CSE 493s/599s
Lecture 13. LM basics and architectures

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Lecture notes

- These lecture notes are based on other courses in LLMs, including
  - CSE493S/599S at UW by Ludwig Schmidt: https://mlfoundations.github.io/advancedml-sp23/
  - EE-628 at EPFL by Volkan Cevher: https://www.epfl.ch/labs/lions/teaching/ee-628-training-large-language-models/ee-628-slides-2025/
  - ECE381V Generative Models at UT Austin by Sujay Sanghavi
  - and various papers and blogs cited at the end of the slide deck

Outline

- Language models
- General LLM framework
  - Token processing
  - Sequence mixing
  - Prediction
- Example architectures

Notations

input data / sample	x
output / label	y
model weight	W
sentence	S
word	w

Language Model (LM) basics

- **Example** language task:
  - What is the most ***likely*** next word [nw] given the following S_{source}?

    S_{source}: "On January 1 people usually say happy new [nw]."
- This task is fundamental in language modeling.
  - check spelling & grammar correction
  - machine translation
  - sentence classification
  - speech recognition
  - chatbot
  - reasoning, planning, coding, math, etc.

$$\mathbb{P}(\text{year} \mid S_{\text{source}}) > \mathbb{P}(\text{years} \mid S_{\text{source}})$$

$$\mathbb{P}(S_{\text{translation 1}} \mid S_{\text{source}}) > \mathbb{P}(S_{\text{translation 2}} \mid S_{\text{source}})$$

$$\mathbb{P}(\text{good} \mid S_{\text{source}}) > \mathbb{P}(\text{bad} \mid S_{\text{source}})$$

$$\mathbb{P}(\text{Wreck a nice beach}) < \mathbb{P}(\text{Recognize speech})$$

$$\mathbb{P}(nw \mid S_{\text{source}})$$

- **Definition** (Language Model [Jurafsky and Martin,2023])
  - Models that assign probability to sequences of words are called language models.
- Next word prediction model is sufficient to assign probability to the entire sentence, $S = (w_1, w_2, \cdots, w_T)$, of T words with the **chain rule**:

$$\mathbb{P}(S) = \mathbb{P}(w_{1:T}) = \mathbb{P}(w_1)\mathbb{P}(w_2 \,|\, w_1)\mathbb{P}(w_3 \,|\, w_1, w_2)\cdots\mathbb{P}(w_T \,|\, w_{1:T-1})$$

  without any assumptions.
- For example, if $S = w_{1:3} =$ "happy new year", then
  $\mathbb{P}(S) = \mathbb{P}(\text{happy})\mathbb{P}(\text{new} \,|\, \text{happy})\mathbb{P}(\text{year} \,|\, \text{happy new})$
- Fundamental question in LM: how can we model and learn $\mathbb{P}(w_t \,|\, w_{1:t-1})$?

- how can we model and learn $\mathbb{P}(w_t \mid w_{1:t-1})$?
- A trivial but memory-inefficient model uses **empirical frequency** on a large corpus:

$$\mathbb{P}(\text{year} \mid S_{\text{source}}) = \frac{\mathbb{P}(S_{\text{source}} + \text{year})}{\mathbb{P}(S_{\text{source}})} \simeq \frac{\#(\text{On January 1 people usually say happy new year})}{\#(\text{On January 1 people usually say happy new})}$$

  - All semantic relations are lost, e.g., "On January **first** people usually say happy new year."
  - A lot of zero counts for rare (combinations of) words.

- A more efficient alternative is N**-gram LM,** that uses past $(N-1)$ words to predict the next one:

$$\mathbb{P}(w_t \mid w_{1:t-1}) = \mathbb{P}(w_t \mid w_{t-N+1:t-1}) \simeq \frac{\#(w_{t-N+1:t})}{\#(w_{t-N+1:t-1})} \qquad \text{e.g., } \frac{\#(\text{happy new year})}{\#(\text{happy new})}$$

  - Bi-gram model, $N = 2$, uses only $\mathbb{P}(w_t \mid w_{t-1})$.

- Terminologies
  - Language models operate on **tokens** and not words, as we learned in the previous lecture, but we will use them interchangeably, unless we need to be precise.
  - **Vocabulary** of tokens is denoted by $\mathscr{V}$ and its size by $|\mathscr{V}|$.
  - Special tokens are used to indicate the beginning and end of sentences:

    For example, 5 tokens to represent “<BOS> Happy new year <EOS>”
  - **Pre-training** is building an LM from scratch on a large corpus of text.
  - **Inference** is using a trained LM to do next word prediction.

- Bi-gram LM **pre-training** requires memory of size $O(|\mathscr{V}|^2)$ and the model is sparse**:**

  - **For** all pair of words in the corpus
    - count $\#(w)$ and $\#(w, w')$
  - compute and store $\mathbb{P}(w \mid w') = \dfrac{\#(w, w')}{\#(w)}$ for all $(w, w') \in \mathscr{V} \times \mathscr{V}$

- Bi-gram LM **inference** generates sequence of words:

  - Initialize: $w_1 \leftarrow$ <BOS>, $t = 1$
  - **While True**
    - $w_{t+1} \leftarrow \arg\max_{w \in \mathscr{V}} \mathbb{P}(w \mid w_t)$
    - **If** $w_{t+1} =$ <EOS>: **Output** $(w_{1:t+1})$; **Break**
    - $t \leftarrow t + 1$

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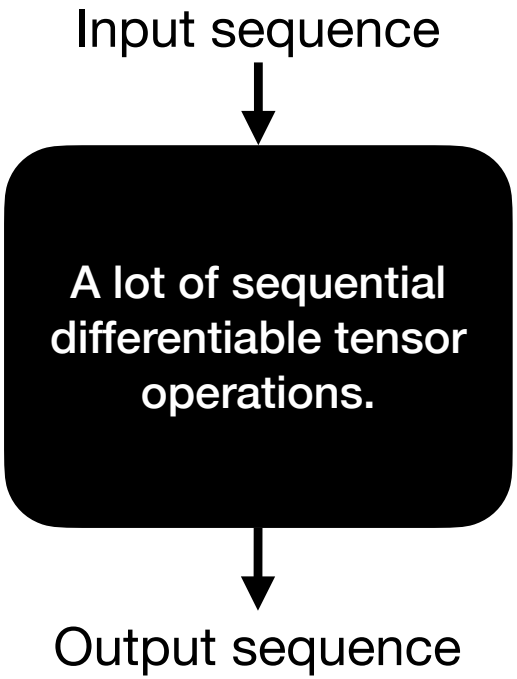
    - $t \leftarrow t + 1$

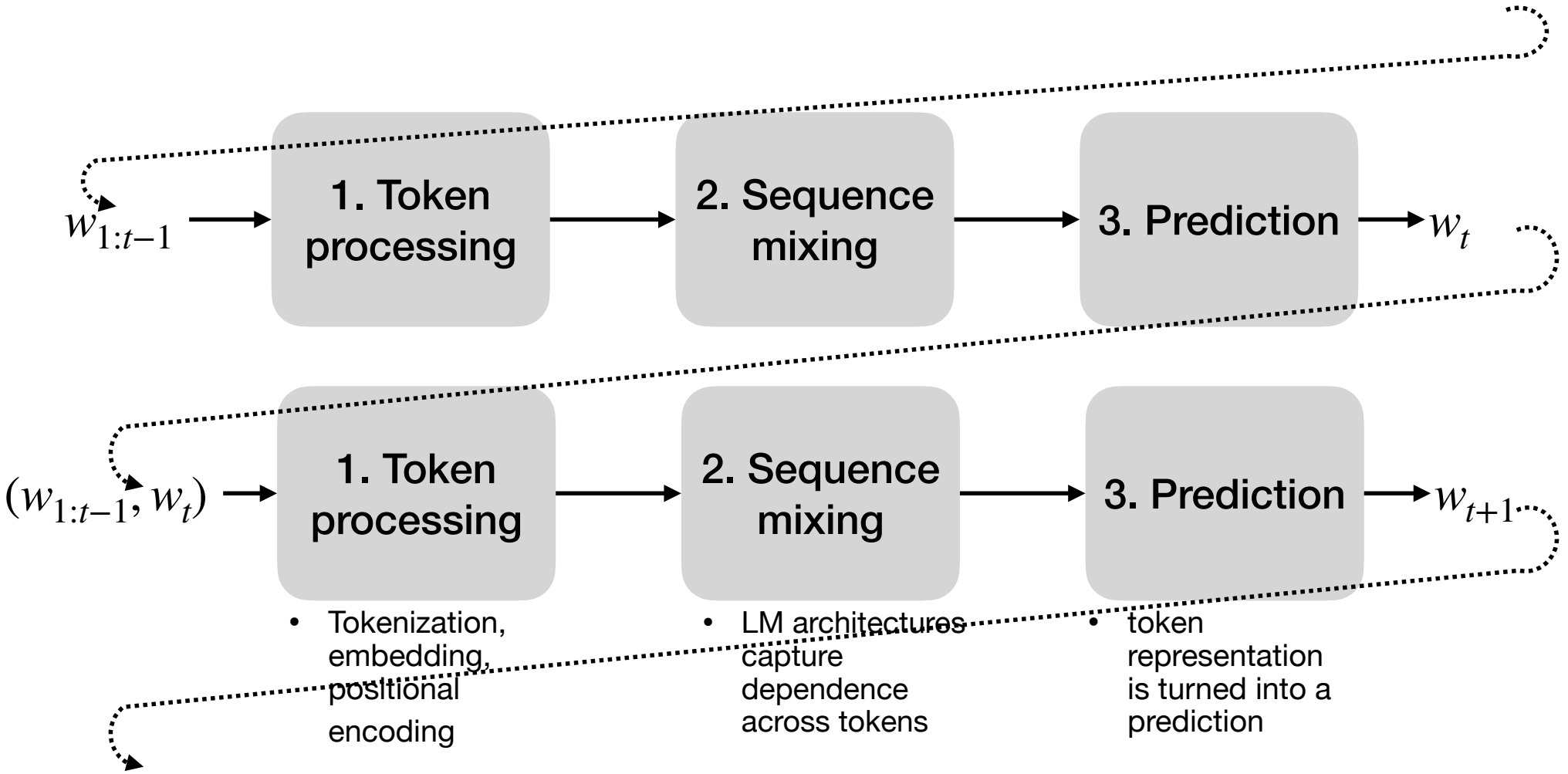
- This toy example generates the same deterministic sequence all the time.
- We can instead start with a given prompt, and continue completing the given prompt using the same techniques, that we will learn more about later.
- Later, we will learn random sampling that makes the output better.

Modern language models

https://xkcd.com/1838/

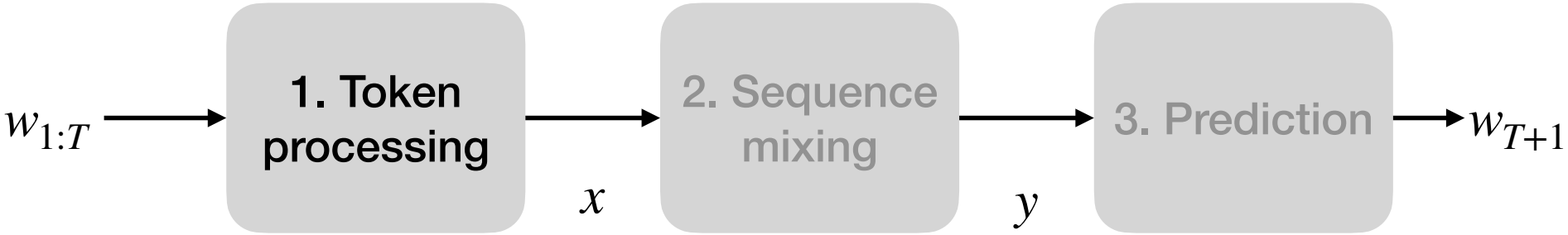


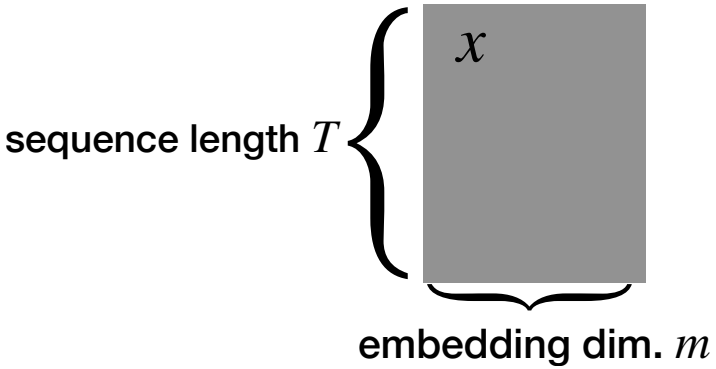
General (Large Language Model) LLM framework



Token processing

- Token processing turns a word or a token into a combination of a **word embedding** and a **positional embedding**





- Fundamental question: what is the useful numerical representation of a word/token?
- **Word (token) embeddings** are the vectors representing words (tokens).
- **One-hot representation** uses standard basis vectors in $\mathbb{R}^{|\mathscr{V}|}$.
    - Relations between words are lost and similarity cannot be measured.
    - Dimension of the vector is too large, e.g., for prediction

Vocab	**one-hot representation**
a	[1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, …]
ability	[0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, …]
able	[0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, …]
about	[0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, …]
above	[0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, …]
acarus	[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, …]
⋮	vocab size $=\|\mathscr{V}\|$

- Breakthrough in word embeeding: **Word2Vec** [Mikolov et al. 2013] and **Glove** [Pennington et al. 2014]
  - capture similarities and relations between words via training on a text corpus
  - gave significant gain for older model architectures like RNN and LSTM
- Word2vec is trained to predict a missing word given a context window, for example, of size 2, before and after the masked word:

$$\cdots \underbrace{\text{people usually}}_{\text{context window}} \quad \underbrace{\text{say}}_{\text{target word}} \quad \underbrace{\text{happy new}}_{\text{context window}} \quad \cdots$$

- The i-th word in the vocabulary is associated with two vector embeddings in m-dimensions,
  - $t_i \in \mathbb{R}^m$ is the embedding when it is a target word, and
  - $c_i \in \mathbb{R}^m$ is the embedding when it is used as a context.
- After these embeddings are learned, the word is represented by either the summation or the concatenation of these two vectors, when used in an LM.
- m is about 200~300 in practice.

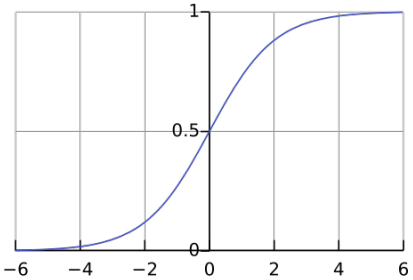
- example sample:

w_p w_t w_n

$\cdots$ people usually say happy new $\cdots$

context window target word context window

- Word2vec embeddings are trained on a corpus of text, where for each context window the loss is defined over a tuple (w_t, w_c, w_n), where
  - w_t is the target word in the sample with embedding t_{w_t}
  - w_p is one of the context words (called positive sample) with c_{w_p}
  - w_n is not a context word (called negative sample) with c_{w_n}
  - Word2vec is learned from text corpus to minimize a loss function:



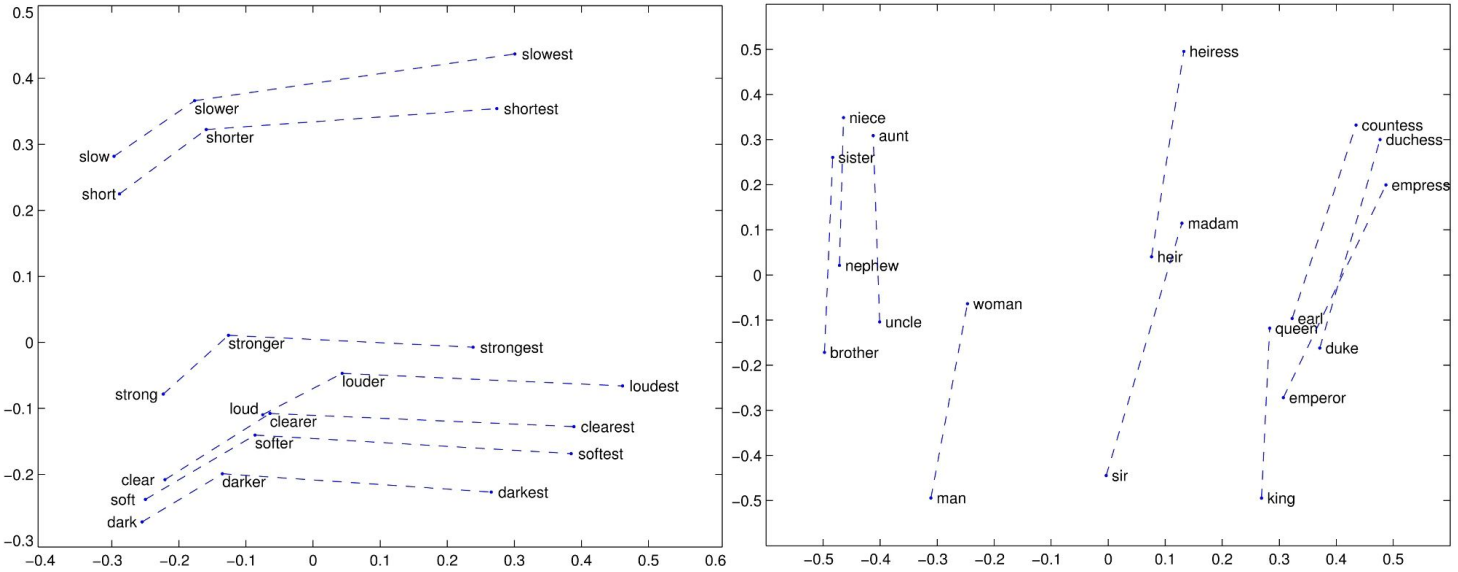
$$L = -\log\left(\mathbb{P}(w_t \text{ is a true target of } w_p) \cdot \mathbb{P}(w_t \text{ is not a true target of } w_n)\right)$$
$$= -\log(\sigma(\langle t_{w_t}, c_{w_p}\rangle)) - \log(1 - \sigma(\langle t_{w_t}, c_{w_n}\rangle))$$
$$= -\log\left(\frac{1}{1+\exp(-\langle t_{w_t}, c_{w_p}\rangle)}\right) - \log\left(1 - \frac{1}{1+\exp(-\langle t_{w_t}, c_{w_n}\rangle)}\right)$$

- Word2vec was a breakthrough due to its simplicity, efficient training, semantic quality, and vector arithmetic (which was not intended at all!)
  - for example, other than the missing word prediction that it was trained to solve, word2vec was amazing at analogy tasks:

    king : man = ? : woman,

    can be solved by **vector arithmetic**, outputting the embedding that is the **nearest neighbor**

$$x_{\text{king}} + x_{\text{woman}} - x_{\text{man}} \simeq x_{\text{queen}}$$



- It is quite surprising that **local rules** of co-occurences, i.e., how often do two words appear together in a context window, are enough to learn such rich semantic relations.

- Building upon the success of Word2vec, for a while, the typical approach has been to use these word vectors, and train specific model for each task: translation, sentiment analysis, etc.
- Typical embedding dimension is 300, which is not too large, but the number of words can be huge, perhaps in the order of 1,000,000.
- No longer used, as subword tokenization is more popular than words, and embeddings learned together with model weights perform better than word2vec.
- Modern embeddings for subword tokens are **learned** in pretraining stage, with an exception of a part of the embedding called the **positional embedding,** which contains information about the word's position in the sentence.
- token vocab ~ 200k, and token embedding vectors of dimension 700~12000 are trained.

- **Positional embeddings** are designed to capture the positions of the input sequence of words to a transformer.

- Without positional encoding, the original transformer treats the input as a set, in which case the following two inputs are treated the same (more to follow when we learn architectures):

"I am happy" vs. "Am I happy"

- To take into account the order of the input, **absolute positional embedding** represents a word by concatenating its learned semantic embedding with an absolute position of the word, for example,

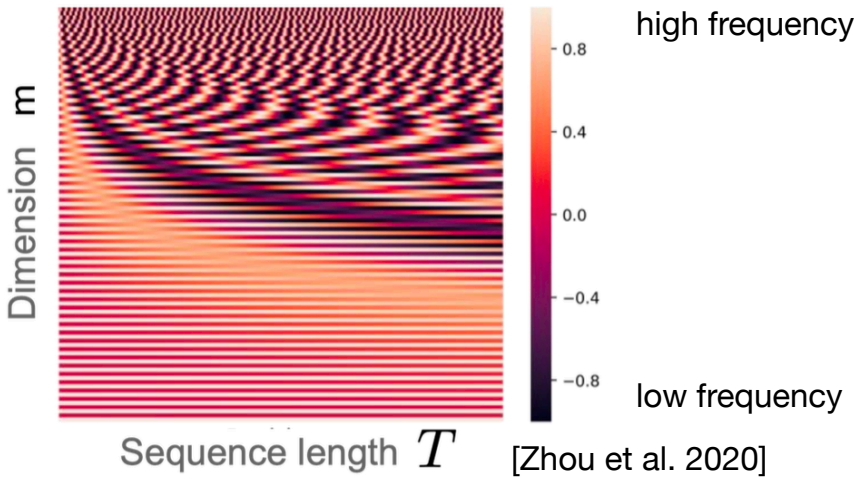
- Attempt 1: its position index t can be used as absolute position embedding:

$$x_t = (\text{WordEmb}(w_t), t)$$

but this is challenging to generalize to unseen sequence lengths, and the range of the value of the embedding increases with t.

- Absolute positional embedding **solution 1**: original transformer paper [Vaswani et al. 2017] proposes using alternating sin() and cos() functions of decreasing frequencies at position index t, **added** to the vector word embedding:

$$x_t = \text{WordEmb}(w_t) + \underbrace{\begin{bmatrix} \sin(t/10000^{0/m}) \\ \cos(t/10000^{0/m}) \\ \sin(t/10000^{2/m}) \\ \cos(t/10000^{2/m}) \\ \vdots \\ \sin(t/10000^{1/2}) \\ \cos(t/10000^{1/2}) \end{bmatrix}}_{\text{position embedding}}$$



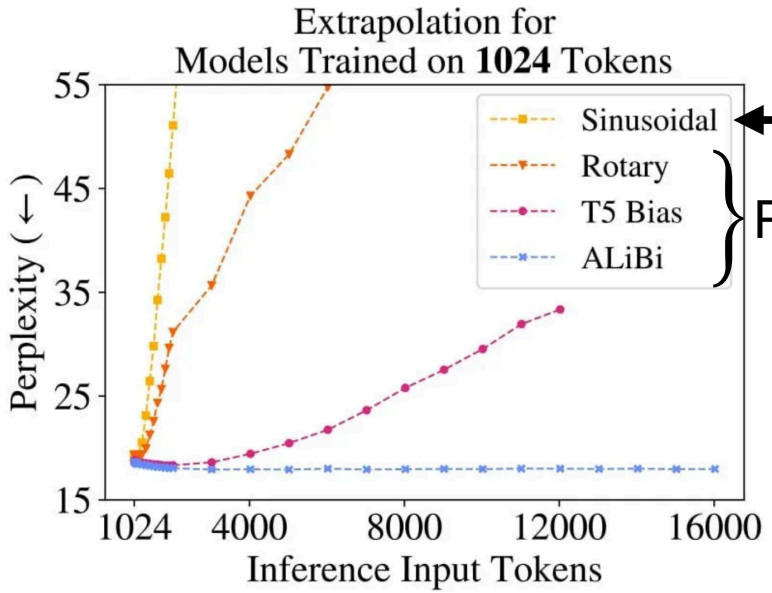
[Zhou et al. 2020]

- Absolute positional embedding **solution 2**: learned positional embedding.

$$x_t = \text{WordEmb}(w_t) + \text{PosEmb}(t)$$

- Empirical performance is similar for the two absolute positional embeddings
- Learned positional embedding is popular in vision transformers

- Absolute positional embeddings encode the absolute position of the word in the sequence, which has two problems:
  - it is hard to extrapolate to sequence lengths unseen during training,



← Absolute positional embedding

} Relative positional embedding

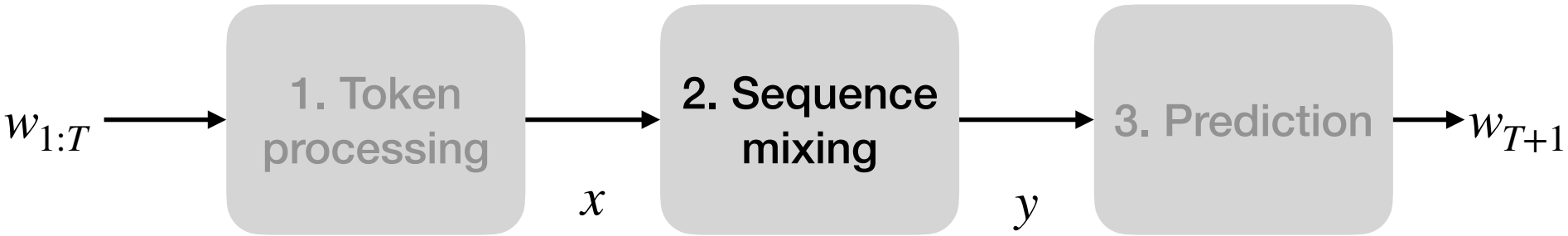
[Press et al. 2022]

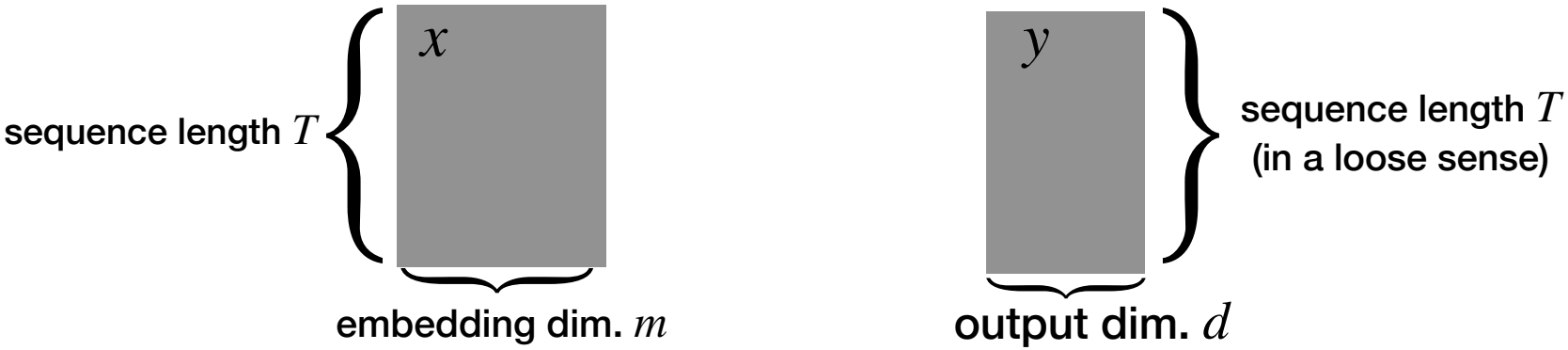
  - and relative position is as important as absolute position, for example, “happy new ?” appearing in positions (1,2,3) have similar meaning as appearing in positions (500,501,502).

- Relative positional encoding addresses both: generalize to sequences of unseen lengths by relying on the pairwise distances between two words. We will revisit after we learn transformers.

Sequence mixing

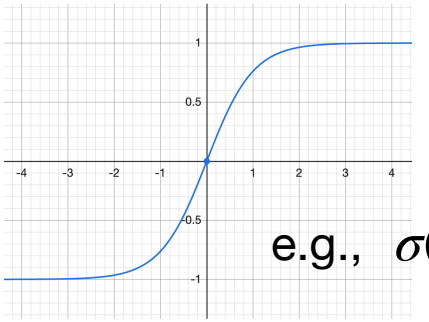
- Sequence mixing is one of the most well studied part of an LM that captures the dependencies across tokens.





- **Sequence mixer** captures the hidden correlation across the tokens in a sequence.
- Markov in 1913 [Markov 1913] used **Markov chains** to predict whether an upcoming letter would be a vowel or a consonant.
    - Markov assumption: the probability of a next word only depends on the past $N-1$ words, i.e., $\mathbb{P}(w_t \mid w_{1:t-1}) = \mathbb{P}(w_t \mid w_{t-N+1:t-1})$.
- Markov chain with a window size of N can be captured by the N-gram models we just studied a few slides ago. But the learning of the Markov chain $\mathbb{P}(w_t \mid w_{t-N+1:t-1})$ can be made efficient using neural networks, for example **Feed Forward Neural networks** that computes

$$\mathrm{Emb}(w_t) = \sigma\left(W \begin{bmatrix} \mathrm{Emb}(w_{t-N+1}) \\ \vdots \\ \mathrm{Emb}(w_{t-1}) \end{bmatrix} \right)$$



e.g., $\sigma(x) = \tanh(x) = \dfrac{e^{2x}-1}{e^{2x}+1}$

for an input of past $N-1$ word embeddings, weight matrix $W \in \mathbb{R}^{m \times m(N-1)}$, entry-wise scalar activation function $\sigma(\cdot)$ of choice, and output of a word embedding.

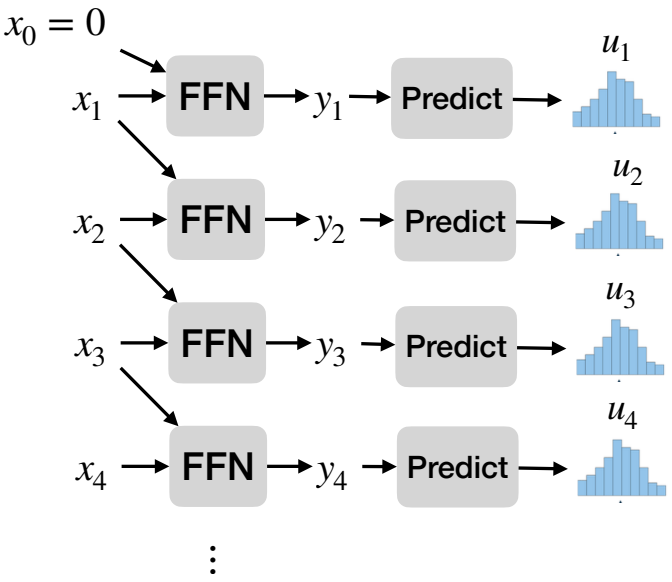
- **Feed Forward Neural networks (FFNs)** as a sequence mixer uses N recent tokens to predict next token similar to N-gram models (differentiable and parametric version of N-gram models).
    - Model weight $W \in \mathbb{R}^{d \times Nm}$ takes as input N token embeddings, each of dimension m, and outputs a d -dimensional representation.
- **Forward pass in pre-training**, the pseudocode is for a single sentence (using only two recent tokens, i.e., $N = 2$), and training proceeds by taking gradient of this loss and using stochastic gradient descent (SGD)

- Initialize the input $x_0 \leftarrow 0$ and loss $L \leftarrow 0$
- **For** $t = 1, \ldots, T$

$$y_t \leftarrow \sigma\left(W \begin{bmatrix} x_{t-1} \\ x_t \end{bmatrix}\right), \qquad \text{FFN}$$

$$u_t \leftarrow \text{Predict}(y_t), \qquad \text{probability}$$

$$L \mathrel{+}= \left(\sum_{i \in \mathcal{V}} -\mathbf{I}_{(i=w_{t+1})} \log(u_t^{[i]})\right) \qquad \text{cross-entropy loss}$$



- A longer window with larger N increases the model size, and short N cannot capture longer dependencies.

- Motivated by HMMs, **Recurrent Neural Networks (RNNs)** as a sequence mixer tracks a hidden state $h_t \in \mathbb{R}^{d_h}$ to capture the temporal dependencies, and uses it to compute the output $y_t \in \mathbb{R}^d$ as

$$\begin{aligned} h_t &= g(x_t, h_{t-1}), \text{ and} \\ y_t &= f(h_t) \end{aligned}$$

where $g(\cdot)$ and $f(\cdot)$ are parametric learnable functions, e.g., multiple layers of FFNs, that are *the same across all positions in the sequence,* motivated by hidden Markov models.

- Forward pass in **pre-training** for RNN, the pseudocode is for a single sentence

- Initialize the state $h_0 \leftarrow 0$ and loss $L \leftarrow 0$
- **For** $t = 1, \ldots, T$

$$h_t \leftarrow g(x_t, h_{t-1}),$$

RNN

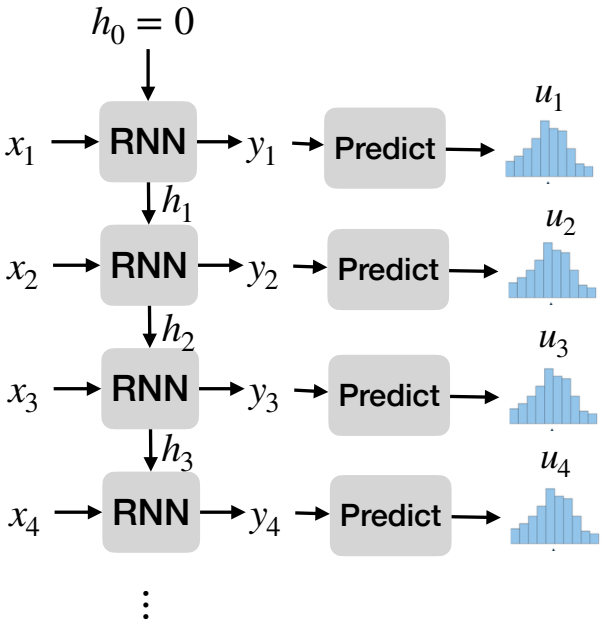
$$y_t \leftarrow f(h_t),$$

$$u_t \leftarrow \text{Predict}(y_t),$$

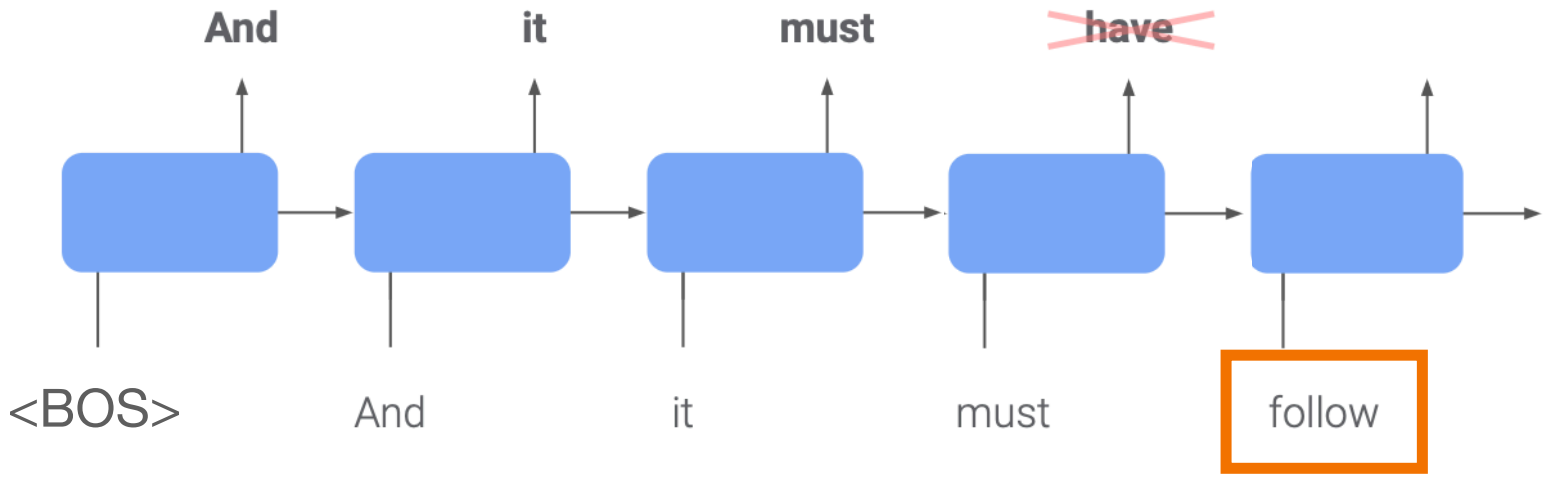
probability

$$L \mathrel{+}= \left(\sum_{i \in \mathcal{V}} - \mathbf{I}_{(i = w_{t+1})} \log(u_t^{[i]}) \right)$$

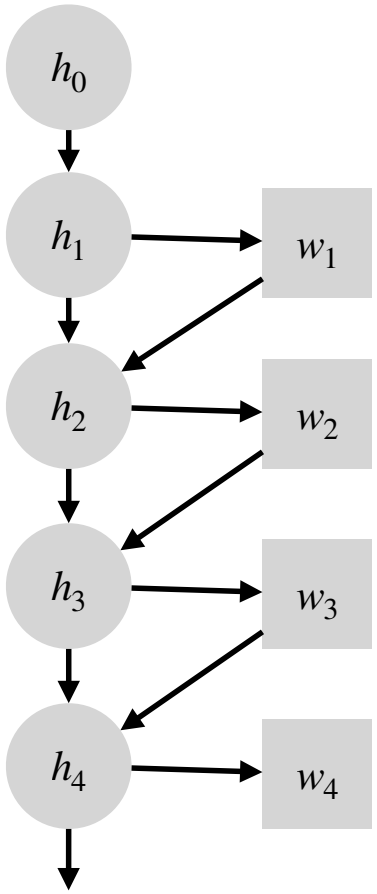
cross-entropy loss



- RNNs are trained via **teacher forcing** on next word prediction:
  - Given a training sequence of words, [And, it, must, follow, …], each RNN step predicts the next word, which is compared to the ground truths and the loss is computed.
  - The next RNN step is fed the ground truth word as an input, and not the prediction from the previous step.



- RNN-based LM is motivated by **Hidden Markov Models (HMMs),** which can capture temporal dependencies in a hidden state, h_t, that is not observed.
  - The probability of a next word only depends on the hidden state as $\mathbb{P}(w_t \mid w_{1:t-1}, h_{1:t-1}) = \mathbb{P}(w_t \mid h_{t-1})$ and $\mathbb{P}(h_t \mid w_{1:t}, h_{1:t-1}) = \mathbb{P}(h_t \mid w_t, h_{t-1})$.



- At **inference-time**, RNN models run auto-regressive inference over forward passes

- Initialize the state $h_0 \leftarrow 0$, $t \leftarrow 1$, and $x_1 \leftarrow \text{Emb}(\langle \text{BOS} \rangle)$
- **While True:**

$$h_t \leftarrow g(x_t, h_{t-1}),$$

RNN

$$y_t \leftarrow f(h_t),$$

$$u_t \leftarrow \text{Predict}(y_t),$$

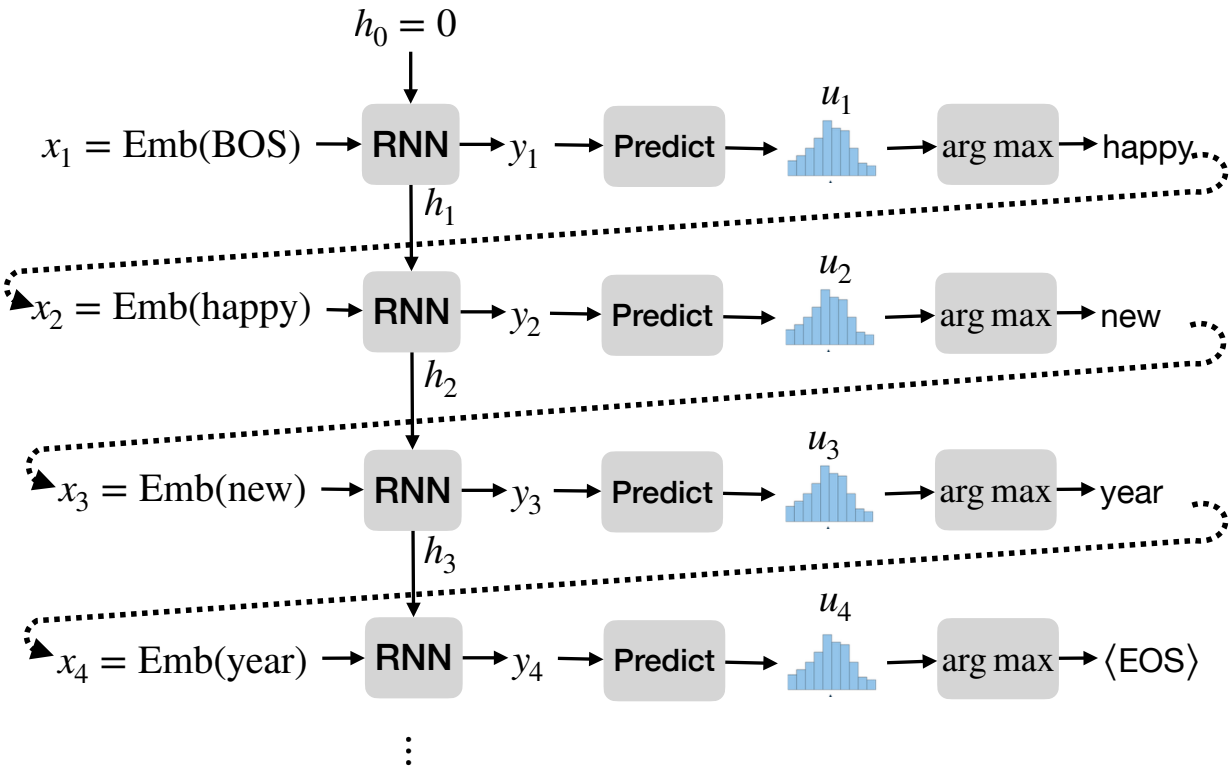
probability

Set x_{t+1} as the embedding of the token for $\arg\max_i u_t^{[i]} x_3 = \text{Emb}(\text{new})$

If x_{t+1} is $\text{Emb}(\langle \text{EOS} \rangle)$: **break**

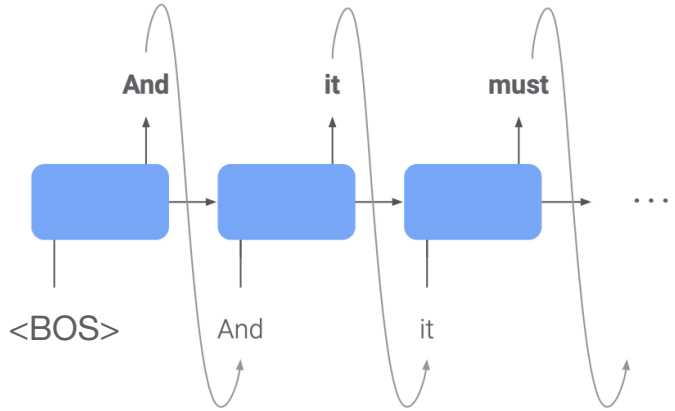
$$t \leftarrow t + 1$$

- **Output**: $x = [x_1, \cdots, x_t], y = [y_1, \ldots, y_{t+1}]$.

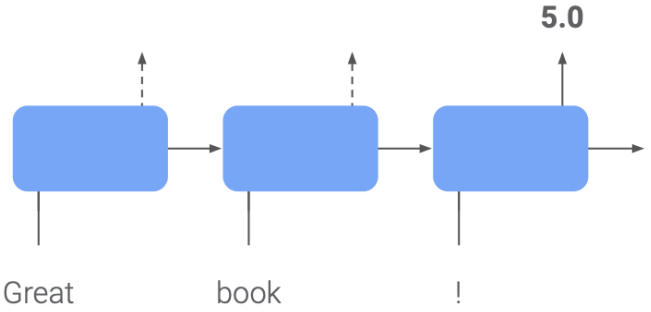


- Next word prediction is a fundamental task central to various other linguistic tasks, but how do we use RNN-based LMs to solve them?

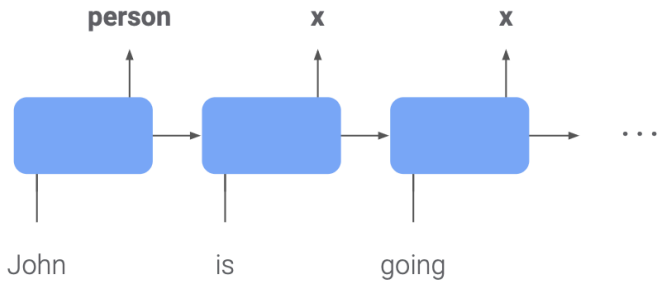
Next word prediction



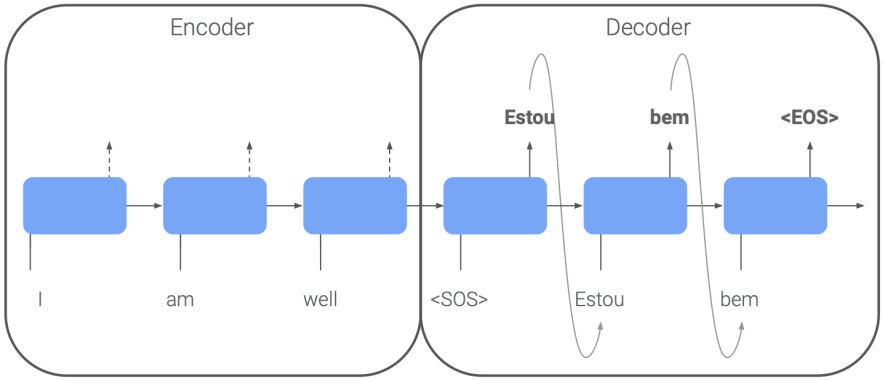
Sentiment analysis in 5 star rating



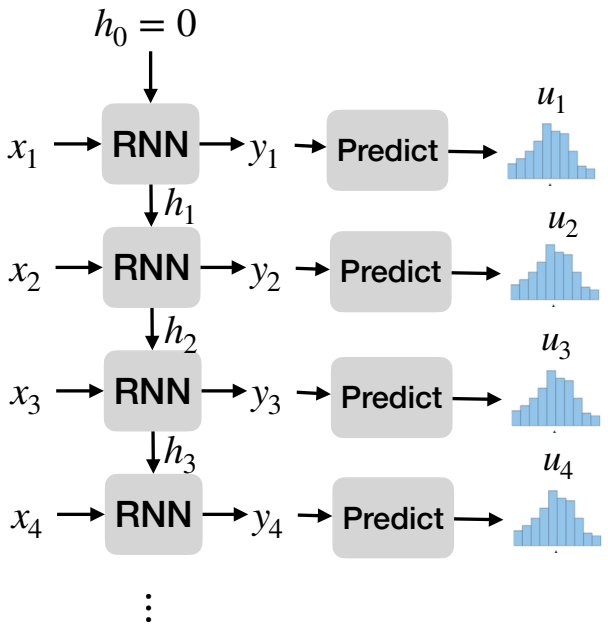
Names entity recognition



Machine translation



- RNN architectures only partially address the long-range dependency problem, since information in the hidden state gets diluted over time.
- Following problems persist:
  - long sequences have vanishing or exploding gradients [Pascanu et al. 2013, Hochreiter et al. 1997],
  - mode collapse (i.e., generating repetitive outputs),
  - struggle with highly variable input sizes due to limited memory in the hidden state, acting as a bottleneck.
- Resource considerations:
  - Inference memory: O(d).
  - Training complexity: O(Td)
  - Training time: no parallelization O(T) due to non-linearities.
- Many attempts to tackle these problems: LSTM [Hochreiter et al. 1997], GRUs [Cho et al. 2014]



Sources

- Other courses in LLMs that the lecture slides are based on
  - CSE493S/599S at UW by Ludwig Schmidt: https://mlfoundations.github.io/advancedml-sp23/
  - EE-628 at EPFL by Volkan Cevher: https://www.epfl.ch/labs/lions/teaching/ee-628-training-large-language-models/ee-628-slides-2025/
- Useful blog posts
  - https://azizbelaweid.substack.com/p/complete-summary-of-absolute-relative
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