Compiling For Parallelism

Input: sequential code w/ loops

Output: parallel code

- vector machine
- multi-processor
- ...

Step 1: analyze loop nests to determine interdependence of stmts

Step 2: transform loops to minimize dependences, overhead

Step 3: generate parallel code

- e.g. if loop iterations are independent, can run in parallel

Dependence

Flow/true dependence: read-after-write

- $s_1 \; \delta \; s_2$

Antidependence: write-after-read

- $s_1 \; \bar{\delta} \; s_2$

Output dependence: write-after-write

- $s_1 \; \delta^o \; s_2$

Control dependence:

- $s_1 \; \delta^c \; s_2$

Easy to see in straight-line code

Harder in loops

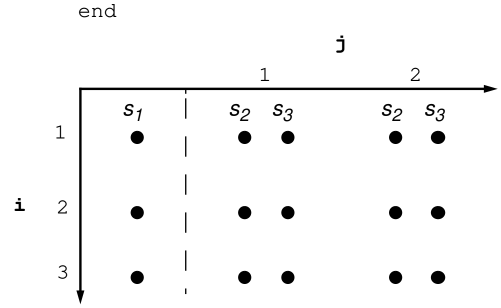
Iteration space

Think about loop dependences by “unrolling” loop

Represent each dynamic iteration in **iteration space**

- N+1-dimensional space if N-nested loops
- each point is an execution of a stmt in loop
- plot dependence between iteration space points

```
    for i := 1 to 3
s1:   a[i] := b[i]
      for j := 1 to 2
s2:     c[i,j] := a[i] + b[j]
s3:     d[i,j] := d[i,j+1] + d[i,j-1]
      end
    end
```



Summarizing dependences in iteration space

Summarize dependence between pts in iteration space

- $s_1 \; \delta \; s_2$ iff some execution of $s_1 \; \delta$ some execution of s_2

Somewhat more precise summary: add direction to dependence

- $s_1 \; \delta_= \; s_2$ iff each execution of $s_1 \; \delta$ an execution of s_2 *in the same iteration*
- $s_1 \; \delta_< \; s_2$ iff each execution of $s_1 \; \delta$ an execution of s_2 *in a previous iteration*
  - **loop-carried dependence**
- $\delta_>$?

Even more precise summary: add distance to dependence

- $s_1 \; \delta_i \; s_2$ iff each execution of $s_1 \; \delta$ an execution of s_2 *i iterations before*
- $\delta_= = \delta_0$
- $\delta_<$ summarizes all possible δ_n, n>0

Examples

```
for i := 2 to 4
  a[i] := a[i-1] * 2
  b[i-1] := a[i] + b[i]
end
```

```
for i := 2 to 4
  a[i+1] := b[i]
  x := a[i] * a[i-1]
  b[i] := x
  b[i+2] := x / 2
end
```


Loop nests

If multiple loops, use multiple direction subscripts

- e.g., $s_1\ \delta_{=,<}\ s_2$
- outermost iteration direction/distance first
- subscripts: **dependence vectors**

```
for i := 1 to 2
  for j := 2 to 5
    a[i,j] := a[i,j-1] + b[i,j]
    c[i,j] := a[i,j] + d[i+1,j-1]
    d[i,j] := 1.0
  end
end
```

Legal dependence vectors are **lexicographically non-negative**

Testing array reference dependence

Simple example: GCD test

- many more sophisticated tests have been developed

Given two array references of the forms
`a[c*I+j]` and `a[d*I+k]`,
where `c`,`d`,`j`,`k` are constants and `I` is a loop index var

For dependence to exist then there must exist some `I` s.t.
`c*I+j = d*I+k`

Can only exist if GCD(`c`,`d`) divides `k-j` evenly

- if doesn't divide, then references independent

Can improve test by taking loop bounds into account

Compiling for a multiprocessor

Example target architecture:

- lots of powerful processors
- costly communication, synchronization between processors

Exploit hardware best by creating large independent tasks

For loops,
want loops with many independent iterations
with long-running bodies of equal duration

Example

```
for i := 1 to N
  a[i,1] := 1.0
  for j := 2 to N
    a[i,j] := a[i,j-1] + 1
    b[i] := b[i] + a[i,j]
  end
end
```


Another example

```
for i := 2 to N
  for j := 1 to N
    a[i-1,j] := a[i,j] + b[i,j]
    b[i,j] := b[i-1,j] * 2
  end
end
```


Loop interchange

Sometimes can reorder loop nest to eliminate loop-carried dependences on outermost loop

When is this legal?
⇒ when data flow relationships are not changed

A test:

- reorder the entries in the dependences' distance vectors
- check that they remain lexicographically non-negative

Example

```
for i := 2 to N
  for j := 1 to N
    a[i-1,j] := a[i,j] + b[i,j]
    b[i,j] := b[i-1,j] * 2
  end
end
```

Another example

```
for i := 1 to 3
  for j := 2 to 5
    a[i+1,j-1] := a[i,j] + b[i,j]
  end
end
```