Optimizing Loops

Some goals:

- compiling for parallel machines
- compiling for memory hierarchies

Compiling loops for parallelism

Input: sequential code w/ loops

Output: parallel code

- vector machine
- multi-processor
- ...

Step 1: analyze loop nests to determine interdependence of stmts

Step 2: transform loops to minimize dependences, overhead

Step 3: generate parallel code

- e.g. if loop iterations are independent, can run in parallel

Dependence

Flow/true dependence: read-after-write

- $s_1 \, \delta \, s_2$

Antidependence: write-after-read

- $s_1 \, \bar{\delta} \, s_2$

Output dependence: write-after-write

- $s_1 \, \delta^o \, s_2$

Control dependence:

- $s_1 \, \delta^c \, s_2$

Easy to see in straight-line code

Harder in loops

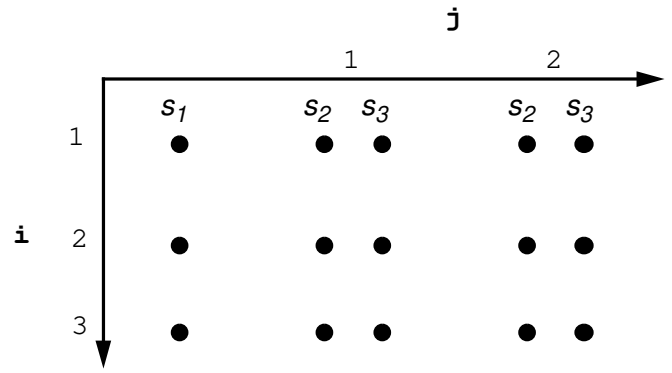
Iteration space

Think about loop dependences by “unrolling” loop

Represent each dynamic iteration in **iteration space**

- N+1-dimensional space if N-nested loops
- each point is an execution of a stmt in loop
- plot dependence between iteration space points

```
    for i := 1 to 3
s1:   a[i] := b[i]
      for j := 1 to 2
s2:     c[i,j] := a[i] + b[j]
s3:     d[i,j] := d[i,j+1] + d[i,j-1]
      end
    end
```



Summarizing dependences in iteration space

Summarize dependence between pts in iteration space

- $s_1 \, \delta \, s_2$ iff some execution of $s_1 \, \delta$ some execution of s_2

Somewhat more precise summary: add direction to dependence

- $s_1 \, \delta_= \, s_2$ iff each execution of $s_1 \, \delta$ an execution of s_2 *in the same iteration*
- $s_1 \, \delta_< \, s_2$ iff each execution of $s_1 \, \delta$ an execution of s_2 *in a previous iteration*
  - **loop carried dependence**

Even more precise summary: add distance to dependence

- $s_1 \, \delta_i \, s_2$ iff each execution of $s_1 \, \delta$ an execution of s_2 *i iterations before*
- $\delta_= = \delta_0$

Examples

```
for i := 2 to 4
  a[i] := a[i-1] * 2
  b[i-1] := a[i] + b[i]
end
```

```
for i := 2 to 4
  a[i+1] := b[i]
  x := a[i] * a[i-1]
  b[i] := x
  b[i+2] := x / 2
end
```

Loop nests

If multiple loops, use multiple direction subscripts

- e.g., $s_1 \, \delta_{=,<} \, s_2$
- outermost iteration direction/distance first
- subscripts: **dependence vectors**

```
for i := 1 to 2
  for j := 2 to 5
    a[i,j] := a[i,j-1] + b[i,j]
    c[i,j] := a[i,j] + d[i+1,j-1]
    d[i,j] := 1.0
  end
end
```


Testing array reference dependence

GCD test:

Given two array references of the forms

`a[c*I+j]` and `a[d*I+k]`,

where `c`,`d`,`j`,`k` are constants and `I` is a loop index var

For dependence to exist then there must exist some `I` s.t.

`c*I+j = d*I+k`

Can only exist if GCD(`c`,`d`) divides `k-j` evenly

- if doesn't divide, then references independent

Can improve test by taking loop bounds into account

Compiling for a multiprocessor

Example target architecture:

- lots of powerful processors
- costly communication, synchronization between processors

Exploit hardware best by creating large independent tasks

For loops,
want loops with many independent iterations
with long-running bodies of equal duration

Example

```
for i := 1 to N
  a[i,1] := 1.0
  for j := 2 to N
    a[i,j] := a[i,j-1] + 1
    b[i] := b[i] + a[i,j]
  end
end
```

Another example

```
for i := 2 to N
  for j := 1 to N
    a[i-1,j] := a[i,j] + b[i,j]
    b[i,j] := b[i-1,j] * 2
  end
end
```


Loop interchange

Sometimes can transform loop to get the right one on the outside

- reorder loop nest

When is this legal?

$\Rightarrow$ when data flow relationships are not changed

A test:

- reorder the entries in the dependences' distance vectors
- check that they remain **lexicographically non-negative**

Example

```
for i := 2 to N
  for j := 1 to N
    a[i-1,j] := a[i,j] + b[i,j]
    b[i,j] := b[i-1,j] * 2
  end
end
```


Another example

```
for i := 1 to 3
  for j := 2 to 5
    a[i+1,j-1] := a[i,j] + b[i,j]
  end
end
```

Optimizing loops for the memory hierarchy

A typical loop walks over lots of data (long arrays)

Enclosing loops lead to repeated traversals of same data

Want to restructure loops to revisit data while it's in cheap memory

Example: matrix multiply

```
for i := 1 to N
  for j := 1 to N
    for k := 1 to N
      c[i,j] += a[i,k] * b[k,j]
    end
  end
end
```

`a[i,*]` visited repeatedly, but usually flushed from cache

`b[*,j]` similar, though less extreme

Blocking

A.k.a. **tiling**

Restructure loop so that it revisits a chunk that fits in registers/cache/page before moving on

```
for k' := 1 to N by ChunkSize
  for i := 1 to N
    for j := 1 to N
      for k := k' to k'+ChunkSize
        c[i,j] += a[i,k] * b[j,k]
      end
    end
  end
end
```

Implement transformation with combination of **strip mining** and **loop interchange**

Strip mining

Strip mining: turn single loop into 2 loops

- inner loop up to some chunk size
- outer loop repeats chunks till done

Before:

```
for k := 1 to N
  c[i,j] += a[i,k] * b[k,j]
end
```

After:

```
for k' := 1 to N by ChunkSize
  for k := k' to k'+ChunkSize
    c[i,j] += a[i,k] * b[k,j]
  end
end
```


Loop interchange

Apply loop interchange to move outer strip-mined outside of enclosing loop(s)

Before:

```
for i := 1 to N
  for j := 1 to N
    for k' := 1 to N by ChunkSize
      for k := k' to k'+ChunkSize
        c[i,j] += a[i,k] * b[j,k]
      end
    end
  end
end
```

After:

```
for k' := 1 to N by ChunkSize
  for i := 1 to N
    for j := 1 to N
      for k := k' to k'+ChunkSize
        c[i,j] += a[i,k] * b[j,k]
      end
    end
  end
end
```

Do again

Block the `j` loop, too

```
for k' := 1 to N by ChunkSize
  for j' := 1 to N by ChunkSize
    for i := 1 to N
      for j := j' to j'+ChunkSize
        for k := k' to k'+ChunkSize
          c[i,j] += a[i,k] * b[j,k]
        end
      end
    end
  end
end
```


Blocking automatically

[Wolf & Lam 91]

Analyze the loop to identify reuse

- spatial reuse
- temporal reuse

Direct blocking to maximize these kinds of reuse

Results: good speed-up curves on LU decomposition, SOR

- some hand-massaging of generated code included