# Graduate Programming Languages: Reference Sheet for Lecture 3

$$
\begin{array}{rcl}
s & ::= & \mathsf{skip} \mid x := e \mid s; s \mid \mathsf{if}\ e\ s\ s \mid \mathsf{while}\ e\ s \\
e & ::= & c \mid x \mid e + e \mid e * e \\
(c & \in & \{\ldots, -2, -1, 0, 1, 2, \ldots\}) \\
(x & \in & \{\mathtt{x}_1, \mathtt{x}_2, \ldots, \mathtt{y}_1, \mathtt{y}_2, \ldots, \mathtt{z}_1, \mathtt{z}_2, \ldots, \ldots\})
\end{array}
$$

- Semantics for expressions
  1. Informal idea; the need for *heaps*
  2. Definition of heaps
  3. The evaluation *judgment* (a relation form)
  4. The evaluation *inference rules* (the relation definition)
  5. Using inference rules
     - *Derivation trees* as interpreters
     - Or as *proofs* about expressions
  6. *Metatheory*: Proofs about the semantics
- Then semantics for statements
  - ...

$$
H(x) = \begin{cases} c & \text{if} \quad H = H', x \mapsto c \\ H'(x) & \text{if} \quad H = H', y \mapsto c' \\ 0 & \text{if} \quad H = \cdot \end{cases}
$$

$\boxed{H\ ;\ e \Downarrow c}$

$$
\text{CONST}\ \frac{}{H\ ;\ c \Downarrow c} \qquad \text{VAR}\ \frac{}{H\ ;\ x \Downarrow H(x)} \qquad \text{ADD}\ \frac{H\ ;\ e_1 \Downarrow c_1 \qquad H\ ;\ e_2 \Downarrow c_2}{H\ ;\ e_1 + e_2 \Downarrow c_1 + c_2} \qquad \text{MULT}\ \frac{H\ ;\ e_1 \Downarrow c_1 \qquad H\ ;\ e_2 \Downarrow c_2}{H\ ;\ e_1 * e_2 \Downarrow c_1 * c_2}
$$

$\boxed{H_1\ ;\ s_1 \to H_2\ ;\ s_2}$

$$
\text{ASSIGN}\ \frac{H\ ;\ e \Downarrow c}{H\ ;\ x := e \to H, x \mapsto c\ ;\ \mathsf{skip}} \qquad \text{SEQ1}\ \frac{}{H\ ;\ \mathsf{skip}; s \to H\ ;\ s} \qquad \text{SEQ2}\ \frac{H\ ;\ s_1 \to H'\ ;\ s_1'}{H\ ;\ s_1; s_2 \to H'\ ;\ s_1'; s_2}
$$

$$
\text{IF1}\ \frac{H\ ;\ e \Downarrow c \qquad c > 0}{H\ ;\ \mathsf{if}\ e\ s_1\ s_2 \to H\ ;\ s_1} \qquad \text{IF2}\ \frac{H\ ;\ e \Downarrow c \qquad c \leq 0}{H\ ;\ \mathsf{if}\ e\ s_1\ s_2 \to H\ ;\ s_2} \qquad \text{WHILE}\ \frac{}{H\ ;\ \mathsf{while}\ e\ s \to H\ ;\ \mathsf{if}\ e\ (s; \mathsf{while}\ e\ s)\ \mathsf{skip}}
$$