In this assignment, you are going to add a record type to the simply typed lambda calculus, using the records of the previous assignment. For example, the type of

\[ \{l_1 = (\lambda x : \text{Bool}.x), l_2 = \text{true}, l_3 = (\lambda x : \text{Bool} \rightarrow \text{Bool}.(x \text{ true})) \}\]

is

\[ \{l_1 : \text{Bool} \rightarrow \text{Bool}, l_2 : \text{Bool}, l_3 : (\text{Bool} \rightarrow \text{Bool}) \rightarrow \text{Bool} \}\]

The new syntax of types is as follows:

\[
T \ ::= \ldots \\
\{l_1 : T_1, \ldots, l_n : T_n\} \quad n \geq 1
\]

1 Typing Rules

Show the new typing rules involving records.

2 Type Soundness

In this question, you will prove the correspondence between the typing rules of the previous question and the operational semantics for records. For the operational semantics, you may either use the rules you created in the previous homework assignment or the rules given in the sample solution. Please specify which rules are being used.

a. State (no proof necessary) any new parts of the Canonical Forms lemma that are necessary in the proofs below.

b. Add the relevant cases to the proof of the Progress theorem:

Theorem (Progress): If \( \vdash e : T \), then either \( e \) is a value or there exists \( e' \) such that \( e \rightarrow e' \).

c. Add the relevant cases to the proof of the Type Preservation theorem:

Theorem (Type Preservation): If \( \Gamma \vdash e : T \) and \( e \rightarrow e' \), then \( \Gamma \vdash e' : T \).