CSE 493s/599s
Lecture 17.

Sewoong Oh

Lecture notes

- These lecture notes are based on other courses in LLMs, including
  - CSE493S/599S at UW by Ludwig Schmidt: https://mlfoundations.github.io/advancedml-sp23/
  - EE-628 at EPFL by Volkan Cevher: https://www.epfl.ch/labs/lions/teaching/ee-628-training-large-language-models/ee-628-slides-2025/
  - ECE381V Generative Models at UT Austin by Sujay Sanghavi
  - and various papers and blogs cited at the end of the slide deck

Outline

- Language models
- General LLM framework
  - Token processing
  - Sequence mixing
  - Prediction
- Prompting techniques at inference time
  - In-context learning
  - Chain-of-thought prompting
- Fine-tuning
- Alignment

Parameter Efficient Fine-Tuning (PEFT)

Fine-tuning

- When given a small data to adopt to a new task domain, there are two ways to use that data
    - In-context learning: put the examples in the prompt
    - Fine-tuning: optimize (part of) model weights
- Supervised Fine-Tuning (SFT) is a common practice to adapt a given base LM to the target domain of interest, given labeled fine-tuning samples.
    - e.g., sentiment analysis, or named-entity classification.
- With the increasing scale of LLMs, oftentimes full-scale fine-tuning of the model weights is prohibitively expensive.
- **Parameter Efficient Fine-Tuning** (PEFT) corresponds to a family of approaches that freezes most of the parameters of the original pertained network and only trains a small subset of parameters.

Low Rank Adaptation (LoRA)

- Given a layer of pretained weight matrix W, fine-tuning results in an updated weight matrix

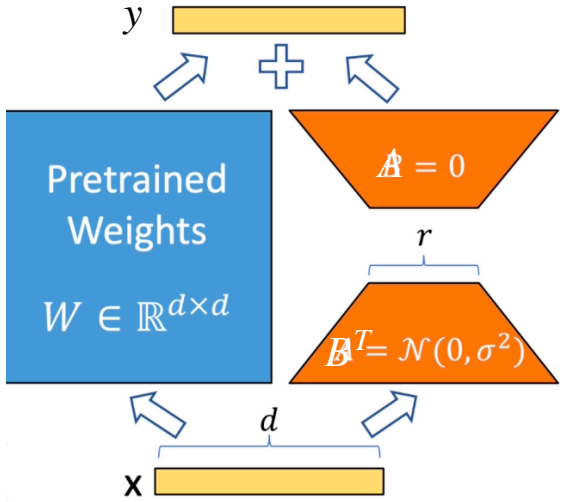
$$W' \leftarrow \underbrace{W}_{\text{pre-trained weight}} + \underbrace{\Delta W}_{\text{fine-tuned update}} .$$

- Instead of a unrestricted, full-rank update ΔW, LoRA parametrizes the update to be low-rank using a bi-linear form:

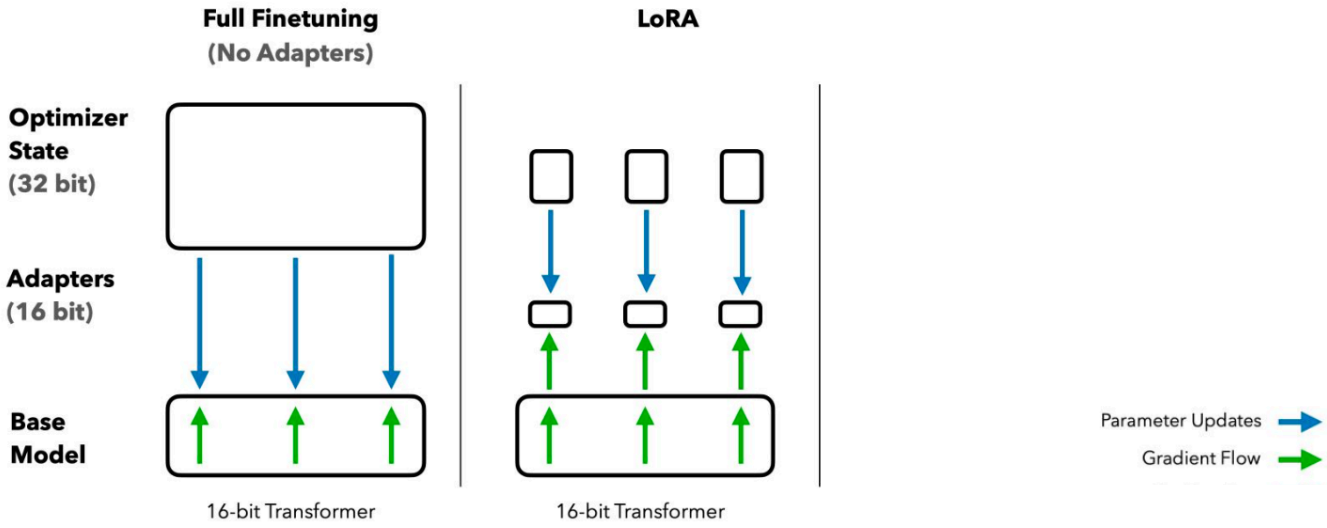
$$\Delta W = AB^T \text{ such that}$$

$$W' \leftarrow \underbrace{W}_{\text{this is frozen}} + \underbrace{AB^T}_{\text{this is optimized}} ,$$

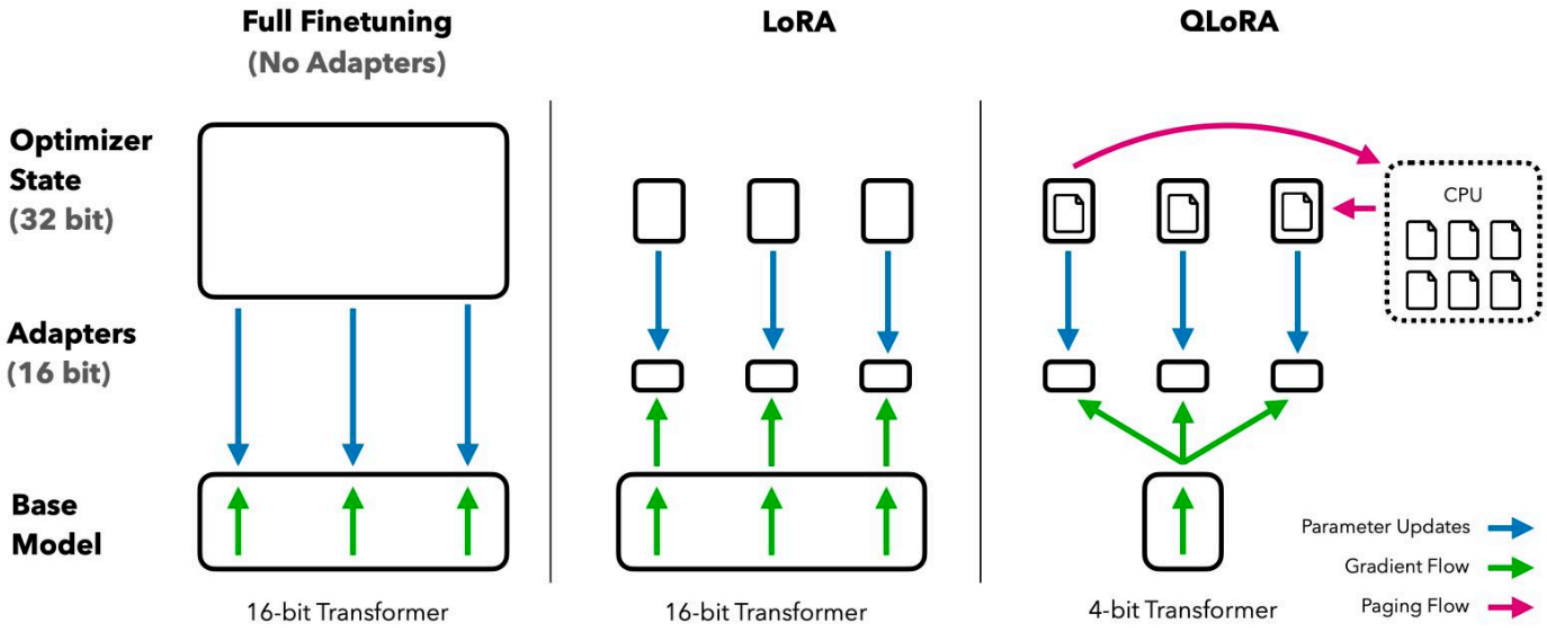
where $A \in \mathbb{R}^{d_{\text{out}} \times r}, B \in \mathbb{R}^{d_{\text{in}} \times r}$ are trainable parameters during fine-tuning.



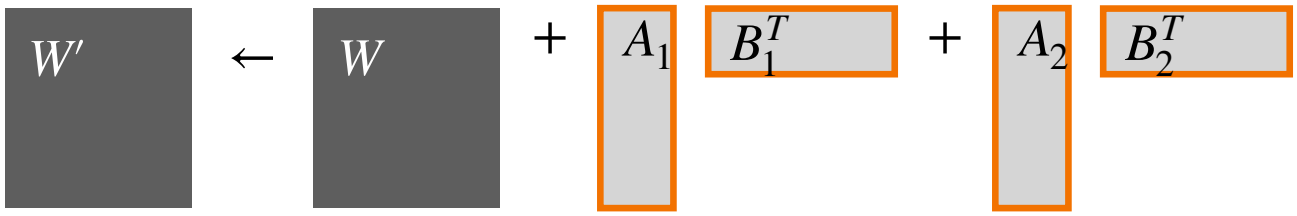
- LoRA Fine-tuning has the following advantages:
  - LoRA requires significantly fewer trainable parameters, requiring reduced **memory** usage: $r \cdot (d_{\text{in}} + d_{\text{out}}) \ll d_{\text{in}} \times d_{\text{out}}$.
  - LoRA avoids making multiple copies of the full parameter, since the base model is frozen.
  - LoRA updates can be applied selectively to specific layers, reducing **computational** overhead.
  - LoRA updates are **modular**, and can be plugged in and out at inference time.



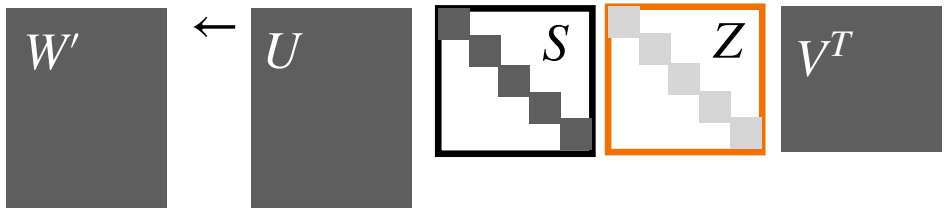
- **QLoRA** [Dettmers et al. 2023] significantly reduces the memory requirement further, by quantizing the frozen base model weights to 4bit precision, while maintaining the fine-tuned performance.
- Average memory requirement for fine-tuning 65B LLM reduces from 780GB of memory to 48GB, which enables fine-tuning on a single GPU.



- One down side, is that when we train multiple LoRA adapters on different tasks, say code and math, and merge them to get both skills, the **merged adapter** use twice the memory.



- This parameter increase when merging can be avoided, if the subspaces spanned by A_1 and A_2 are the same and the subspaces spanned by B_1 and B_2 are the same. But, how do we enforce that?
  - **SVF**: Use the subspace of the SVD(W), and train a diagonal Z, initialized at I



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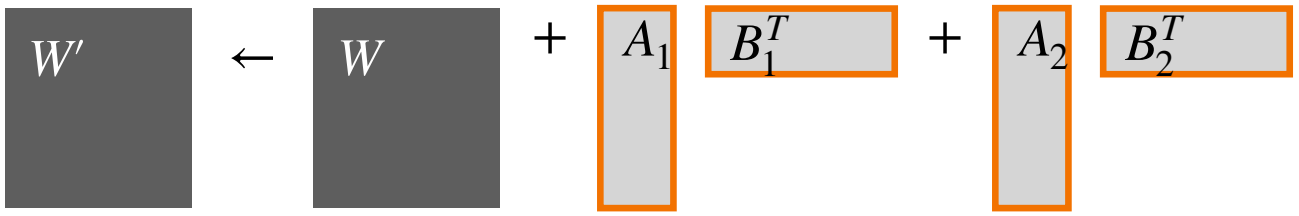
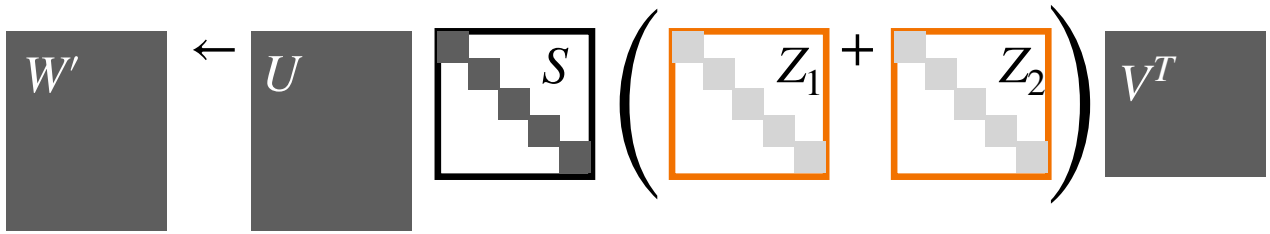
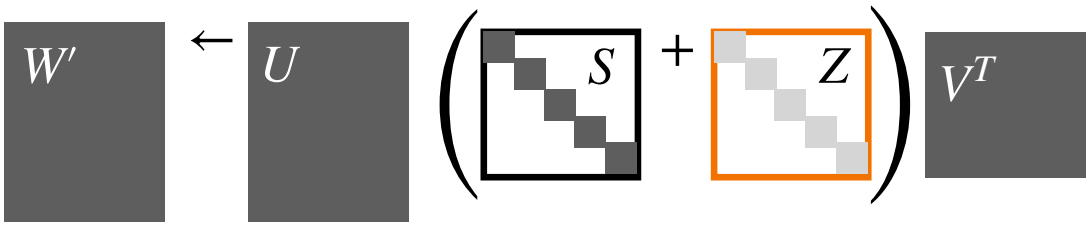


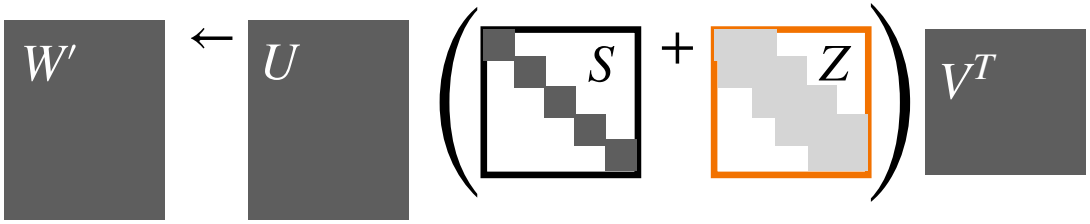
- This parameter increase when merging can be avoided, if the subspaces spanned by A_1 and A_2 are the same and the subspaces spanned by B_1 and B_2 are the same. But, how do we enforce that?
  - SVF: Use the subspace of the SVD(W), and train a diagonal Z, initialized at I
  - When merged, the number of parameters stay the same.



- One can traverse the parameter size vs. downstream accuracy trade-off keeping the SVD subspace frozen:
  - Plain:



  - Banded:



  - Random:

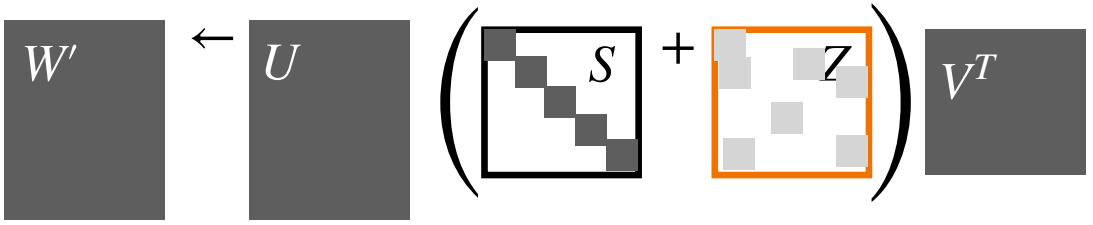
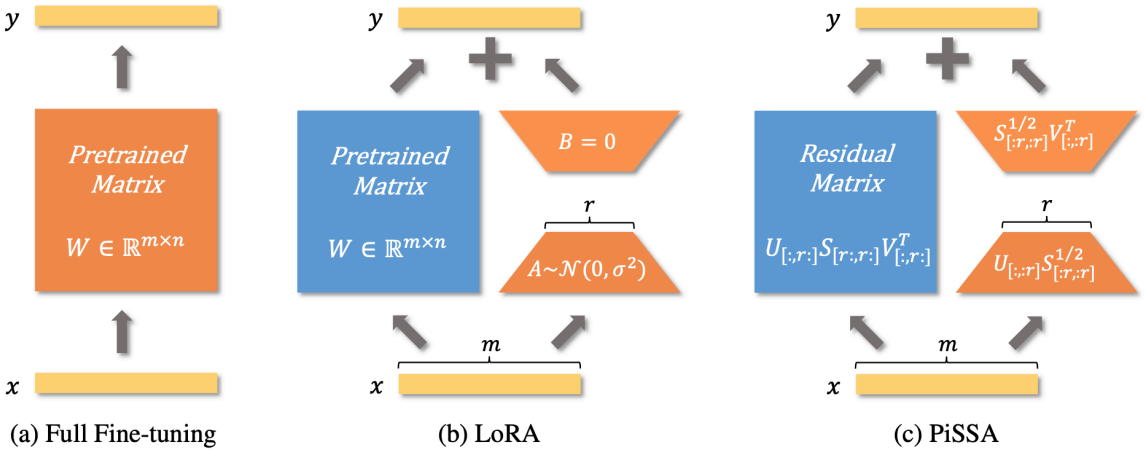


Table 5: Results on fine-tuning Gemma-2B with SVFT using different M parameterizations.

Structure	#Params	GSM-8K	MATH
Plain(diagonal)	0.2M	40.34	14.38
Banded	3.3M	46.47	**16.04**
	6.4M	47.84	15.68
Random	3.3M	47.76	15.98
	6.4M	**50.03**	15.56

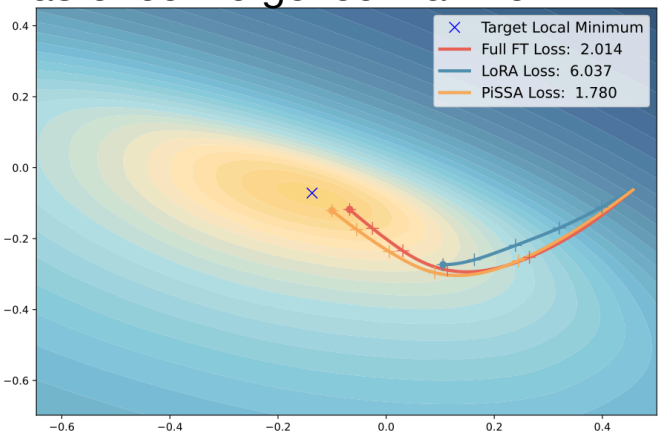
- PISSA (Principal Singular values and Singular vectors Adaptation) [Meng et al. 2024]
  - freezes only the **minor components**, and
  - initializes LoRA with the **principal components**

$$W = \begin{bmatrix} U_p & U_m \end{bmatrix} \begin{bmatrix} S_p & \\ & S_m \end{bmatrix} \begin{bmatrix} V_p^T \\ V_m^T \end{bmatrix} = \underbrace{U_p S_p V_p^T}_{\text{large singular values}} + \underbrace{U_m S_m V_m^T}_{\text{small singular values}}$$



(a) Full Fine-tuning (b) LoRA (c) PiSSA

Better initialization leads to training along PCA directions and faster convergence than LoRA



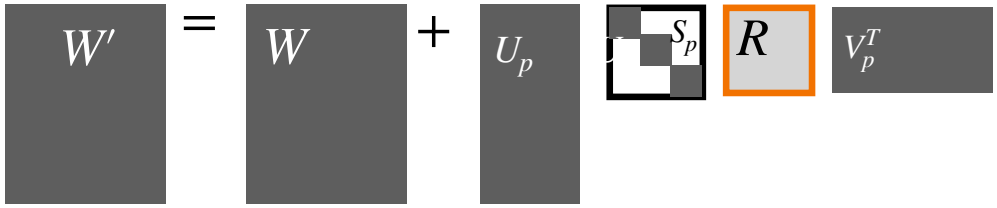
- MiLoRA [Wang et al. 2025] on the other hand
  - freezes only the **principal components**, and
  - initializes LoRA with the **minor components**
- When the pretrained tasks are aligned with the target task, the principal components are well-aligned and training the minor components are more effective

PEFT	GSM8K	MATH	Avg.
LoRA	56.6	10.8	33.7
PiSSA	51.3	10.4	30.8
MiLoRA	**58.6**	**11.6**	**35.1**

Table 9: Math reasoning evaluation results for LLaMA2- 7B

- Theoretical analysis of [Li et al. 2025] suggests that initializing with the PCA subspace gives faster convergence.

- LoRA-XS achieves even further parameter reduction than LoRA [Balazy et.al. 2024]

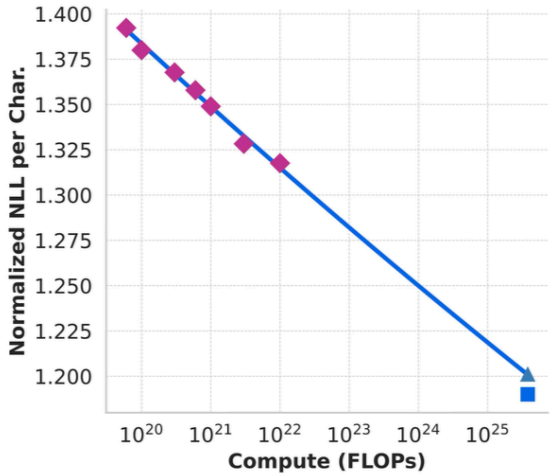


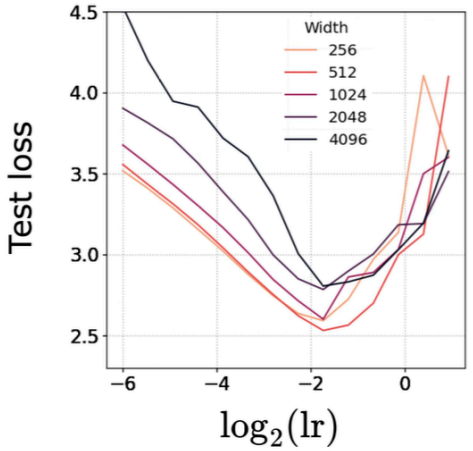
$$W' = W + U_p S_p R V_p^T$$

Model	Method	Rank	# Trainable Parameters	GSM8K	MATH
Mistral (7B)	Full FT	-	7242M	67.02	18.60
	LoRA	64	168M	67.70	19.68
	LoRA-XS	64	0.92M	68.01	17.86
		32	0.23M	63.23	15.88
		16	0.057M	57.92	14.44
Gemma (7B)	Full FT	-	8538M	71.34	22.74
	LoRA	64	200M	74.90	31.28
	LoRA-XS	64	0.80M	74.22	27.62
		32	0.20M	71.72	27.32
		16	0.050M	68.46	26.38

Scaling Laws in LLMs

- More compute in pre-training, through larger models, more training data, and longer training, can improve performance
    - if optimal hyperparameters are chosen
- On the other hand, suboptimal combinations of choices can lead to performance degradation with scale
- We need guidance for hyper parameter choices when scaling up the model training
    - for optimal performance , and
    - for predicting that performance
- $\Longrightarrow$ Scaling Law





- **Definition**. **Neural Scaling Laws** describe how neural network performance changes as key factors are scaled up or down.
- We consider the following four factors:
  - Size of the model N: number of parameters
  - Size of the training dataset D: number of samples or tokens
  - Compute C: measured in FLOPs (FLoating-point OPerations)
  - Generalization performance L: test loss after training

- In the middle regime of dataset size D, it is conjectured that some power-law governs the scaling

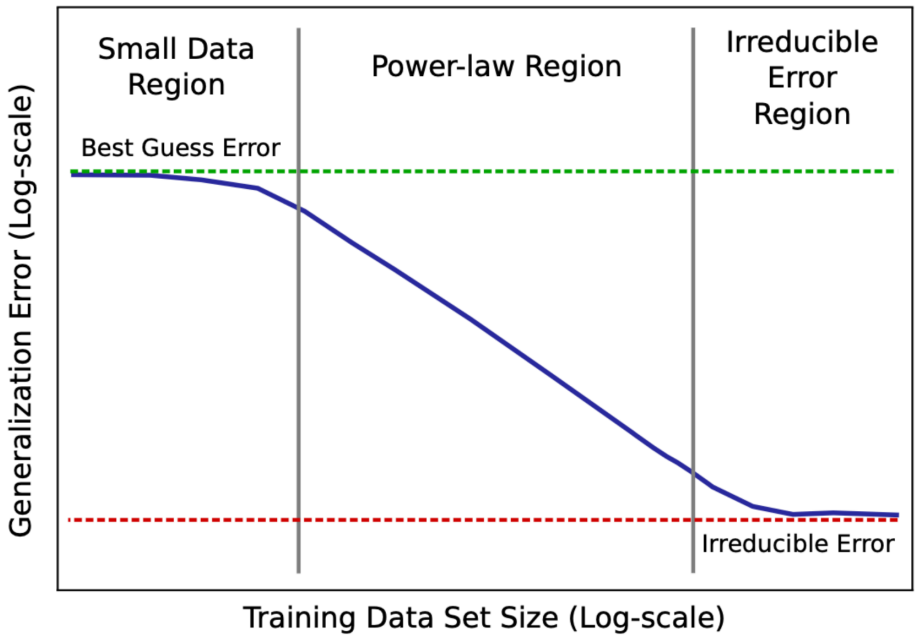


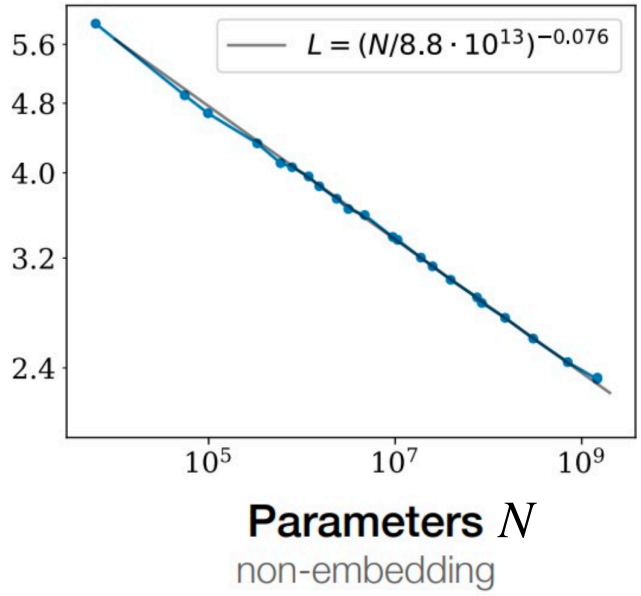
Figure 6: Sketch of power-law learning curves

- Precisely, Kaplan's (empirical) scaling law [Kaplan et al. 2020] predicts that

  - when **the number of parameters** N is limited, for sufficiently large datasets,

$$L(N) = \left(\frac{N_c}{N}\right)^{\alpha_N}, \text{ where } \alpha_N \sim 0.076, \; N_c \sim 8.8 \times 10^{13} \text{ (parameters)}$$

test loss $L(N)$



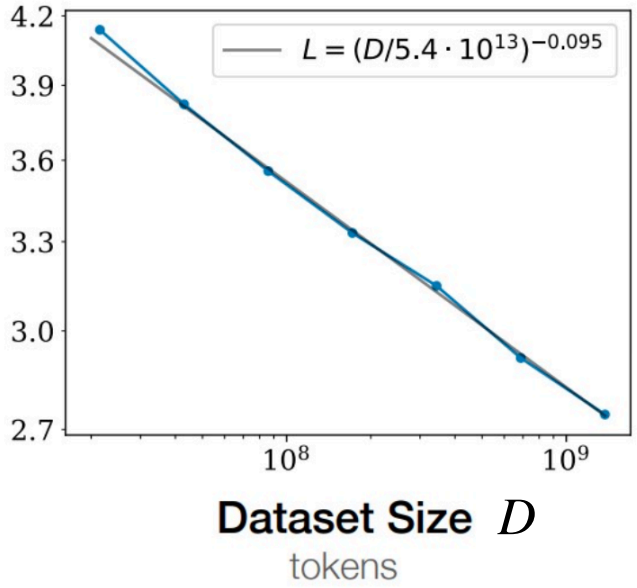
each point on the curve is achieved by a different data size D_N hyperparameter tuned for each of model size N

- Precisely, Kaplan's (empirical) scaling law [Kaplan et al. 2020] predicts that

  - when **the dataset size** D is limited, for sufficiently large models,

$$L(D) = \left(\frac{D_c}{D}\right)^{\alpha_D}, \text{ where } \alpha_N \sim 0.095, \; N_c \sim 5.4 \times 10^{13} \text{ (samples)}$$

test loss $L(D)$



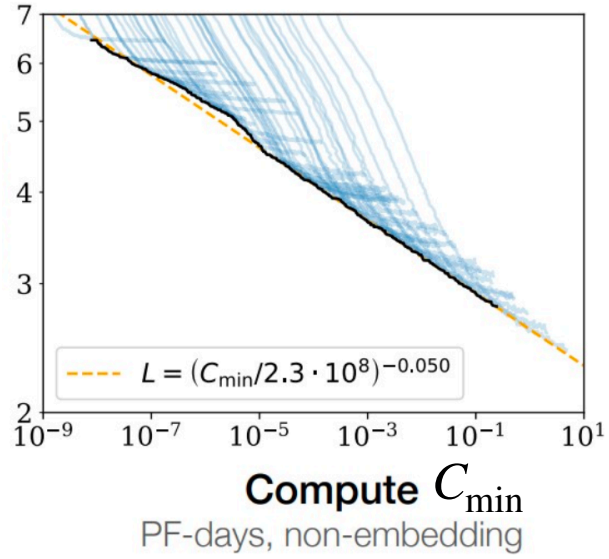
each point on the curve is achieved by a model size N_D hyperparameter tuned for each of dataset size D

- Precisely, Kaplan's (empirical) scaling law [Kaplan et al. 2020] predicts that

  - when **the compute** C is limited, for sufficiently large dataset, the minimum compute $C_{\min}$ needed to achieve a target test loss scales as

$$L(C_{\min}) = \left(\frac{C_{\min,c}}{C_{\min}}\right)^{\alpha_{C_{\min}}}, \text{ where } \alpha_{C_{\min}} \sim 0.050, \; C_{\min,c} \sim 2.3 \times 10^8 \text{ (PF-days)}$$

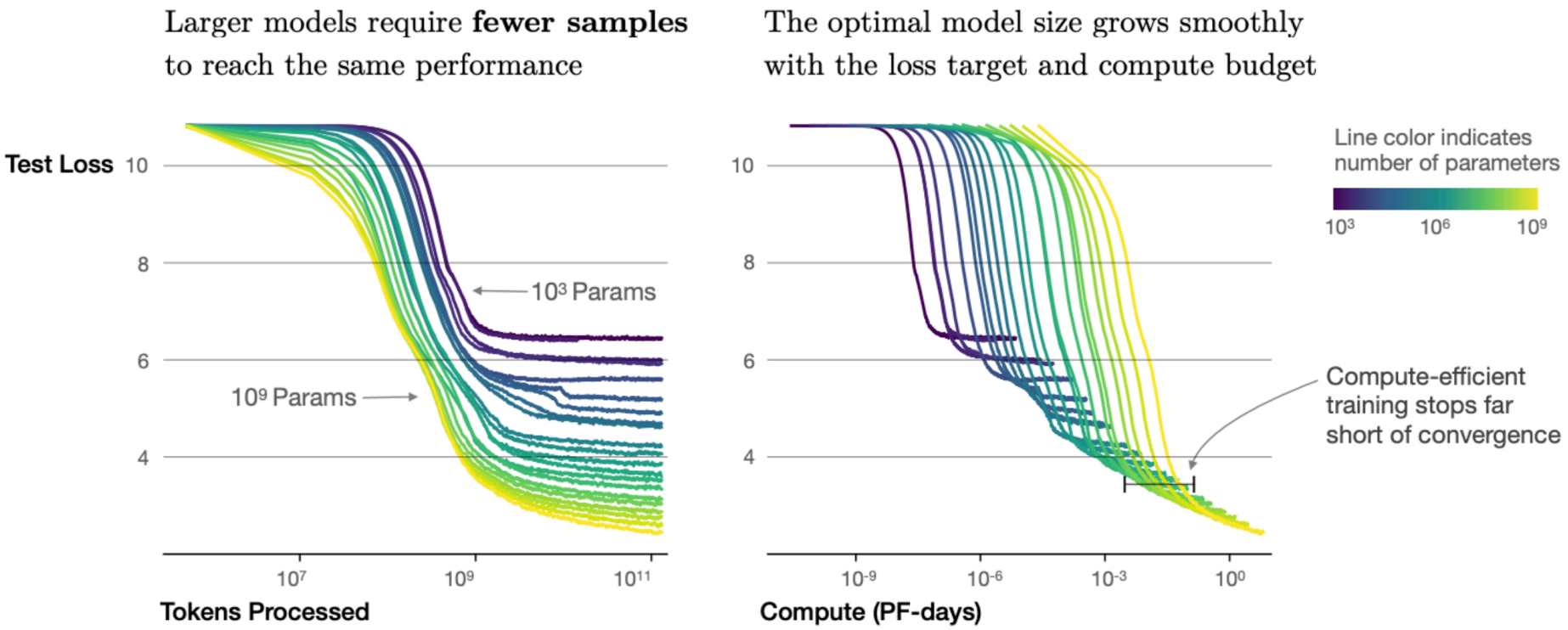
test loss $L(C_{\min})$



- the scaling law is for the pareto frontier (in black solid curve)
- each blue curve is for a fixed model size and changing the dataset size

1 PF-day = 10^{15} FLOPs/second x 24 hours x 3600 seconds/hour

- For optimal performance, all three factors (dataset, compute, model size) need to scale together
- Larger models need fewer samples to achieve the same loss
- Large models are compute-optimal when undertrained
- Train on **larger model with fewer samples** (than training smaller model to convergence)



- Kaplan's scaling law suggests that for fixed compute budget, we should prioritize larger model size
- However, it fails to predict compute optimal scaling in large compute regime, because there is not enough data to train all the parameters

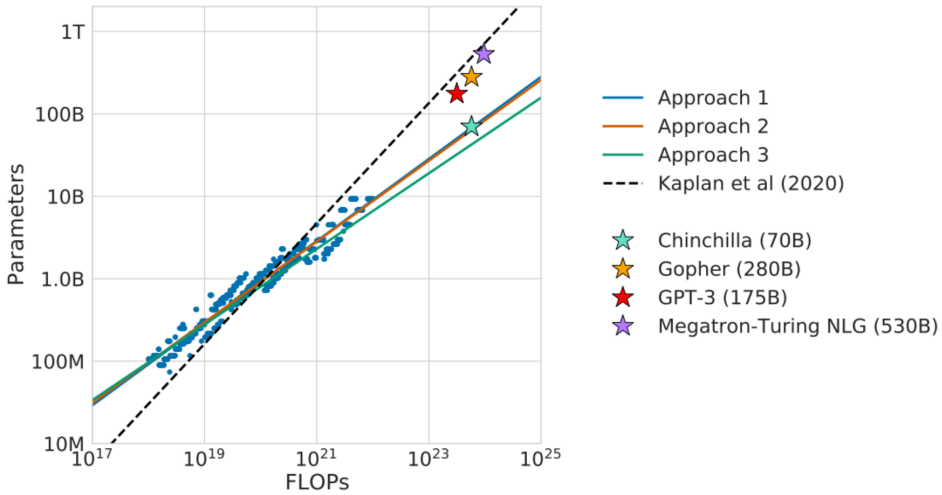


Figure 1 | **Overlaid predictions.** We overlay the predictions from our three different approaches, along with projections from Kaplan et al. (2020). We find that all three methods predict that current large models should be substantially smaller and therefore trained much longer than is currently done. In Figure A3, we show the results with the predicted optimal tokens plotted against the optimal number of parameters for fixed FLOP budgets. ***Chinchilla* outperforms *Gopher* and the other large models (see Section 4.2).**

- Chinchila [Hoffman et al. 2022] scaling law suggests that model size N and data size D should scale together, at the same rate

Table 2 | **Estimated parameter and data scaling with increased training compute.** The listed values are the exponents, a and b, on the relationship $N_{opt} \propto C^a$ and $D_{opt} \propto C^b$. Our analysis suggests a near equal scaling in parameters and data with increasing compute which is in clear contrast to previous work on the scaling of large models. The 10^{th} and 90^{th} percentiles are estimated via bootstrapping data (80% of the dataset is sampled 100 times) and are shown in parenthesis.

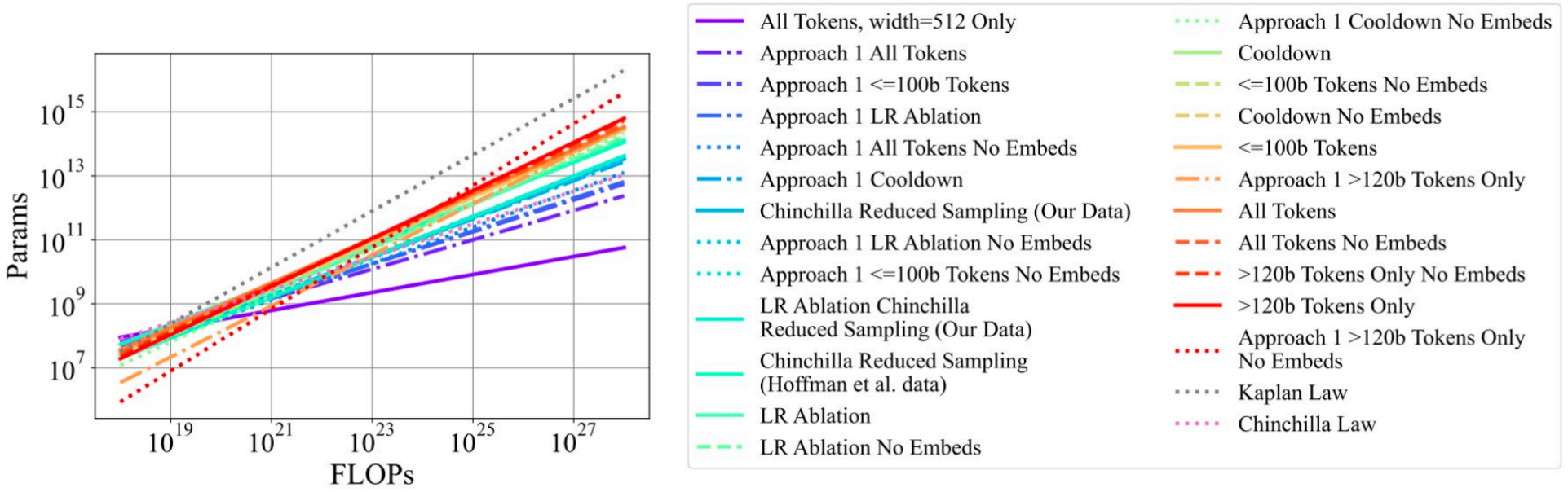
Approach	Coeff. a where $N_{opt} \propto C^a$	Coeff. b where $D_{opt} \propto C^b$
1. Minimum over training curves	0.50 (0.488, 0.502)	0.50 (0.501, 0.512)
2. IsoFLOP profiles	0.49 (0.462, 0.534)	0.51 (0.483, 0.529)
3. Parametric modelling of the loss	0.46 (0.454, 0.455)	0.54 (0.542, 0.543)
Kaplan et al. (2020)	0.73	0.27

- it was empirically discovered that to scale up compute, N and D need to increase together: 20 training tokens is optimal per parameter

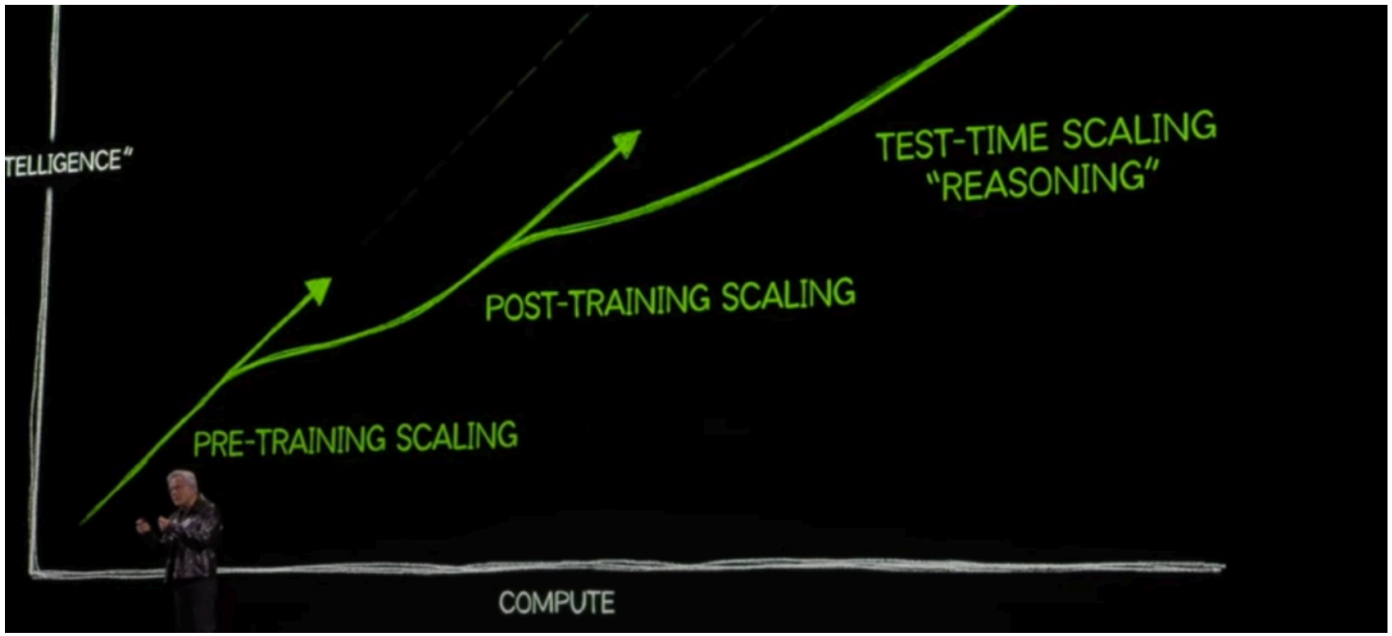
Optimal number of tokens for various sized models

Parameters	FLOPs	FLOPs (in *Gopher* unit)	Tokens
400 Million	1.92e+19	1/29,968	8.0 Billion
1 Billion	1.21e+20	1/4,761	20.2 Billion
10 Billion	1.23e+22	1/46	205.1 Billion
67 Billion	5.76e+23	1	1.5 Trillion
175 Billion	3.85e+24	6.7	3.7 Trillion
280 Billion	9.90e+24	17.2	5.9 Trillion

- Later experiments [McLeish et al. 2025] show that scaling law slope is sensitive to many small experimental design choices, including
  - parameter counting rule: whether to count the embedding layer or not
  - width and depth ratio
  - learning rate scheduler
- Kaplan and Chinchilla extract different scaling laws but neither of them are wrong



- Beyond pretraining scaling laws, there is increasing interest in post-training scaling and test-time scaling. Techniques to improve scaling include
  - Post training: fine-tuning, quantization, pruning, sdistillation, etc.
  - Test time: best of N sampling, MCMC sampling, thinking time, etc.



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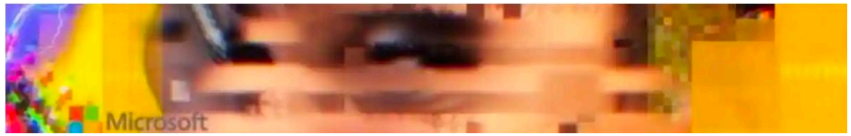
- Alignment

Alignment

- AI model responses can be misaligned with what we want them to do, which can sometimes cause real harm.

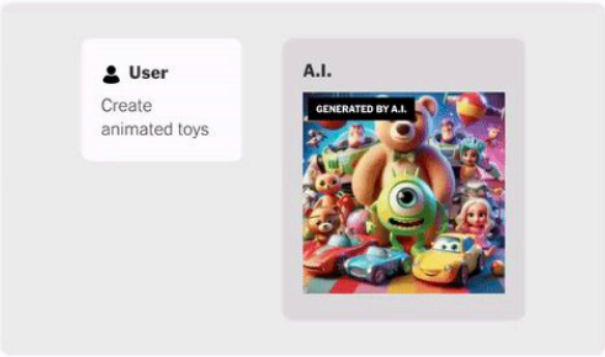
Microsoft 'deeply sorry' for racist and sexist tweets by AI chatbot

Company finally apologises after 'Tay' quickly learned to produce offensive posts, forcing the tech giant to shut it down after just 16 hours



The New York Times

Artificial Intelligence › | A.I. Faces Quiz | How the A.I. Race Began | Key Figures in the Field | One Year of ChatGPT



We Asked A.I. to Create the Joker. It Generated a Copyrighted Image.

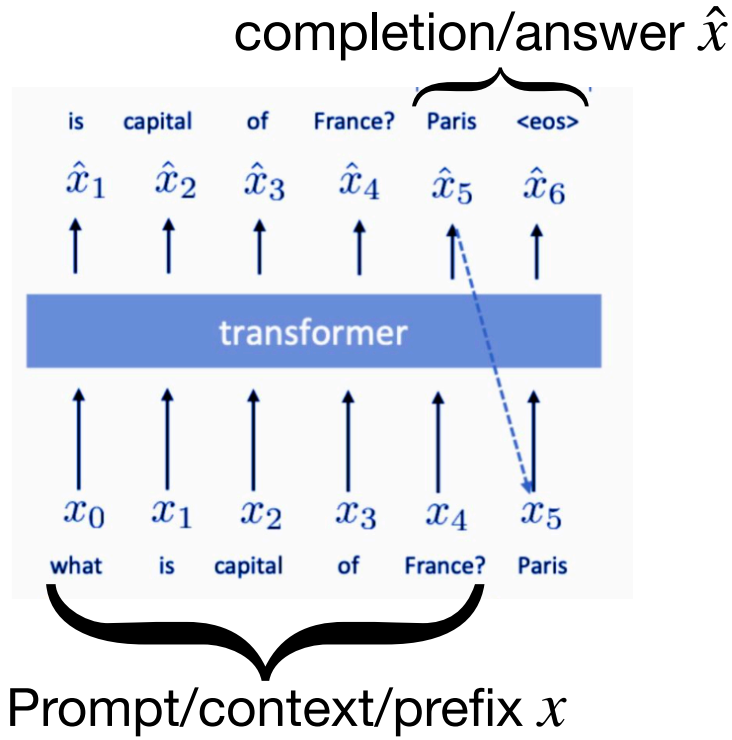
By Stuart A. Thompson Jan. 25, 2024

The Washington Post
Democracy Dies in Darkness

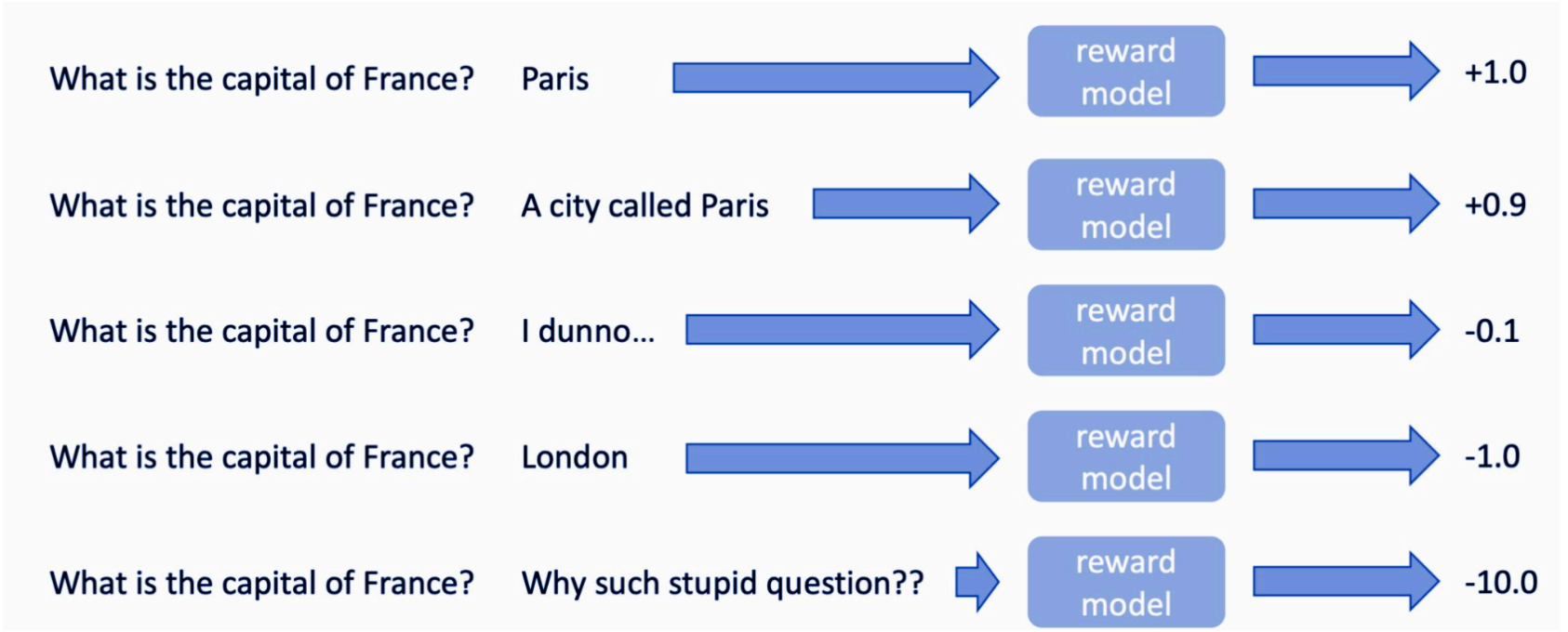
Chatbots' inaccurate, misleading responses about U.S. elections threaten to keep voters from polls

By Garance Burke | AP

- Scaling pertaining does not address the challenges in **alignment** with human values and intent. We need post-training based on **RLHF (Reinforcement Learning with Human Feedback)**.
- We do not cover reinforcement learning in this class in any depths, but we will learn as much as we need along the way.
- Given a prompt/context/prefix x and its completion y, a **reward model** assigns a scalar value on the quality of the completion: $r(x, \hat{x}) \in \mathbb{R}$

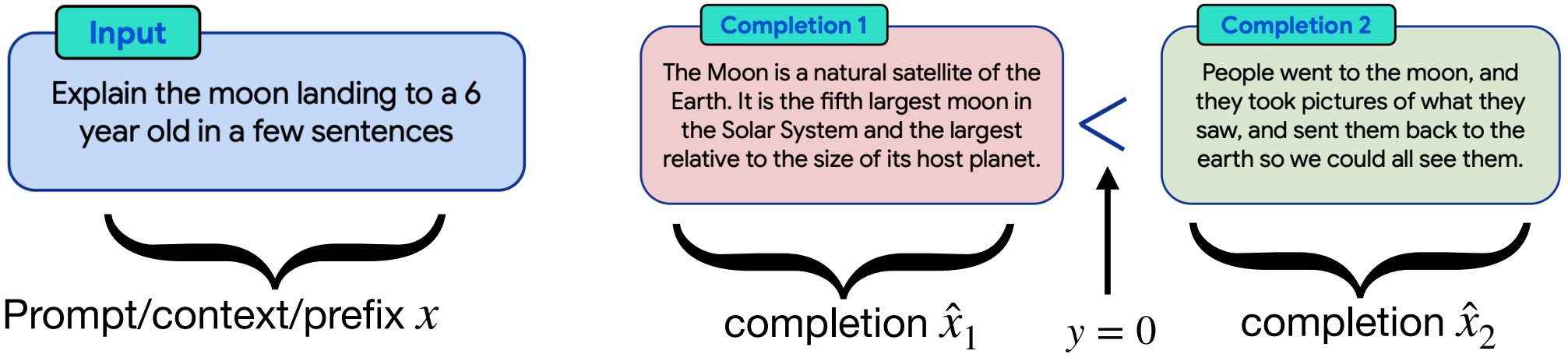


- consider a neural network function $r(x, \hat{x})$, that we want to train on **human labelled preference**, such that it provides useful "reward"



- However, it is challenging to collect reliable reward, so instead we collect data consisting of pairwise comparisons on which completion is preferred. This does not fit questions where exact solutions exist, like math and coding, where we use RLVF (Reinforcement Learning with Verifiable Feedback) instead.

- Each sample $(x, \hat{x}_1, \hat{x}_2, y)$ consists of prompt x, two different completions $\hat{x}_1$ and $\hat{x}_2$, and preference label $y = 1$ if $\hat{x}_1$ is preferred over $\hat{x}_2$, which we write as $\hat{x}_1 \succ \hat{x}_2$, and $y = 0$, otherwise.



- To model such preferences so that we can learn the model, we borrow mathematical foundations of **choice models,** in particular, **Random Utility Models (RUMs)**.
- Under RUM, each option has corresponding utility, and when we make a choice, we observe a randomly perturbed utility and choose the one that has maximum observed utility. In the context of LLM post-training, the completions are our options to choose from and utility of an option is the reward of a completion.
  - **Random Utility Model**
    - hidden true rewards of the two completions: $r^*(x, \hat{x}_1)$ and $r^*(x, \hat{x}_2)$
    - observed rewards: $r^*(x, \hat{x}_1) + z_1$ and $r^*(x, \hat{x}_2) + z_2$
    - preference: $\mathbb{P}(\hat{x}_1 \succ \hat{x}_2) = \mathbb{P}(r^*(x, \hat{x}_1) + z_1 > r^*(x, \hat{x}_2) + z_2)$
  - Different choices of the noise gives different models. When the noise z_i's follow independent **Gumbel distribution**, the resulting distribution of the preference simplifies to a logistic distribution, which is called **Bradley-Terry model**.

- **Bradley-Terry (BT) model** or Bradley-Terry-Luce (BTL) model
  - hidden true rewards of the two completions: $r^*(x, \hat{x}_1)$ and $r^*(x, \hat{x}_2)$
  - observed rewards: $r^*(x, \hat{x}_1) + z_1$ and $r^*(x, \hat{x}_2) + z_2$
  - **preference**:

$$\begin{aligned}\mathbb{P}(\hat{x}_1 \succ \hat{x}_2) &= \mathbb{P}(r^*(x, \hat{x}_1) + z_1 \geq r^*(x, \hat{x}_2) + z_2) \\ &= \frac{1}{1 + \exp\{-(r^*(x, \hat{x}_1) - r^*(x, \hat{x}_2))\}}\end{aligned}$$

- Learning a NN reward function $r(\cdot, \cdot)$ given data $\mathcal{D} = \{(x_i, \hat{x}_{i,1}, \hat{x}_{i,2}, y_i)\}$:

$$\max_r \sum_{(x_i, \hat{x}_{i,1}, \hat{x}_{i,2}, y_i)} \log \frac{1}{1 + \exp\{-(\text{sign}(y_i - 0.5))(r(x_i, \hat{x}_{i,1}) - r(x_i, \hat{x}_{i,2}))\}}$$

Sources

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