Denotational semantics, with function declarations

Semantic domains:

`Bool`, `Int`, `Loc`, `Store` as before

Use `Identifier` in semantic rules, too

$\texttt{Value} = \texttt{Int} \cup \texttt{Bool}$

$\texttt{Env} = \texttt{Identifier} \rightarrow \texttt{Meaning}$

$\texttt{Meaning} = \texttt{Store} \rightarrow \texttt{Value}$

Semantic rules, with function declarations, part 1

$\mathcal{N}$: $\texttt{Numeral} \rightarrow \texttt{Int}$

$\mathcal{N}\ [\![\pi \vdash n : int]\!] = n$

$\mathcal{L}$: $\texttt{Location} \rightarrow \texttt{Loc}$

$\mathcal{L}\ [\![\pi \vdash loc_i : intloc]\!] = loc_i$

$\mathcal{E}$: $\texttt{Expression} \rightarrow \texttt{Env} \rightarrow \texttt{Store} \rightarrow \texttt{Value}$

$\mathcal{E}\ [\![\pi \vdash \texttt{N} : intexp]\!]\ \texttt{e s} = \mathcal{N}\ [\![\pi \vdash \texttt{N} : int]\!]$

$\mathcal{E}\ [\![\pi \vdash \texttt{@L} : intexp]\!]\ \texttt{e s} = \texttt{lookup}(\mathcal{L}\ [\![\pi \vdash \texttt{L} : intloc]\!],\ \texttt{s})$

$\mathcal{E}\ [\![\pi \vdash \texttt{E}_1 + \texttt{E}_2 : intexp]\!]\ \texttt{e s} =$
$\quad \texttt{plus}(\mathcal{E}\ [\![\pi \vdash \texttt{E}_1 : intexp]\!]\ \texttt{e s},\ \mathcal{E}\ [\![\pi \vdash \texttt{E}_2 : intexp]\!]\ \texttt{e s})$

$\mathcal{E}\ [\![\pi \vdash \texttt{not E} : boolexp]\!]\ \texttt{e s} =$
$\quad \texttt{not}(\mathcal{E}\ [\![\pi \vdash \texttt{E} : boolexp]\!]\ \texttt{e s})$

$\mathcal{E}\ [\![\pi \vdash \texttt{E}_1 = \texttt{E}_2 : boolexp]\!]\ \texttt{e s} =$
$\quad \texttt{equalint}(\mathcal{E}\ [\![\pi \vdash \texttt{E}_1 : intexp]\!]\ \texttt{e s},\ \mathcal{E}\ [\![\pi \vdash \texttt{E}_2 : intexp]\!]$
$\quad \texttt{e s})$

$\mathcal{E}\ [\![\pi \vdash \texttt{E}_1 = \texttt{E}_2 : boolexp]\!]\ \texttt{e s} =$
$\quad \texttt{equalbool}(\mathcal{E}\ [\![\pi \vdash \texttt{E}_1 : boolexp]\!]\ \texttt{e s},\ \mathcal{E}[\![\pi \vdash$
$\quad \texttt{E}_2 : boolexp]\!]\ \texttt{e s})$

$\mathcal{E}\ [\![\texttt{I} : \tau exp]\!]\ \texttt{e s} = \texttt{f(s)}\ \texttt{where}\ \texttt{f} = \texttt{e(I)}$

Semantic rules, with function declarations, part 2

$\mathcal{C}$: Command $\rightarrow$ Env $\rightarrow$ Store $\rightarrow$ Store

$\mathcal{C}$ $[\![\pi \vdash$ L := E:$\mathit{comm}]\!]$ e s =
  update($\mathcal{L}$ $[\![\pi \vdash$ L:$\mathit{intloc}]\!]$, $\mathcal{E}$ $[\![\pi \vdash$ E:$\mathit{intexp}]\!]$ e s, s)

$\mathcal{C}$ $[\![\pi \vdash C_1 ; C_2 : \mathit{comm}]\!]$ e s =
  $\mathcal{C}$ $[\![\pi \vdash C_2 : \mathit{comm}]\!]$ e ($\mathcal{C}$ $[\![\pi \vdash C_1 : \mathit{comm}]\!]$ e s)

$\mathcal{C}$ $[\![\pi \vdash$ if E then C_1 else $C_2 : \mathit{comm}]\!]$ e s =
  if($\mathcal{E}$ $[\![\pi \vdash$ E:$\mathit{boolexp}]\!]$ e s,
    $\mathcal{C}$ $[\![\pi \vdash C_1 : \mathit{comm}]\!]$ e s,
    $\mathcal{C}$ $[\![\pi \vdash C_2 : \mathit{comm}]\!]$ e s)

$\mathcal{C}$ $[\![\pi \vdash$ while E do C:$\mathit{comm}]\!]$ e s = w(s)
  where w(s') =
    if($\mathcal{E}$ $[\![\pi \vdash$ E:$\mathit{boolexp}]\!]$ e s',
      w($\mathcal{C}$ $[\![\pi \vdash$ C:$\mathit{comm}]\!]$ e s'),
      s')

$\mathcal{C}$ $[\![\pi \vdash$ skip:$\mathit{comm}]\!]$ e s = s

Semantic rules, with function declarations, part 3

$\mathcal{P}$: Program $\rightarrow$ Store $\rightarrow$ Store

$\mathcal{P}$ $[\![$D in C:$\mathit{prog}]\!]$ s =
  let e = $\mathcal{D}$ $[\![\varnothing \vdash$ D:$\pi\mathit{dec}]\!]$ $\varnothing$ s in
    $\mathcal{C}$ $[\![\pi \vdash C_2 : \mathit{comm}]\!]$ e s

$\mathcal{D}$: Declaration $\rightarrow$ Env $\rightarrow$ Store $\rightarrow$ Env

$\mathcal{D}$ $[\![\pi \vdash$ fun I = E:{I:$\tau\mathit{exp}$}$]\!]$ e s = {(I,f)}
  where f(s') = $\mathcal{E}$ $[\![\pi \vdash$ E:$\tau\mathit{exp}]\!]$ e s'

$\mathcal{D}$ $[\![\pi \vdash D_1 , D_2 : (\pi_1 \cup \pi_2)\mathit{dec}]\!]$ e s =
  ($\mathcal{D}[\![\pi \vdash D_1 : \pi_1\mathit{dec}]\!]$ e s) $\cup$ ($\mathcal{D}$ $[\![\pi \vdash D_2 : \pi_2\mathit{dec}]\!]$ e s)

$\mathcal{D}$ $[\![\pi \vdash D_1 ; D_2 : (\pi_1 \cup \pi_2)\mathit{dec}]\!]$ e s =
  let e_1 = ($\mathcal{D}[\![\pi \vdash D_1 : \pi_1\mathit{dec}]\!]$ e s) in
  let e_2 = ($\mathcal{D}$ $[\![(\pi \cup \pi_1) \vdash D_2 : \pi_2\mathit{dec}]\!]$ ($e \cup e_1$) s) in
  $e_1 \cup e_2$