Where are we

Done:
- Formal definition of evaluation contexts and first-class continuations
- Continuation-passing style as a programming idiom
- The CPS transform

Now:
- Implement an efficient lambda-calculus interpreter using little more than malloc and a single while-loop
  - Explicit evaluation contexts (i.e., continuations) is essential
  - Key novelty is maintaining the current context incrementally
  - letcc and throw can be $O(1)$ operations (homework problem)

See the code

See lec14code.ml for four interpreters where each is:
- More efficient than the previous one and relies on less from the meta-language
- Close enough to the previous one that equivalence among them is tractable to prove

The interpreters:
1. Plain-old small-step with substitution
2. Evaluation contexts, re-decomposing at each step
3. Incremental decomposition, made efficient by representing evaluation contexts (i.e., continuations) as a linked list with “shallow end” of the stack at the beginning of the list
4. Replacing substitution with environments

The last interpreter is trivial to port to assembly or C

Example

Small-step (first interpreter):

Decomposition (second interpreter):

$E = \lambda a. a$
$H = \lambda b. b$
$A = \lambda c. c$
$\lambda d. d$
$\lambda e. e$

$E = \lambda a. a$
$H = \lambda b. b$
$A = \lambda c. c$
$\lambda d. d$
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Example

Decomposition (second interpreter):

\[ E = \lambda.a \quad e = \lambda.b \quad A = \lambda.c \quad H = \lambda.d \quad A = \lambda.e \]

Decomposition rewritten with linked list (hole implicit at front):

\[ c = L(A(\lambda.d, \lambda.e)) :: R(\lambda.a) :: [] \quad e = A(\lambda.b, \lambda.c) \]

Some loop iterations of third interpreter:

\[ \quad \]

Fourth interpreter: replace substitution with environment/closures

The end result

The last interpreter needs just:

- A loop
- Lists for contexts and environments
- Tag tests

Moreover:

- Function calls execute in \( O(1) \) time
- Variable look-ups don’t, but that’s fixable
  - (e.g., de Bruijn indices and arrays for environments)
- Other operations, including pairs, conditionals, letcc, and throw also all work in \( O(1) \) time
  - Need new kinds of contexts and values
  - Left as a homework exercise as a way to understand the code

Making evaluation contexts explicit data structures was key