Data flow analysis

Want to compute some info about program

- at **program points**
- to determine opportunities for improving transformations

Can model data flow analysis as solving a system of constraints

- each node in CFG places a constraint relating info at predecessor and successor points
- solution to constraints is result of analysis

Solution must be **safe**/**sound**, can be **conservative**

Key issues:

- how to know if constraint system defines the analysis correctly?
- how to represent info efficiently?
- how to represent & solve constraints efficiently?
  - how long does constraint solving take? finite time?
- what if multiple solutions are possible?
- how to synchronize transformations with analysis?

Example: reaching definitions

For each program point,
want to compute set of definitions (statements) that *may reach* that point

- reach: are the last definition of some variable
- can use info to build def-use chains, do constant propagation, etc.

Info ≡ set of *var*→*rtl* bindings

E.g.:
$\{x \to s_1, y \to s_5, y \to s_8\}$

Safety rule:
can have more bindings than the "true" answer, but can't miss any

Initial conditions: {}

- or maybe bindings for formals

Reaching definitions, continued

Main constraints:

- a simple assignment removes any old reaching defs for the lhs and replaces them with this stmt:

  s: `x := ...`:
  $\text{info}_{succ} = \text{info}_{pred} - \{x \to *\} \cup \{x \to s\}$
- a pointer assignment may modify anything, but doesn't definitely replace anything

  s: `*p := ...`:
  $\text{info}_{succ} = \text{info}_{pred} \cup \{* \to s\}$
- other statements: do nothing
  $\text{info}_{succ} = \text{info}_{pred}$

At branches:

- pass through reaching defs to both successors

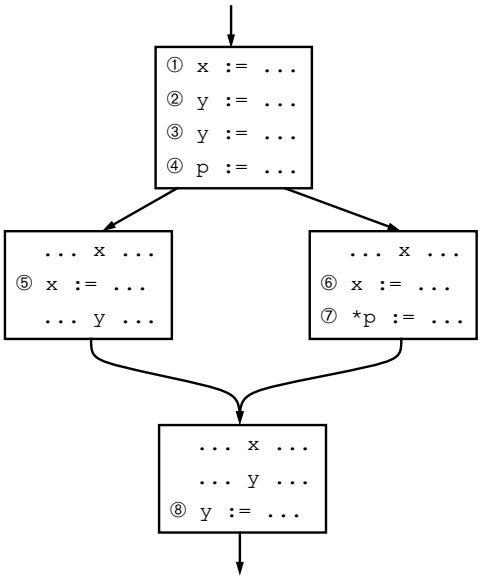
  $\text{info}_{succ[i]} = \text{info}_{pred}$

At merges:

- take the union of all incoming reaching defs (we don't know which path is being taken at run-time)

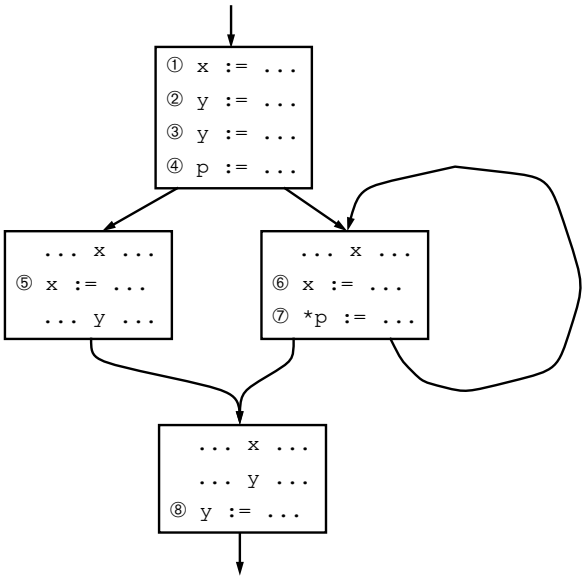
  $\text{info}_{succ} = \bigcup_i \text{info}_{pred[i]}$

Example



Simple constraint-solving algorithm:
traverse instructions in forward topological order, computing successor info from predecessor info

Another example



Topological order not defined!

Loops

If CFG has a loop, data flow constraints are recursively defined:

$info_{loop\text{-}head} = info_{loop\text{-}entry} \cup info_{back\text{-}edge}$

$info_{back\text{-}edge} = ... info_{loop\text{-}head} ...$

Summarizing set of constraints in loop body as *F*:

$info_{loop\text{-}head} = F(info_{loop\text{-}entry})$

Legal solution to constraints is a **fixpoint** of *F*

- want **least**/**greatest** fixpoint, corresponding to most precise answer

How to find fixpoint?

- for restricted CFGs can use specialized methods
  - e.g. **interval analysis** for **reducible** CFGs
- for arbitrary CFGs, can use **iterative** approximation

Loop terminology

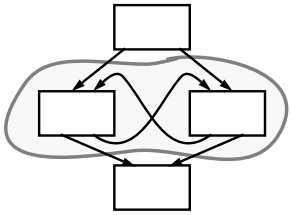
loop: strongly-connected component in CFG with single entry

loop entry edge: source not in loop, target in loop
loop exit edge: the reverse
back edge: target is loop head node

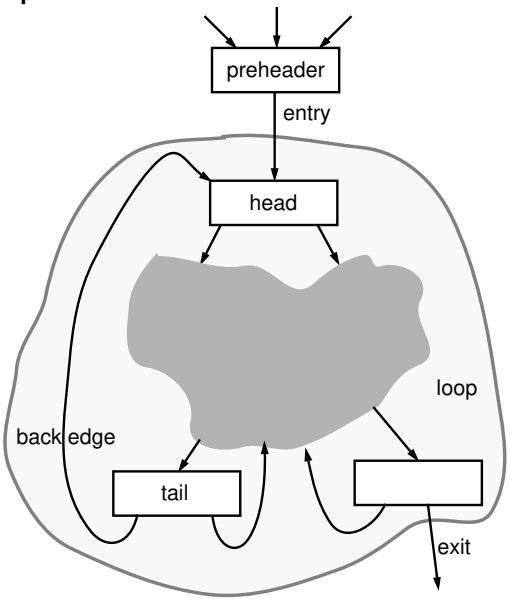
loop head node: target of loop entry edge
loop tail node: source of back edge
loop preheader node:
single node that's source of loop entry edge

nested loop: loop that's a proper subset of another loop

reducible flow graph: all SCC's have single entry



Example



Iterative data flow analysis

1. Start with initial guess of info at end of loop:
   $info_{back\text{-}edge}$ = *guess*
   $info_{loop\text{-}head} = info_{loop\text{-}entry} \cup info_{back\text{-}edge}$

2. Solve equations for loop:
   $info_{back\text{-}edge}' = F(info_{loop\text{-}head})$
   $info_{loop\text{-}head}' = info_{loop\text{-}entry} \cup info_{back\text{-}edge}'$

3. Test if found fixpoint:
   $info_{loop\text{-}head}' = info_{loop\text{-}head}$?

   A. if so, then done

   B. if not, then adopt result as (better) guess and repeat:

   $info_{back\text{-}edge}'' = F(info_{loop\text{-}head}')$
   $info_{loop\text{-}head}'' = info_{loop\text{-}entry} \cup info_{back\text{-}edge}''$
   $info_{loop\text{-}head}'' = info_{loop\text{-}head}'$?
   ...

When does iterating work?

1. need to be able to make an initial guess
2. $info^{n+1}$ must be closer to the fixpoint than $info^n$
   (constraints must be **monotonic**)
3. must eventually reach the fixpoint in a finite # of iterations
   (info must be drawn from a **finite-height domain**)

To reach best fixpoint, initial guess for loop head should be **optimistic**

- standard choice: $info_{loop\text{-}head} = info_{loop\text{-}entry}$

To speed iterative analysis, want to test guess ASAP

- ideal: solve constraints along shortest path from loop head to loop tail
- practical: don't solve any constraints outside of loop until fixpoint is reached

Direction of dataflow analysis

In what order are constraints solved, in general?

Constraints are declarative, not directional/procedural, so may require mixing forward & backward solving, or other more global solution methods

But often constraints can be solved by (directional) propagation & iteration

- may be **forward** or **backward** propagation of info
- topological traversals of acyclic graph components are best

Constraints of simple statements called **flow functions**

- often written as functions on input info to compute output

$RD_{s:\,x := \ldots}(in) = in - \{x \rightarrow *\} \cup \{x \rightarrow s\}$
$RD_{s:\,*p := \ldots}(in) = in \cup \{* \rightarrow s\}$

GEN and KILL sets

Can often think of flow functions in terms of each's GEN set and KILL set

- GEN = new information added
- KILL = old information removed

Then
$F_{instr}(in) = in - KILL_{instr} \cup GEN_{instr}$

E.g., for reaching defs:
$RD_{s:\,x := \ldots}(in) = in - \{x \rightarrow *\} \cup \{x \rightarrow s\}$
$RD_{s:\,*p := \ldots}(in) = in \cup \{* \rightarrow s\}$

Bit vectors

Can sometimes represent info/KILL/GEN sets as **bit vectors**

- if can express abstractly as set of things
  (e.g. statements, vars),
  each thing getting a bit position

Each flow function is then represented by a pair of bit vectors,
and bit vector operations are used for set difference & union

Can summarize effect of basic block as single flow function
by combining KILL & GEN sets for flow functions in block,
for faster iteration

- perhaps can do this even if not using bit vectors