Abstract syntax, with function declarations

Syntactic domains:

```
P ∈ Program
D ∈ Declaration
C ∈ Command
E ∈ Expression
L ∈ Location
N ∈ Numeral
I ∈ Identifier
```

Syntactic rules:

```
P ::= D in C
D ::= fun I = E | D1, D2 | D1; D2
C ::= ...
E ::= ... | I
L ::= ...
N ::= ...
```


Type system, with function declarations, part 1

For `Program`:

$$\frac{\emptyset \vdash D:\pi\,dec \qquad \pi \vdash C:comm}{D \text{ in } C:prog}$$

For `Declaration`:

$$\frac{\pi \vdash E:\tau\,exp}{\pi \vdash \text{fun } I = E: \{I:\tau\,exp\}\ dec}$$

$$\frac{\pi \vdash D_1:\pi_1 dec \qquad \pi \vdash D_2:\pi_2 dec}{\pi \vdash D_1, D_2: (\pi_1 \dot{\cup} \pi_2)\ dec}$$

$$\frac{\pi \vdash D_1:\pi_1 dec \qquad (\pi \dot{\cup} \pi_1) \vdash D_2:\pi_2 dec}{\pi \vdash D_1;D_2: (\pi_1 \cup \pi_2)\ dec}$$

$$\left(\begin{array}{l} \pi_1 \dot{\cup} \pi_2 \equiv \pi_1 \cup \pi_2, \text{ if } \{I \mid (I:\theta) \in \pi_1\} \cap \{I \mid (I:\theta) \in \pi_2\} = \emptyset \\ \\ \qquad\qquad \text{undefined, otherwise} \end{array}\right)$$

Type system, with function declarations, part 2

For `Numeral`:

$$\pi \vdash n{:}int$$

For `Location`:

$$\pi \vdash loc_i{:}intloc$$

For `Expression`:

$$\frac{\pi \vdash N{:}int}{\pi \vdash N{:}intexp} \qquad \frac{\pi \vdash L{:}intloc}{\pi \vdash @L{:}intexp}$$

$$\frac{\pi \vdash E_1{:}intexp \qquad \pi \vdash E_2{:}intexp}{\pi \vdash E_1 + E_2{:}intexp} \qquad \frac{\pi \vdash E{:}boolexp}{\pi \vdash \text{not } E{:}boolexp}$$

$$\frac{\pi \vdash E_1{:}\tau exp \qquad \pi \vdash E_2{:}\tau exp}{\pi \vdash E_1 = E_2{:}boolexp} \quad \text{if } \tau \in \{int, bool\}$$

$$\pi \vdash I{:}\tau exp \quad \text{if } (I{:}\tau exp) \in \pi$$

For `Command`:

(just pass π around as with expressions)