CSE-505: Programming Languages

Lecture 19 — Type-and-Effect Systems

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Type-and-effect systems

New topic: An elegant framework to extend type systems to track “things that may happen” (effects) during evaluation

Plain-old type systems have judgments like $\Gamma \vdash e : \tau$ to mean:

- $e$ won’t get stuck
- If $e$ produces a value, that value has type $\tau$

Adding *effects* reuses the “plumbing” of typing rules to compute something about “how $e$ executes”

- There are many things we may want to conservatively approximate
  - Example: What exceptions might get thrown
- All effect systems are very similar, especially treatment of functions
  - Example: All values have no effect since their “computation” does nothing
First a type system

(In this example, exceptions raise constant strings $s$)

$$
\tau ::= \text{bool} \mid \tau \rightarrow \tau \mid \tau \ast \tau
$$

$$
e ::= x \mid \text{true} \mid \text{false} \mid \lambda x. \ e \mid e \ e \mid (e, e) \mid e.1 \mid e.2
\mid \text{if } e \ e \ e \mid \text{raise } s \mid \text{try } e \ \text{handle } s \ e
$$

$$
\Gamma \vdash e : \tau
\quad \text{if } \Gamma \vdash x : \Gamma(x)
\quad \text{if } \Gamma \vdash \text{true} : \text{bool}
\quad \text{if } \Gamma \vdash \text{false} : \text{bool}
\quad \Gamma, x : \tau_1 \vdash e : \tau_2
\quad \Gamma \vdash \lambda x. \ e : \tau_1 \rightarrow \tau_2
\quad \Gamma \vdash e_1 : \tau_2 \rightarrow \tau_1
\quad \Gamma \vdash e_2 : \tau_2
\quad \Gamma \vdash e_1 \ e_2 : \tau_1
\quad \Gamma \vdash (e_1, e_2) : \tau_1 \ast \tau_2
\quad \Gamma \vdash e : \tau_1 \ast \tau_2
\quad \Gamma \vdash e : \tau_1 \ast \tau_2
\quad \Gamma \vdash e_1 : \tau_1
\quad \Gamma \vdash e_2 : \tau_2
\quad \Gamma \vdash e_1 : \text{bool}
\quad \Gamma \vdash e_2 : \tau
\quad \Gamma \vdash e_3 : \tau
\quad \Gamma \vdash \text{if } e_1 \ e_2 \ e_3 : \tau
\quad \Gamma \vdash \text{raise } s : \tau
\quad \Gamma \vdash \text{try } e_1 \ \text{handle } s \ e_2 : \tau
$$
Add effects

\[ \epsilon ::= \text{...sets of strings...} \]

\[ \tau ::= \text{bool} \mid \tau \rightarrow \tau \mid \tau \ast \tau \]

\[ e ::= x \mid \text{true} \mid \text{false} \mid \lambda x. \ e \mid e \ e \mid (e, e) \mid \text{e.1} \mid \text{e.2} \]

\[ \Gamma \vdash e : \tau; \epsilon \]

\[ \Gamma \vdash x : \Gamma(x); \emptyset \]

\[ \Gamma \vdash \text{true} : \text{bool}; \emptyset \]

\[ \Gamma \vdash \text{false} : \text{bool}; \emptyset \]

\[ \Gamma, x : \tau_1 \vdash e : \tau_2; \epsilon \]

\[ \Gamma \vdash \lambda x. \ e : \tau_1 \rightarrow \tau_2; \emptyset \]

\[ \Gamma \vdash e_1 : \tau_2 \rightarrow \tau_1; \epsilon_1 \]

\[ \Gamma \vdash e_2 : \tau_2; \epsilon_2 \]

\[ \Gamma \vdash e_1 \ e_2 : \tau_1; \epsilon_1 \cup \epsilon_2 \cup \epsilon_3 \]

\[ \Gamma \vdash e_1 : \tau_1 \rightarrow \tau_2; \epsilon \]

\[ \Gamma \vdash e_2 : \tau_2; \epsilon \]

\[ \Gamma \vdash (e_1, e_2) : \tau_1 \ast \tau_2; \epsilon_1 \cup \epsilon_2 \]

\[ \Gamma \vdash e.1 : \tau_1; \epsilon \]

\[ \Gamma \vdash e.2 : \tau_2; \epsilon \]

\[ \Gamma \vdash \text{if } e_1 \ e_2 \ e_3 : \tau; \epsilon_1 \cup \epsilon_2 \cup \epsilon_3 \]

\[ \Gamma \vdash \text{raise } s : \tau; \{s\} \]

\[ \Gamma \vdash \text{try } e_1 \ \text{handle } s \ e_2 : \tau; (\epsilon_1 - \{s\}) \cup \epsilon_2 \]
Key facts

Soundness: If $\vdash e : \tau; \epsilon$ and $e$ raises uncaught exception $s$, then $s \in \epsilon$

- Corollary to Preservation and Progress (once you define the operational semantics for exceptions)

All effect systems work this way:

- Values effectless
- Functions have *latent effects*
- Conservative due to control-flow (if and try/handle)
- Often some way to *mask effects* (here, catch an exception)

Only a couple rules special to this effect system

- Also, not always sets and $\cup$
More general rules

Every effect system also substantially more expressive via appropriate subsumption:

- Typing rule for subeffecting (also useful for Preservation)
- Subtyping of function types is covariant in latent effects

\[
\begin{align*}
\Gamma \vdash \tau : e; \epsilon & \quad \epsilon \subseteq \epsilon' \\
\Gamma \vdash \tau : e; \epsilon'
\end{align*}
\]

\[
\begin{align*}
\tau_3 & \leq \tau_1 \\
\tau_2 & \leq \tau_4 \\
\epsilon & \subseteq \epsilon'
\end{align*}
\]

\[
\begin{align*}
\tau_1 \rightarrow \tau_2 & \leq \tau_3 \rightarrow \tau_4
\end{align*}
\]

Not shown: Also want effect polymorphism (type variables ranging over effects) for higher-order functions like map
Other examples

- Definitely terminates (true) or possibly diverges (false)
  - Give `fix e` effect `false`
  - Give values effect `true`
  - Treat `∪` as `and`
  - No change to rules for functions, pairs, conditionals, etc.

- What type casts might occur
- Are certain variables always accessed in critical sections
- Does code obey a locking protocol
- Does code only access memory regions that haven’t been deallocated
  - ...

Really a general way to lift static analysis to higher-order functions

- Key is recognizing “from a mile away” when an effect system is the right tool