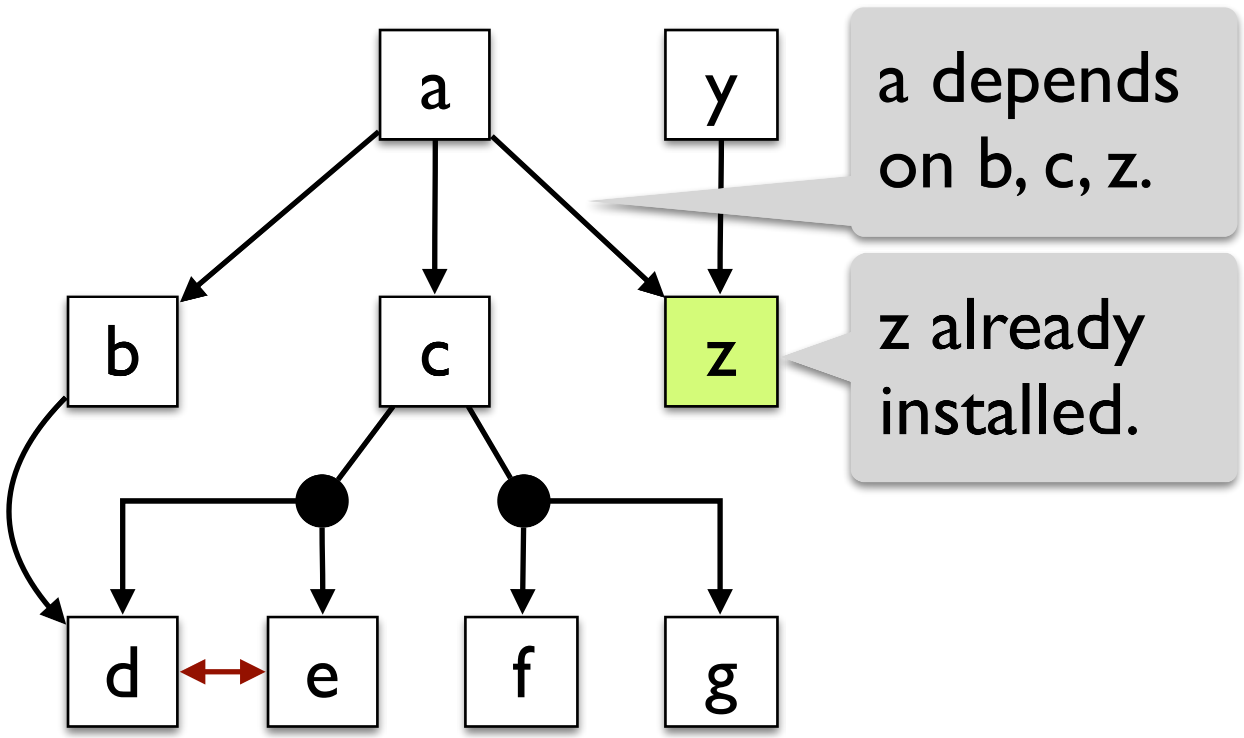

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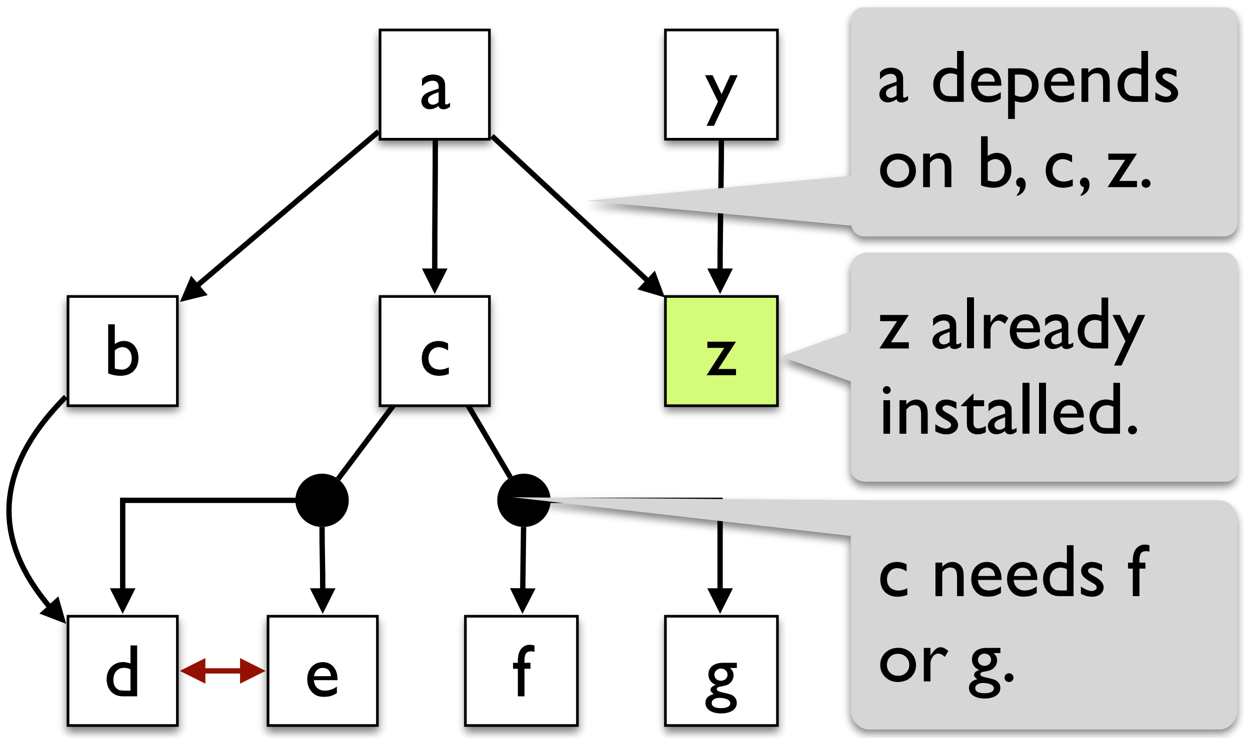

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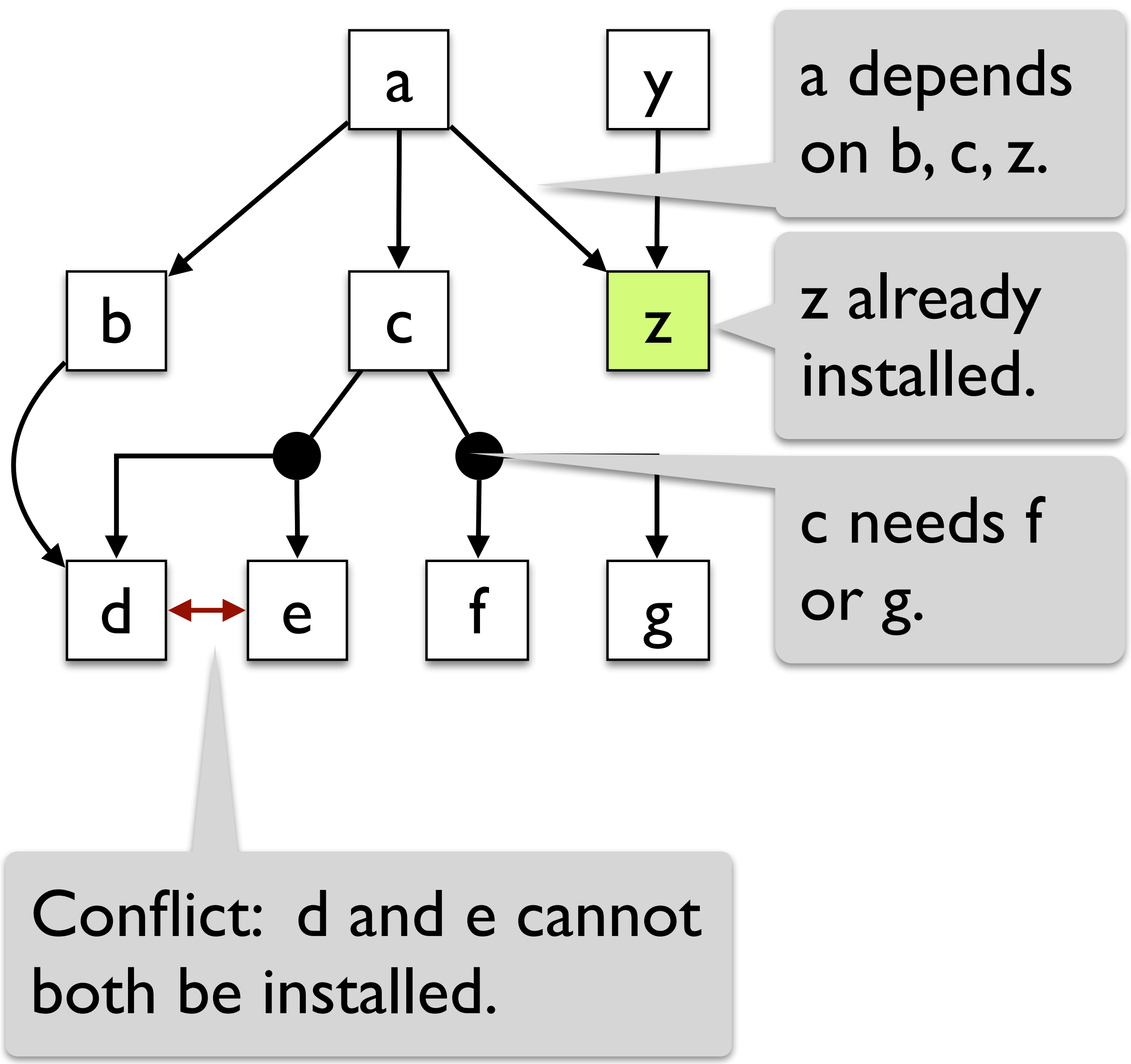

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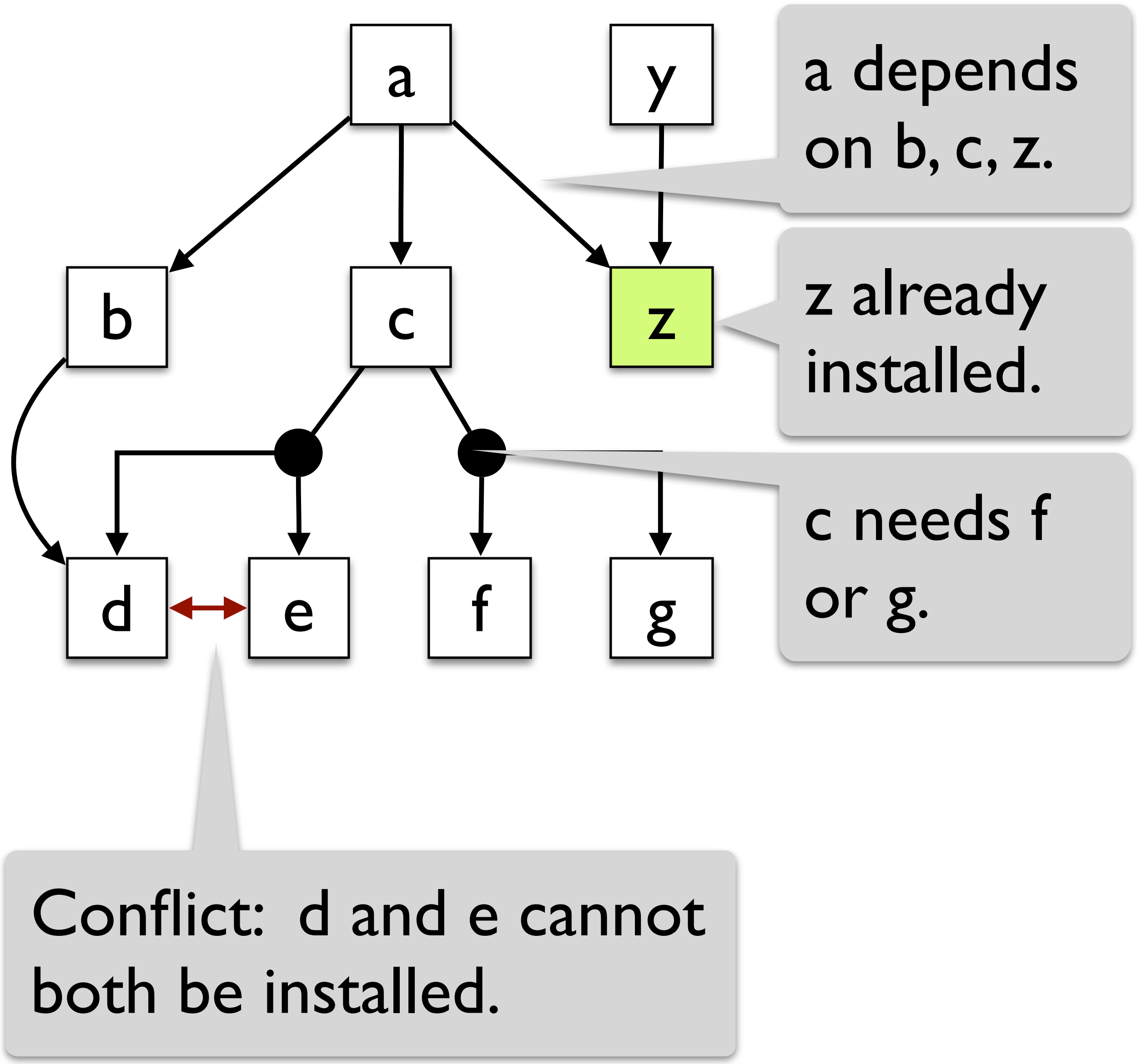
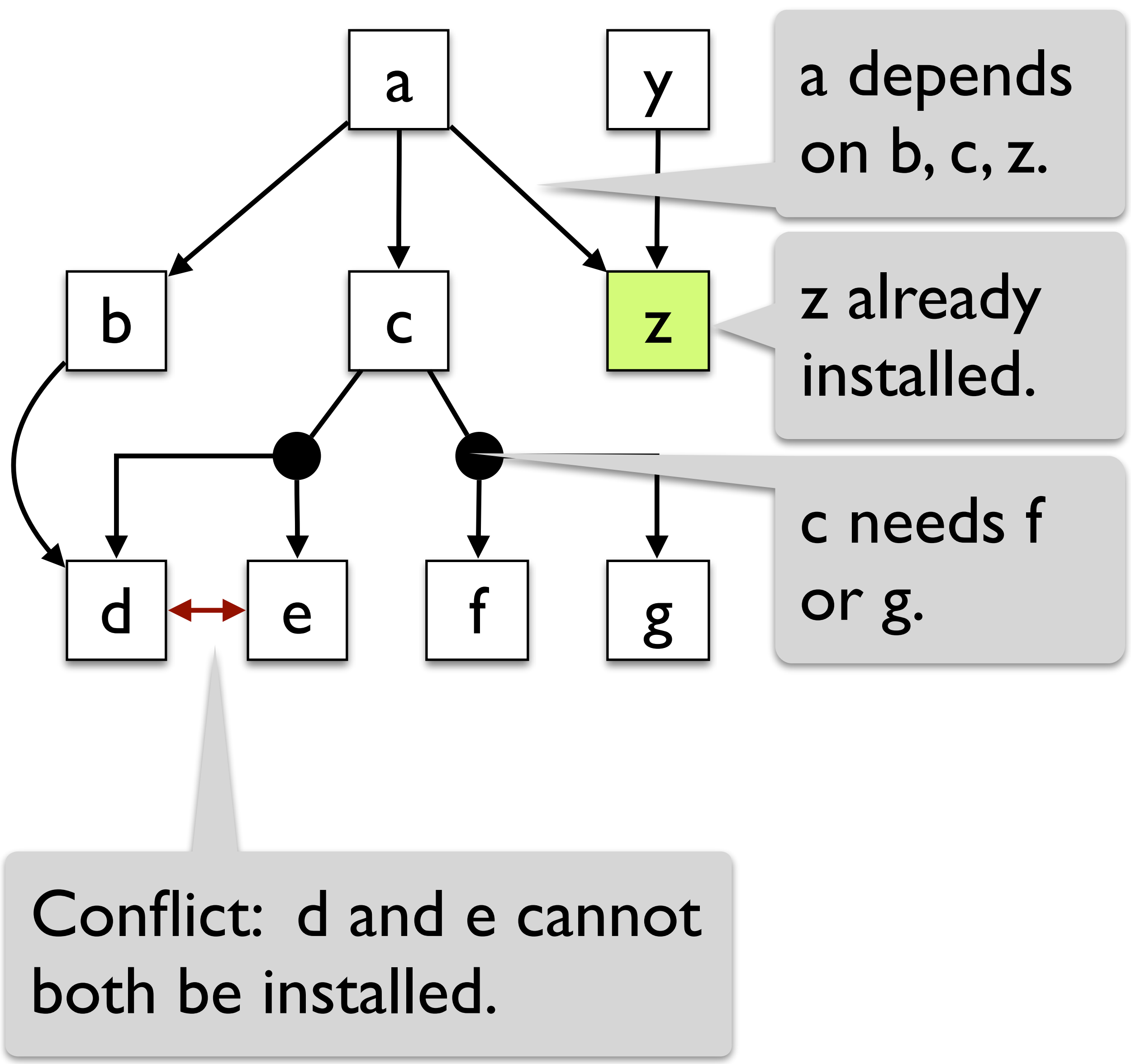


To install a, CNF constraints are:

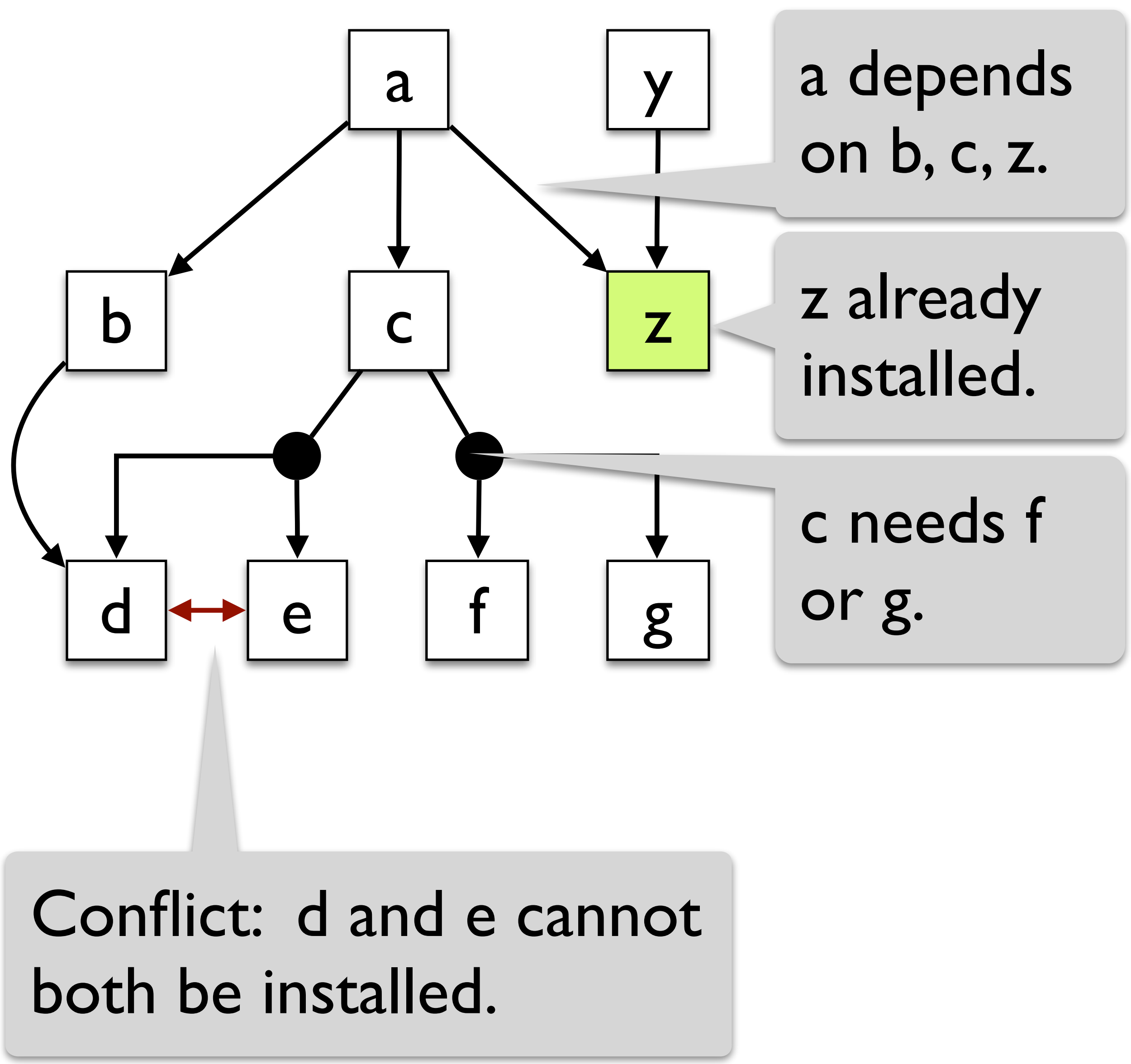
Deciding if a component can be installed



To install a, CNF constraints are:

$(\neg a \lor b) \land (\neg a \lor c) \land (\neg a \lor z) \land$

Deciding if a component can be installed



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$(\neg b \lor d) \land$

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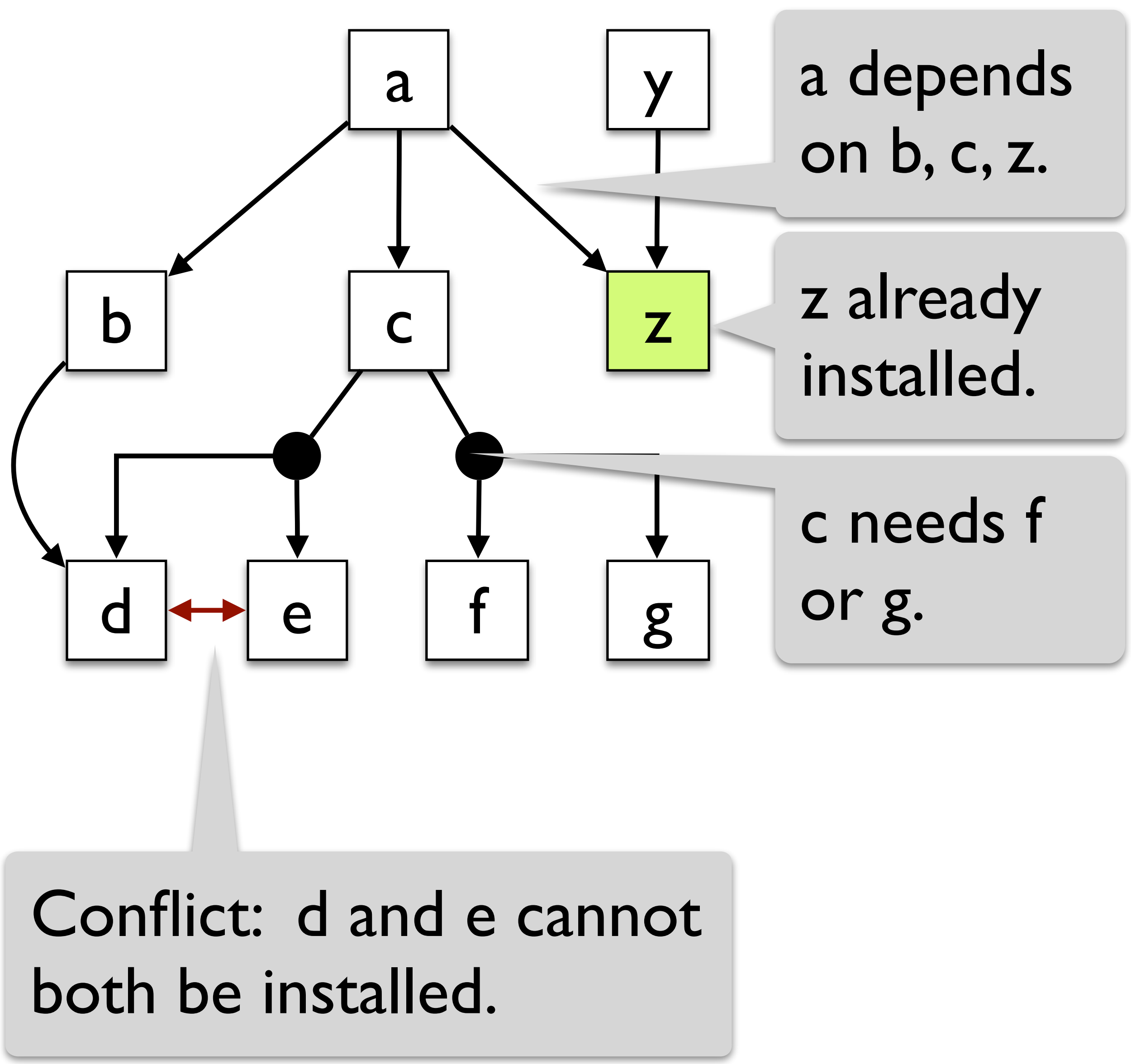

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$(\neg a \lor b) \land (\neg a \lor c) \land (\neg a \lor z) \land$
$(\neg b \lor d) \land$
$(\neg c \lor d \lor e) \land (\neg c \lor f \lor g) \land$

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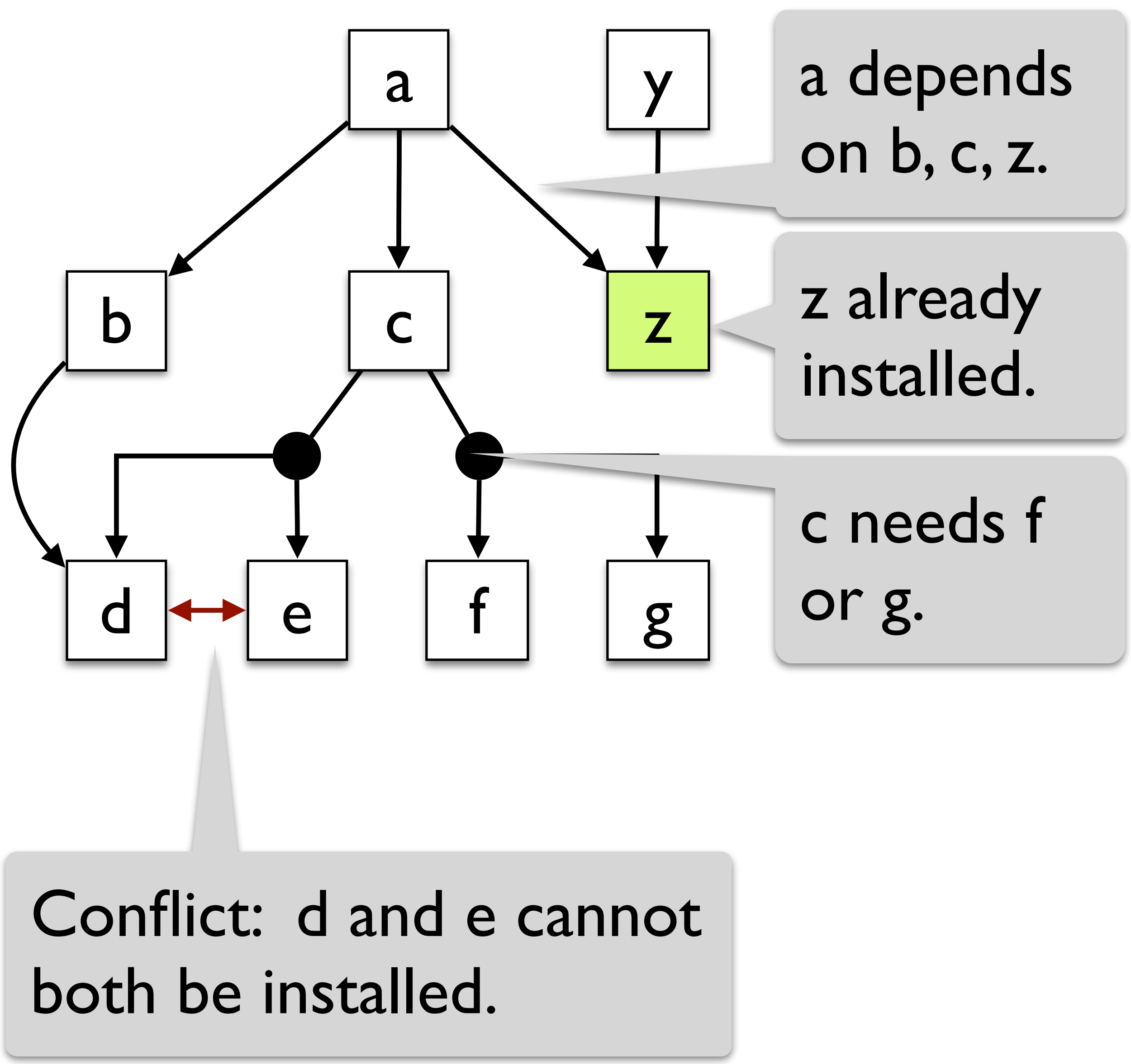

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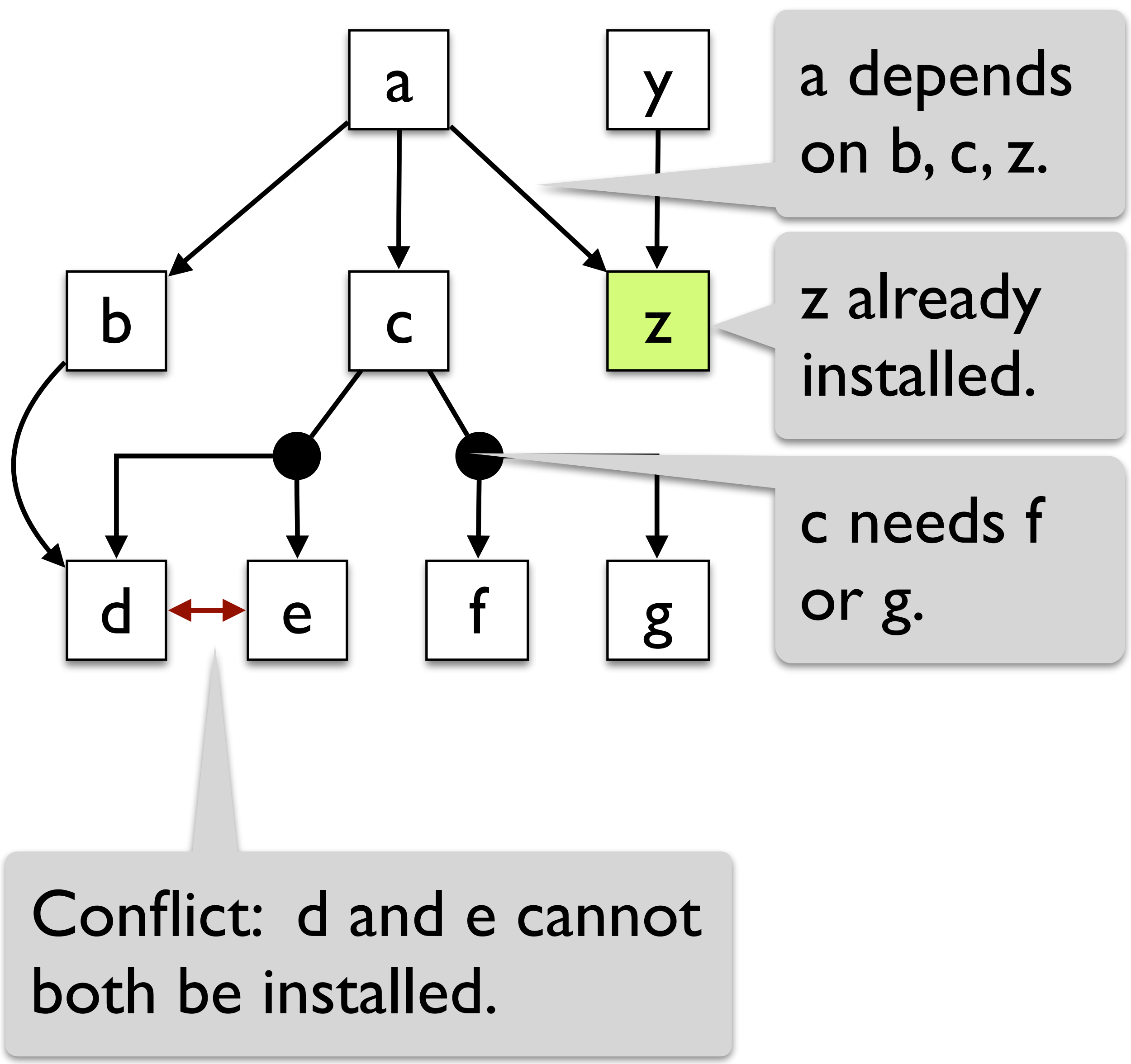
$(\neg a \lor b) \land (\neg a \lor c) \land (\neg a \lor z) \land$
$(\neg b \lor d) \land$
$(\neg c \lor d \lor e) \land (\neg c \lor f \lor g) \land$
$(\neg d \lor \neg e) \land$

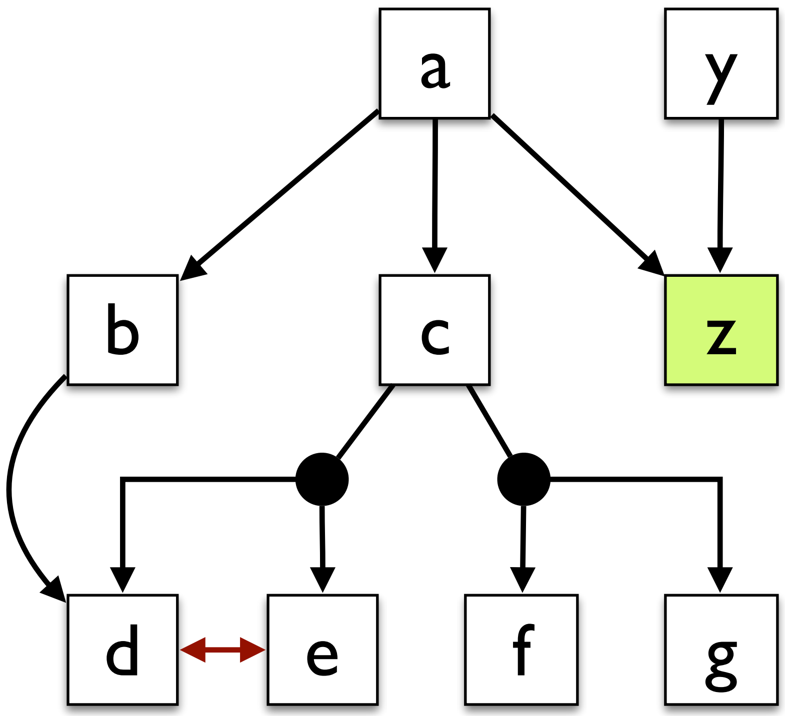
Deciding if a component can be installed



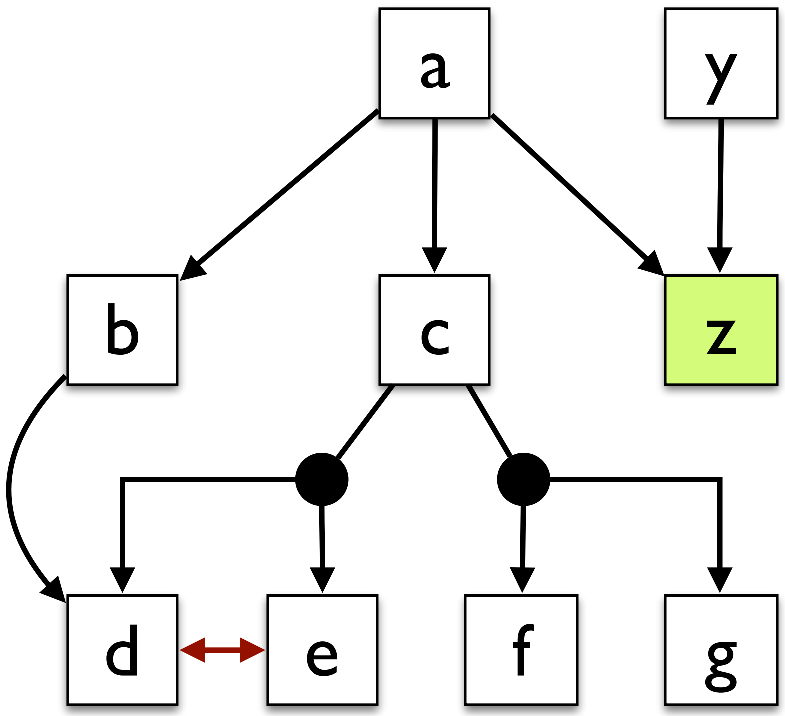
To install a, CNF constraints are:

$(\neg a \vee b) \wedge (\neg a \vee c) \wedge (\neg a \vee z) \wedge$
$(\neg b \vee d) \wedge$
$(\neg c \vee d \vee e) \wedge (\neg c \vee f \vee g) \wedge$
$(\neg d \vee \neg e) \wedge$
$a \wedge z$

Optimal installation



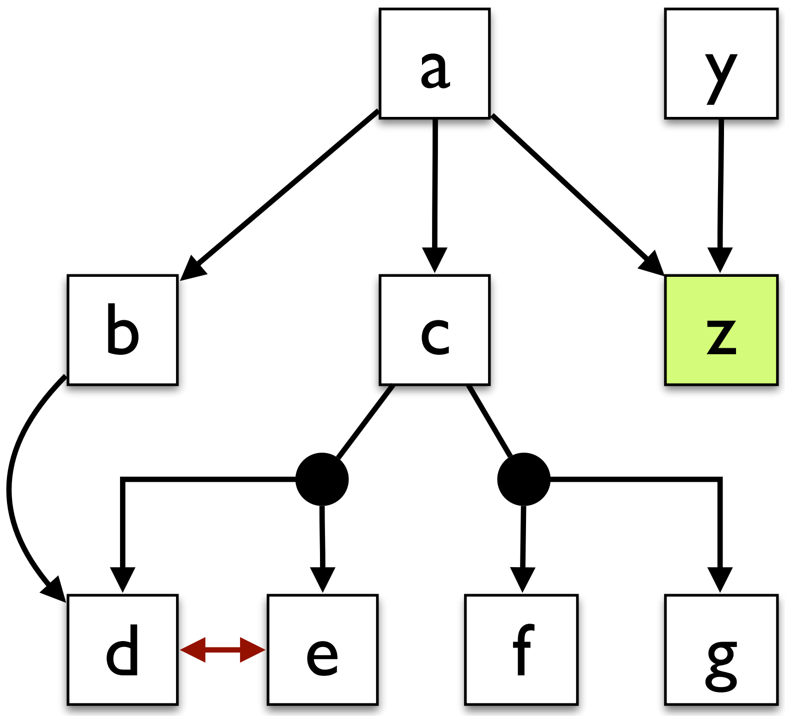
Optimal installation



Pseudo-boolean solvers accept a linear function to minimize, in addition to a (weighted) CNF.

Assume f and g are 5MB and 2MB each, and all other components are 1MB. To install a, while minimizing total size, pseudo-boolean constraints are:

Optimal installation



$$\min\ c_1x_1 + \ldots + c_nx_n$$

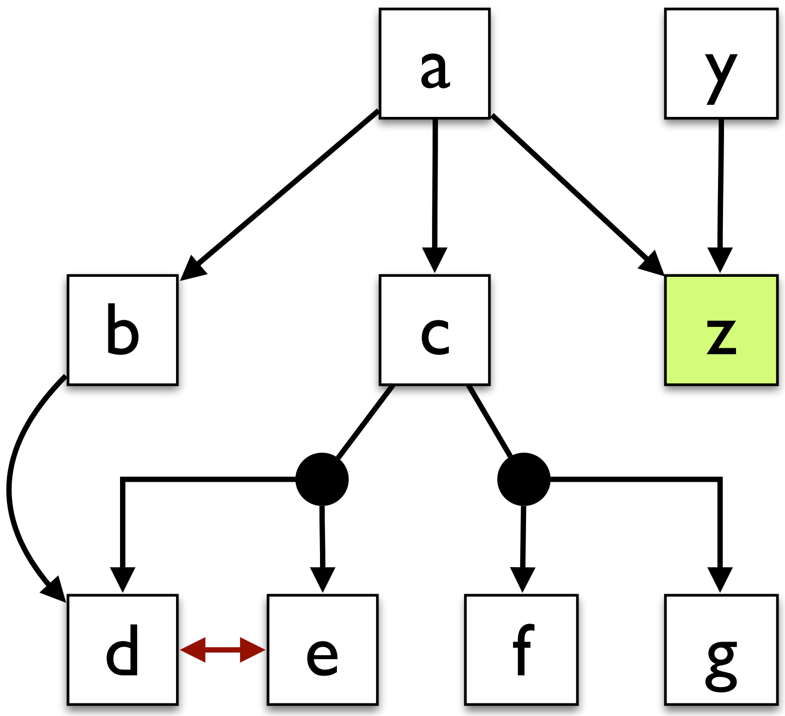
$$a_{11}x_1 + \ldots + a_{1n}x_n \geq b_1$$

$$\ldots$$

$$a_{k1}x_1 + \ldots + a_{kn}x_n \geq b_k$$

Assume f and g are 5MB and 2MB each, and all other components are 1MB. To install a, while minimizing total size, pseudo-boolean constraints are:

Optimal installation



$$\min c_1x_1 + \ldots + c_nx_n$$

$$a_{11}x_1 + \ldots + a_{1n}x_n \geq b_1$$

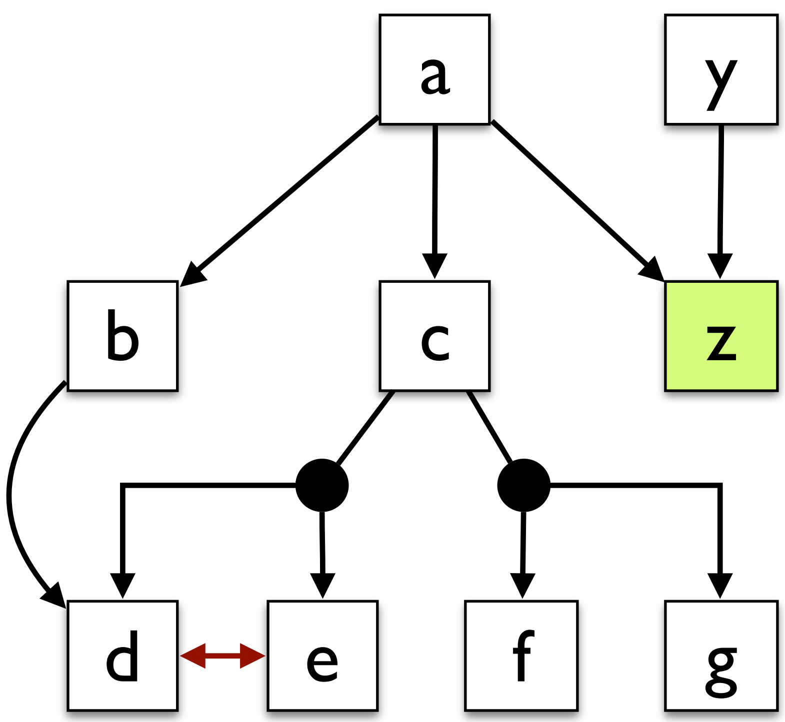
$$\ldots$$

$$a_{k1}x_1 + \ldots + a_{kn}x_n \geq b_k$$

Assume f and g are 5MB and 2MB each, and all other components are 1MB. To install a, while minimizing total size, pseudo-boolean constraints are:

min a + b + c + d + e + 5f + 2g + y

Optimal installation



$$\min\ c_1x_1 + \ldots + c_nx_n$$

$$a_{11}x_1 + \ldots + a_{1n}x_n \geq b_1$$

$$\ldots$$

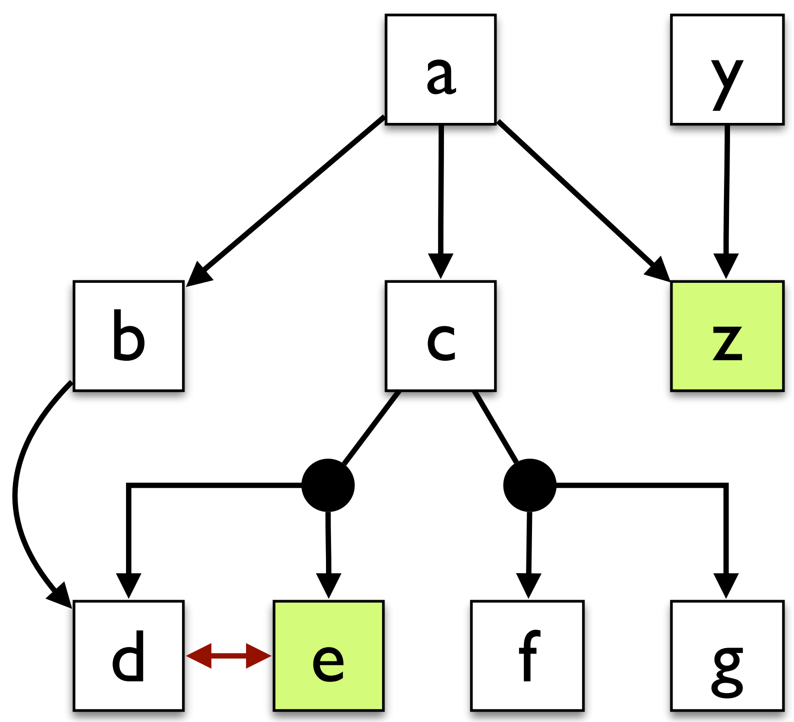
$$a_{k1}x_1 + \ldots + a_{kn}x_n \geq b_k$$

Assume f and g are 5MB and 2MB each, and all other components are 1MB. To install a, while minimizing total size, pseudo-boolean constraints are:

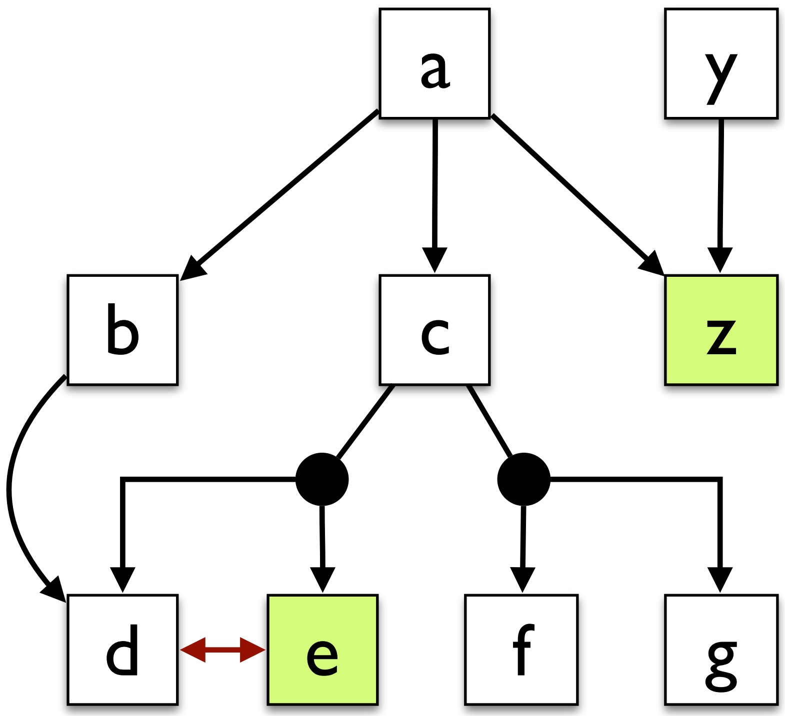
$$\min\ a + b + c + d + e + 5f + 2g + y$$

$$(-a + b \geq 0) \wedge (-a + c \geq 0) \wedge (-a + z \geq 0) \wedge$$
$$(-b + d \geq 0) \wedge$$
$$(-c + d + e \geq 0) \wedge (-c + f + g \geq 0) \wedge$$
$$(-d + -e \geq -1) \wedge$$
$$(a \geq 1) \wedge (z \geq 1)$$

Installation in the presence of conflicts

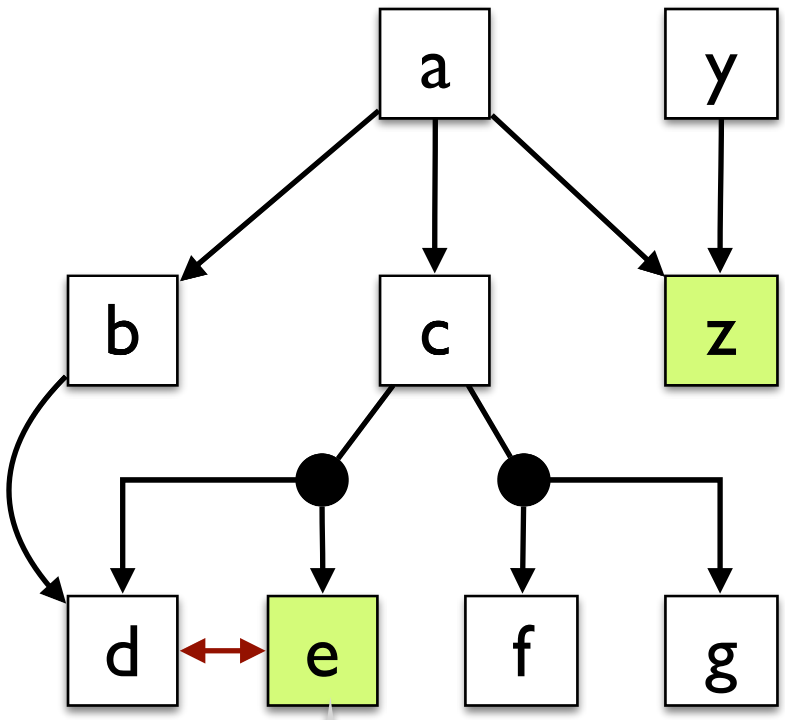


Installation in the presence of conflicts



a cannot be installed because it requires b, which requires d, which conflicts with e.

Installation in the presence of conflicts



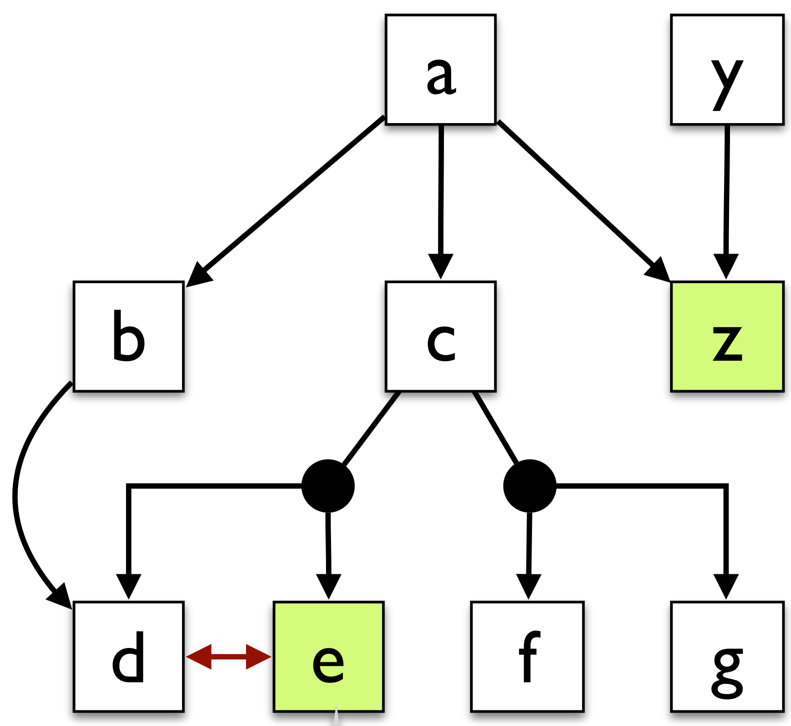
To install a, while minimizing the number of removed components, Partial MaxSAT constraints are:

hard: (¬a ∨ b) ∧ (¬a ∨ c) ∧ (¬a ∨ z) ∧
(¬b ∨ d) ∧
(¬c ∨ d ∨ e) ∧ (¬c ∨ f ∨ g) ∧
(¬d ∨ ¬e) ∧ a

soft: e ∧ z

Partial MaxSAT solver takes as input a set of **hard** clauses and a set of **soft** clauses, and it produces an assignment that satisfies all hard clauses and the greatest number of soft clauses.

Installation in the presence of conflicts



To install a, while minimizing the number of removed components, Partial MaxSAT constraints are:

hard: $(\neg a \vee b) \wedge (\neg a \vee c) \wedge (\neg a \vee z) \wedge$
$(\neg b \vee d) \wedge$
$(\neg c \vee d \vee e) \wedge (\neg c \vee f \vee g) \wedge$
$(\neg d \vee \neg e) \wedge a$

soft: $e \wedge z$

Partial MaxSAT solver takes as input a set of **hard** clauses and a set of **soft** clauses, and it produces an assignment that satisfies all hard clauses and the greatest number of soft clauses.

Summary

Today

- SAT solvers have been used successfully in many applications & domains
- But reducing problems to SAT is a lot like programming in assembly …
- We need higher-level logics!

Next lecture

- On to richer logics: introduction to Satisfiability Modulo Theories (SMT)