Computer-Aided Reasoning for Software

CSE507

Bounded Verification

courses.cs.washington.edu/courses/cse507/14au/

Emina Torlak
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Today

Today

Last lecture

- Full functional verification with Dafny, Boogie, and Z3

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- Full functional verification with Dafny, Boogie, and Z3

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- Bounded verification with Kodkod (Forge, Miniatur, TACO)

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Last lecture

- Full functional verification with Dafny, Boogie, and Z3

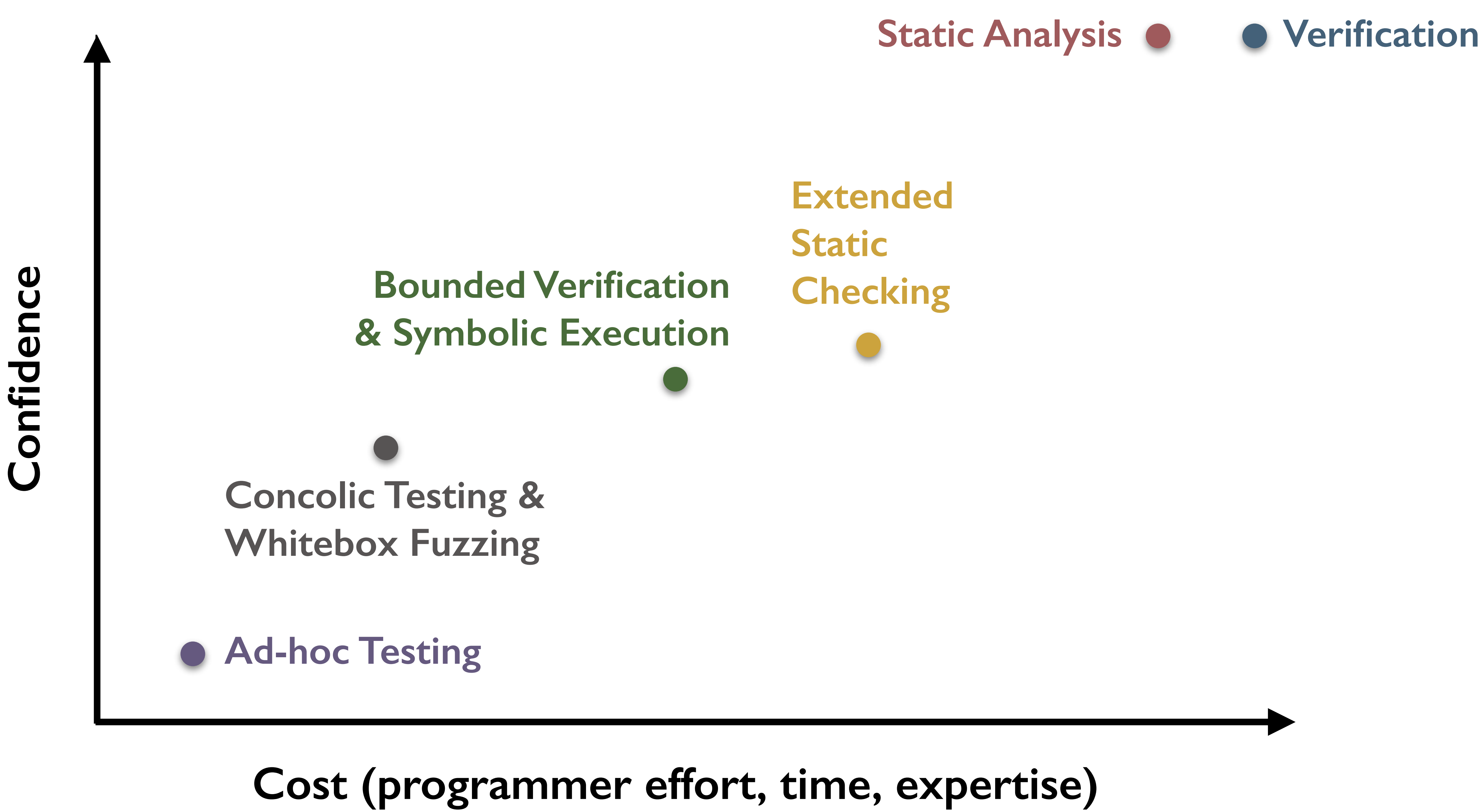
Today

- Bounded verification with Kodkod (Forge, Miniatur, TACO)

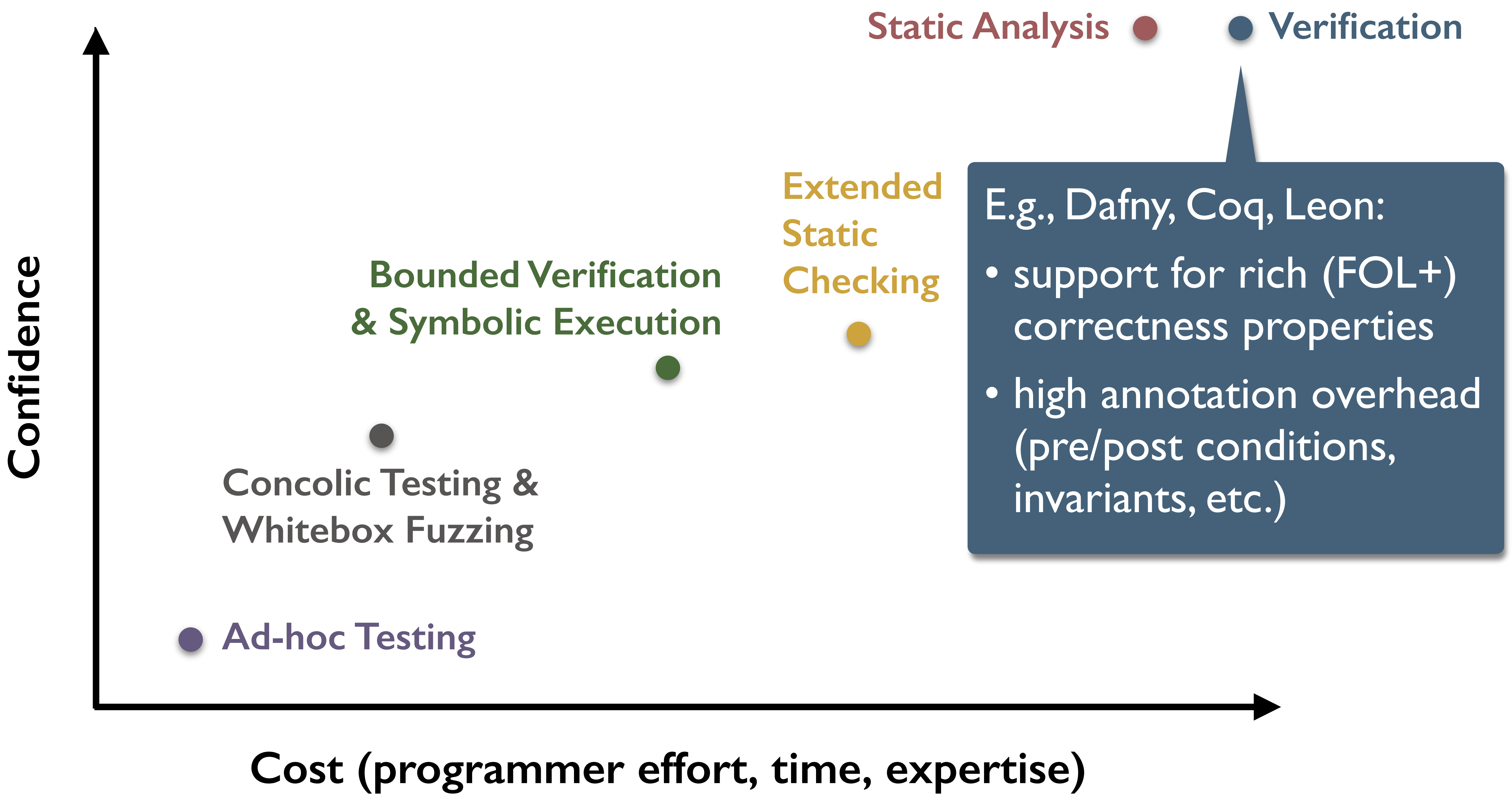
Announcements

- Homework 2 is due today at 11pm
- Homework 3 has been released

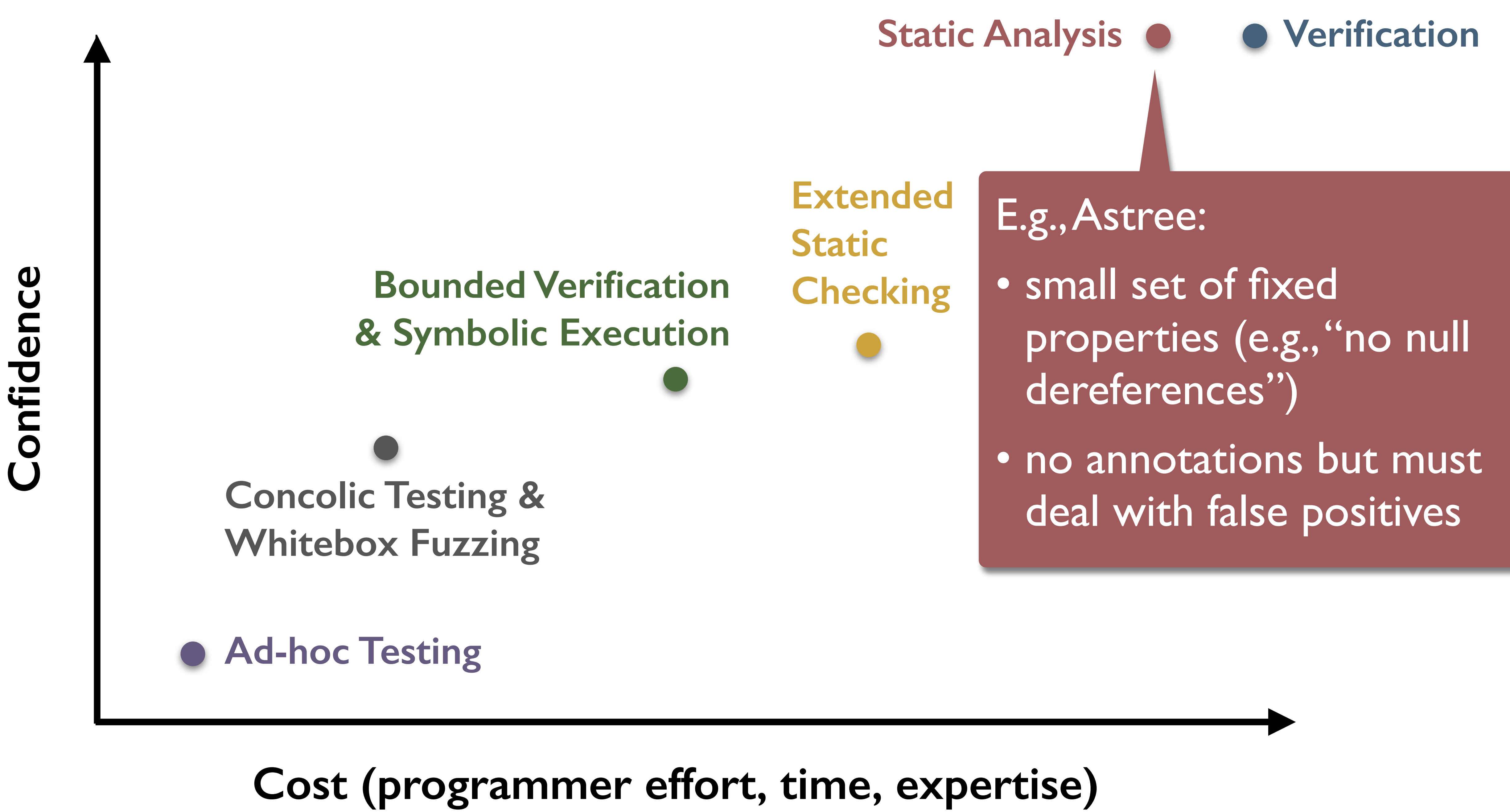
The spectrum of program validation tools



The spectrum of program validation tools



The spectrum of program validation tools



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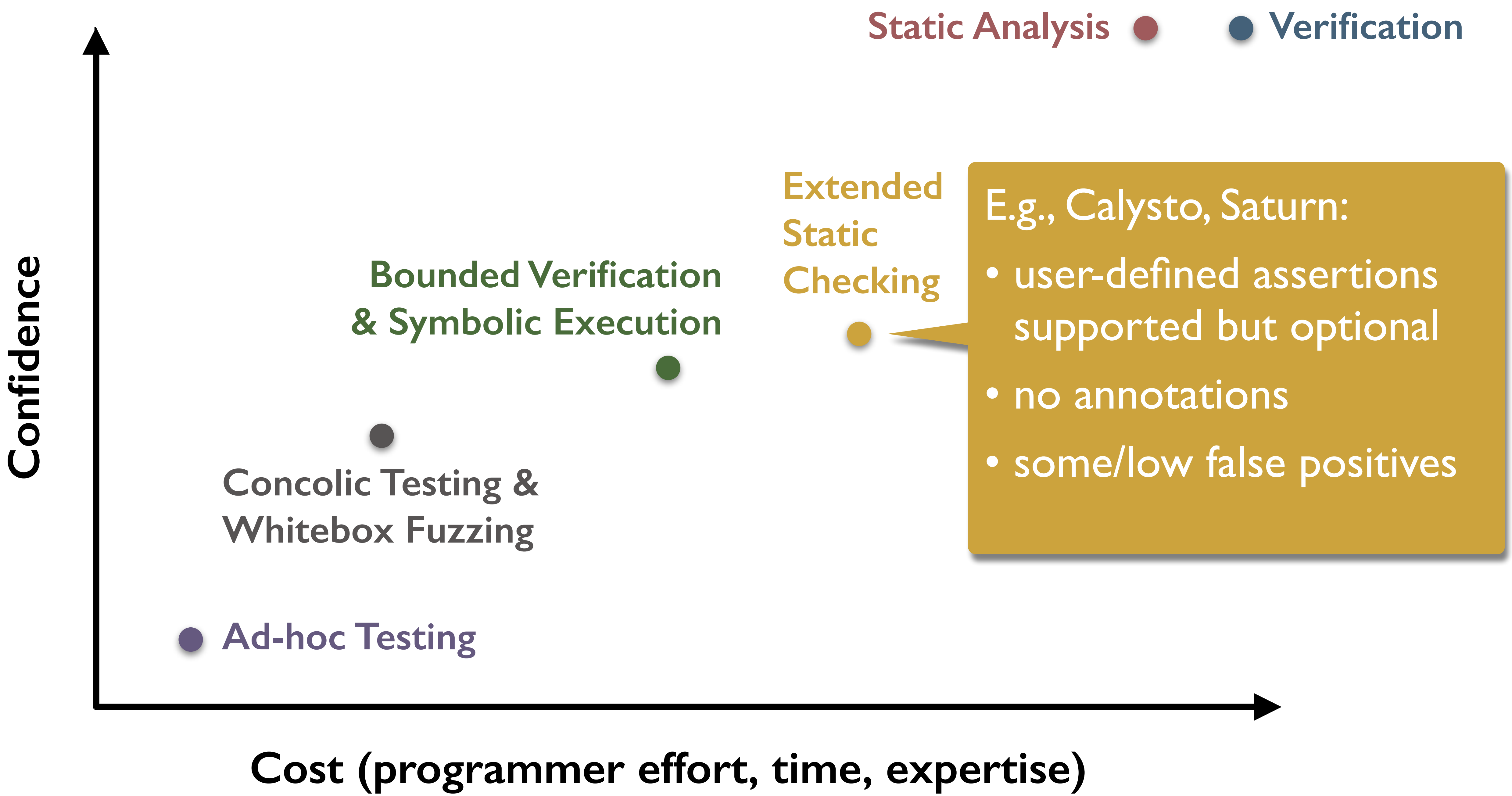
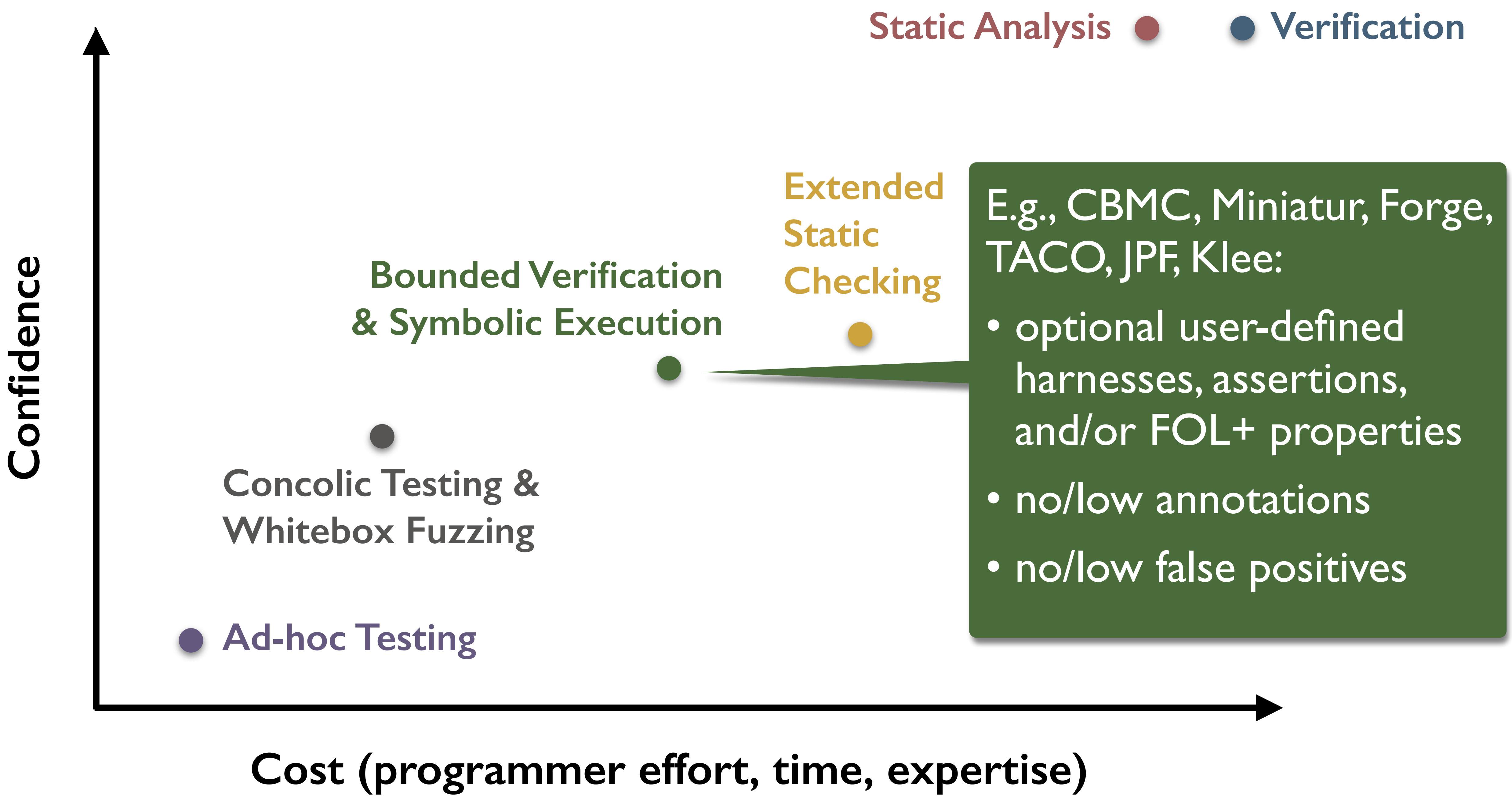


The spectrum of program validation tools



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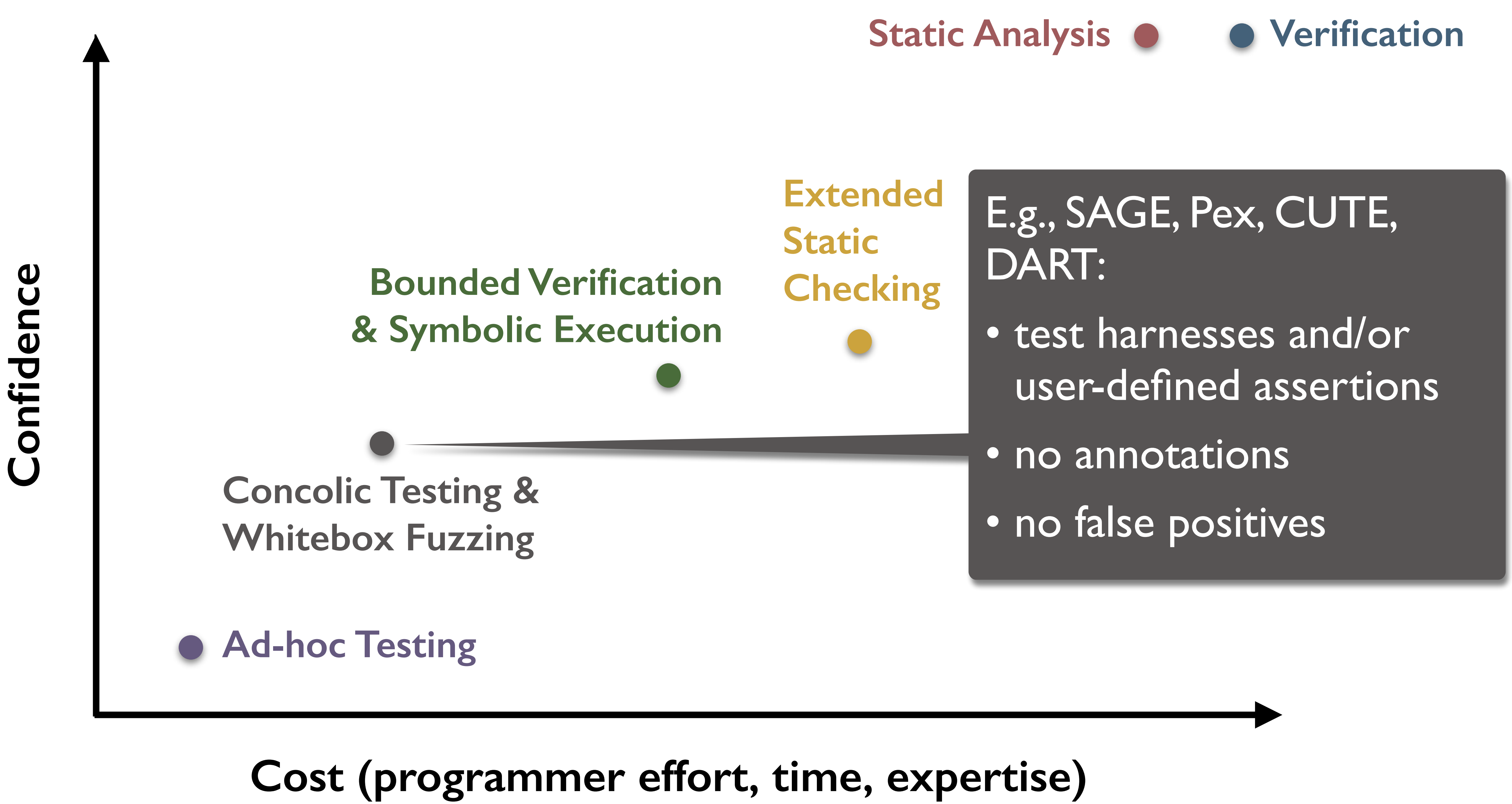

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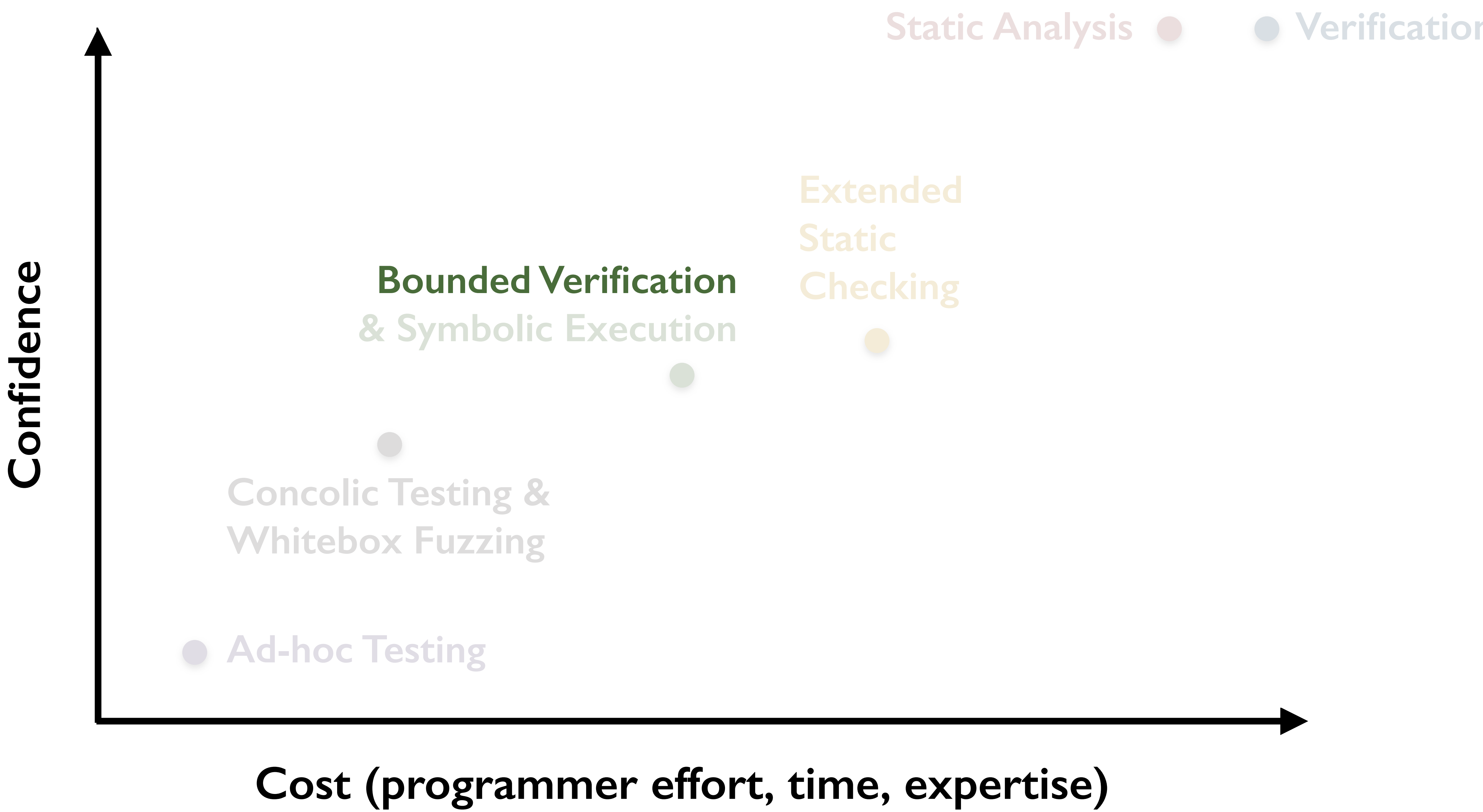
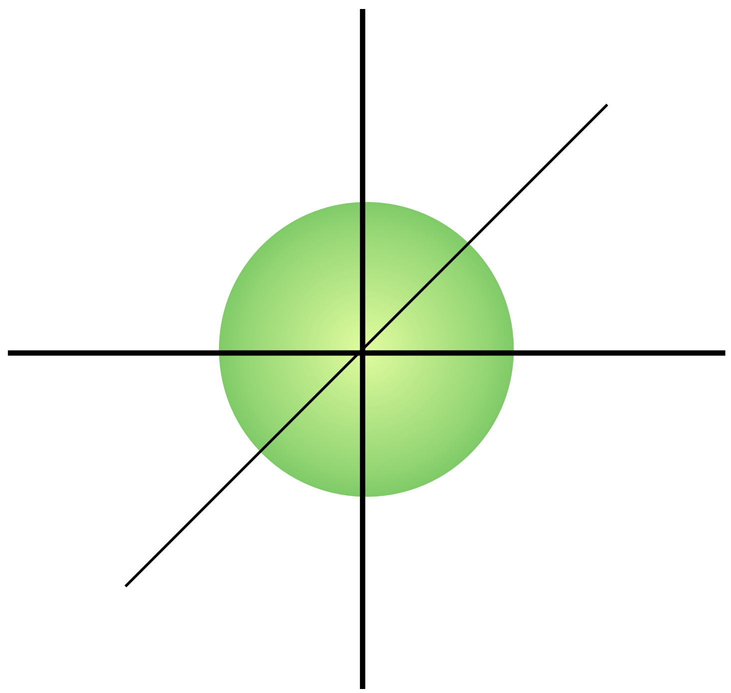


Bounded verification

Bound *everything*

- Execution length
- Bitwidth
- Heap size (number of objects per type)



Sound counterexamples but no proof

- Exhaustive search within bounded scope

Empirical "small-scope hypothesis"

- Bugs usually have small manifestations

Bounded verification by example

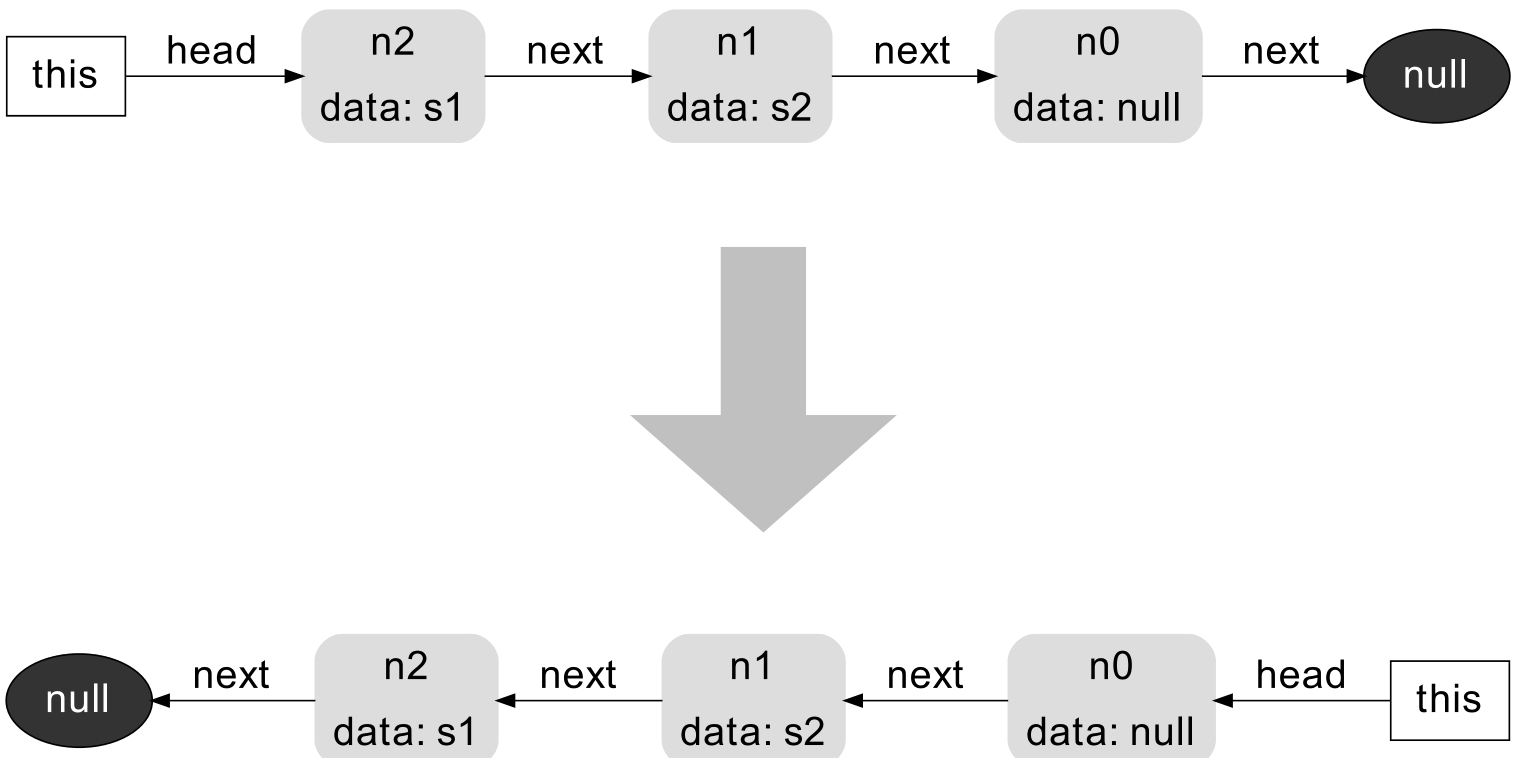
```
class List {
   Node head;

   void reverse() {
      Node near = head;
      Node mid = near.next;
      Node far = mid.next;

      near.next = far;
      while (far != null) {
         mid.next = near;
         near = mid;
         mid = far;
         far = far.next;
      }

      mid.next = near;
      head = mid;
   }
}

class Node {
   Node next;
   String data; }
```



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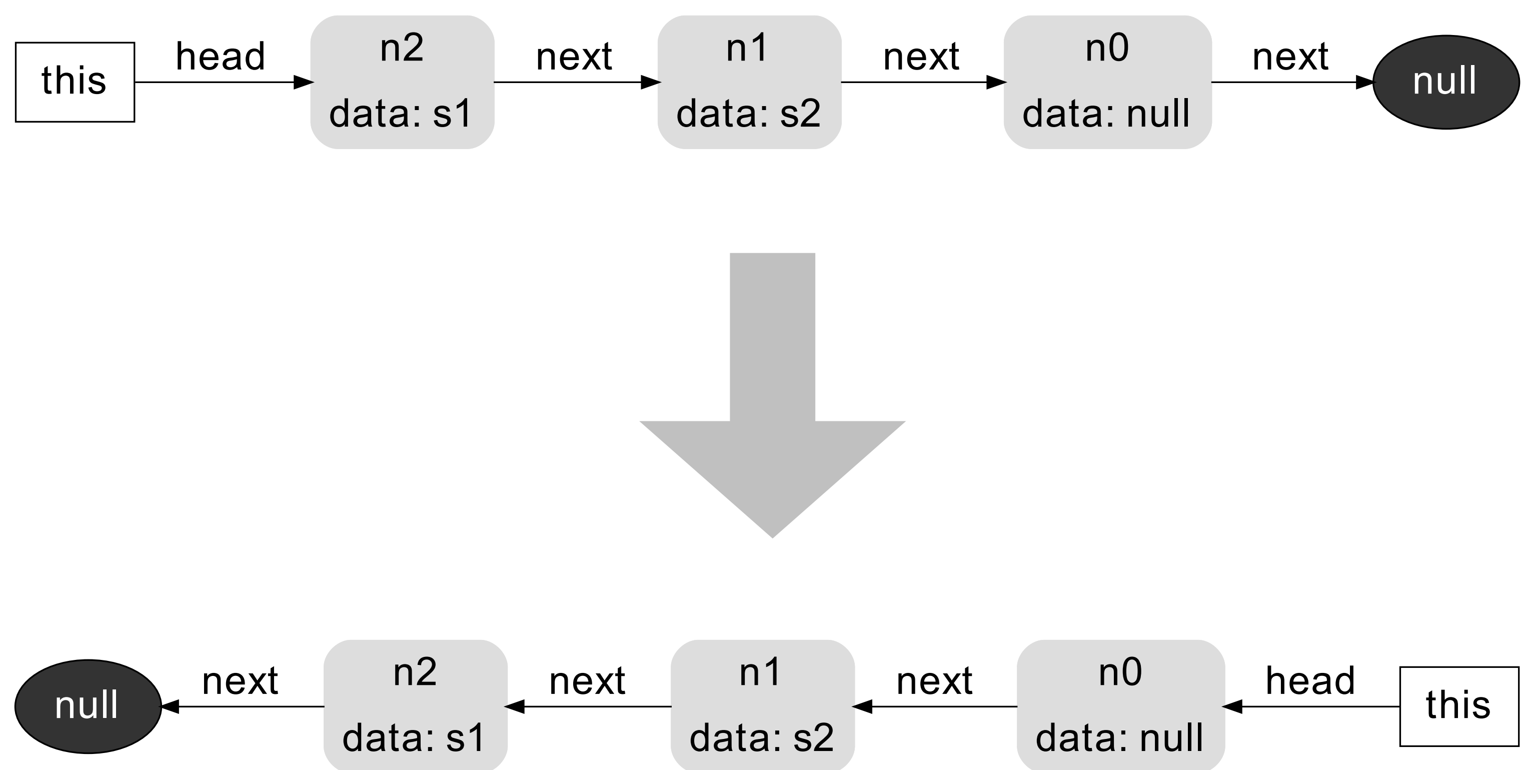
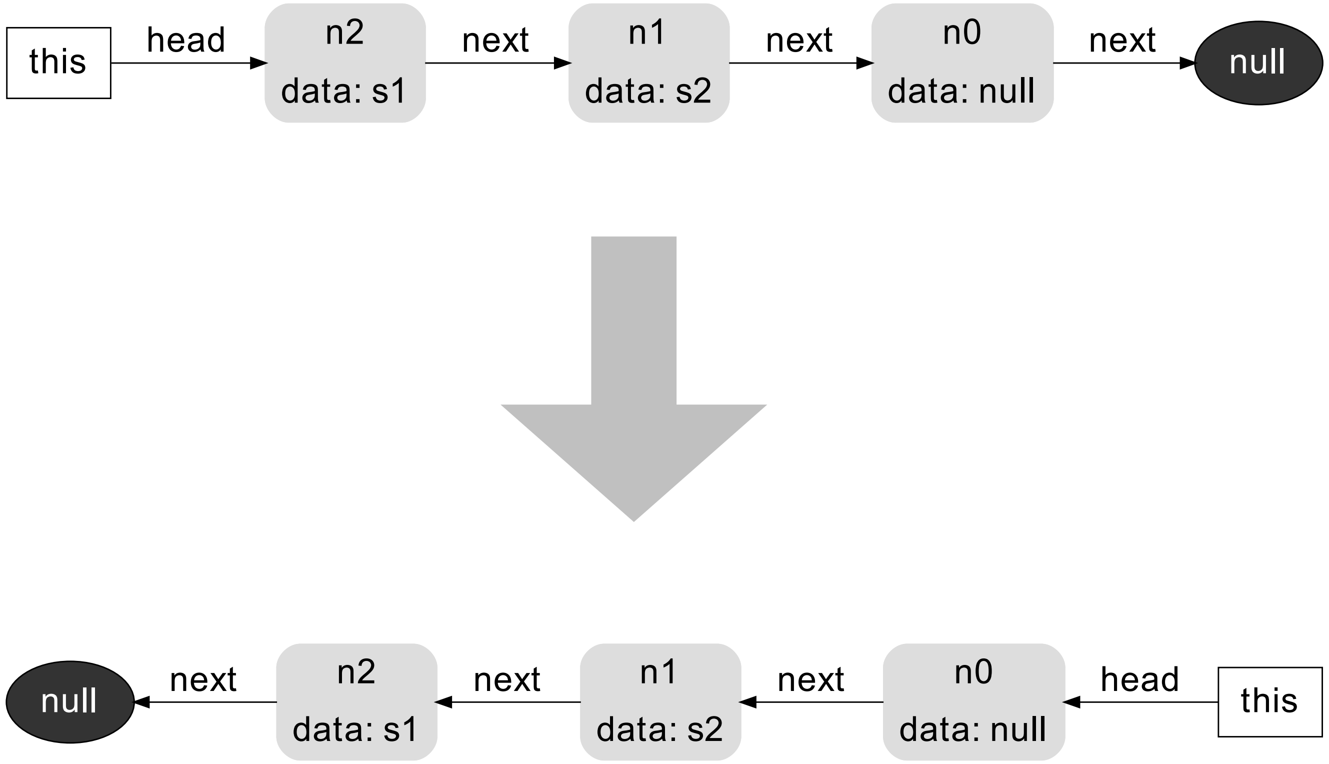


Express the property either by writing a test harness or by providing FOL+ *contracts*.

Pre/post/frame conditions & data invariants

```
class List {
   Node head;

   void reverse() {
      Node near = head;
      Node mid = near.next;
      Node far = mid.next;

      near.next = far;
      while (far != null) {
         mid.next = near;
         near = mid;
         mid = far;
         far = far.next;
      }

      mid.next = near;
      head = mid;
   }
}

class Node {
   Node next;
   String data;
}
```

@requires
   this.head != null &&
   this.head.next != null



Pre/post/frame conditions & data invariants

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class List {
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    mid.next = near;
    head = mid;
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class Node {
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```

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this.head != null &&
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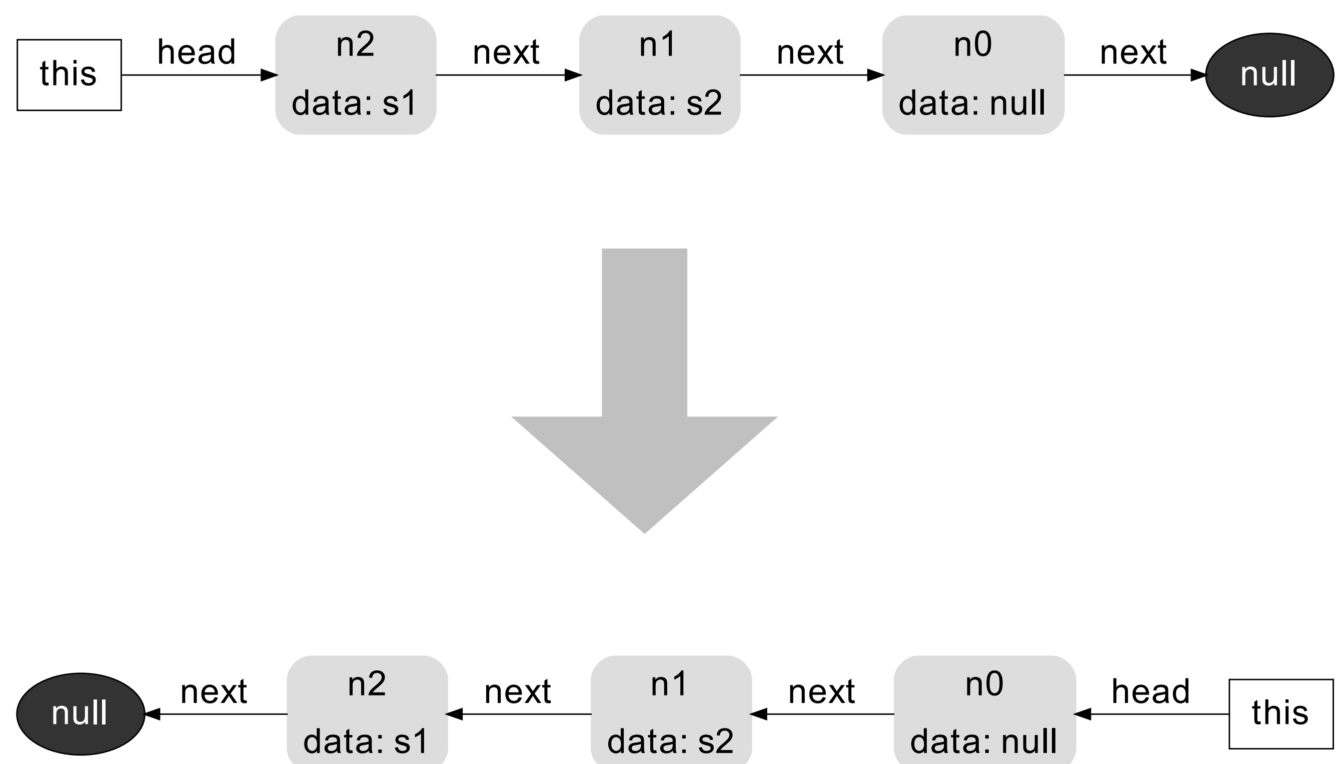

@invariant **no ^next ∩ iden**

Pre/post/frame conditions & data invariants

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      mid = far;
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@requires
this.head != null &&
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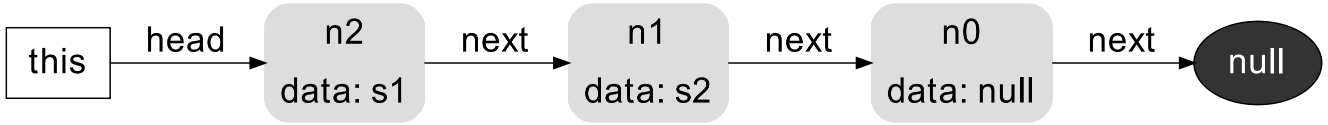
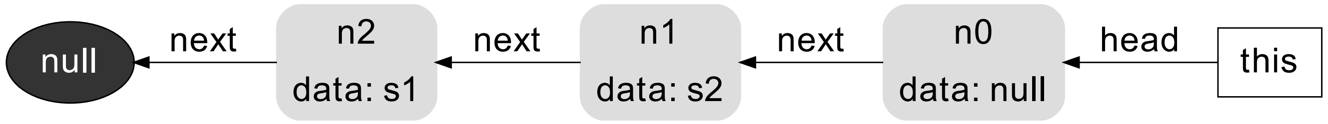


@ensures
this.head.*next = this.old(head).*old(next) &&



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Pre/post/frame conditions & data invariants

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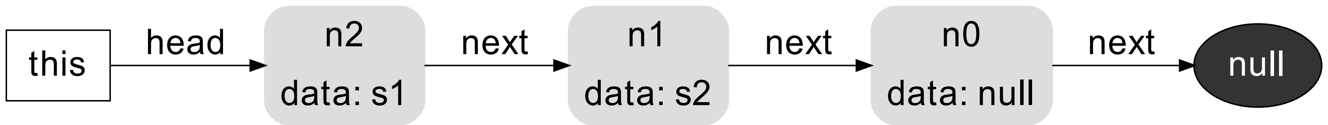
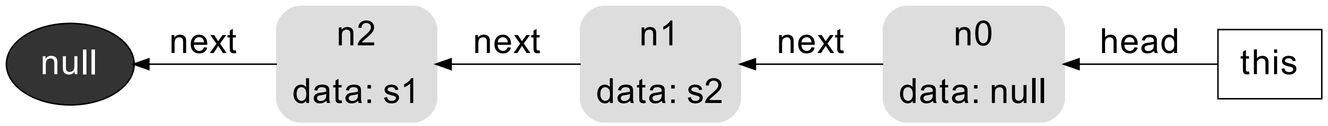
@requires
this.head != null &&
this.head.next != null



@ensures
this.head.*next = this.old(head).*old(next) &&
let N = this.old(head).*old(next) - null |
next = old(next) ++ this.old(head)×null ++
~(old(next) ∩ N×N)



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Pre/post/frame conditions & data invariants

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class Node {
  Node next;
  String data;
}
```

@requires Pre(this, head, next)

@ensures Post(this, old(head), head, old(next), next)

@invariant Inv(next)

A relational model of memory (heap)

@invariant Inv(next)
@requires Pre(this, head, next)
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void reverse() {
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      near = mid;
      mid = far;
      far = far.next;
   }

   mid.next = near;
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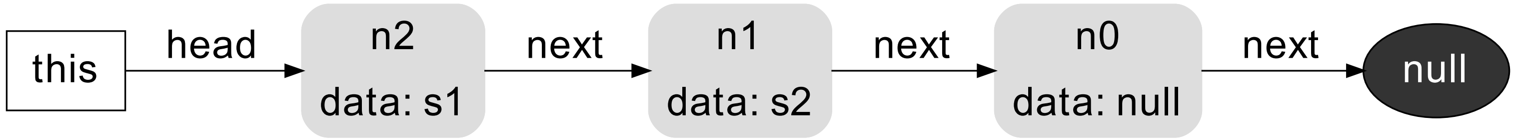
```



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      mid.next = near;
      near = mid;
      mid = far;
      far = far.next;
   }

   mid.next = near;
   head = mid;
}
```
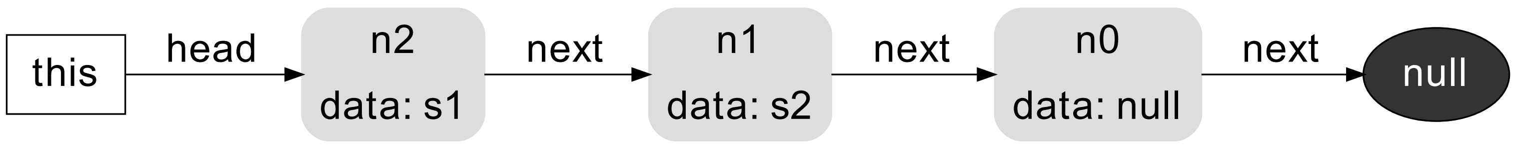

Fields as binary relations

- head : { ⟨this, n2⟩ }, next : { ⟨n2, n1⟩, … }



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  while (far != null) {
    mid.next = near;
    near = mid;
    mid = far;
    far = far.next;
  }

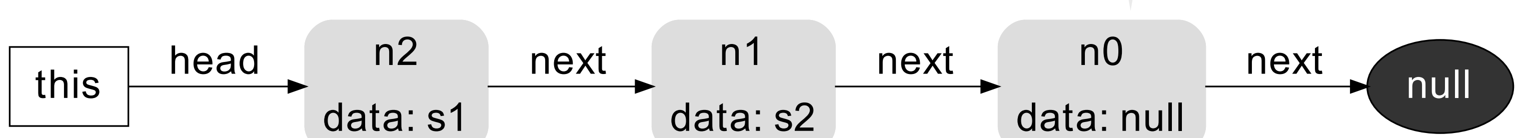
  mid.next = near;
  head = mid;
}
```

Fields as binary relations

- head : { ⟨this, n2⟩ }, next : { ⟨n2, n1⟩, … }

Types as sets (unary relations)

- List : { ⟨this⟩ }, Node : { ⟨n0⟩, ⟨n1⟩, ⟨n2⟩ }



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  Node far = mid.next;

  near.next = far;
  while (far != null) {
    mid.next = near;
    near = mid;
    mid = far;
    far = far.next;
  }

  mid.next = near;
  head = mid;
}
```

Fields as binary relations

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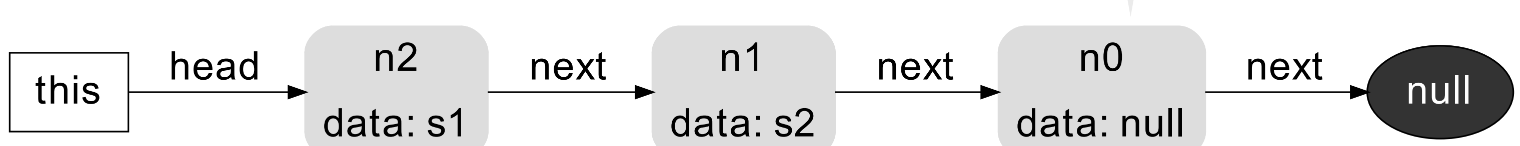
Types as sets (unary relations)

- List : { ⟨this⟩ }, Node : { ⟨n0⟩, ⟨n1⟩, ⟨n2⟩ }

Objects as scalars (singleton sets)

- this : { ⟨this⟩ }, null : { ⟨null⟩ }



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   near.next = far;
   while (far != null) {
      mid.next = near;
      near = mid;
      mid = far;
      far = far.next;
   }

   mid.next = near;
   head = mid;
}
```

Fields as binary relations

- head : { ⟨this, n2⟩ }, next : { ⟨n2, n1⟩, … }

Types as sets (unary relations)

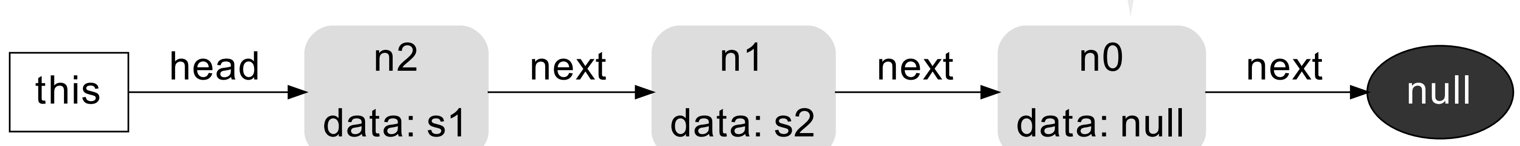
- List : { ⟨this⟩ }, Node : { ⟨n0⟩, ⟨n1⟩, ⟨n2⟩ }

Objects as scalars (singleton sets)

- this : { ⟨this⟩ }, null : { ⟨null⟩ }

Field read as relational join (.)

- this.head : { ⟨this⟩ } . { ⟨this, n2⟩ } = { ⟨n2⟩ }



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void reverse() {
   Node near = head;
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   Node far = mid.next;

   near.next = far;
   while (far != null) {
      mid.next = near;
      near = mid;
      mid = far;
      far = far.next;
   }

   mid.next = near;
   head = mid;
}
```

Fields as binary relations

- head : { ⟨this, n2⟩ }, next : { ⟨n2, n1⟩, … }

Types as sets (unary relations)

- List : { ⟨this⟩ }, Node : { ⟨n0⟩, ⟨n1⟩, ⟨n2⟩ }

Objects as scalars (singleton sets)

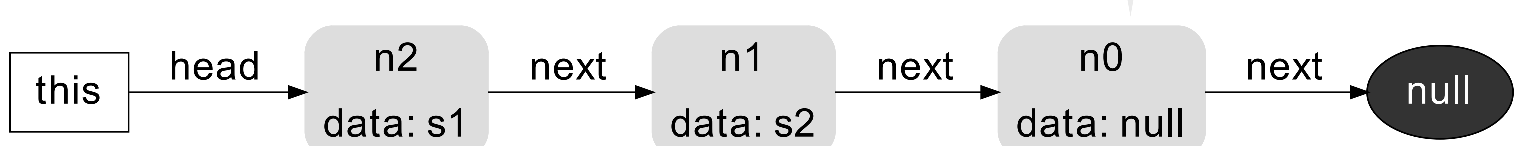
- this : { ⟨this⟩ }, null : { ⟨null⟩ }

Field read as relational join (.)

- this.head : { ⟨this⟩ } . { ⟨this, n2⟩ } = { ⟨n2⟩ }

Field write as relational override (++)

- this.head = null : head ++ (this × null) = { ⟨this, n2⟩ } ++ { ⟨this, null⟩ } = { ⟨this, null⟩ }



Bounded verification: step 1/4

@invariant Inv(next)
@requires Pre(this, head, next)
@ensures Post(this, old(head), head, old(next), next)

```
void reverse() {
   Node near = head;
   Node mid = near.next;
   Node far = mid.next;

   near.next = far;
   while (far != null) {
      mid.next = near;
      near = mid;
      mid = far;
      far = far.next;
   }

   mid.next = near;
   head = mid;
}
```

Bounded verification: step 1/4

```
@invariant Inv(next)
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void reverse() {
   Node near = head;
   Node mid = near.next;
   Node far = mid.next;

   near.next = far;
   if (far != null) {
      mid.next = near;
      near = mid;
      mid = far;
      far = far.next;
   }
   assume far == null;

   mid.next = near;
   head = mid;
}
```

Execution finitization (inlining, unrolling, SSA)

Bounded verification: step 1/4

@invariant Inv(next)
@requires Pre(this, head, next)
@ensures Post(this, old(head), head, old(next), next)

```
void reverse() {
   Node near0 = this.head;
   Node mid0 = near0.next;
   Node far0 = mid0.next;

   next0 = update(next, near0, far0);
   boolean guard = (far0 != null);
   next1 = update(next0, mid0, near0);
   near1 = mid0;
   mid1 = far0;
   far1 = far0.next1;

   near2 = phi(guard, near1, near0);
   mid2 = phi(guard, mid1, mid0);
   far2 = phi(guard, far1, far0);
   next2 = phi(guard, next1, next0);

   assume far2 == null;

   next3 = update(next2, mid2, near2);
   head0 = update(head, this, mid2);
}
```

Execution finitization (inlining, unrolling, SSA)

Bounded verification: step 2/4

```
@invariant Inv(next)
@requires Pre(this, head, next)
@ensures Post(this, old(head), head, old(next), next)

void reverse() {
   Node near0 = this.head;
   Node mid0 = near0.next;
   Node far0 = mid0.next;

   next0 = update(next, near0, far0);
   boolean guard = (far0 != null);
   next1 = update(next0, mid0, near0);
   near1 = mid0;
   mid1 = far0;
   far1 = far0.next1;

   near2 = phi(guard, near1, near0);
   mid2 = phi(guard, mid1, mid0);
   far2 = phi(guard, far1, far0);
   next2 = phi(guard, next1, next0);

   assume far2 == null;

   next3 = update(next2, mid2, near2);
   head0 = update(head, this, mid2);
}
```

Execution finitization (inlining, unrolling, SSA)

↓

Forward VCG

Symbolic interpretation of the code with respect to the relational heap model.

Bounded verification: step 2/4

```
@invariant Inv(next)
@requires Pre(this, head, next)
@ensures Post(this, old(head), head, old(next), next)

void reverse() {
   Node near₀ = this.head;
   Node mid₀ = near₀.next;
   Node far₀ = mid₀.next;

   next₀ = update(next, near₀, far₀);
   boolean guard = (far₀ != null);
   next₁ = update(next₀, mid₀, near₀);
   near₁ = mid₀;
   mid₁ = far₀;
   far₁ = far₀.next₁;

   near₂ = phi(guard, near₁, near₀);
   mid₂ = phi(guard, mid₁, mid₀);
   far₂ = phi(guard, far₁, far₀);
   next₂ = phi(guard, next₁, next₀);

   assume far₂ == null;

   next₃ = update(next₂, mid₂, near₂);
   head₀ = update(head, this, mid₂);
}
```

$\text{this} \subseteq \text{List} \wedge \textbf{one}\ \text{this} \wedge$

$\text{head} \subseteq \text{List} \mapsto (\text{Node} \cup \text{null}) \wedge$

$\text{next} \subseteq \text{Node} \mapsto (\text{Node} \cup \text{null}) \wedge$

$\text{data} \subseteq \text{Node} \mapsto (\text{String} \cup \text{null}) \wedge$

let $\text{near}_0 = \text{this.head}$,
$\text{mid}_0 = \text{near}_0.\text{next}$,
$\text{far}_0 = \text{mid}_0.\text{next}$,

$\text{next}_0 = \text{next} \mathbin{++} (\text{near}_0 \times \text{far}_0)$,
$\text{guard} = (\text{far}_0 \mathrel{!=} \text{null})$,
$\text{next}_1 = \text{next}_0 \mathbin{++} (\text{mid}_0 \times \text{near}_0)$,
$\text{near}_1 = \text{mid}_0$,
$\text{mid}_1 = \text{far}_0$,
$\text{far}_1 = \text{far}_0.\text{next}_1$,

$\text{near}_2 = \textbf{if}\ \text{guard}\ \textbf{then}\ \text{near}_1\ \textbf{else}\ \text{near}_0$,
$\text{mid}_2 = \textbf{if}\ \text{guard}\ \textbf{then}\ \text{mid}_1\ \textbf{else}\ \text{mid}_0$,
$\text{far}_2 = \textbf{if}\ \text{guard}\ \textbf{then}\ \text{far}_1\ \textbf{else}\ \text{far}_0$,
$\text{next}_2 = \textbf{if}\ \text{guard}\ \textbf{then}\ \text{next}_1\ \textbf{else}\ \text{next}_0$,
$\text{next}_3 = \text{next}_2 \mathbin{++} (\text{mid}_2 \times \text{near}_2)$
$\text{head}_0 = \text{head} \mathbin{++} (\text{this} \times \text{mid}_2)\ \ |$

$\text{far}_2 = \text{null} \wedge \text{Inv}(\text{next}) \wedge \text{Pre}(\text{this}, \text{head}, \text{next}) \wedge$
$\neg\, (\text{Inv}(\text{next}_3) \wedge \text{Post}(\text{this}, \text{head}, \text{head}_0, \text{next}, \text{next}_3))$

Bounded verification: step 3/4

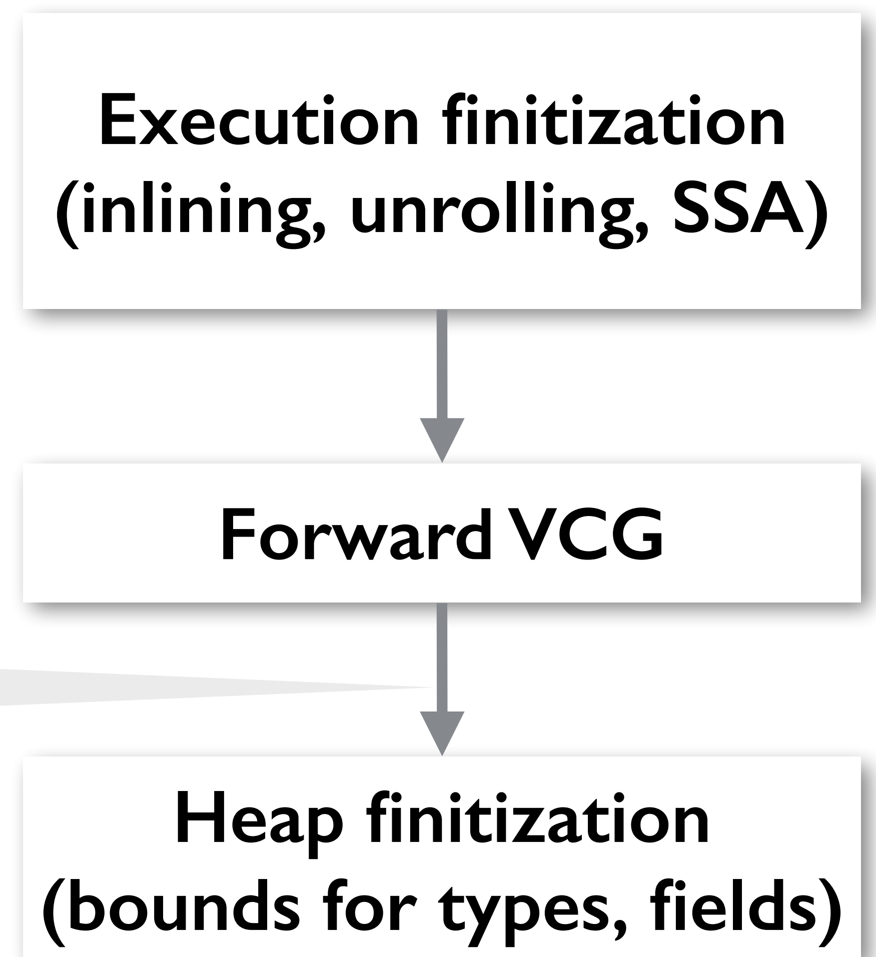
$\text{this} \subseteq \text{List} \wedge \textbf{one } \text{this} \wedge$
$\text{head} \subseteq \text{List} \mapsto (\text{Node} \cup \text{null}) \wedge$
$\text{next} \subseteq \text{Node} \mapsto (\text{Node} \cup \text{null}) \wedge$
$\text{data} \subseteq \text{Node} \mapsto (\text{String} \cup \text{null}) \wedge$

$\textbf{let } near_0 = \text{this.head},$
$mid_0 = near_0.\text{next},$
$far_0 = mid_0.\text{next},$

$next_0 = \text{next} \mathbin{++} (near_0 \times far_0),$
$guard = (far_0 \mathrel{!=} \text{null}),$
$next_1 = next_0 \mathbin{++} (mid_0 \times near_0),$
$near_1 = mid_0,$
$mid_1 = far_0,$
$far_1 = far_0.next_1,$

$near_2 = \textbf{if } guard \textbf{ then } near_1 \textbf{ else } near_0,$
$mid_2 = \textbf{if } guard \textbf{ then } mid_1 \textbf{ else } mid_0,$
$far_2 = \textbf{if } guard \textbf{ then } far_1 \textbf{ else } far_0,$
$next_2 = \textbf{if } guard \textbf{ then } next_1 \textbf{ else } next_0,$
$next_3 = next_2 \mathbin{++} (mid_2 \times near_2)$
$head_0 = \text{head} \mathbin{++} (\text{this} \times mid_2) \;|$

$far_2 = \text{null} \wedge \text{Inv}(\text{next}) \wedge \text{Pre}(\text{this}, \text{head}, \text{next}) \wedge$
$\neg (\text{Inv}(next_3) \wedge \text{Post}(\text{this}, \text{head}, head_0, \text{next}, next_3))$



Bounded verification: step 3/4

$\text{this} \subseteq \text{List} \wedge \textbf{one}\ \text{this} \wedge$
$\text{head} \subseteq \text{List} \mapsto (\text{Node} \cup \text{null}) \wedge$
$\text{next} \subseteq \text{Node} \mapsto (\text{Node} \cup \text{null}) \wedge$
$\text{data} \subseteq \text{Node} \mapsto (\text{String} \cup \text{null}) \wedge$

$\textbf{let}\ \text{near}_0 = \text{this.head},$
$\text{mid}_0 = \text{near}_0\text{.next},$
$\text{far}_0 = \text{mid}_0\text{.next},$

$\text{next}_0 = \text{next} \mathbin{++} (\text{near}_0 \times \text{far}_0),$
$\text{guard} = (\text{far}_0 \mathrel{!=} \text{null}),$
$\text{next}_1 = \text{next}_0 \mathbin{++} (\text{mid}_0 \times \text{near}_0),$
$\text{near}_1 = \text{mid}_0,$
$\text{mid}_1 = \text{far}_0,$
$\text{far}_1 = \text{far}_0\text{.next}_1,$

$\text{near}_2 = \textbf{if}\ \text{guard}\ \textbf{then}\ \text{near}_1\ \textbf{else}\ \text{near}_0,$
$\text{mid}_2 = \textbf{if}\ \text{guard}\ \textbf{then}\ \text{mid}_1\ \textbf{else}\ \text{mid}_0,$
$\text{far}_2 = \textbf{if}\ \text{guard}\ \textbf{then}\ \text{far}_1\ \textbf{else}\ \text{far}_0,$
$\text{next}_2 = \textbf{if}\ \text{guard}\ \textbf{then}\ \text{next}_1\ \textbf{else}\ \text{next}_0,$
$\text{next}_3 = \text{next}_2 \mathbin{++} (\text{mid}_2 \times \text{near}_2)$
$\text{head}_0 = \text{head} \mathbin{++} (\text{this} \times \text{mid}_2) \mid$

$\text{far}_2 = \text{null} \wedge \text{Inv}(\text{next}) \wedge \text{Pre}(\text{this}, \text{head}, \text{next}) \wedge$
$\neg (\text{Inv}(\text{next}_3) \wedge \text{Post}(\text{this}, \text{head}, \text{head}_0, \text{next}, \text{next}_3))$

$\{\ \text{this}, \text{n0}, \text{n1}, \text{n2}, \text{s0}, \text{s1}, \text{s2}, \text{null}\ \}$

$\{\ \langle\text{null}\rangle\ \} \subseteq \text{null} \subseteq \{\ \langle\text{null}\rangle\ \}$

$\{\} \subseteq \text{this} \subseteq \{\ \langle\ \text{this}\ \rangle\ \}$
$\{\} \subseteq \text{List} \subseteq \{\ \langle\ \text{this}\ \rangle\ \}$
$\{\} \subseteq \text{Node} \subseteq \{\ \langle\text{n0}\rangle, \langle\text{n1}\rangle, \langle\text{n2}\rangle\ \}$
$\{\} \subseteq \text{String} \subseteq \{\ \langle\text{s0}\rangle, \langle\text{s1}\rangle, \langle\text{s2}\rangle\ \}$

$\{\} \subseteq \text{head} \subseteq \{\ \text{this}\ \} \times \{\ \text{n0}, \text{n1}, \text{n2}, \text{null}\ \}$
$\{\} \subseteq \text{next} \subseteq \{\ \text{n0}, \text{n1}, \text{n2}\ \} \times \{\ \text{n0}, \text{n1}, \text{n2}, \text{null}\ \}$
$\{\} \subseteq \text{data} \subseteq \{\ \text{n0}, \text{n1}, \text{n2}\ \} \times \{\ \text{s0}, \text{s1}, \text{s2}, \text{null}\ \}$

Bounded verification: step 3/4

$\text{this} \subseteq \text{List} \land \textbf{one}\ \text{this} \land$
$\text{head} \subseteq \text{List} \mapsto (\text{Node} \cup \text{null}) \land$
$\text{next} \subseteq \text{Node} \mapsto (\text{Node} \cup \text{null}) \land$
$\text{data} \subseteq \text{Node} \mapsto (\text{String} \cup \text{null}) \land$

let $\text{near}_0 = \text{this.head}$,
$\text{mid}_0 = \text{near}_0.\text{next}$,
$\text{far}_0 = \text{mid}_0.\text{next}$,

$\text{next}_0 = \text{next} \mathbin{++} (\text{near}_0 \times \text{far}_0)$,
$\text{guard} = (\text{far}_0 \mathrel{!=} \text{null})$,
$\text{next}_1 = \text{next}_0 \mathbin{++} (\text{mid}_0 \times \text{near}_0)$,
$\text{near}_1 = \text{mid}_0$,
$\text{mid}_1 = \text{far}_0$,
$\text{far}_1 = \text{far}_0.\text{next}_1$,

$\text{near}_2 = \textbf{if}\ \text{guard}\ \textbf{then}\ \text{near}_1\ \textbf{else}\ \text{near}_0$,
$\text{mid}_2 = \textbf{if}\ \text{guard}\ \textbf{then}\ \text{mid}_1\ \textbf{else}\ \text{mid}_0$,
$\text{far}_2 = \textbf{if}\ \text{guard}\ \textbf{then}\ \text{far}_1\ \textbf{else}\ \text{far}_0$,
$\text{next}_2 = \textbf{if}\ \text{guard}\ \textbf{then}\ \text{next}_1\ \textbf{else}\ \text{next}_0$,
$\text{next}_3 = \text{next}_2 \mathbin{++} (\text{mid}_2 \times \text{near}_2)$
$\text{head}_0 = \text{head} \mathbin{++} (\text{this} \times \text{mid}_2)\ \mid$

$\text{far}_2 = \text{null} \land \text{Inv}(\text{next}) \land \text{Pre}(\text{this}, \text{head}, \text{next}) \land$
$\neg\, (\text{Inv}(\text{next}_3) \land \text{Post}(\text{this}, \text{head}, \text{head}_0, \text{next}, \text{next}_3))$

Finite **universe** of uninterpreted symbols.

$\{\ \text{this}, \text{n0}, \text{n1}, \text{n2}, \text{s0}, \text{s1}, \text{s2}, \text{null}\ \}$

$\{\ \langle\text{null}\rangle\ \} \subseteq \text{null} \subseteq \{\ \langle\text{null}\rangle\ \}$

$\{\} \subseteq \text{this} \subseteq \{\ \langle\ \text{this}\ \rangle\ \}$
$\{\} \subseteq \text{List} \subseteq \{\ \langle\ \text{this}\ \rangle\ \}$
$\{\} \subseteq \text{Node} \subseteq \{\ \langle\text{n0}\rangle, \langle\text{n1}\rangle, \langle\text{n2}\rangle\ \}$
$\{\} \subseteq \text{String} \subseteq \{\ \langle\text{s0}\rangle, \langle\text{s1}\rangle, \langle\text{s2}\rangle\ \}$

$\{\} \subseteq \text{head} \subseteq \{\ \text{this}\ \} \times \{\ \text{n0}, \text{n1}, \text{n2}, \text{null}\ \}$
$\{\} \subseteq \text{next} \subseteq \{\ \text{n0}, \text{n1}, \text{n2}\ \} \times \{\ \text{n0}, \text{n1}, \text{n2}, \text{null}\ \}$
$\{\} \subseteq \text{data} \subseteq \{\ \text{n0}, \text{n1}, \text{n2}\ \} \times \{\ \text{s0}, \text{s1}, \text{s2}, \text{null}\ \}$

Bounded verification: step 3/4

this ⊆ List ∧ **one** this ∧
head ⊆ List ↦ (Node ∪ null) ∧
next ⊆ Node ↦ (Node ∪ null) ∧
data ⊆ Node ↦ (String ∪ null) ∧

let $near_0$ = this.head,
mid_0 = $near_0$.next,
far_0 = mid_0.next,

$next_0$ = next ++ ($near_0$ × far_0),
guard = (far_0 != null),
$next_1$ = $next_0$ ++ (mid_0 × $near_0$),
$near_1$ = mid_0,
mid_1 = far_0,
far_1 = far_0.$next_1$,

$near_2$ = **if** guard **then** $near_1$ **else** $near_0$,
mid_2 = **if** guard **then** mid_1 **else** mid_0,
far_2 = **if** guard **then** far_1 **else** far_0,
$next_2$ = **if** guard **then** $next_1$ **else** $next_0$,
$next_3$ = $next_2$ ++ (mid_2 × $near_2$)
$head_0$ = head ++ (this × mid_2) |

far_2 = null ∧ Inv(next) ∧ Pre(this, head, next) ∧
¬ (Inv($next_3$) ∧ Post(this, head, $head_0$, next, $next_3$))

Finite **universe** of uninterpreted symbols.

{ this, n0, n1, n2, s0, s1, s2, null }

{ ⟨null⟩ } ⊆ null ⊆ { ⟨null⟩ }

{} ⊆ this ⊆ { ⟨ this ⟩ }
{} ⊆ List ⊆ { ⟨ this ⟩ }
{} ⊆ Node ⊆ { ⟨n0⟩, ⟨n1⟩, ⟨n2⟩ }
{} ⊆ String ⊆ { ⟨s0⟩, ⟨s1⟩, ⟨s2⟩ }

{} ⊆ head ⊆ { this } × { n0, n1, n2, null }
{} ⊆ next ⊆ { n0, n1, n2 } × { n0, n1, n2, null }
{} ⊆ data ⊆ { n0, n1, n2 } × { s0, s1, s2, null }

Upper bound on each relation: tuples it *may* contain.

Bounded verification: step 3/4

this ⊆ List ∧ **one** this ∧
head ⊆ List ↦ (Node ∪ null) ∧
next ⊆ Node ↦ (Node ∪ null) ∧
data ⊆ Node ↦ (String ∪ null) ∧

let $near_0$ = this.head,
mid_0 = $near_0$.next,
far_0 = mid_0.next,

$next_0$ = next ++ ($near_0$ × far_0),
guard = (far_0 != null),
$next_1$ = $next_0$ ++ (mid_0 × $near_0$),
$near_1$ = mid_0,
mid_1 = far_0,
far_1 = far_0.$next_1$,

$near_2$ = **if** guard **then** $near_1$ **else** $near_0$,
mid_2 = **if** guard **then** mid_1 **else** mid_0,
far_2 = **if** guard **then** far_1 **else** far_0,
$next_2$ = **if** guard **then** $next_1$ **else** $next_0$,
$next_3$ = $next_2$ ++ (mid_2 × $near_2$)
$head_0$ = head ++ (this × mid_2) |

far_2 = null ∧ Inv(next) ∧ Pre(this, head, next) ∧
¬ (Inv($next_3$) ∧ Post(this, head, $head_0$, next, $next_3$))

Finite **universe** of uninterpreted symbols.

{ this, n0, n1, n2, s0, s1, s2, null }

{ ⟨null⟩ } ⊆ null ⊆ { ⟨null⟩ }

{} ⊆ this ⊆ { ⟨ this ⟩ }
{} ⊆ List ⊆ { ⟨ this ⟩ }
{} ⊆ Node ⊆ { ⟨n0⟩, ⟨n1⟩, ⟨n2⟩ }
{} ⊆ String ⊆ { ⟨s0⟩, ⟨s1⟩, ⟨s2⟩ }

{} ⊆ head ⊆ { this } × { n0, n1, n2, null }
{} ⊆ next ⊆ { n0, n1, n2 } × { n0, n1, n2, null }
{} ⊆ data ⊆ { n0, n1, n2 } × { s0, s1, s2, null }

Lower bound on each relation: tuples it *must* contain.

Upper bound on each relation: tuples it *may* contain.

Bounded verification: step 4/4

this ⊆ List ∧ **one** this ∧
head ⊆ List ↦ (Node ∪ null) ∧
next ⊆ Node ↦ (Node ∪ null) ∧
data ⊆ Node ↦ (String ∪ null) ∧

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ne

{ this, n0, n1, n2, s0, s1, s2, null }

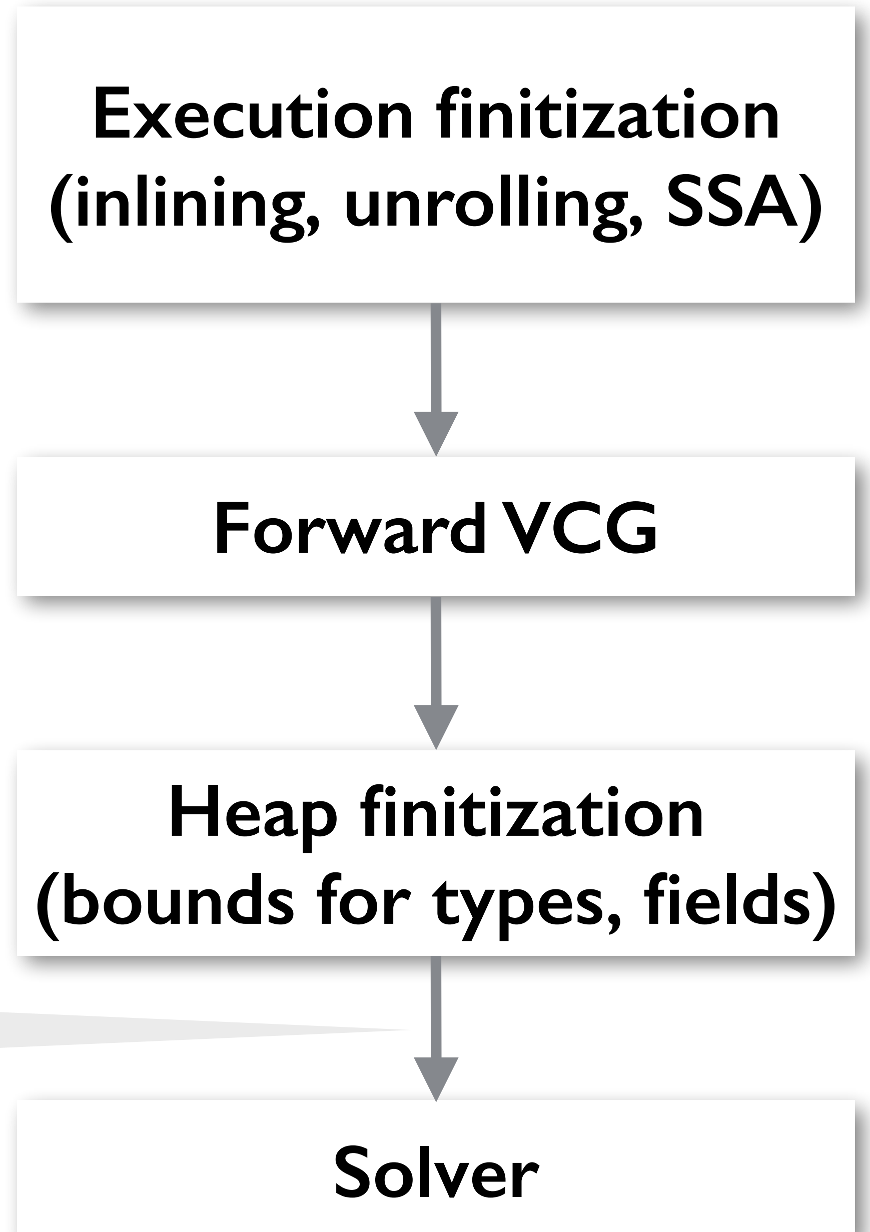
{ ⟨null⟩ } ⊆ null ⊆ { ⟨null⟩ }

{} ⊆ this ⊆ { ⟨ this ⟩ }
{} ⊆ List ⊆ { ⟨ this ⟩ }
{} ⊆ Node ⊆ { ⟨n0⟩, ⟨n1⟩, ⟨n2⟩ }
{} ⊆ String ⊆ { ⟨s0⟩, ⟨s1⟩, ⟨s2⟩ }

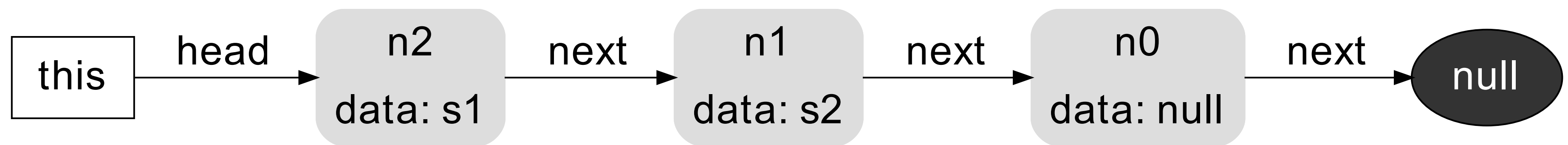
{} ⊆ head ⊆ { this } × { n0, n1, n2, null }
{} ⊆ next ⊆ { n0, n1, n2 } × { n0, n1, n2, null }
{} ⊆ data ⊆ { n0, n1, n2 } × { s0, s1, s2, null }

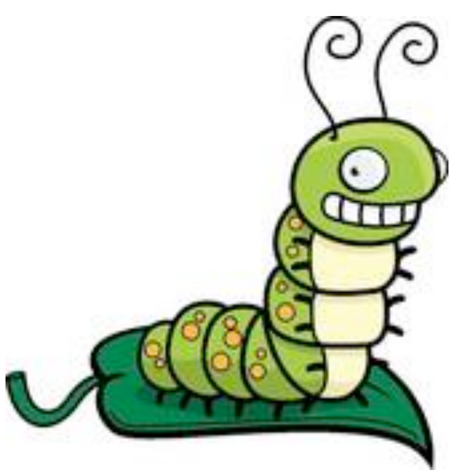
$next_3$ = $next_2$ ++ (mid_2 × $near_2$)
$head_0$ = head ++ (this × mid_2) |

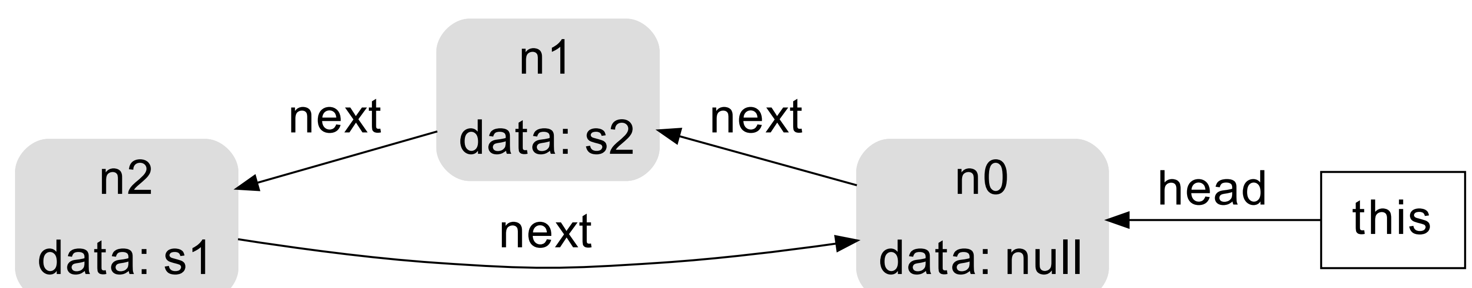
far_2 = null ∧ Inv(next) ∧ Pre(this, head, next) ∧
¬ (Inv($next_3$) ∧ Post(this, head, $head_0$, next, $next_3$))



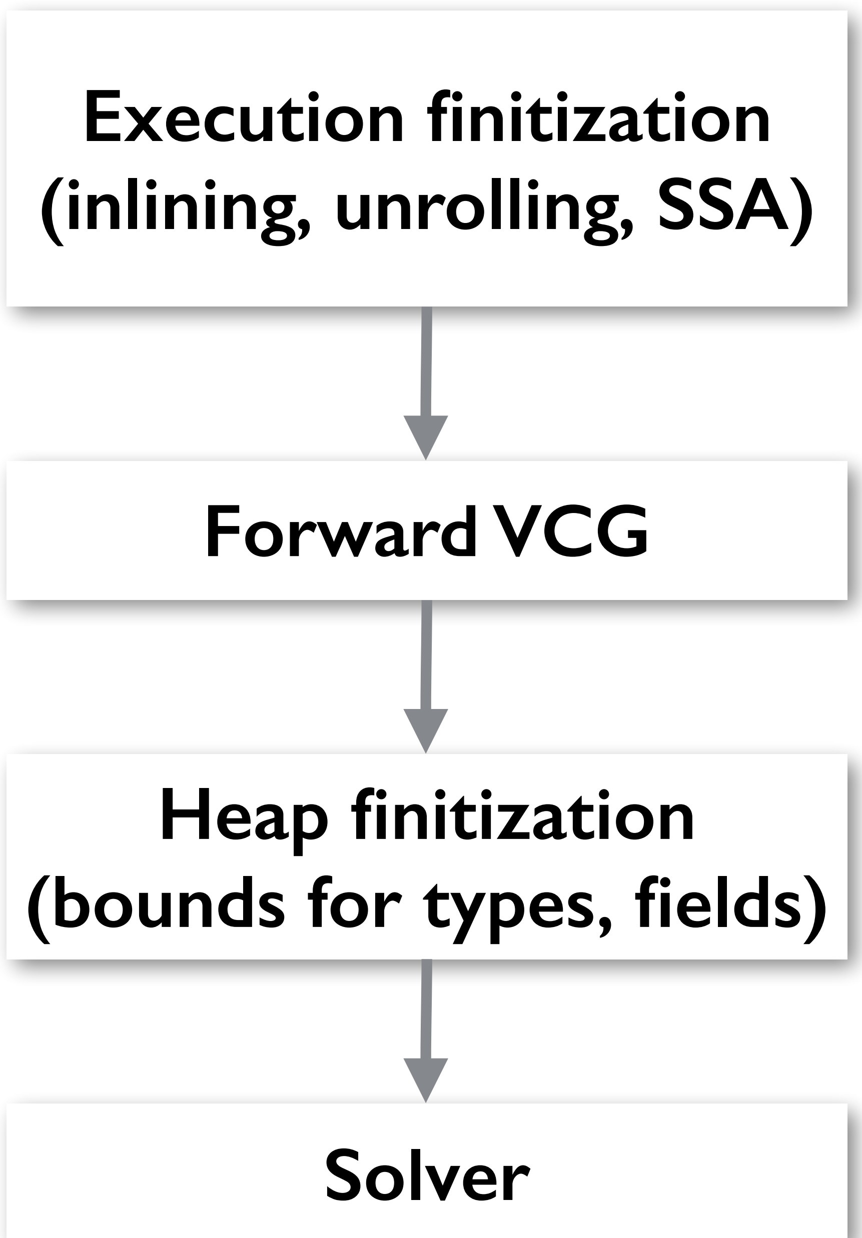
Bounded verification: counterexample



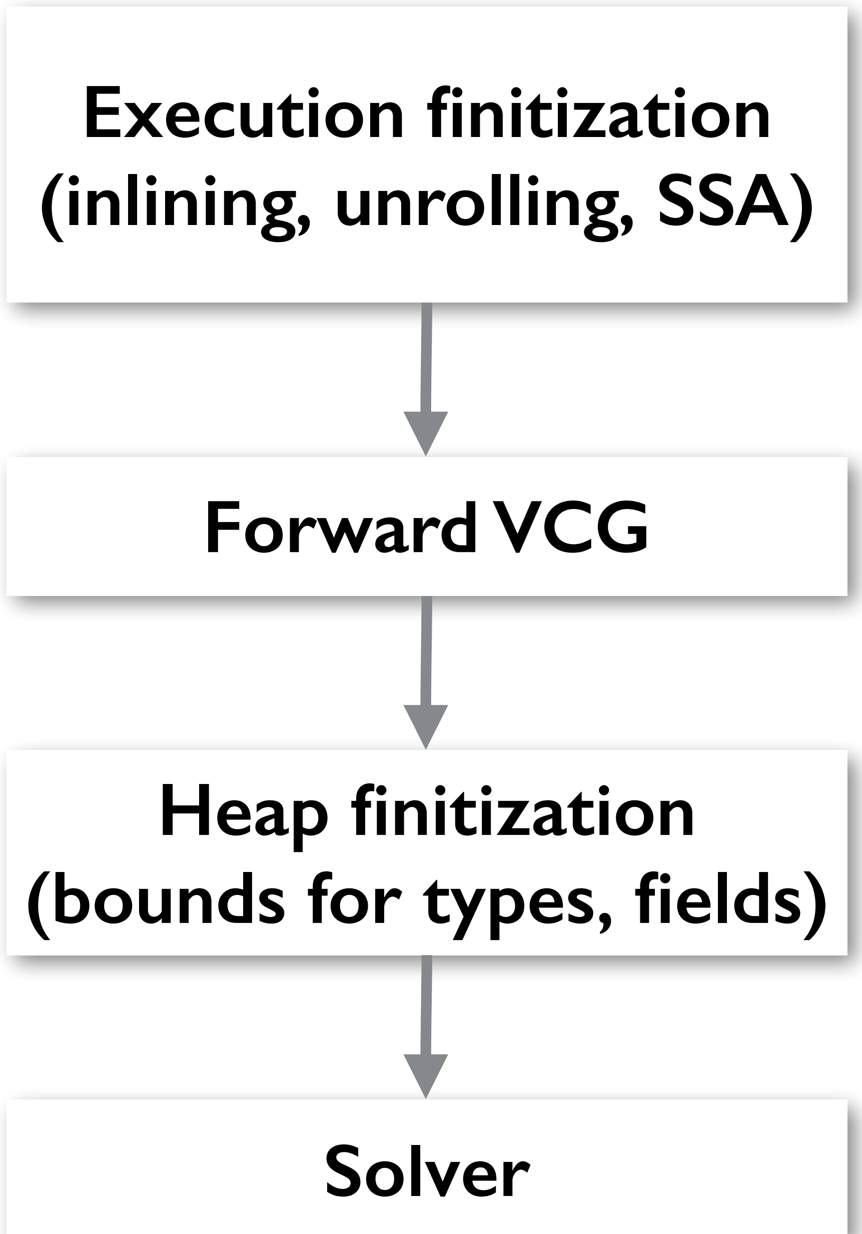




Bounded verification: optimization

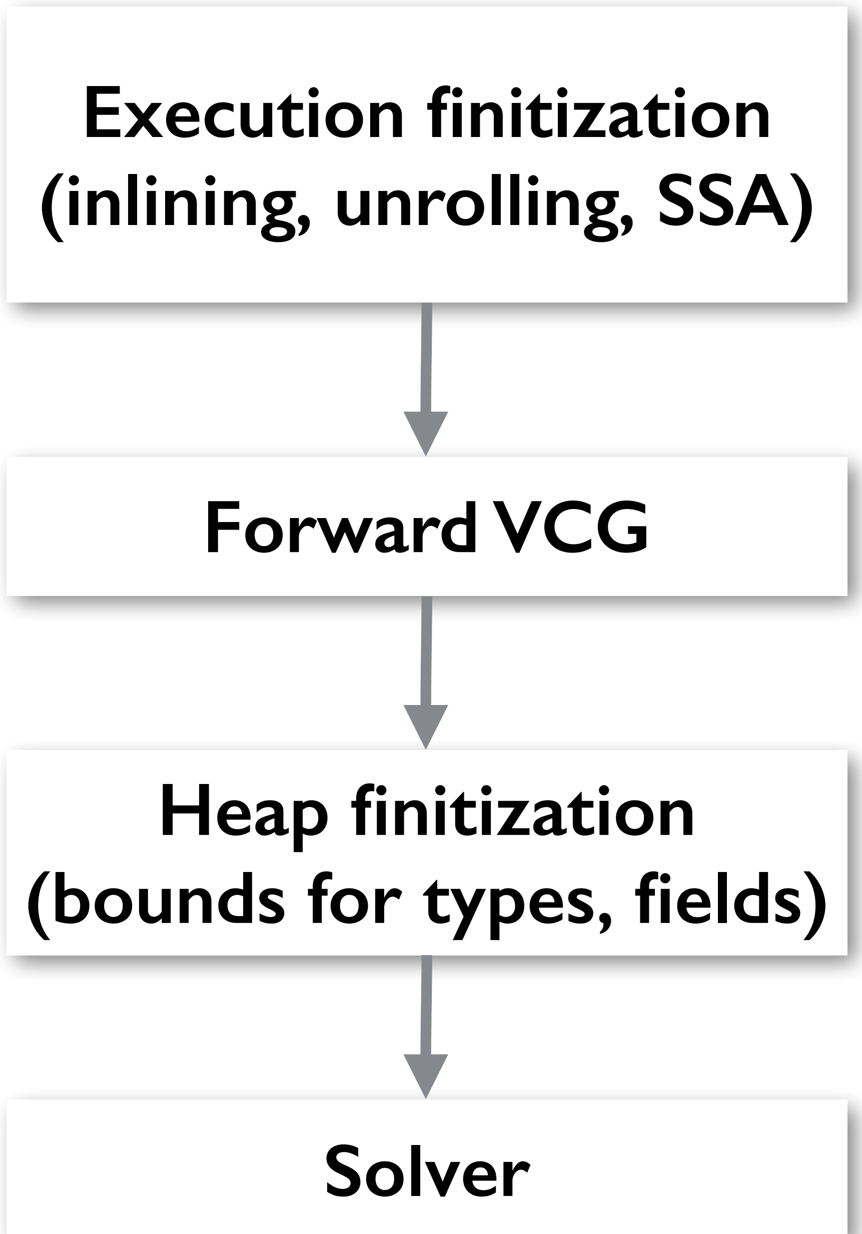


Bounded verification: optimization



Finitized program after inlining may be huge.

Bounded verification: optimization



Finitized program after inlining may be huge.

Full inlining is rarely needed to check partial correctness.

Bounded verification: optimization

Execution finitization (inlining, unrolling, SSA)

↓

Forward VCG

↓

Heap finitization (bounds for types, fields)

↓

Solver

Finitized program after inlining may be huge.

Full inlining is rarely needed to check partial correctness.

Optimization: Counterexample-Guided Abstraction Refinement with Unsatisfiable Cores [Taghdiri, 2004]

From bounded verification to fault localization

```
@invariant Inv(next)
@requires Pre(this, head, next)
@ensures Post(this, old(head), head, old(next), next)

void reverse() {
  Node near₀ = this.head;
  Node mid₀ = near₀.next;
  Node far₀ = mid₀.next;

  next₀ = update(next, near₀, far₀);
  boolean guard = (far₀ != null);
  next₁ = update(next₀, mid₀, near₀);
  near₁ = mid₀;
  mid₁ = far₀;
  far₁ = far₀.next₁;

  near₂ = phi(guard, near₁, near₀);
  mid₂ = phi(guard, mid₁, mid₀);
  far₂ = phi(guard, far₁, far₀);
  next₂ = phi(guard, next₁, next₀);

  assume far₂ == null;

  next₃ = update(next₂, mid₂, near₂);
  head₀ = update(head, this, mid₂);
}
```

From bounded verification to fault localization

```
@invariant Inv(next)
@requires Pre(this, head, next)
@ensures Post(this, old(head), head, old(next), next)

void reverse() {
   Node near0 = this.head;
   Node mid0 = near0.next;
   Node far0 = mid0.next;

   next0 = update(next, near0, far0);
   boolean guard = (far0 != null);
   next1 = update(next0, mid0, near0);
   near1 = mid0;
   mid1 = far0;
   far1 = far0.next1;

   near2 = phi(guard, near1, near0);
   mid2 = phi(guard, mid1, mid0);
   far2 = phi(guard, far1, far0);
   next2 = phi(guard, next1, next0);

   assume far2 == null;

   next3 = update(next2, mid2, near2);
   head0 = update(head, this, mid2);
}
```

Given a buggy program and a valid input and the expected output, find a minimal subset of program statements that prevents the execution on the given input from reaching a valid output state.

From bounded verification to fault localization

@invariant Inv(next)
@requires Pre(this, head, next)
@ensures Post(this, old(head), head, old(next), next)

```
void reverse() {
   Node near0 = this.head;
   Node mid0 = near0.next;
   Node far0 = mid0.next;

   next0 = update(next, near0, far0);
   boolean guard = (far0 != null);
   next1 = update(next0, mid0, near0);
   near1 = mid0;
   mid1 = far0;
   far1 = far0.next1;

   near2 = phi(guard, near1, near0);
   mid2 = phi(guard, mid1, mid0);
   far2 = phi(guard, far1, far0);
   next2 = phi(guard, next1, next0);

   assume far2 == null;

   next3 = update(next2, mid2, near2);
   head0 = update(head, this, mid2);
}
```

Given a buggy program and a valid input and the expected output, find a minimal subset of program statements that prevents the execution on the given input from reaching a valid output state.

Introduce additional "indicator" relations into the encoding.

From bounded verification to fault localization

```
@invariant Inv(next)
@requires Pre(this, head, next)
@ensures Post(this, old(head), head, old(next), next)

void reverse() {
   Node near0 = this.head;
   Node mid0 = near0.next;
   Node far0 = mid0.next;

   next0 = update(next, near0, far0);
   boolean guard = (far0 != null);
   next1 = update(next0, mid0, near0);
   near1 = mid0;
   mid1 = far0;
   far1 = far0.next1;

   near2 = phi(guard, near1, near0);
   mid2 = phi(guard, mid1, mid0);
   far2 = phi(guard, far1, far0);
   next2 = phi(guard, next1, next0);

   assume far2 == null;

   next3 = update(next2, mid2, near2);
   head0 = update(head, this, mid2);
}
```

Given a buggy program and a valid input and the expected output, find a minimal subset of program statements that prevents the execution on the given input from reaching a valid output state.

Introduce additional "indicator" relations into the encoding.

The resulting formula, together with the input partial model, is unsatisfiable.

From bounded verification to fault localization

```
@invariant Inv(next)
@requires Pre(this, head, next)
@ensures Post(this, old(head), head, old(next), next)

void reverse() {
  Node near0 = this.head;
  Node mid0 = near0.next;
  Node far0 = mid0.next;

  next0 = update(next, near0, far0);
  boolean guard = (far0 != null);
  next1 = update(next0, mid0, near0);
  near1 = mid0;
  mid1 = far0;
  far1 = far0.next1;

  near2 = phi(guard, near1, near0);
  mid2 = phi(guard, mid1, mid0);
  far2 = phi(guard, far1, far0);
  next2 = phi(guard, next1, next0);

  assume far2 == null;

  next3 = update(next2, mid2, near2);
  head0 = update(head, this, mid2);
}
```

Given a buggy program and a valid input and the expected output, find a minimal subset of program statements that prevents the execution on the given input from reaching a valid output state.

Introduce additional “indicator” relations into the encoding.

The resulting formula, together with the input partial model, is unsatisfiable.

A minimal unsatisfiable core of this formula represents an irreducible cause of the program’s failure to meet the specification.

Fault localization: encoding

@invariant Inv(next)
@requires Pre(this, head, next)
@ensures Post(this, old(head), head, old(next), next)

```
void reverse() {
   Node near_0 = this.head;
   Node mid_0 = r
   Node far_0 = m

   next_0 = updat
   boolean guar
   next_1 = update(next_0, mid_0, near_0);
   near_1 = mid_0;
   mid_1 = far_0;
   far_1 = far_0.next_1;

   near_2 = phi(guard, near_1, near_0);
   mid_2 = phi(guard, mid_1, mid_0);
   far_2 = phi(guard, far_1, far_0);
   next_2 = phi(guard, next_1, next_0);

   assume far_2 == null;

   next_3 = update(next_2, mid_2, near_2);
   head_0 = update(head, this, mid_2);
}
```

Start with the encoding for bounded verification.

$\text{this} \subseteq \text{List} \wedge \textbf{one}\ \text{this} \wedge$
$\text{head} \subseteq \text{List} \mapsto (\text{Node} \cup \text{null}) \wedge$
$\text{next} \subseteq \text{Node} \mapsto (\text{Node} \cup \text{null}) \wedge$
$\text{data} \subseteq \text{Node} \mapsto (\text{String} \cup \text{null}) \wedge$

$\textbf{let}\ \text{near}_0 = \text{this.head},$
$\quad \text{mid}_0 = \text{near}_0.\text{next},$
$\quad \text{far}_0 = \text{mid}_0.\text{next},$

$\quad \text{next}_0 = \text{next} \mathbin{++} (\text{near}_0 \times \text{far}_0),$
$\quad \text{guard} = (\text{far}_0 \mathrel{!=} \text{null}),$
$\quad \text{next}_1 = \text{next}_0 \mathbin{++} (\text{mid}_0 \times \text{near}_0),$
$\quad \text{near}_1 = \text{mid}_0,$
$\quad \text{mid}_1 = \text{far}_0,$
$\quad \text{far}_1 = \text{far}_0.\text{next}_1,$

$\quad \text{near}_2 = \textbf{if}\ \text{guard}\ \textbf{then}\ \text{near}_1\ \textbf{else}\ \text{near}_0,$
$\quad \text{mid}_2 = \textbf{if}\ \text{guard}\ \textbf{then}\ \text{mid}_1\ \textbf{else}\ \text{mid}_0,$
$\quad \text{far}_2 = \textbf{if}\ \text{guard}\ \textbf{then}\ \text{far}_1\ \textbf{else}\ \text{far}_0,$
$\quad \text{next}_2 = \textbf{if}\ \text{guard}\ \textbf{then}\ \text{next}_1\ \textbf{else}\ \text{next}_0,$
$\quad \text{next}_3 = \text{next}_2 \mathbin{++} (\text{mid}_2 \times \text{near}_2)$
$\quad \text{head}_0 = \text{head} \mathbin{++} (\text{this} \times \text{mid}_2) \mid$

$\quad \text{far}_2 = \text{null} \wedge \text{Inv}(\text{next}) \wedge \text{Pre}(\text{this}, \text{head}, \text{next}) \wedge$
$\quad \neg\, (\text{Inv}(\text{next}_3) \wedge \text{Post}(\text{this}, \text{head}, \text{head}_0, \text{next}, \text{next}_3))$

Fault localization: encoding

@invariant Inv(next)
@requires Pre(this, head, next)
@ensures Post(this, old(head), head, old(next), next)

```
void reverse() {
   Node near0 = this.head;
   Node mid0 = near0.next;
   Node far0 = mid0.next;

   next0 = update(next, near0, far0);
   boolean guard = (far0 != null);
   next1 = update(next0, mid0, near0);
   near1 = mid0;
   mid1 = far0;
   far1 = far0.n

   near2 = phi(g
   mid2 = phi(gu
   far2 = phi(gu
   next2 = phi(guard, next1, next0);

   assume far2 == null;

   next3 = update(next2, mid2, near2);
   head0 = update(head, this, mid2);
}
```

Introduce fresh relations for source-level expressions.

$\text{this} \subseteq \text{List} \wedge \textbf{one}\ \text{this} \wedge$
$\text{head} \subseteq \text{List} \mapsto (\text{Node} \cup \text{null}) \wedge$
$\text{next} \subseteq \text{Node} \mapsto (\text{Node} \cup \text{null}) \wedge$
$\text{data} \subseteq \text{Node} \mapsto (\text{String} \cup \text{null}) \wedge$

$\text{near}_0 = \text{this.head} \wedge$
$\text{mid}_0 = \text{near}_0\text{.next} \wedge$
$\text{far}_0 = \text{mid}_0\text{.next} \wedge$

$\text{next}_0 = \text{next} \mathbin{++} (\text{near}_0 \times \text{far}_0) \wedge$
$\text{next}_1 = \text{next}_0 \mathbin{++} (\text{mid}_0 \times \text{near}_0) \wedge$
$\text{near}_1 = \text{mid}_0 \wedge$
$\text{mid}_1 = \text{far}_0 \wedge$
$\text{far}_1 = \text{far}_0\text{.next}_1 \wedge$

$\textbf{let}\ \text{guard} = (\text{far}_0\ \text{!=}\ \text{null}),$
$\quad \text{near}_2 = \textbf{if}\ \text{guard}\ \textbf{then}\ \text{near}_1\ \textbf{else}\ \text{near}_0,$
$\quad \text{mid}_2 = \textbf{if}\ \text{guard}\ \textbf{then}\ \text{mid}_1\ \textbf{else}\ \text{mid}_0,$
$\quad \text{far}_2 = \textbf{if}\ \text{guard}\ \textbf{then}\ \text{far}_1\ \textbf{else}\ \text{far}_0,$
$\quad \text{next}_2 = \textbf{if}\ \text{guard}\ \textbf{then}\ \text{next}_1\ \textbf{else}\ \text{next}_0\ |$

$\quad \text{next}_3 = \text{next}_2 \mathbin{++} (\text{mid}_2 \times \text{near}_2) \wedge$
$\quad \text{head}_0 = \text{head} \mathbin{++} (\text{this} \times \text{mid}_2) \wedge$
$\quad \text{far}_2 = \text{null} \wedge \text{Inv}(\text{next}) \wedge \text{Pre}(\text{this}, \text{head}, \text{next}) \wedge$
$\quad \text{Inv}(\text{next}_3) \wedge \text{Post}(\text{this}, \text{head}, \text{head}_0, \text{next}, \text{next}_3)$

Fault localization: bounds

$this \subseteq List \wedge$ **one** $this \wedge$
$head \subseteq List \mapsto (Node \cup null) \wedge$
$next \subseteq Node \mapsto (Node \cup null) \wedge$
$data \subseteq Node \mapsto (String \cup null) \wedge$

$near_0 = this.head \wedge$
$mid_0 = near_0.next \wedge$
$far_0 = mid_0.next \wedge$

$next_0 = next \mathbin{++} (near_0 \times far_0) \wedge$
$next_1 = next_0 \mathbin{++} (mid_0 \times near_0) \wedge$
$near_1 = mid_0 \wedge$
$mid_1 = far_0 \wedge$
$far_1 = far_0.next_1 \wedge$

let guard = (far_0 != null),
  $near_2$ = **if** guard **then** $near_1$ **else** $near_0$,
  mid_2 = **if** guard **then** mid_1 **else** mid_0,
  far_2 = **if** guard **then** far_1 **else** far_0,
  $next_2$ = **if** guard **then** $next_1$ **else** $next_0$ |

  $next_3 = next_2 \mathbin{++} (mid_2 \times near_2) \wedge$
  $head_0 = head \mathbin{++} (this \times mid_2) \wedge$
  $far_2 = null \wedge Inv(next) \wedge Pre(this, head, next) \wedge$
  $Inv(next_3) \wedge Post(this, head, head_0, next, next_3)$

Input expressed as a partial model.

$\{ this, n0, n1, n2, s0, s1, s2, null \}$

$null = \{ \langle null \rangle \}$
$this = \{ \langle this \rangle \}$
$List = \{ \langle this \rangle \}$
$Node = \{ \langle n0 \rangle, \langle n1 \rangle, \langle n2 \rangle \}$
$String = \{ \langle s1 \rangle, \langle s2 \rangle \}$

$head = \{ \langle this, n2 \rangle \}$
$next = \{ \langle n2, n1 \rangle, \langle n1, n0 \rangle, \langle n0, null \rangle \}$
$data = \{ \langle n2, s1 \rangle, \langle n1, s2 \rangle, \langle n0, null \rangle \}$

$\{\} \subseteq head_0 \subseteq \{ this \} \times \{ n0, n1, n2, null \}$
$\{\} \subseteq next_0 \subseteq \{ n0, n1, n2 \} \times \{ n0, n1, n2, null \}$
$\{\} \subseteq next_1 \subseteq \{ n0, n1, n2 \} \times \{ n0, n1, n2, null \}$
$\{\} \subseteq next_3 \subseteq \{ n0, n1, n2 \} \times \{ n0, n1, n2, null \}$
$\{\} \subseteq near_0 \subseteq \{ n0, n1, n2, null \}$
$\{\} \subseteq near_1 \subseteq \{ n0, n1, n2, null \}$
$\{\} \subseteq mid_0 \subseteq \{ n0, n1, n2, null \}$
$\{\} \subseteq mid_1 \subseteq \{ n0, n1, n2, null \}$
$\{\} \subseteq far_0 \subseteq \{ n0, n1, n2, null \}$
$\{\} \subseteq far_1 \subseteq \{ n0, n1, n2, null \}$

Fault localization: bounds

$this \subseteq List \wedge$ **one** $this \wedge$
$head \subseteq List \mapsto (Node \cup null) \wedge$
$next \subseteq Node \mapsto (Node \cup null) \wedge$
$data \subseteq Node \mapsto (String \cup null) \wedge$

$near_0 = this.head \wedge$
$mid_0 = near_0.next \wedge$
$far_0 = mid_0.next \wedge$

$next_0 = next \mathbin{++} (near_0 \times far_0) \wedge$
$next_1 = next_0 \mathbin{++} (mid_0 \times near_0) \wedge$
$near_1 = mid_0 \wedge$
$mid_1 = far_0 \wedge$
$far_1 = far_0.next_1 \wedge$

let guard = $(far_0 \mathrel{!=} null)$,
  $near_2 =$ **if** guard **then** $near_1$ **else** $near_0$,
  $mid_2 =$ **if** guard **then** mid_1 **else** mid_0,
  $far_2 =$ **if** guard **then** far_1 **else** far_0,
  $next_2 =$ **if** guard **then** $next_1$ **else** $next_0$ |

  $next_3 = next_2 \mathbin{++} (mid_2 \times near_2) \wedge$
  $head_0 = head \mathbin{++} (this \times mid_2) \wedge$
  $far_2 = null \wedge Inv(next) \wedge Pre(this, head, next) \wedge$
  $Inv(next_3) \wedge Post(this, head, head_0, next, next_3)$

Fault localization: minimal unsat core

$this \subseteq List \wedge$ **one** $this \wedge$
$head \subseteq List \mapsto (Node \cup null) \wedge$
$next \subseteq Node \mapsto (Node \cup null) \wedge$
$data \subseteq Node \mapsto (String \cup null) \wedge$

$near_0 = this.head \wedge$
$mid_0 = near_0.next \wedge$
$far_0 = mid_0.next \wedge$

$next_0 = next \mathbin{++} (near_0 \times far_0) \wedge$
$next_1 = next_0 \mathbin{++} (mid_0 \times near_0) \wedge$
$near_1 = mid_0 \wedge$
$mid_1 = far_0 \wedge$
$far_1 = far_0.next_1 \wedge$

let guard = (far_0 != null),
  $near_2$ = **if** guard **then** $near_1$ **else** $near_0$,
  mid_2 = **if** guard **then** mid_1 **else** mid_0,
  far_2 = **if** guard **then** far_1 **else** far_0,
  $next_2$ = **if** guard **then** $next_1$ **else** $next_0$ |

  $next_3 = next_2 \mathbin{++} (mid_2 \times near_2) \wedge$
  $head_0 = head \mathbin{++} (this \times mid_2) \wedge$
  $far_2 = null \wedge Inv(next) \wedge Pre(this, head, next) \wedge$
  $Inv(next_3) \wedge Post(this, head, head_0, next, next_3)$

{ this, n0, n1, n2, s0, s1, s2, null }

null = { <null> }
this = { <this> }
List = { <this> }
Node = { <n0>, <n1>, <n2> }
String = { <s1>, <s2> }

head = { <this, n2> }
next = { <n2, n1>, <n1, n0>, <n0, null> }
data = { <n2, s1>, <n1, s2>, <n0, null> }

$\{\} \subseteq head_0 \subseteq \{ this \} \times \{ n0, n1, n2, null \}$
$\{\} \subseteq next_0 \subseteq \{ n0, n1, n2 \} \times \{ n0, n1, n2, null \}$
$\{\} \subseteq next_1 \subseteq \{ n0, n1, n2 \} \times \{ n0, n1, n2, null \}$
$\{\} \subseteq next_3 \subseteq \{ n0, n1, n2 \} \times \{ n0, n1, n2, null \}$
$\{\} \subseteq near_0 \subseteq \{ n0, n1, n2, null \}$
$\{\} \subseteq near_1 \subseteq \{ n0, n1, n2, null \}$
$\{\} \subseteq mid_0 \subseteq \{ n0, n1, n2, null \}$
$\{\} \subseteq mid_1 \subseteq \{ n0, n1, n2, null \}$
$\{\} \subseteq far_0 \subseteq \{ n0, n1, n2, null \}$
$\{\} \subseteq far_1 \subseteq \{ n0, n1, n2, null \}$

Fault localization: minimal unsat core

```
@invariant Inv(next)
@requires Pre(this, head, next)
@ensures Post(this, old(head), head, old(next), next)

void reverse() {
   Node near0 = this.head;
   Node mid0 = near0.next;
   Node far0 = mid0.next;

   next0 = update(next, near0, far0);
   boolean guard = (far0 != null);
   next1 = update(next0, mid0, near0);
   near1 = mid0;
   mid1 = far0;
   far1 = far0.next1;

   near2 = phi(guard, near1, near0);
   mid2 = phi(guard, mid1, mid0);
   far2 = phi(guard, far1, far0);
   next2 = phi(guard, next1, next0);

   assume far2 == null;

   next3 = update(next2, mid2, near2);
   head0 = update(head, this, mid2);
}
```

Summary

Today

- Bounded verification
  - A relational model of the heap
  - CEGAR with unsat cores
  - Fault localization

Next lecture

- Symbolic execution and concolic testing