Computer-Aided Reasoning for Software

CSE507

Model Checking I

courses.cs.washington.edu/courses/cse507/14au/

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Today

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Last lecture

- Symbolic execution and concolic testing

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- Introduction to model checking

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Reminders

- Homework 3 is due on Tuesday, November 18, at 11pm

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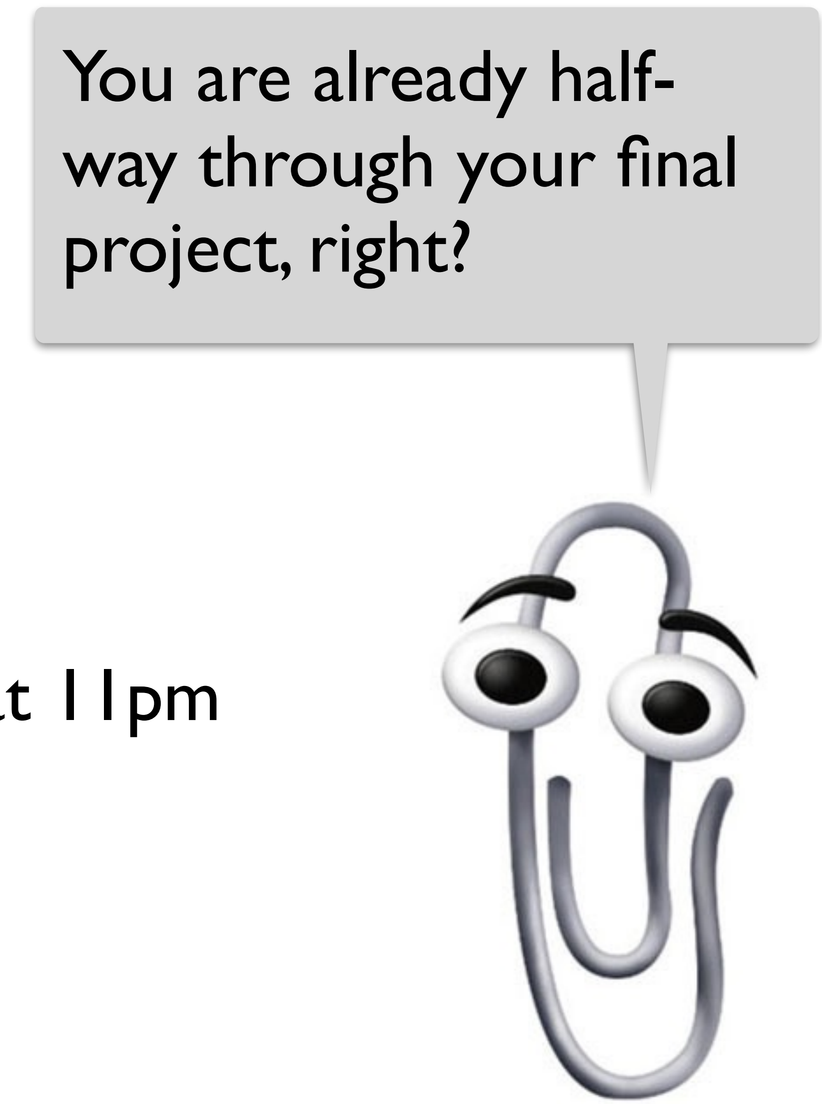

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An automated technique for verifying that a concurrent finite state system satisfies a given temporal property.

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A **temporal logic** formula (e.g., a request is *eventually* acknowledged).

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A mathematical model of the system, given as a **Kripke structure** (a finite state machine).

Why model checking?

Model checking

Classic & bounded verification

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- Deterministic, single-threaded, possibly infinite-state, terminating programs.
- Fully described by their input/output behavior.
- Semi-automatic or bounded-automatic checking of properties in expressive logics (e.g., FOL).

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- *Reactive systems*: concurrent finite-state programs with *ongoing* input/output behavior.
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- Embedded controllers (e.g., cars, planes)
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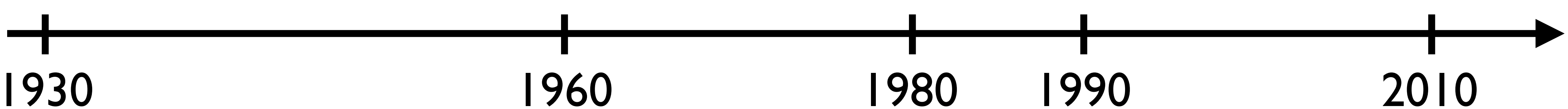
- Libraries and ADT implementations
- Heap-manipulating programs (e.g., OO)
- Tricky deterministic algorithms

A brief history of model checking



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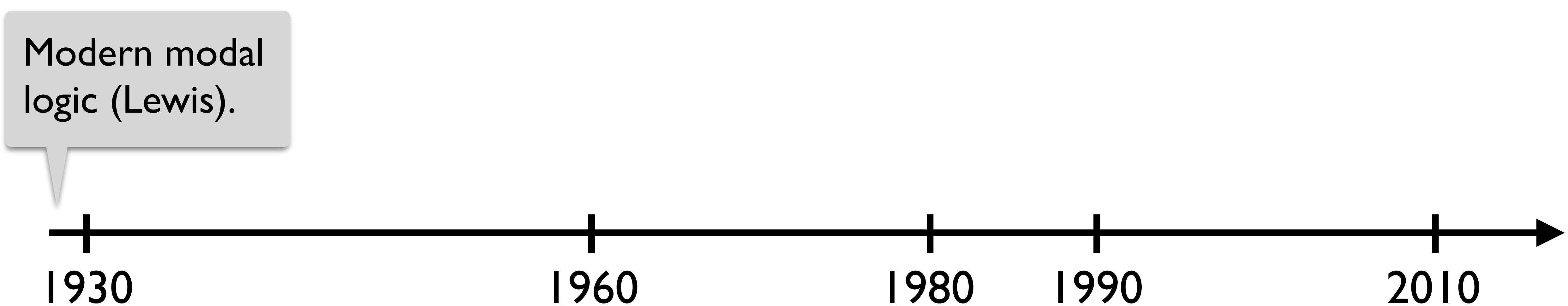

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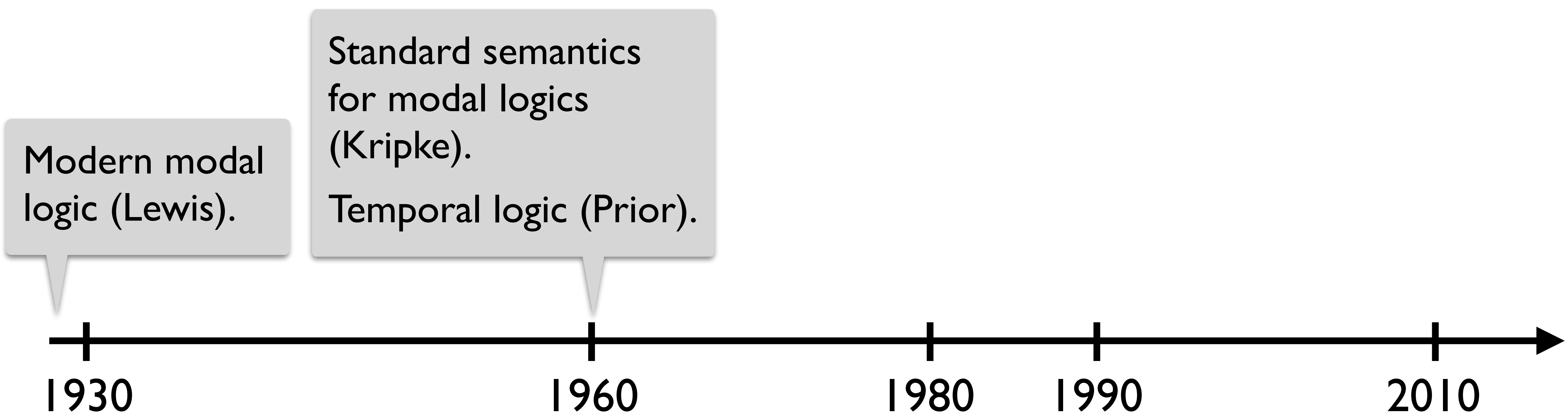

A brief history of model checking

Modern modal logic (Lewis).

Standard semantics for modal logics (Kripke).

Temporal logic (Prior).

1977: Using LTL to reason about concurrent programs (Pnueli).

1981-82: **Explicit-state model checking** for CTL (Emerson & Clarke; Queille & Sifakis).

1985: Automata-theoretic approach for LTL model checking (Vardi & Wolper).

1987: **Symbolic model checking** for CTL (McMillan).

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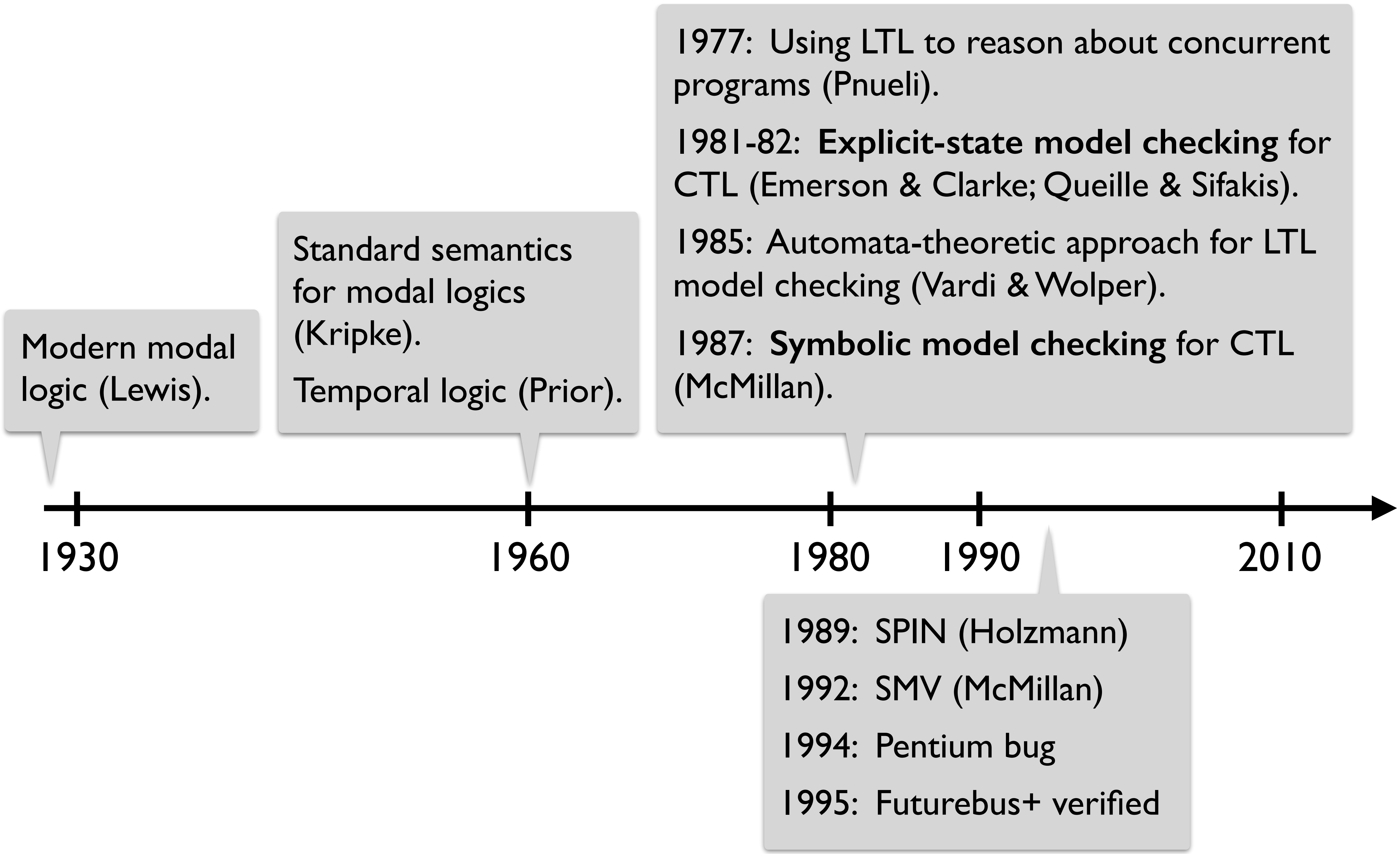

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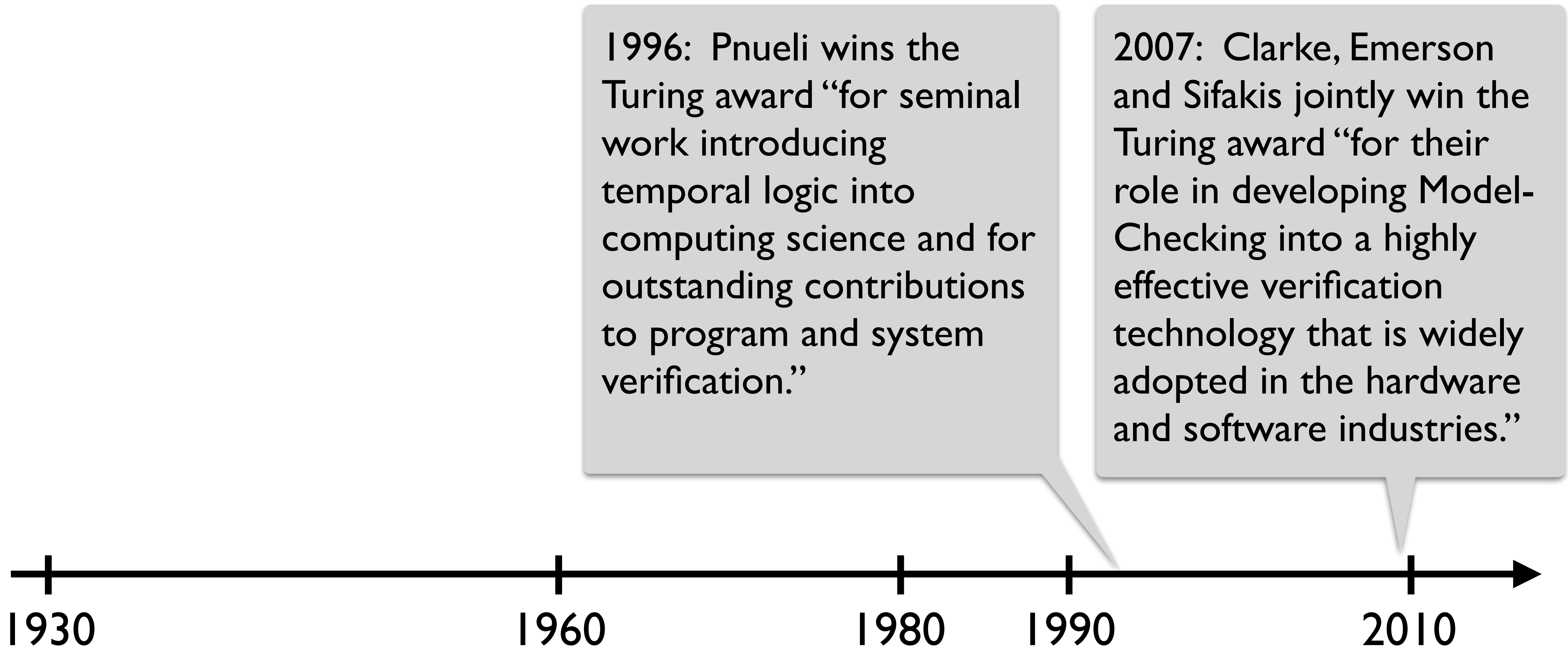

Kripke structures

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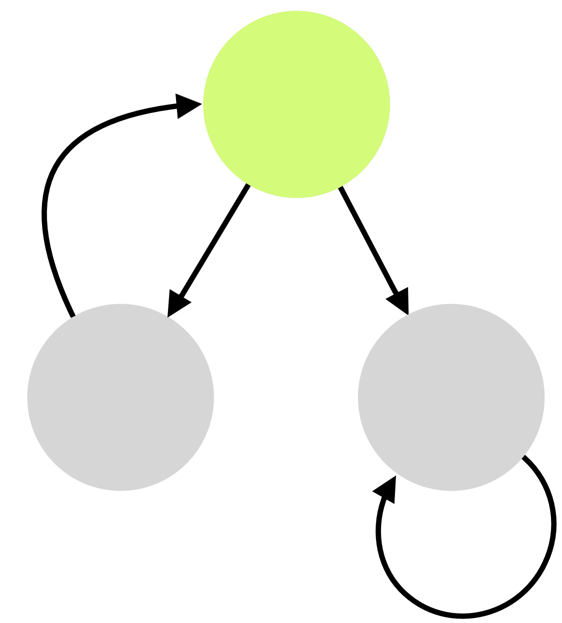

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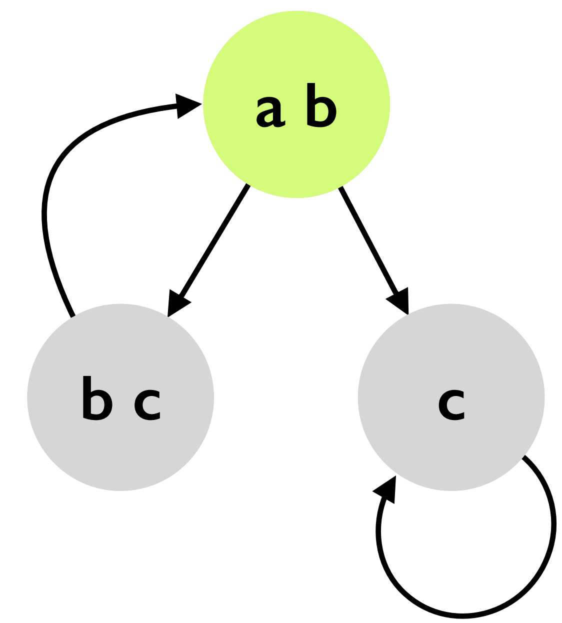

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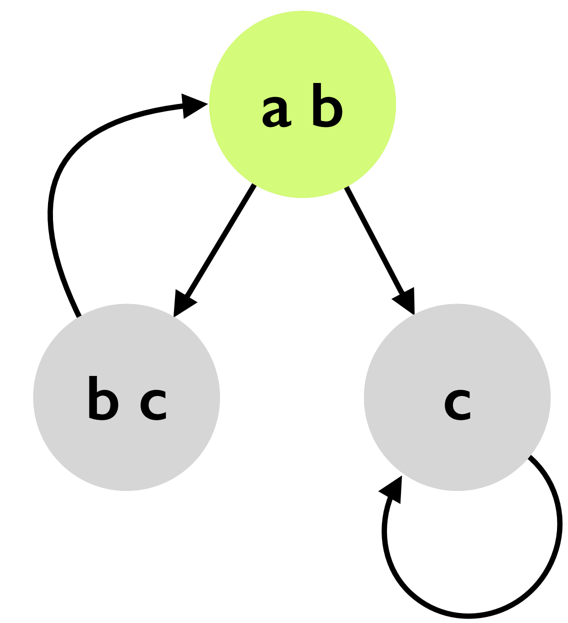

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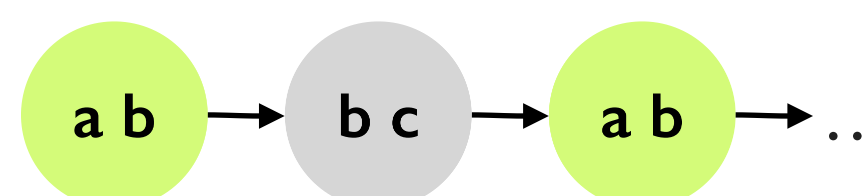
A *path* in M **is an infinite sequence of states** $\pi = s_0 s_1 \ldots$ **such that for all** $i \geq 0$, $(s_i, s_{i+1}) \in R$**.**



Modeling systems with Kripke structures

```
// x==1, y==1
x := (x + y) % 2
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- In a finite-state program, *system variables* V range over a *finite domain* D: V = {x, y} and D = {0, 1}.
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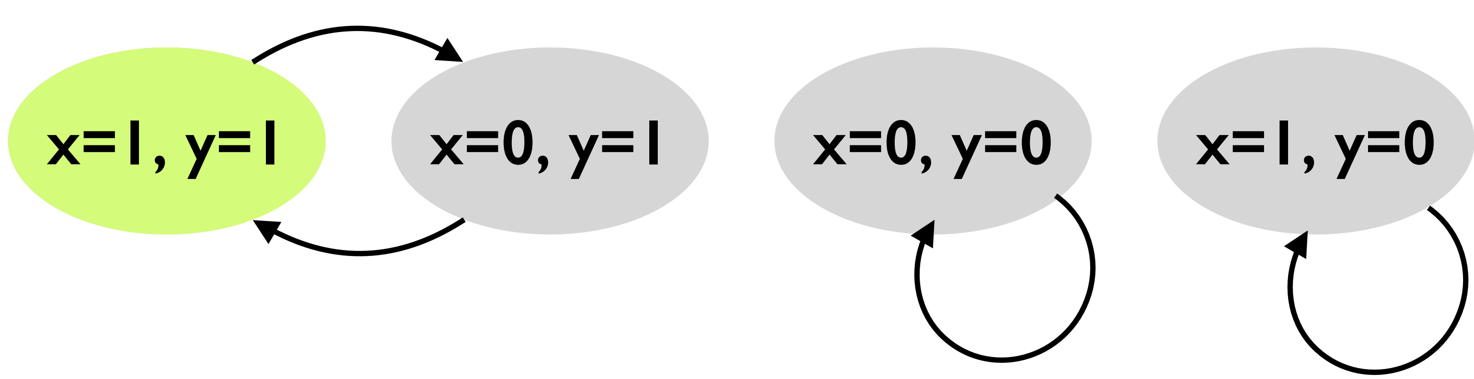

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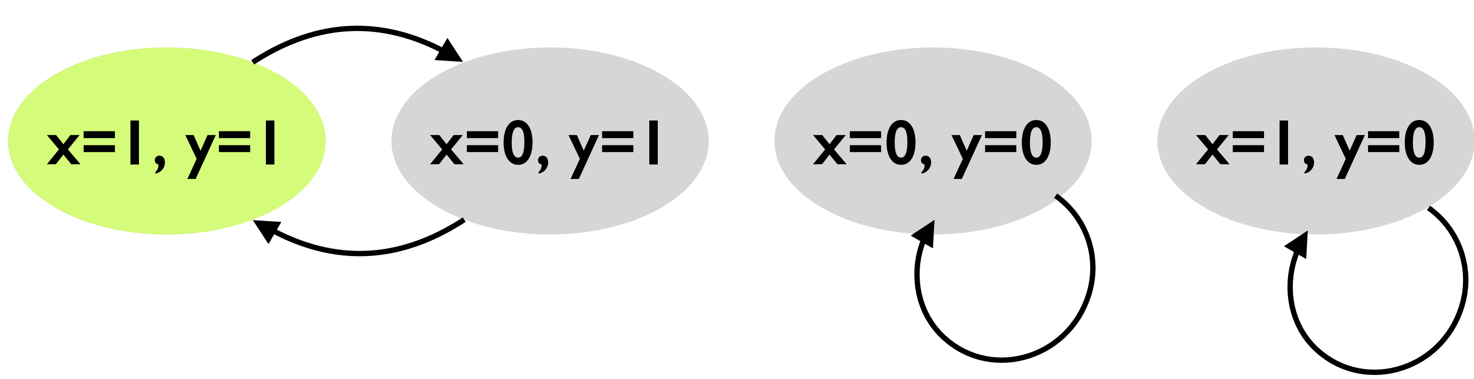

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State explosion: Kripke structure usually exponential in the size of the program.

A Kripke structure for a concurrent program

P_1

```
10  while (true) {
11    wait(turn == 0);
      // critical section
12    turn := 1;
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```

P_2

```
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Two processes executing concurrently and asynchronously, using the shared variable `turn` to ensure ***mutual exclusion***:

They are never in the critical section at the same time.

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State of the program described by the variable `turn` and the *program counters* for the two processes.

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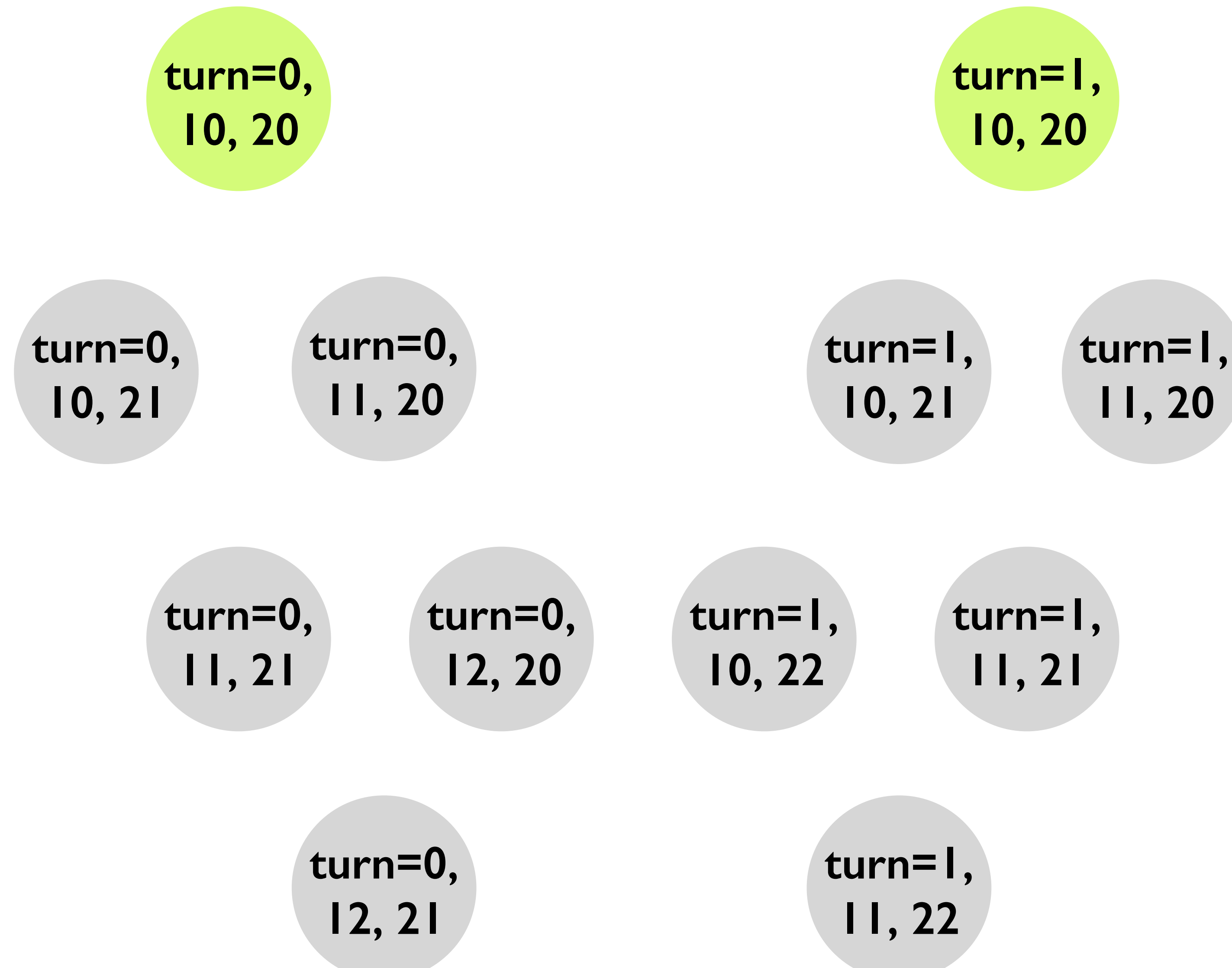

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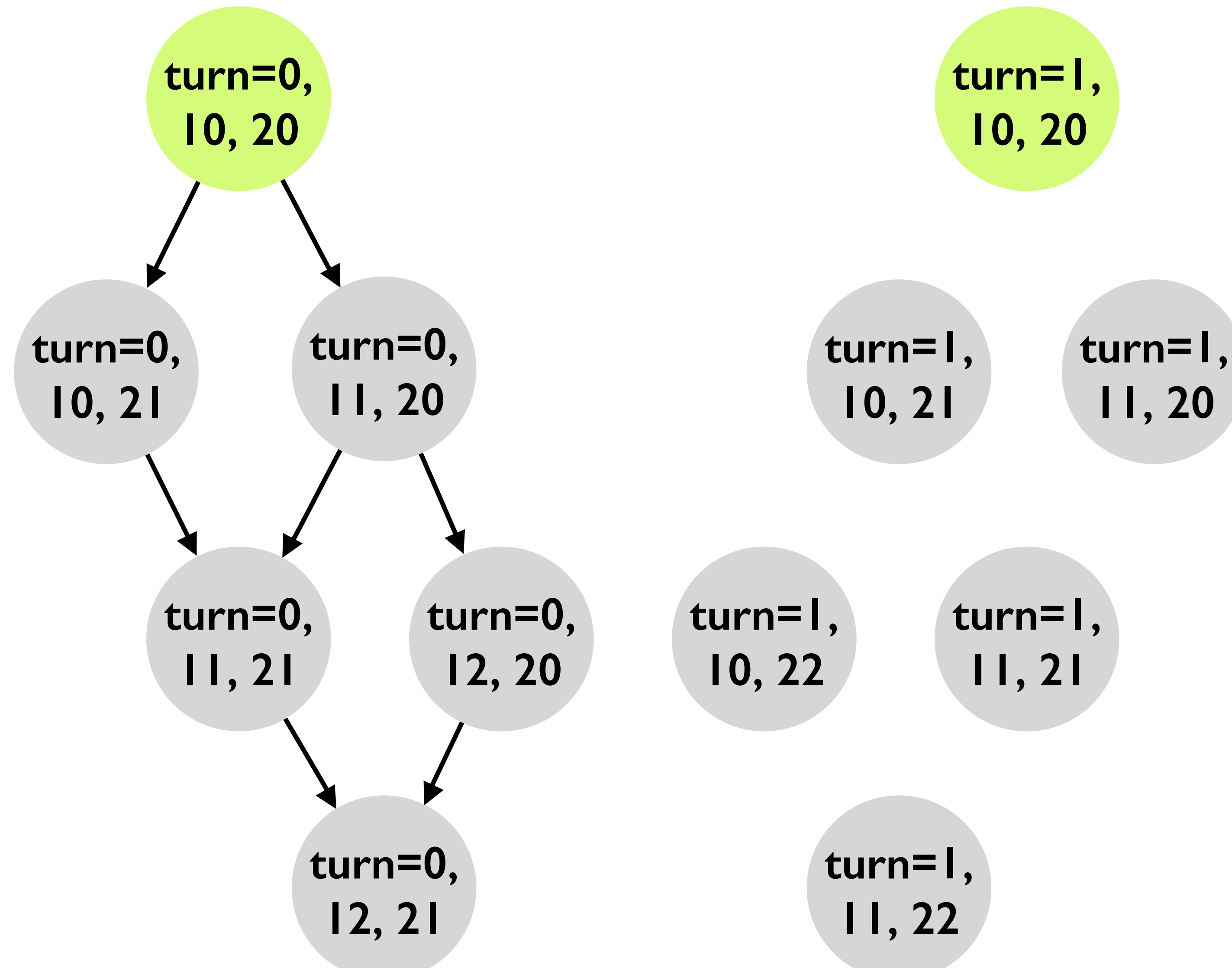

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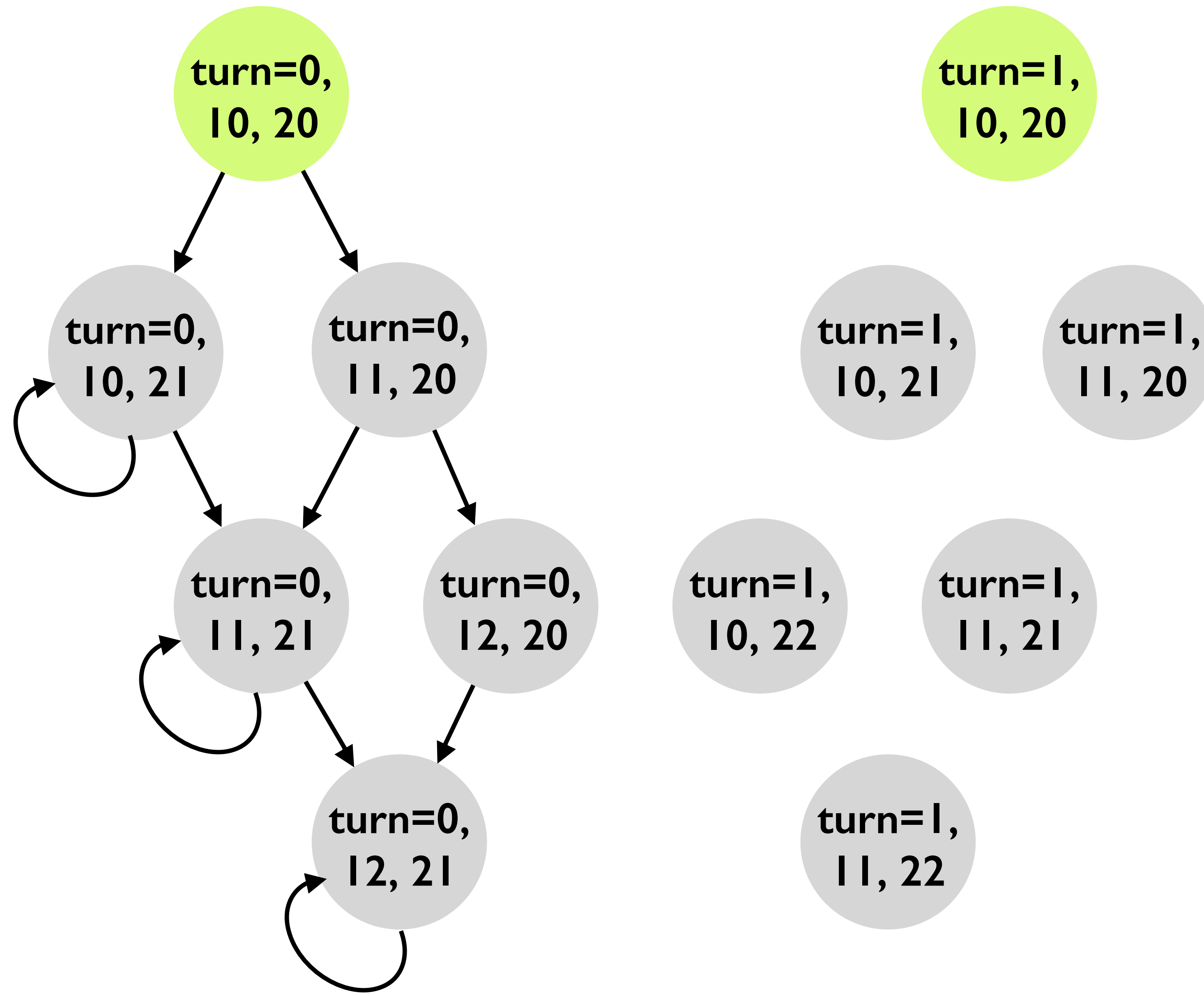

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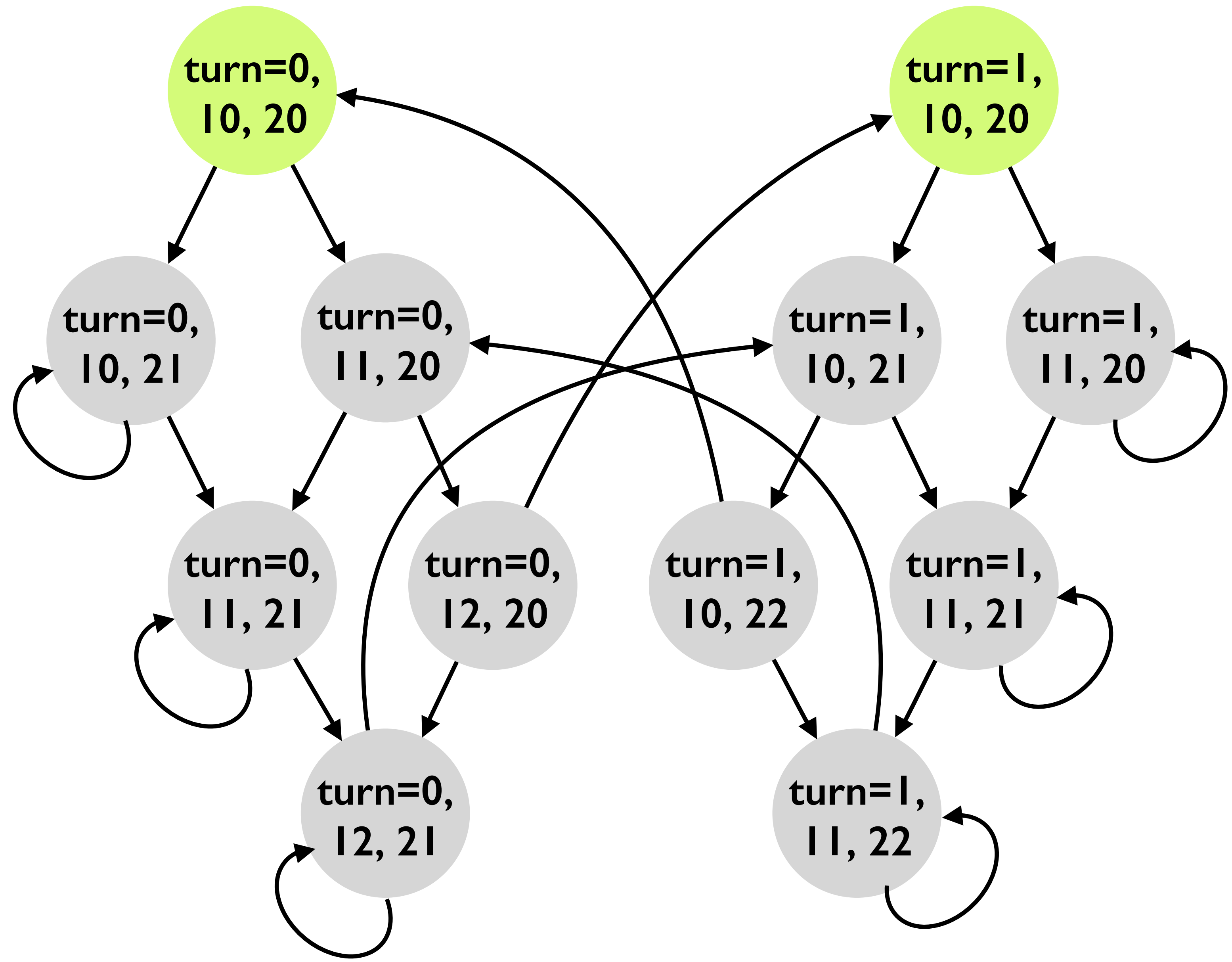

Safety & liveness properties of reactive systems

Safety

- "Nothing bad will happen."
- φ is a safety property iff every infinite path π violating φ has a **finite prefix** π' such that every extension of π' violates φ.

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Finite witnesses (counterexamples).

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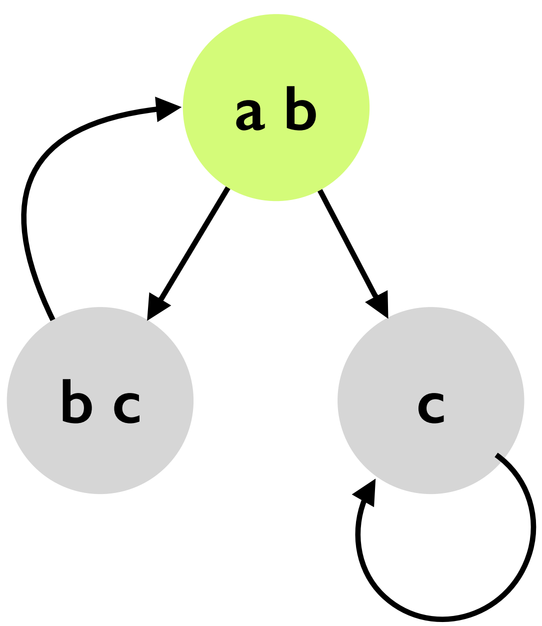
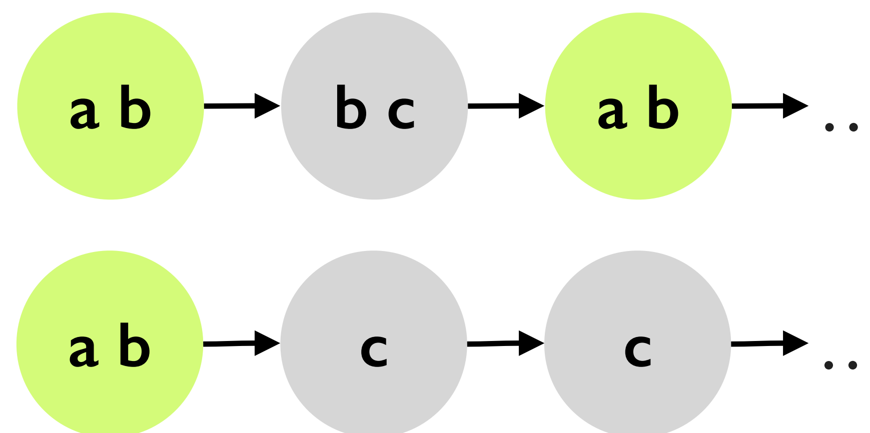
Starvation freedom: whenever P_1 is ready to enter its critical section, it will eventually succeed (provided that the scheduler is *fair* and does not let P_2 stay in its critical section forever).

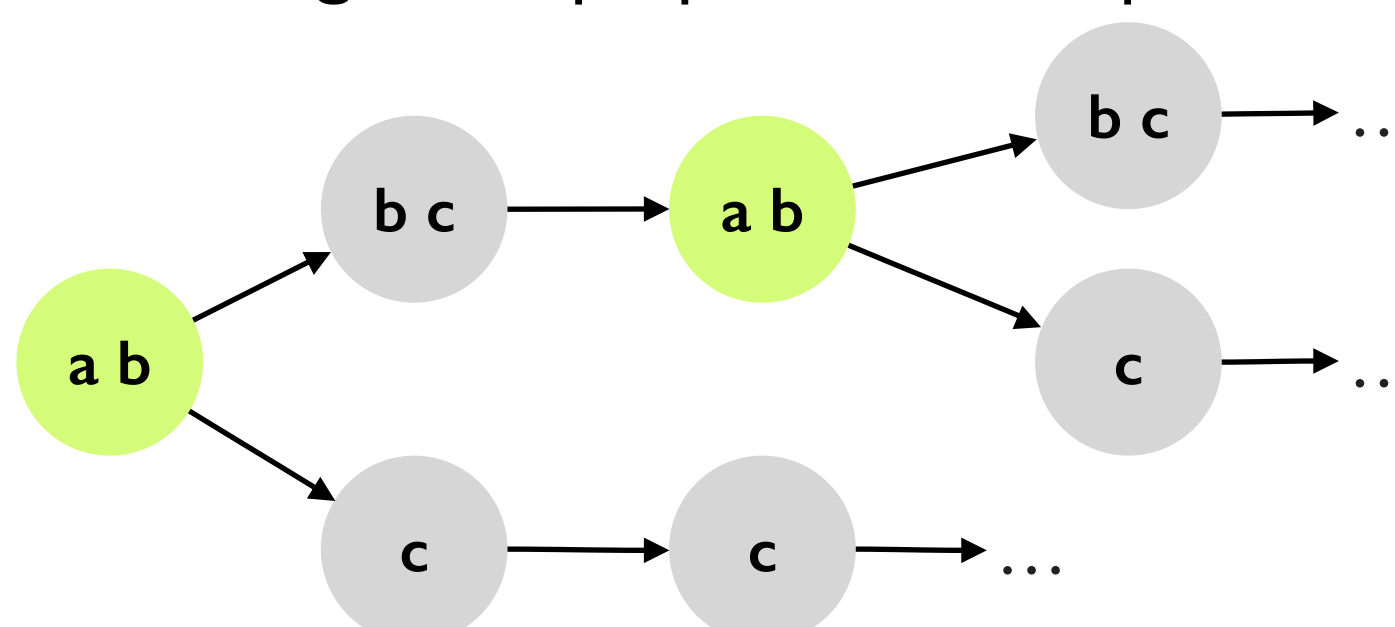
Expressing properties in temporal logics



Linear time: properties of computation paths



Branching time: properties of computation trees



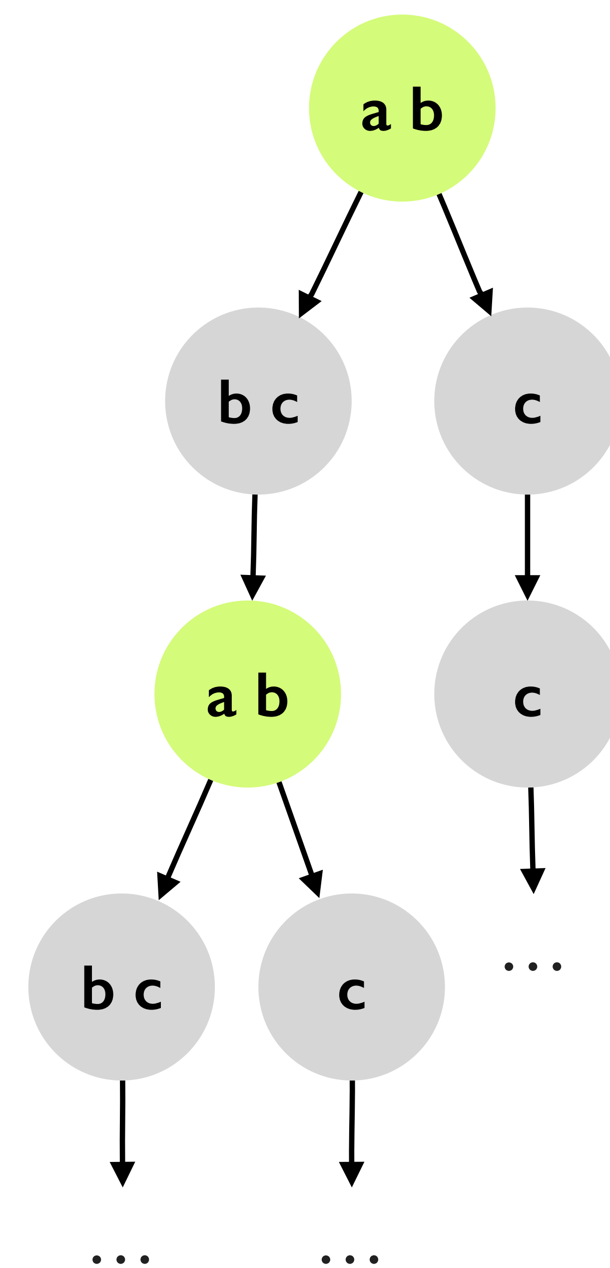
Computation tree logic CTL*

Path quantifiers describe the branching structure of the computation tree:

- **A** (for all paths)
- **E** (there exists a path)

Temporal operators describe properties of a path through a tree:

- **X**p (p holds "next time")
- **F**p (p holds "eventually" or "in the future")
- **G**p (p holds "always" or "globally")
- p **U** q (p holds "until" q holds)



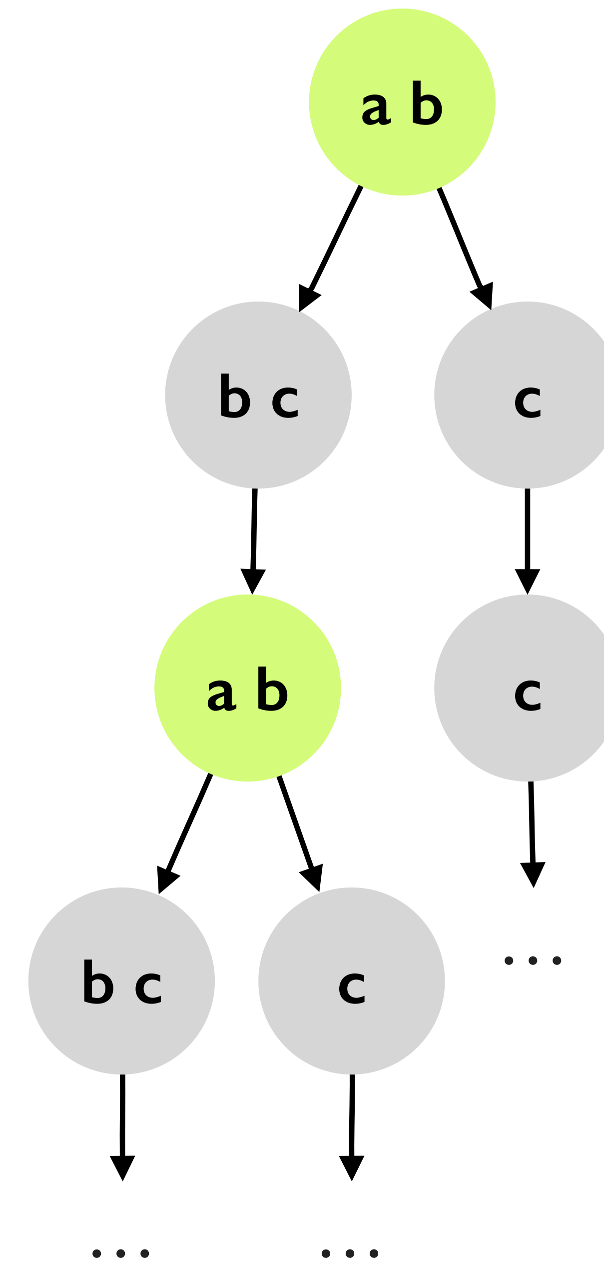
Syntax of CTL*

State formulas

- Atomic propositions: $a \in AP$
- $\neg f$, $f \wedge g$, $f \vee g$, where f and g are state formulas
- **A**p and **E**p, where p is a path formula

Path formulas

- f, where f is a state formula
- $\neg p$, $p \wedge p$, $p \vee q$, where p and q are path formulas
- **X**p, **F**p, **G**p, p **U** q, where p and q are path formulas



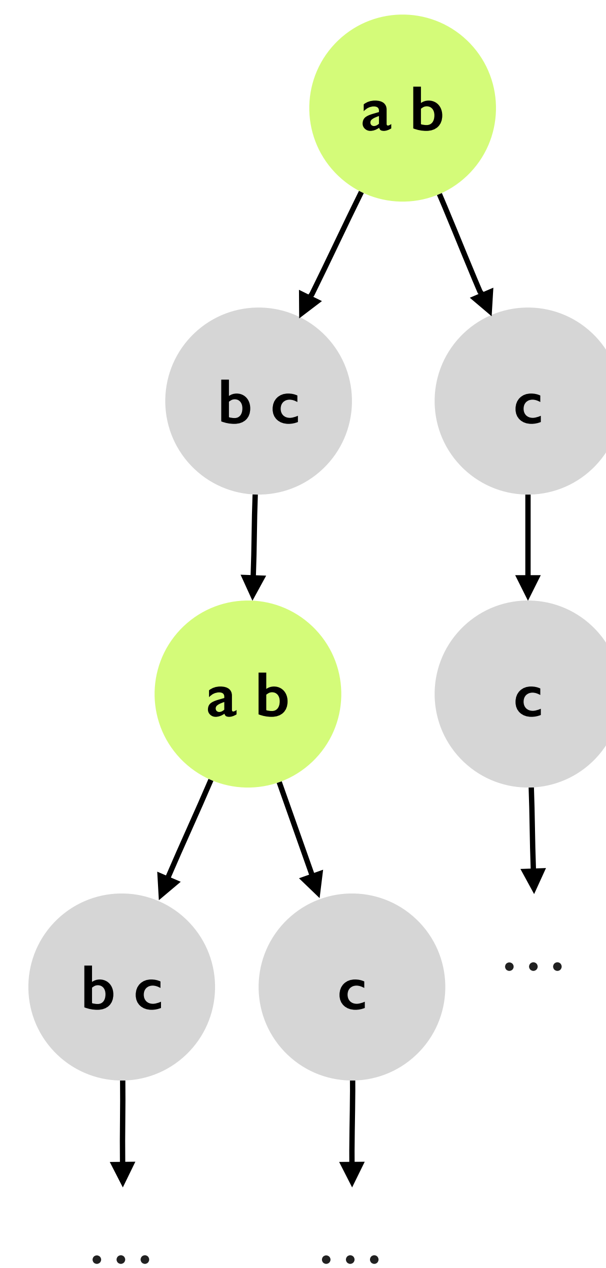
Semantics of CTL*

State formulas

- M, s $\vDash$ a iff a $\in$ L(s)
- M, s $\vDash$ **A**p iff M, $\pi \vDash$ p for all paths π that start at s
- M, s $\vDash$ **E**p iff M, $\pi \vDash$ p for some path π that starts at s

Path formulas (π^k is suffix of π starting at s_k)

- M, $\pi \vDash$ f iff M, s $\vDash$ f and s is the first state of π
- M, $\pi \vDash$ **X**p iff M, $\pi^1 \vDash$ p
- M, $\pi \vDash$ **F**p iff M, $\pi^k \vDash$ p for some k $\geq$ 0
- M, $\pi \vDash$ **G**p iff M, $\pi^k \vDash$ p for all k $\geq$ 0
- M, $\pi \vDash$ p **U** q iff M, $\pi^k \vDash$ q and M, $\pi^j \vDash$ q for some k $\geq$ 0 and for all 0 $\leq$ j < k



CTL and Linear Temporal Logic (LTL)

Computation Tree Logic (CTL)

Linear Temporal Logic (LTL)

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- **AG**(**EF** p): From any state, it is possible to get to a state where p holds.

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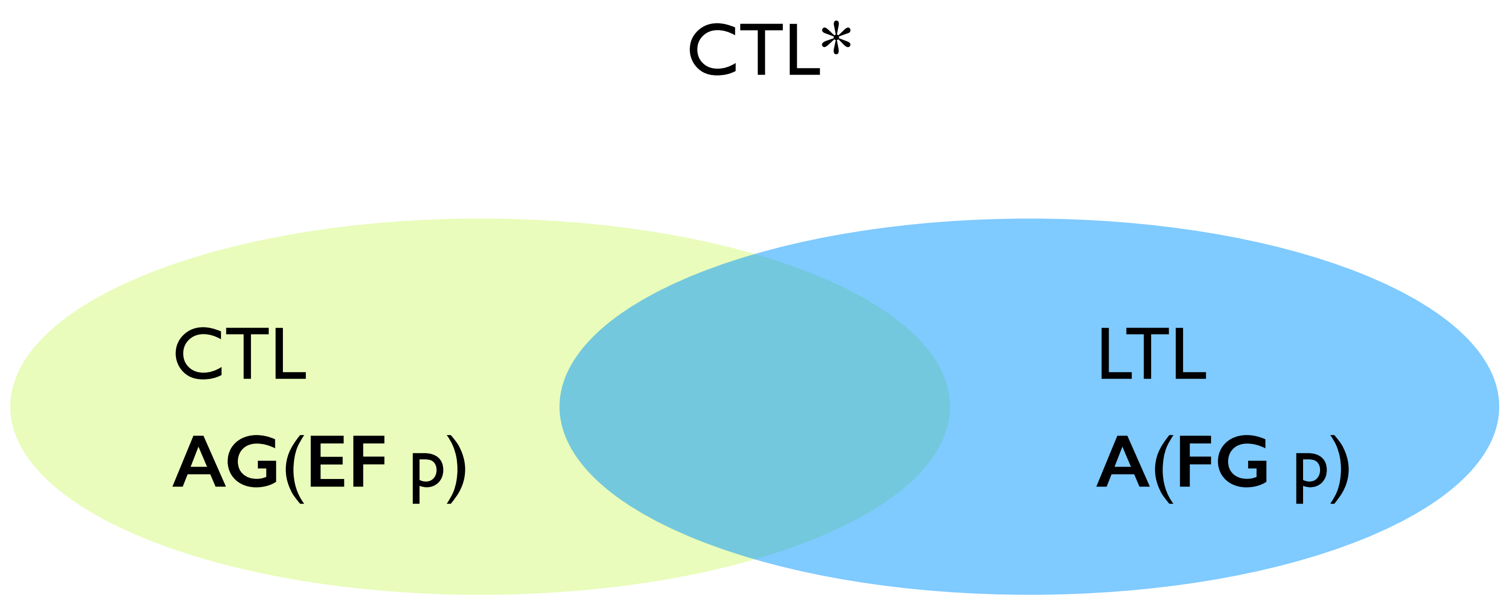
Linear Temporal Logic (LTL)

- Fragment of CTL* with formulas of the form **A**p, where p contains no path quantifiers.
- **A**(**FG** p): Along every path, there is some state from which p will hold forever.

Expressive power of CTL, LTL, and CTL*



Fairness

Cannot be expressed in CTL

Can be expressed in LTL

Fairness

Cannot be expressed in CTL

- Handled by changing the semantics to use fair Kripke structures.
- A *fair* Kripke structure $M = \langle S, S_0, R, L, F \rangle$ includes an additional set of sets of states $F \subseteq 2^S$.
- For each $P \in F$, a *fair path* π includes some states from P infinitely often.
- Path quantifiers interpreted only with respect to fair paths.

Can be expressed in LTL

Fairness

Cannot be expressed in CTL

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- Path quantifiers interpreted only with respect to fair paths.

Can be expressed in LTL

- Absolute fairness: $\mathbf{A}(\mathbf{GF}\ p_{exec})$
- Strong fairness:
  $\mathbf{A}((\mathbf{GF}\ p_{ready}) \Rightarrow (\mathbf{GF}\ p_{ready} \wedge p_{exec}))$
- Weak fairness:
  $\mathbf{A}((\mathbf{FG}\ p_{ready}) \Rightarrow (\mathbf{GF}\ p_{ready} \wedge p_{exec}))$

Model checking complexity for CTL, LTL, CTL*

Polynomial Time for CTL

- Best known algorithm: $O(|M| * |f|)$

$M, s \models f$

PSPACE-complete for LTL

- Best known algorithm: $O(|M| * 2^{|f|})$

PSPACE-complete for CTL*

- Best known algorithm: $O(|M| * 2^{|f|})$

Model checking techniques for CTL and LTL

CTL

- Graph-theoretic explicit-state model checking (EMC)
- Symbolic model checking with Ordered Binary Decision Diagrams (SMV, NuSMV)
- Bounded model checking based on SAT (NuSMV)

LTL

- Automata-theoretic model checking:
  - Explicit-state (SPIN) or
  - Symbolic (NuSMV)

Summary

Today

- Basics of model checking:
  - Kripke structures
  - Temporal logics (CTL, LTL,CTL*)
  - Model checking techniques

Next lecture

- Software model checking