Section 4: Quantization

CSE 493G1, Winter 2026

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Course Logistics

- Project Proposal due yesterday
- A2 due Sunday (2/1)
- W Interest Form due Monday (2/2)

Motivation & Background

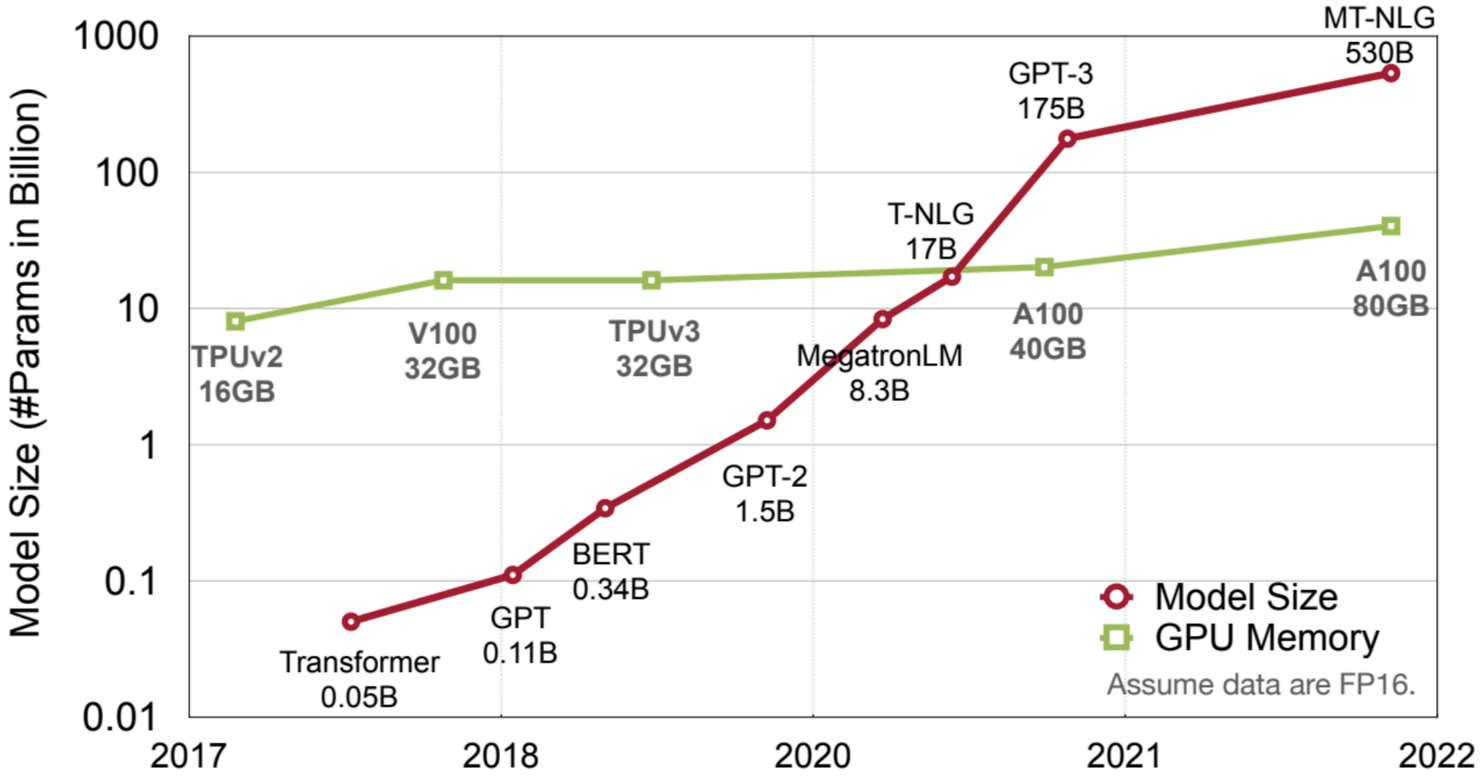
Decreasing model size*
is a worthy goal.

Decreasing model size*
is a worthy goal.

*while preserving performance

Decreasing model size is a worthy goal.

DL models are outgrowing hardware



Decreasing model size is a worthy goal.

AI Data Centers Are Sending Power Bills Soaring

Wholesale electricity costs as much as 267% more than it did five years ago in areas near data centers. That's being passed on to customers.

Cheyenne to host massive AI data center using more electricity than all Wyoming homes combined

AI data center 'frenzy' is pushing up your electric bill — here's why

A $10 Billion Data Center Could Rise Beside Horseshoe Bend, Just Outside of Arizona's Grand Canyon

The town of Page, Arizona, sold 500 acres of land near the Colorado River to a private developer. Some locals fear the facility will drain the region's already stressed water supply.

Children in the Democratic Republic of the Congo are subjected to the worst forms of child labor, including recruitment and use in armed conflict and forced labor mining gold, tin ore (cassiterite), tantalum ore (coltan), and tungsten ore (wolframite). Children also perform dangerous tasks mining cobalt ore (heterogenite) and copper ore.

Bloomberg; CNBC; Outside Magazine; Associated Press; U.S. Bureau of International Labor Affairs

Decreasing model size is a worthy goal.

Model	Precision	bits/param
Gemma 3 (1B, 4B, 12B, 27B)	int4	~ 4 bits/param
GPT OSS (20B, 120B)	mxfp4	~ 4 bits/param
Apple Intelligence (3B)		

model size = (# of parameters) × (parameter size)

$$\text{model size} = (\#\ \text{of parameters}) \times (\text{parameter size})$$

pruning

quantization

Parameter Size

How many bits does each parameter occupy? (“bit width”)

- FP32 ⇒ 4 bytes per parameter
- FP16, BF16 ⇒ 2 bytes per parameter
- FP8, int8 ⇒ 1 byte per parameter
- int4 ⇒ 0.5 bytes per parameter

Parameter Size

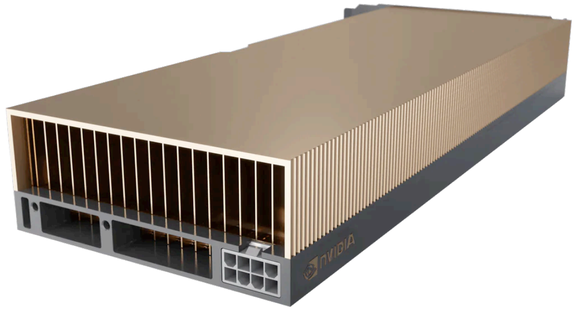
Qwen2.5-VL has 3B, 7B, and 72B versions

NVIDIA 2080 Ti

11GB memory

NVIDIA RTX 6000

24GB memory



NVIDIA A40

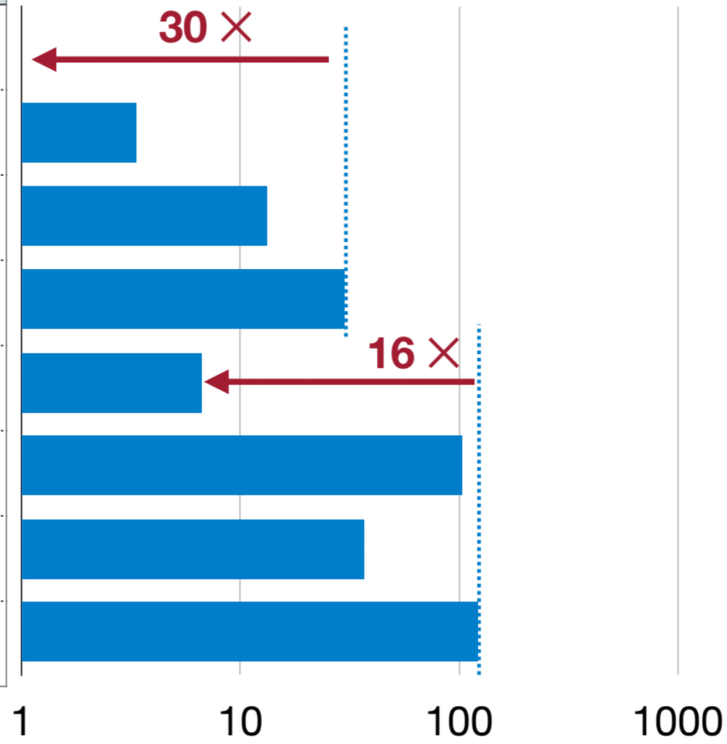
40GB memory

Parameter Size

Operations are cheaper on lower bit widths

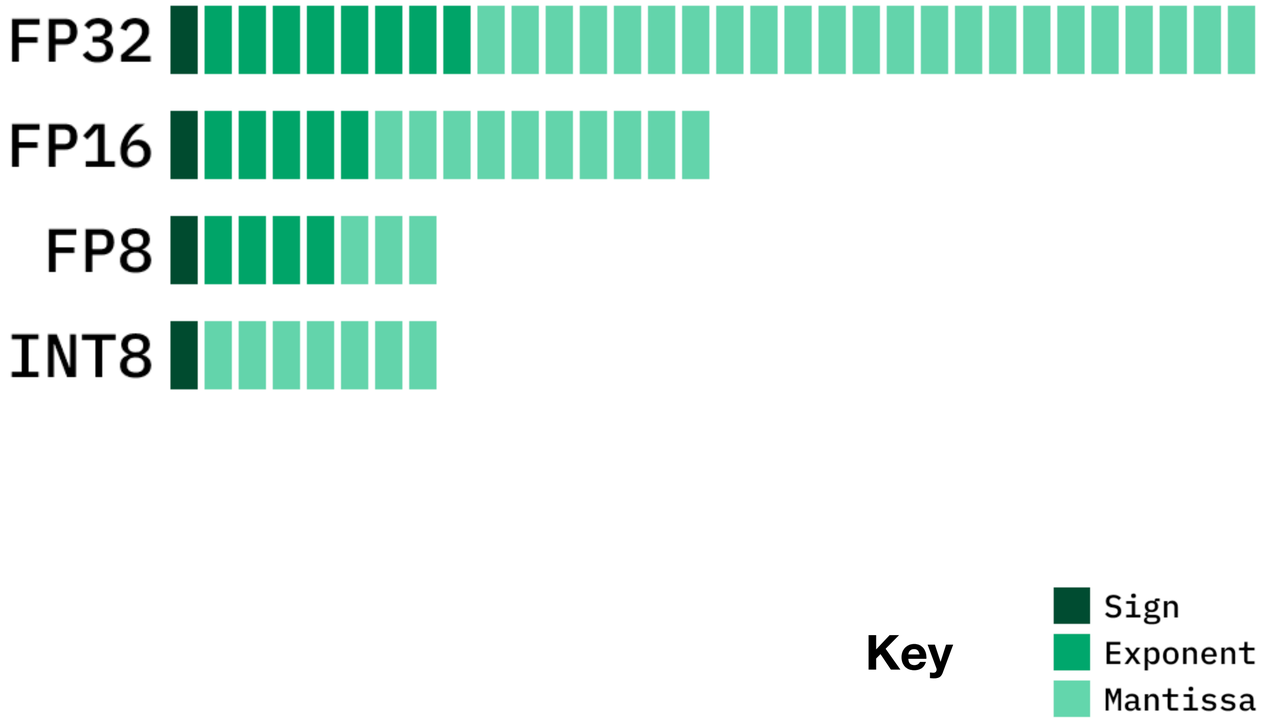
Operation	Energy [pJ]
8 bit int ADD	0.03
32 bit int ADD	0.1
16 bit float ADD	0.4
32 bit float ADD	0.9
8 bit int MULT	0.2
32 bit int MULT	3.1
16 bit float MULT	1.1
32 bit float MULT	3.7

Rough Energy Cost For Various Operations in 45nm 0.9V



Parameter Size

What do the underlying bits represent?



	Min	Max
FP32	3.4×10^{38}	-3.4×10^{38}
FP16	-65504	65504
FP8	-448	448
INT8	-128	127

Baseten, “FP8”

Quantization

- process of constraining an input from a continuous or otherwise large set of values to a smaller discrete set

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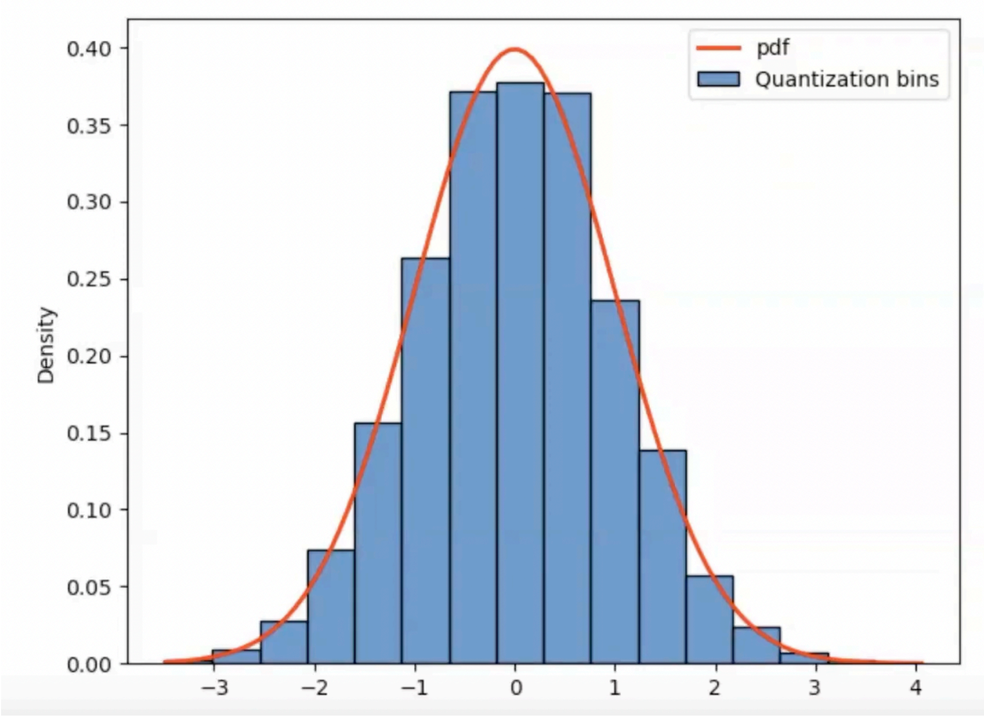

Tim Dettmers, “8-bit Methods”

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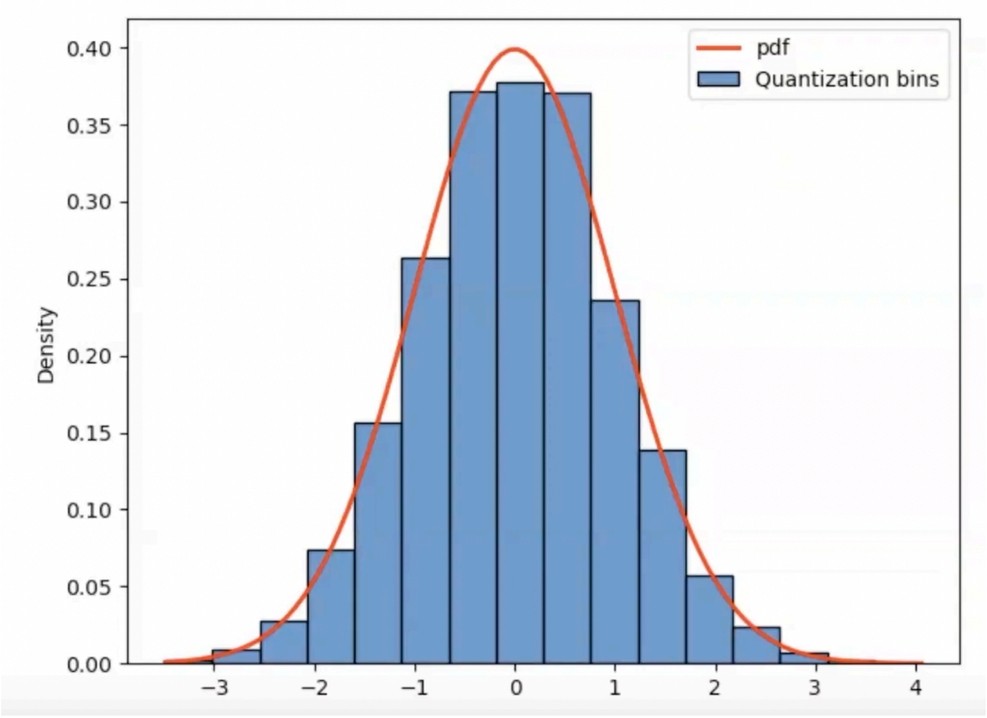

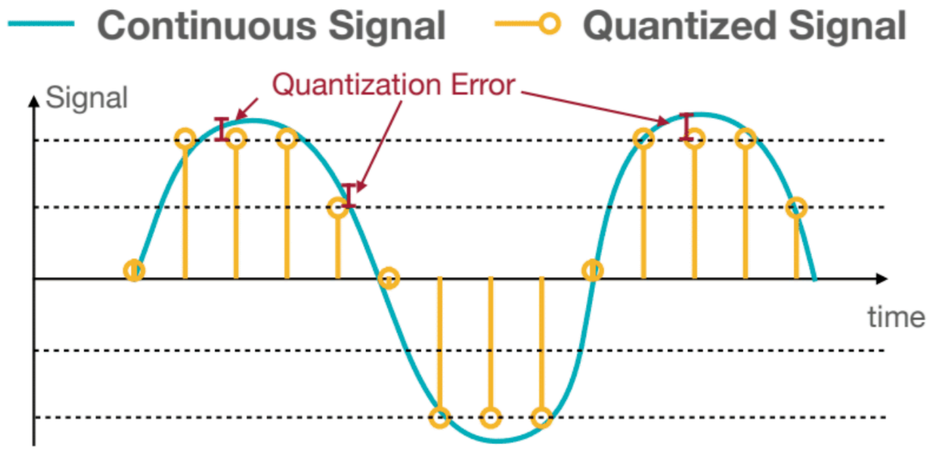

Tim Dettmers, "8-bit Methods"
Song Han, MIT 6.5940, Lecture 5

Quantization

In the context of deep learning

- process of constraining an input from a continuous or otherwise large set of values to a smaller discrete set

fp16 → int8

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Quantization

In the context of deep learning

- process of constraining an input from a continuous or otherwise large set of values to a smaller discrete set

fp16 → int8

while minimizing quantization error

Tim Dettmers, “8-bit Methods”
Song Han, MIT 6.5940, Lecture 5

Mathematical Quantization Techniques

Quantization

Formalizing our problem

- Given a matrix of floating point numbers, we wish to…
  - store it as an integer matrix from which we can re-construct a float matrix
  - minimize the difference between the original matrix and the reconstructed matrix (“quantization error”)

Quantization

Formalizing our problem

- Let q be an integer. We wish to map it to a real number r
  - Intuitively, we want to **shift** and **scale** our integer
  - Mathematically, we want to apply an **affine mapping**
    (that is, a transformation that preserves collinearity and ratio of distances)

Jacobs et al. 2018, "Efficient Integer-Arithmetic-Only Inference"

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    (that is, a transformation that preserves collinearity and ratio of distances)

$$r = S(q - Z) = Sq - Sz$$

S **is the scaling factor (float)**
Z **is the zero-point (int)**

Jacobs et al. 2018, "Efficient Integer-Arithmetic-Only Inference"

Zeropoint Quantization

$$r = S(q - Z)$$

Also known as “asymmetric quantization”

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  - for $q = 1$ we have $r = 1.5(1 - 1) = 1.5(0) = 0 \in \mathbb{R}$
  - for $q = 2$ we have $r = 1.5(2 - 1) = 1.5(1) = 1.5 \in \mathbb{R}$

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  - for $q = 2$ we have $r = 1.5(2 - 1) = 1.5(1) = 1.5 \in \mathbb{R}$
  - for $q = 3$ we have $r = 1.5(3 - 1) = 1.5(2) = 3 \in \mathbb{R}$

Zeropoint Quantization

We can go the other way too

$$r = S(q - Z) \quad \Rightarrow \quad q = \left\lfloor \frac{r}{S} + Z \right\rfloor$$

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- Regardless of which S and Z we choose, $r = 0$ maps **exactly** to $q = Z$

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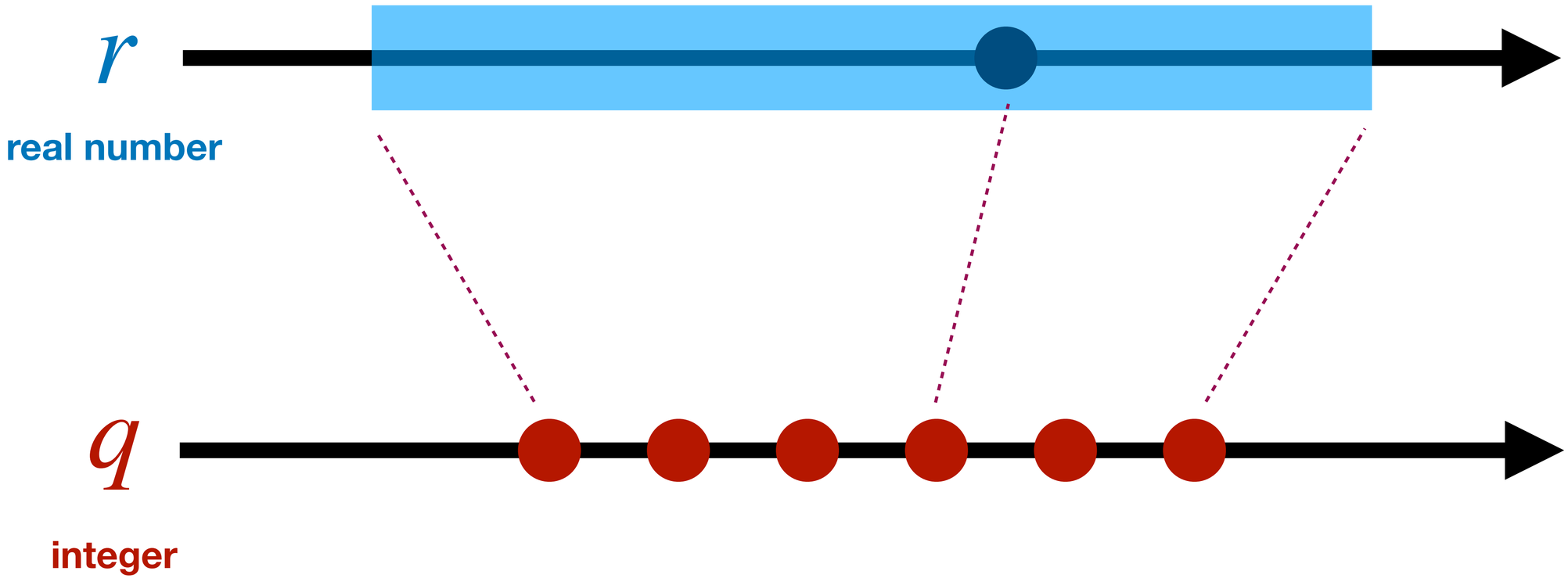

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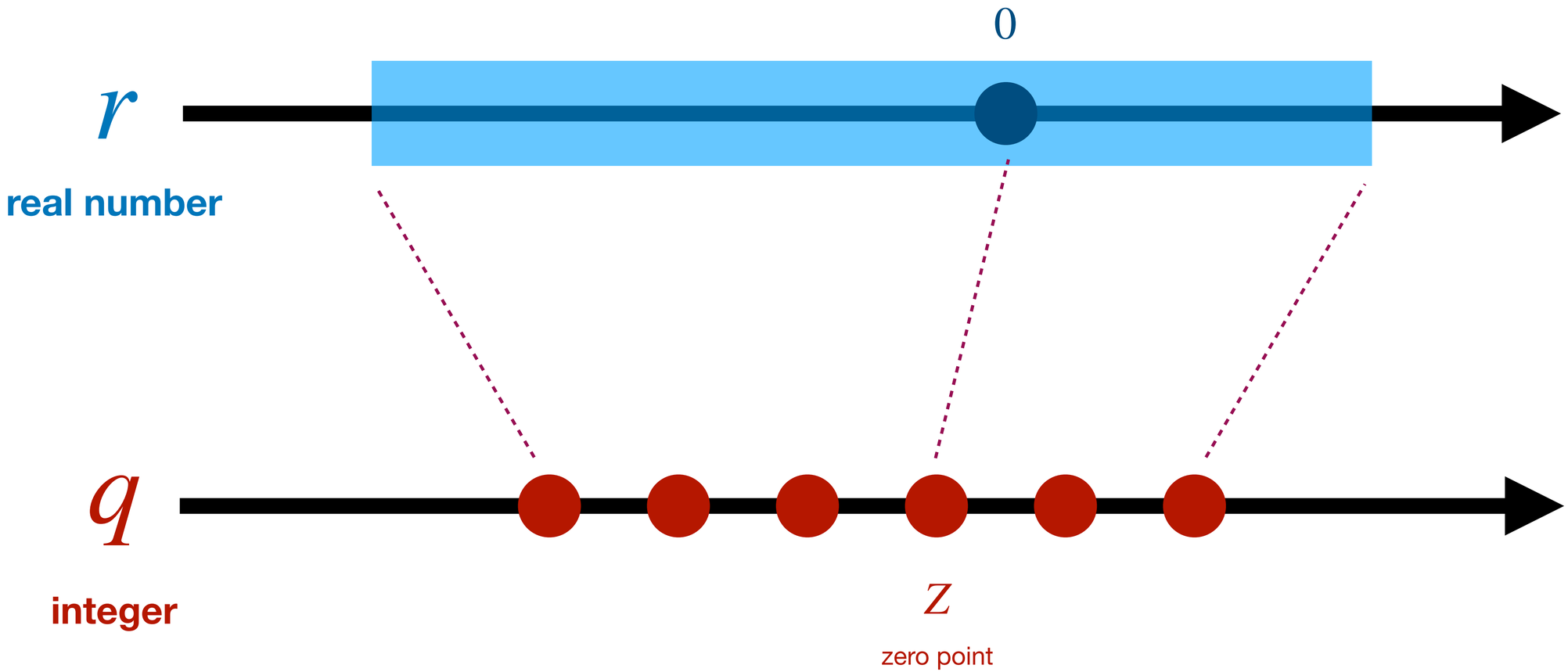

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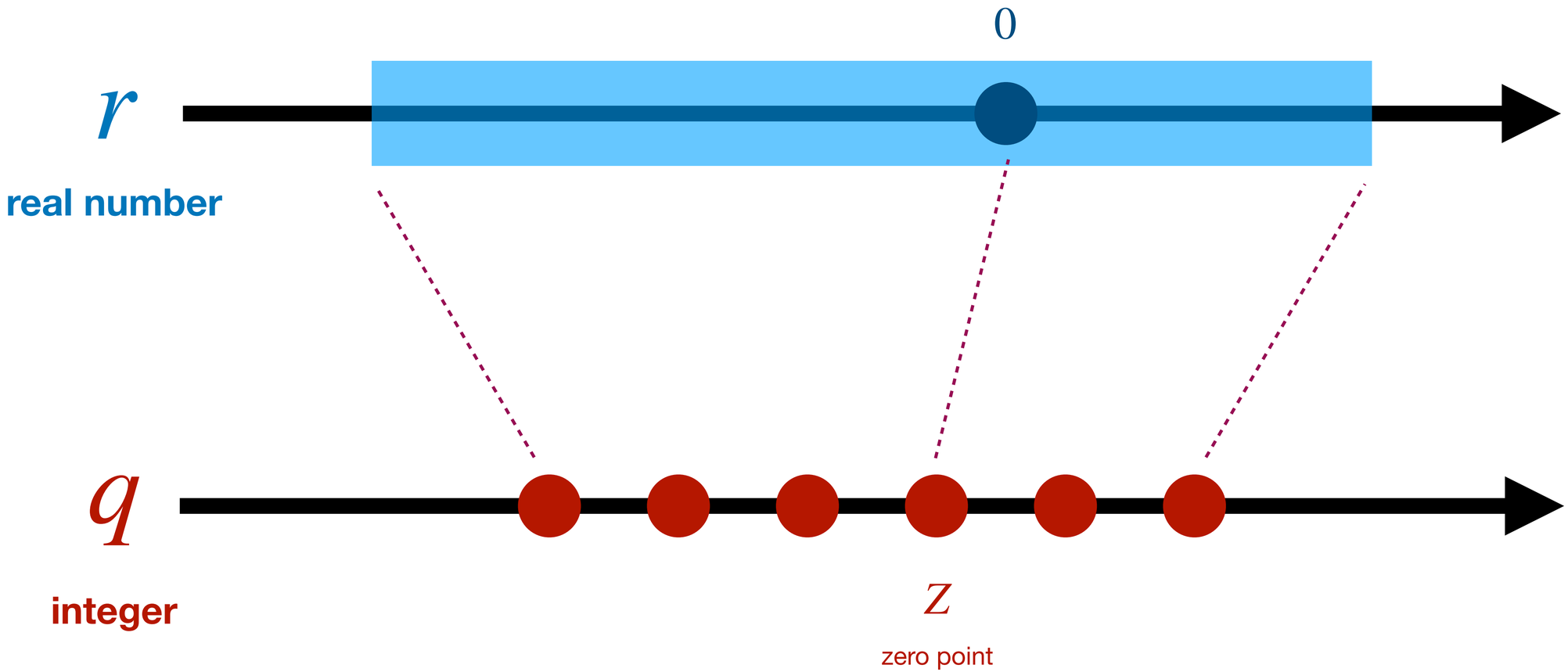
$$r = S(q - Z)$$

How do we determine S and Z ?



Quantization

Formalizing our problem

- Given a **specific** matrix,

fp16 → int8

while minimizing quantization error

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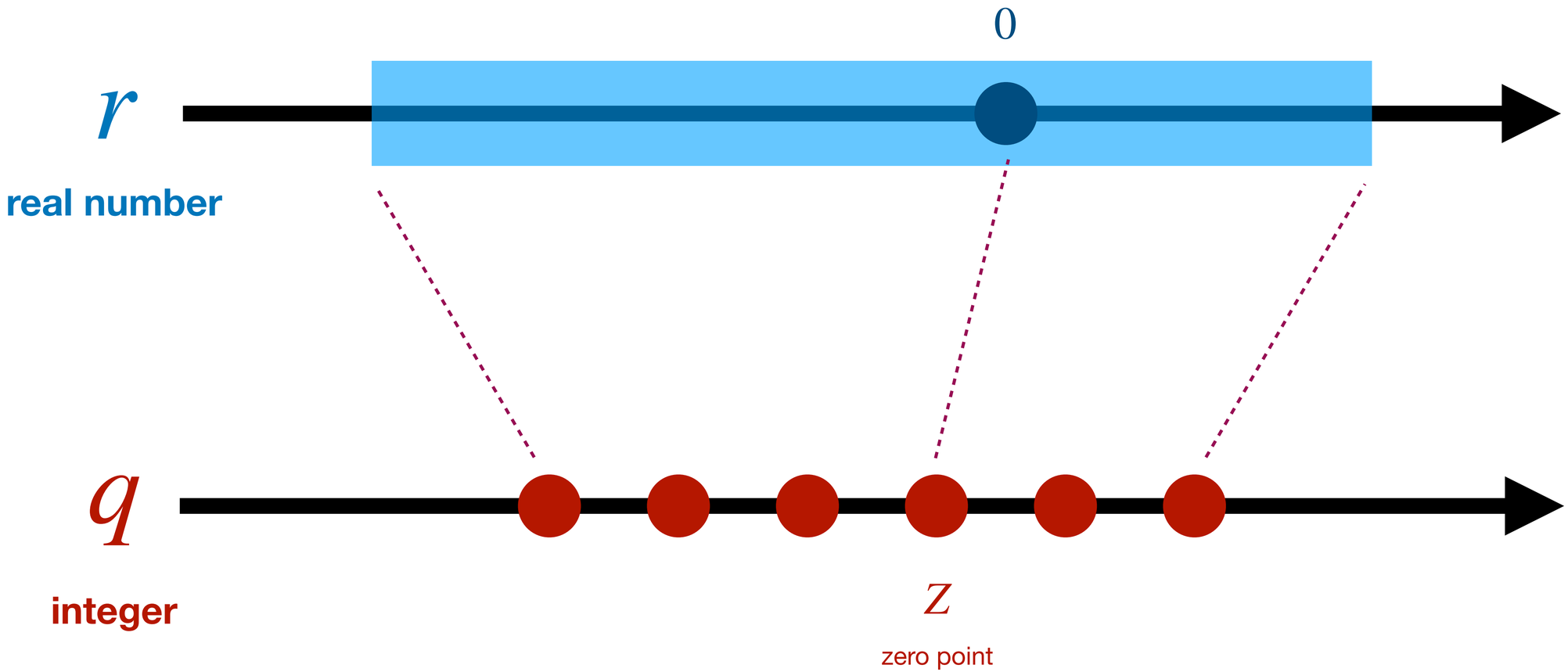

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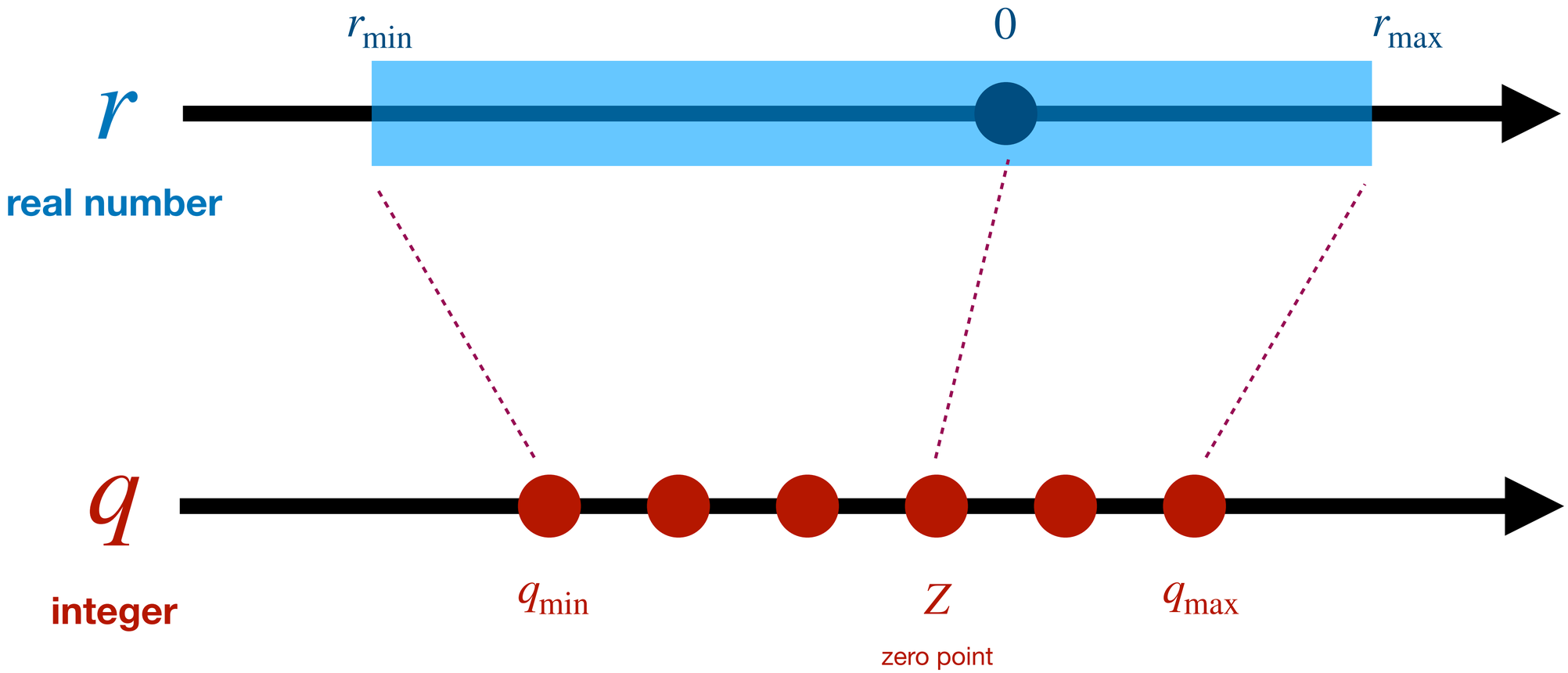

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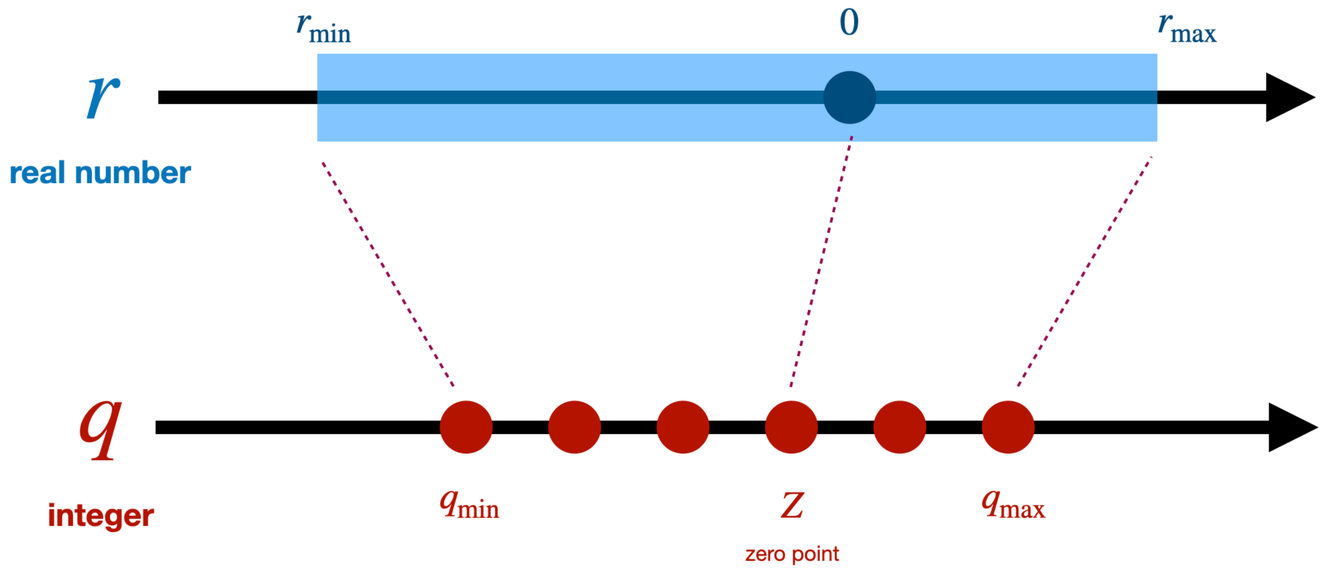

$$r_{min} = S\left(q_{min} - Z\right) \qquad r_{max} = S\left(q_{max} - Z\right)$$

Zeropoint Quantization

How do we determine S and Z ?

$$r_{\min} = S\left(q_{\min} - Z\right) \quad (1)$$

$$r_{\max} = S\left(q_{\max} - Z\right) \quad (2)$$

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Subtracting (1) from (2),

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$$r_{\max} - r_{\min} = S\left(q_{\max} - Z\right) - S\left(q_{\min} - Z\right)$$

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Simplifying,

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$$r_{\max} - r_{\min} = S\left(q_{\max} - q_{\min}\right) \quad \Rightarrow \quad \boxed{S = \frac{r_{\max} - r_{\min}}{q_{\max} - q_{\min}}}$$

Zeropoint Quantization

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$$r_{\min} = S\left(q_{\min} - Z\right) \quad \Rightarrow \quad Z = \left\lfloor q_{\min} - \frac{r_{\min}}{S} \right\rceil$$

Zeropoint Quantization

Quantize the following matrix to 2-bit signed ints

2.09	-0.98	1.48	0.09
0.05	-0.14	**-1.08**	**2.12**
0.91	1.92	0	-1.03
1.87	0	1.53	1.49

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$$r_{\min} = -1.08 \qquad r_{\max} = 2.12$$

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Quantize the following matrix to 2-bit signed ints

Recall from CSE 351 that a N-bit signed integer has the range $[-2^{N-1}, 2^{N-1}-1]$

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$$S = \frac{r_{\max} - r_{\min}}{q_{\max} - q_{\min}} = \frac{2.12 - (-1.08)}{1 - (-2)} = 1.07 \in \mathbb{R}$$

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$$Z = \left\lfloor q_{\min} - \frac{r_{\min}}{S} \right\rceil = \left\lfloor -2 - \frac{-1.08}{1.07} \right\rceil = -1 \in \mathbb{Z}$$

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$$q = \left\lfloor \frac{r}{1.07} - 1 \right\rceil$$

$S = 1.07 \qquad Z = -1$

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Original Weights (float)

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$$q = \left\lfloor \frac{r}{1.07} - 1 \right\rceil$$

Quantized Weights (int2)

1	-2	0	-1
-1	-1	-2	1
-2	1	-1	-2
1	-1	0	0

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-2	1	-1	-2
1	-1	0	0

Examples

Zeropoint Quantization

Quantize the following matrix to 2-bit signed ints

Original Weights (float)

2.09	-0.98	1.48	0.09
0.05	-0.14	-1.08	2.12
0.91	1.92	0	-1.03
1.87	0	1.53	1.49

$$q = \left\lfloor \frac{r}{1.07} - 1 \right\rfloor$$

Quantized Weights (int2)

1	-2	0	-1
-1	-1	-2	1
-2	1	-1	-2
1	-1	0	0

Examples $q = \left\lfloor \frac{r}{1.07} - 1 \right\rfloor = \left\lfloor \frac{1.48}{1.07} - 1 \right\rfloor = \lfloor 1.38 - 1 \rfloor = \lfloor 0.38 \rfloor = 0$

Zeropoint Quantization

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$q = \left\lfloor \frac{r}{1.07} - 1 \right\rceil = \left\lfloor \frac{1.87}{1.07} - 1 \right\rceil = \lfloor 1.75 - 1 \rceil = \lfloor 0.75 \rceil = 1$

Zeropoint Quantization

Determine the quantization error

Quantized Weights (int2)

1	-2	0	-1
-1	-1	-2	1
-2	1	-1	-2
1	-1	0	0

Zeropoint Quantization

Determine the quantization error

Quantized Weights (int2)

1	-2	0	-1
-1	-1	-2	1
-2	1	-1	-2
1	-1	0	0

$$r = 1.07(q + 1)$$

Zeropoint Quantization

Determine the quantization error

Quantized Weights (int2)

1	-2	0	-1
-1	-1	-2	1
-2	1	-1	-2
1	-1	0	0

$$r = 1.07(q + 1)$$

Reconstructed Weights (float)

2.14	-1.07	1.07	0
0	0	-1.07	2.14
-1.07	2.14	0	-1.07
2.14	0	1.07	1.07

Zeropoint Quantization

Determine the quantization error

Quantized Weights (int2)

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Examples

Zeropoint Quantization

Determine the quantization error

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2.14	-1.07	1.07	0
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2.14	0	1.07	1.07

Examples $r = 1.07(q + 1) = 1.07(0 + 1) = 1.07$

Zeropoint Quantization

Determine the quantization error

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$r = 1.07(q + 1)$

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0	0	-1.07	2.14
-1.07	2.14	0	-1.07
2.14	0	1.07	1.07

Examples $r = 1.07(q + 1) = 1.07(0 + 1) = 1.07$ $r = 1.07(q + 1) = 1.07(1 + 1) = 2.14$

Zeropoint Quantization

Determine the quantization error

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-1	-1	-2	1
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Zeropoint Quantization

Determine the quantization error

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Zeropoint Quantization

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Reconstructed Weights (float)

2.14	-1.07	1.07	0
0	0	-1.07	2.14
-1.07	2.14	0	-1.07
2.14	0	1.07	1.07

Quantization Error

-0.05	0.09	0.41	0.09
0.05	-0.14	-0.01	-0.02
0.16	-0.22	0	0.04
-0.27	0	0.46	0.42

Zeropoint Quantization

Practice!

- Quantize the following matrix into 3-bit signed integers
- Reconstruct the weights from the quantized weights
- Calculate the quantization error

18	7	8
3	6	12
0	-3	-2

Zeropoint Quantization

Practice!

- Quantize the following matrix into 3-bit signed integers
- Reconstruct the weights from the quantized weights
- Calculate the quantization error

18	7	8
3	6	12
0	-3	-2

Recall from CSE 351 that a N-bit signed integer has the range $[-2^{N-1}, 2^{N-1} - 1]$

Zeropoint Quantization

Practice!

Original Weights (float)

15	6	9
3	-3	12
0	-6	0

Zeropoint Quantization

Practice!

Original Weights (float)

15	6	9
3	-3	12
0	-6	0

$r_{\text{min}} = -6$ $\quad$ $r_{\text{max}} = 15$

Zeropoint Quantization

Practice!

Original Weights (float)

15	6	9
3	-3	12
0	-6	0

$$r_{\min} = -6 \qquad r_{\max} = 15$$

$$S = 3 \qquad Z = -1$$

Zeropoint Quantization

Practice!

Original Weights (float)

15	6	9
3	-3	12
0	-6	0

$$q = \left\lfloor \frac{r}{3} - 1 \right\rfloor$$

$$r_{\min} = -6 \qquad r_{\max} = 15$$

$$S = 3 \qquad Z = -1$$

Zeropoint Quantization

Practice!

Original Weights (float)

15	6	9
3	-3	12
0	-6	0

$$q = \left\lfloor \frac{r}{3} - 1 \right\rfloor$$

Quantized Weights (int3)

4	1	2
0	-2	3
-1	-3	-1

$r_{\min} = -6$ $\quad$ $r_{\max} = 15$

$S = 3$ $\quad$ $Z = -1$

Zeropoint Quantization

Practice!

Quantized Weights (int3)

4	1	2
0	-2	3
-1	-3	1

Zeropoint Quantization

Practice!

Quantized Weights (int3)

4	1	2
0	-2	3
-1	-3	1

$$r = 3(q + 1)$$

Zeropoint Quantization

Practice!

Quantized Weights (int3)

4	1	2
0	-2	3
-1	-3	1

$$r = 3(q + 1)$$

Reconstructed Weights (float)

15	6	9
3	-3	12
0	-6	0

Zeropoint Quantization

Practice!

Original Weights (float)

15	6	9
3	-3	12
0	-6	0

Zeropoint Quantization

Practice!

Original Weights (float)

15	6	9
3	-3	12
0	-6	0

Reconstructed Weights (float)

15	6	9
3	-3	12
0	-6	0

Zeropoint Quantization

Matmul as an integer-arithmetic-only operation

Zeropoint Quantization

Matmul as an integer-arithmetic-only operation

$$r = S(q - Z)$$

$$q = \left\lfloor \frac{r}{S} + Z \right\rceil$$

Zeropoint Quantization

Matmul as an integer-arithmetic-only operation

- We mostly care about the first equation

$$r = S(q - Z)$$

$$q = \left\lfloor \frac{r}{S} + Z \right\rceil$$

Zeropoint Quantization

Matmul as an integer-arithmetic-only operation

- We mostly care about the first equation
- S is still stored as a float 😔

$$r = S(q - Z)$$

$$q = \left\lfloor \frac{r}{S} + Z \right\rceil$$

Zeropoint Quantization

$$r = S(q - Z)$$

Matmul as an integer-arithmetic-only operation

Let $A, B \in \mathbb{R}^{n \times n}$. Let $\hat{A}, \hat{B} \in \mathbb{Z}^{n \times n}$ be their quantized versions. Let $C = AB$. Find $\hat{C}$.

Notation: for some matrix M, we will refer to the element in its i-th row and j-th column as m_{ij}.

Zeropoint Quantization

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By the definition of matrix multiplication,

$$c_{ij} = \sum_{k=1}^{N} a_{ik} b_{kj}$$

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By the definition of matrix multiplication,

$$c_{ij} = \sum_{k=1}^{N} a_{ik} b_{kj}$$

Applying our quantization schemes,

$$S_C \left(\hat{c}_{ij} - Z_C \right) = \sum_{k=1}^{N} S_A \left(\hat{a}_{ij} - Z_A \right) S_B \left(\hat{b}_{ij} - Z_B \right)$$

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Factoring constants out of the sum,

$$S_C\left(\hat{c}_{ij} - Z_C\right) = S_A S_B \sum_{k=1}^{N} \left(\hat{a}_{ij} - Z_A\right)\left(\hat{b}_{ij} - Z_B\right)$$

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Isolating $\hat{c}_{ij}$ on the left,

$$\hat{c}_{ij} = Z_C + \frac{S_A S_B}{S_C} \sum_{k=1}^{N}\left(\hat{a}_{ij} - Z_A\right)\left(\hat{b}_{ij} - Z_B\right)$$

Jacobs et al. 2018, “Efficient Integer-Arithmetic-Only Inference”

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Matmul as an integer-arithmetic-only operation

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Define $M := \frac{S_A S_B}{S_C}$. Then,

$$\hat{c}_{ij} = Z_C + M \sum_{k=1}^{N} \left(\hat{a}_{ij} - Z_A \right) \left(\hat{b}_{ij} - Z_B \right)$$

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Empirically, $M \in (0,1)$**. Normalizing for some** $M_0 \in [0.5,1)$**,**

$$M = 2^{-n} \cdot M_0$$

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fixed-point multiplier

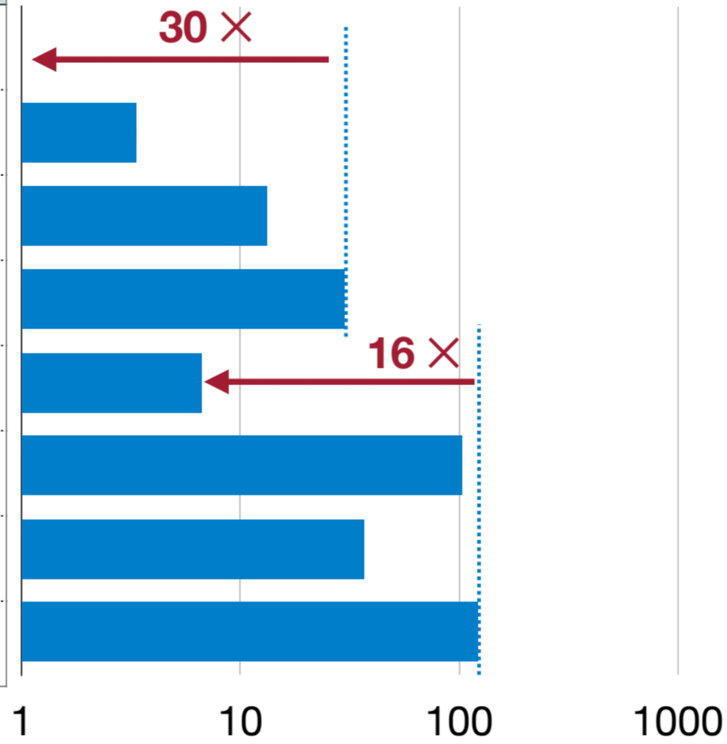
Assuming int32, this will always have 30 bits of relative accuracy. Why?

Zeropoint Quantization

We can now do integer-only matmul!

Operation	Energy [pJ]
8 bit int ADD	0.03
32 bit int ADD	0.1
16 bit float ADD	0.4
32 bit float ADD	0.9
8 bit int MULT	0.2
32 bit int MULT	3.1
16 bit float MULT	1.1
32 bit float MULT	3.7

Rough Energy Cost For Various Operations in 45nm 0.9V



Zeropoint Quantization

Practical Considerations

$$\hat{c}_{ij} = Z_C + M \sum_{k=1}^{N} \left(\hat{a}_{ij} - Z_A \right) \left(\hat{b}_{ij} - Z_B \right)$$

Dettmers et al. 2022, “LLM.int8()”

Zeropoint Quantization

Practical Considerations

$$\hat{c}_{ij} = Z_C + M \sum_{k=1}^{N} \left(\hat{a}_{ij} - Z_A \right) \left(\hat{b}_{ij} - Z_B \right)$$

The bulk of the computation occurs in the sum:

$$\left(\hat{a}_{ij} - Z_A \right) \left(\hat{b}_{ij} - Z_B \right) = \sum_{k=1}^{N} \hat{a}_{ij}\hat{b}_{ij} - \hat{a}_{ij}Z_B - \hat{b}_{ij}Z_A + Z_A Z_B$$

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CPUs contain `PMADDUBSW`, but most GPUs/TPUs don't.

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int8

int16

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int8

int16

CPUs contain `PMADDUBSW`, but most GPUs/TPUs don't.

The zeropoint slows us down tremendously!

Dettmers et al. 2022, "LLM.int8()"

Absolute Maximum Quantization

Also known as “symmetric quantization”

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- What if get rid of the zeropoint?

$$r = S(q - Z) \quad \Rightarrow \quad r = Sq$$

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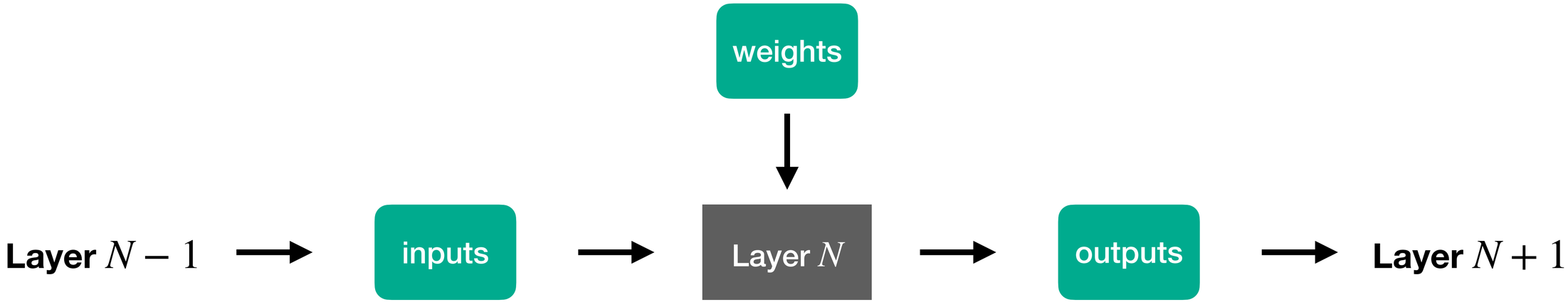
- This forces a symmetric quantization scheme. Why?

$$S = \frac{r_{\max} - r_{\min}}{q_{\max} - q_{\min}} \approx \frac{2 \cdot r_{absmax}}{2 \cdot q_{\max}} = \frac{r_{absmax}}{q_{\max}}$$

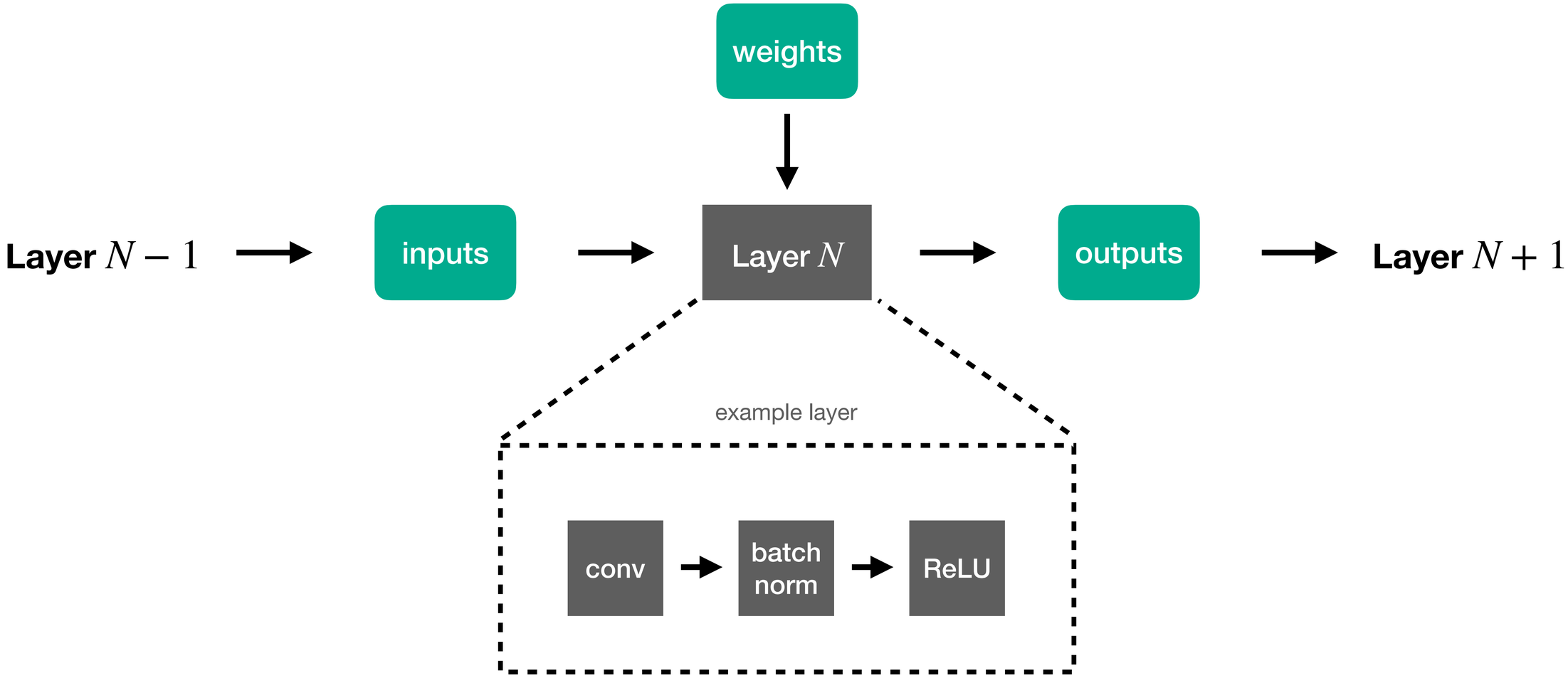
Deep Learning Quantization Paradigms

Quantization-Aware Training

Quantization-Aware Training

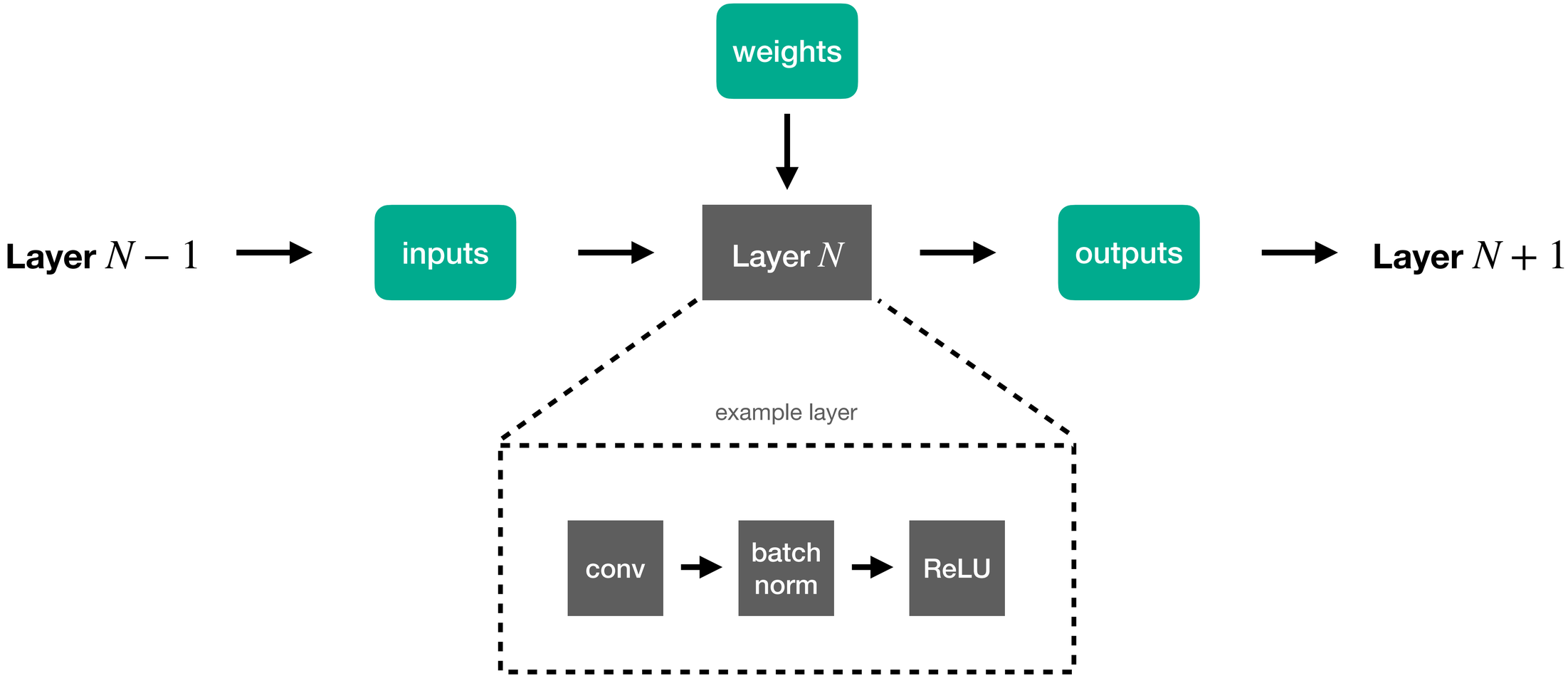


Quantization-Aware Training



Quantization-Aware Training

Simulated / Fake Quantization



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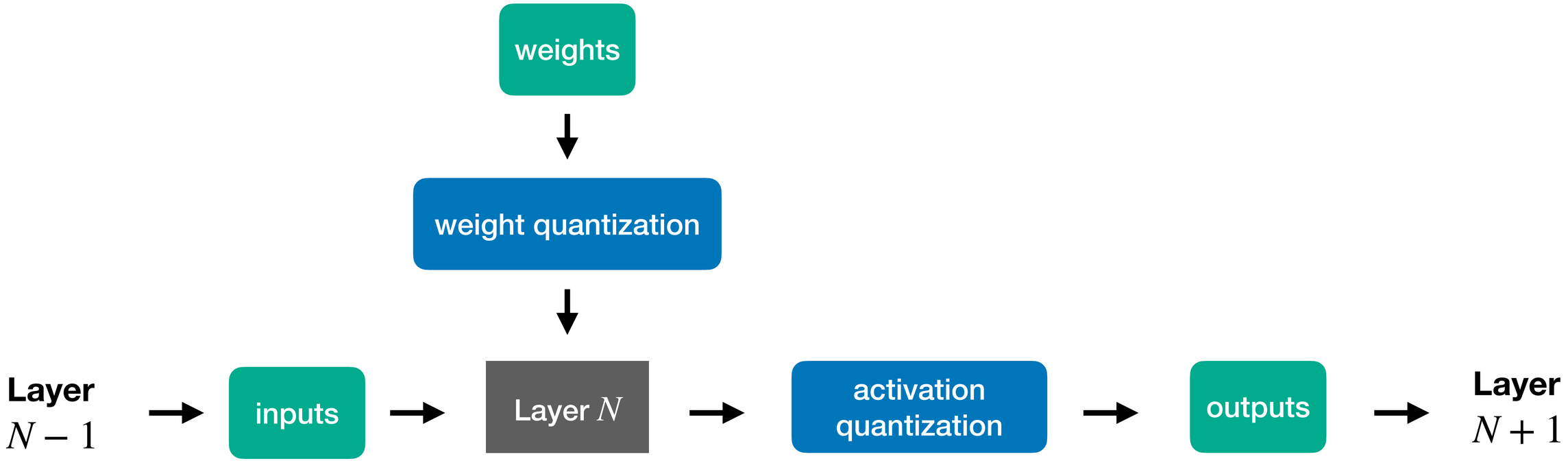

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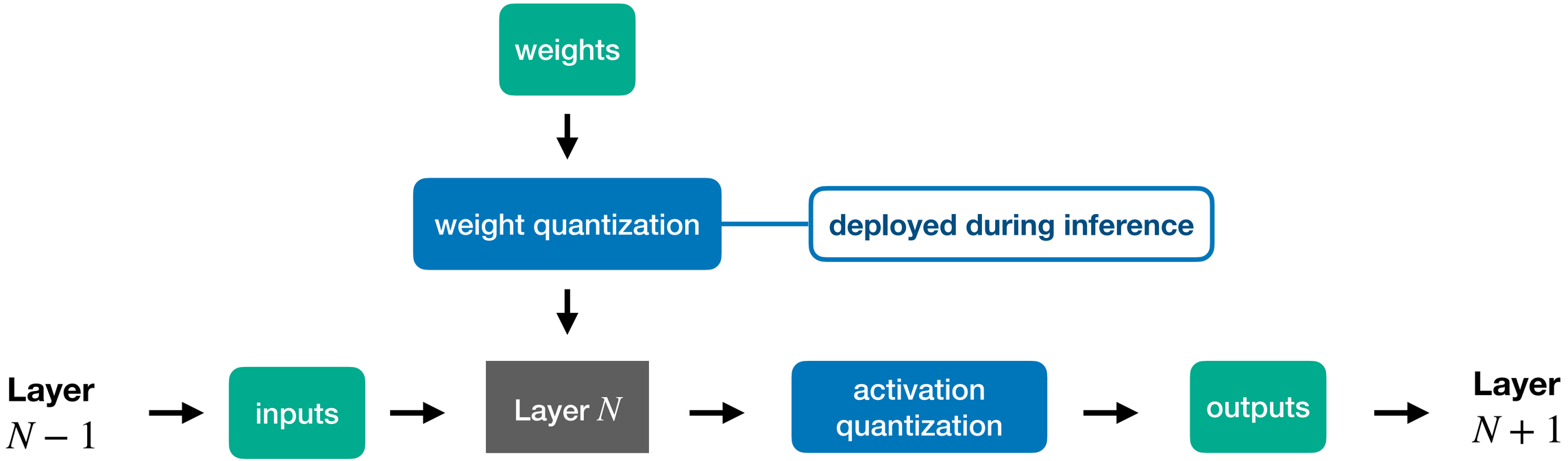

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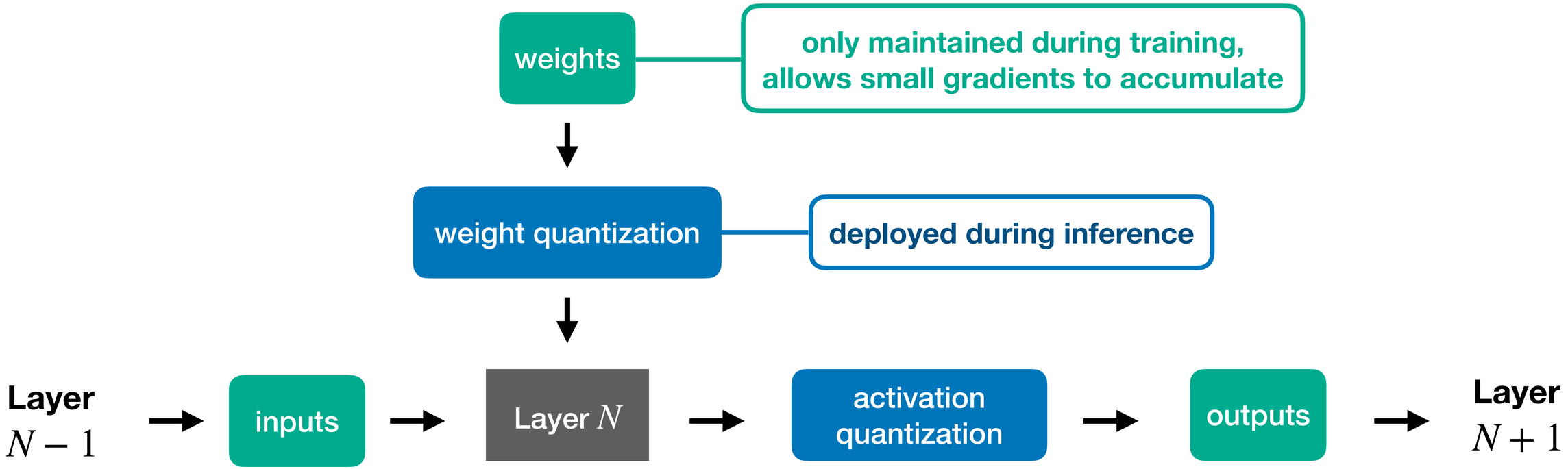

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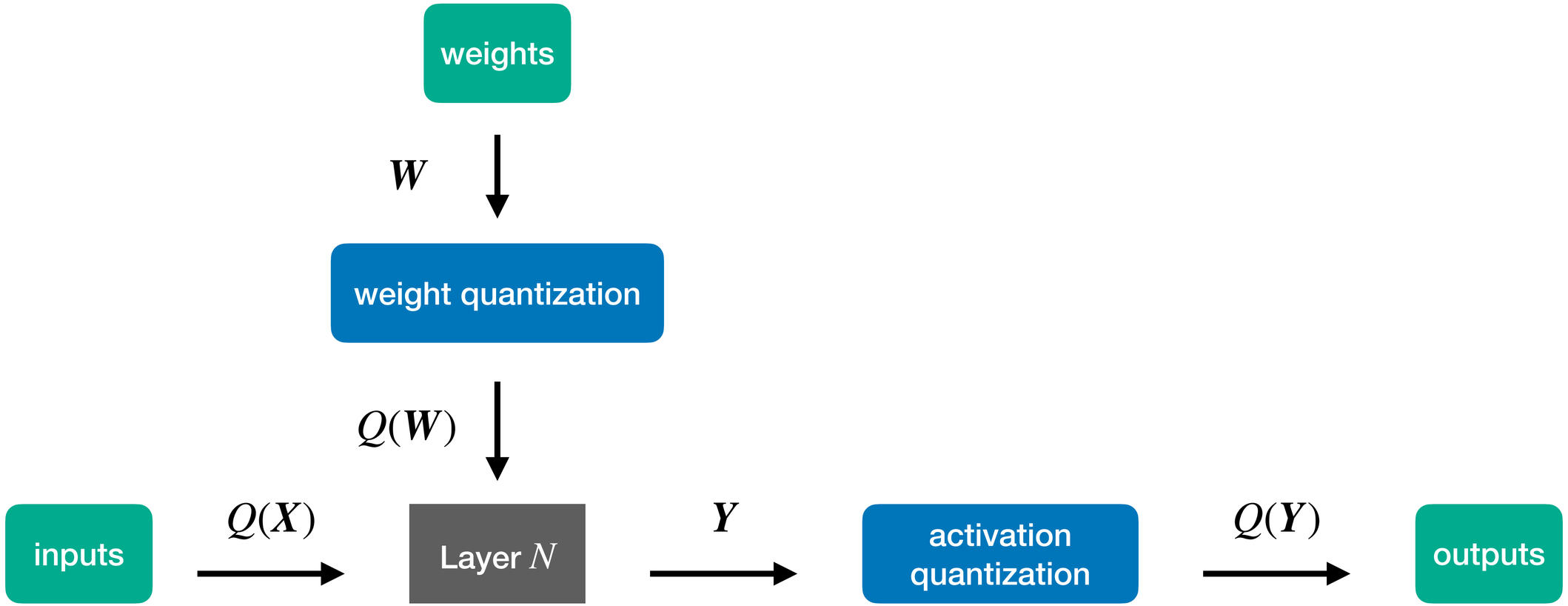
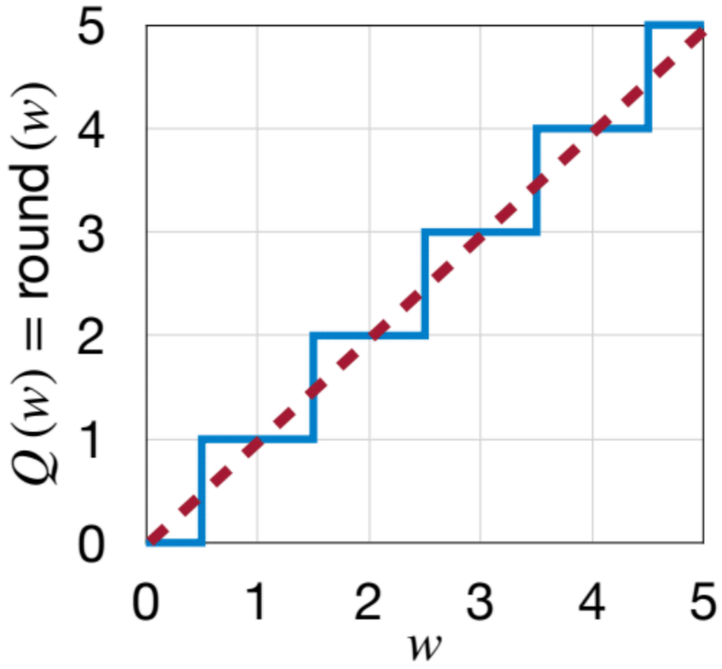


Quantization-Aware Training

Straight-Through Estimator (STE)

- Quantization is a step function ⇒ gradient is almost always 0



Quantization-Aware Training

Straight-Through Estimator (STE)

- Quantization is a step function ⇒ gradient is almost always 0
- STE passes gradients along “as if it had been the identity function”

$$\frac{\partial L}{\partial \boldsymbol{W}} = \frac{\partial L}{\partial Q(\boldsymbol{W})} \cdot \frac{\partial Q(\boldsymbol{W})}{\partial \boldsymbol{W}} = \frac{\partial L}{\partial Q(\boldsymbol{W})} \cdot 1 = \frac{\partial L}{\partial Q(\boldsymbol{W})}$$

Bengio et al. 2013, “Estimating or Propagating Gradients”

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chain rule

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chain rule

straight-through estimator

Bengio et al. 2013, “Estimating or Propagating Gradients”

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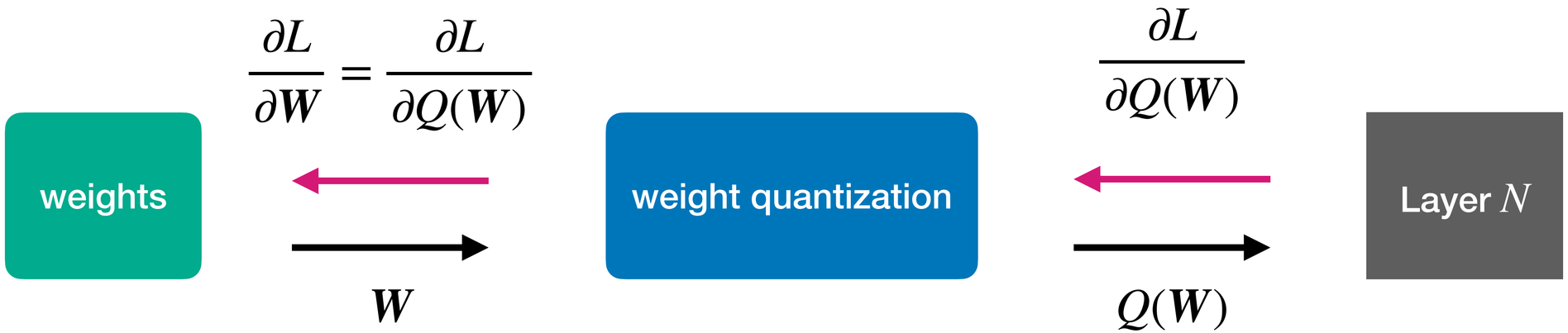

Bengio et al. 2013, “Estimating or Propagating Gradients”

Quantization-Aware Training

Practical Considerations

- Quite effective, especially useful for really large models
  - Requires training (or re-training) the model 😔
- Also called “Mixed Precision Training”

Krishnamoorthi 2018, “Quantizing deep convolutional networks”

Post-Training Quantization

Practical Considerations

Krishnamoorthi 2018, “Quantizing deep convolutional networks”

Post-Training Quantization

Practical Considerations

Model Size	Model Name	Asymmetric PTQ	Symmetric PTQ	Asymmetric QAT	Symmetric QAT	Floating Point
3.5M	MobileNet-v2	0.1%	69.8%	70.9%	71.1%	71.9%
25M	ResNet-50	75%	75%	75%	75%	75.6%
60M	ResNet-152	76.1%	76%	76%	76%	77.8%

Krishnamoorthi 2018, “Quantizing deep convolutional networks”

Post-Training Quantization

Practical Considerations

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60M	ResNet-152	76.1%	76%	76%	76%	77.8%

Smaller models suffer from PQT, likely because they have a smaller representational capacity

Quantization Libraries

& their underlying PTQ methods

LLM.int8()

Seminal work in quantization for LLMs

Dettmers et al. 2022, “LLM.int8()”

LLM.int8()

Seminal work in quantization for LLMs

- Implemented in `bitsandbytes`
- Integrated with 🤗 Transformers

Dettmers et al. 2022, “LLM.int8()”

LLM.int8()

Seminal work in quantization for LLMs

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Parameters	125M	1.3B	2.7B	6.7B	13B
32-bit Float	25.65	15.91	14.43	13.30	12.45
Int8 absmax	87.76	16.55	15.11	14.59	19.08
Int8 zeropoint	56.66	16.24	14.76	13.49	13.94
Int8 absmax row-wise	30.93	17.08	15.24	14.13	16.49
Int8 absmax vector-wise	35.84	16.82	14.98	14.13	16.48
Int8 zeropoint vector-wise	25.72	15.94	14.36	13.38	13.47
Int8 absmax row-wise + decomposition	30.76	16.19	14.65	13.25	12.46
Absmax LLM.int8() (vector-wise + decomp)	25.83	15.93	14.44	**13.24**	**12.45**
Zeropoint LLM.int8() (vector-wise + decomp)	**25.69**	**15.92**	**14.43**	**13.24**	**12.45**

perplexity scores

Dettmers et al. 2022, “LLM.int8()”

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perplexity scores

previously discussed methods

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perplexity scores

previously discussed methods

what we're about to discuss

Dettmers et al. 2022, "LLM.int8()"

LLM.int8()

Absmax Quantization

$$r = S(q - Z) \quad \Rightarrow \quad r = Sq \quad \Rightarrow \quad q = \left\lfloor \frac{r}{S} \right\rceil$$

$$S = \frac{r_{\max} - r_{\min}}{q_{\max} - q_{\min}} \approx \frac{r_{absmax}}{q_{\max}} = \frac{r_{absmax}}{127}$$

int8 $\Rightarrow q_{\max} = 127$

Dettmers et al. 2022, “LLM.int8()”

LLM.int8()

Absmax Quantization

$$r = S(q - Z) \quad \Rightarrow \quad r = Sq \quad \Rightarrow \quad q = \left\lfloor \frac{r}{S} \right\rceil$$

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quantized → real

$$\frac{r_{absmax}}{127}$$

real → quantized

$$\frac{127}{r_{absmax}}$$

we will refer to this value as s

Dettmers et al. 2022, “LLM.int8()”

LLM.int8()

Absmax Quantization

- $\frac{1}{s}$: quantized → real

- s : real → quantized

quantized → real

$$\frac{r_{absmax}}{127}$$

real → quantized

$$\frac{127}{r_{absmax}}$$

Dettmers et al. 2022, “LLM.int8()”

LLM.int8()

int8 matmul with fp16 inputs and outputs

- Given inputs $\boldsymbol{X}_{f16} \in \mathbb{R}^{s \times h}$ and $\boldsymbol{W}_{f16} \in \mathbb{R}^{h \times o}$, we compute $\boldsymbol{C}_{f16} \in \mathbb{R}^{s \times o}$

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$$\boldsymbol{X}_{f16}\boldsymbol{W}_{f16} = \boldsymbol{C}_{f16}$$

Dettmers et al. 2022, “LLM.int8()”

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$$\boldsymbol{X}_{f16}\boldsymbol{W}_{f16} = \boldsymbol{C}_{f16} \approx \frac{1}{s_{x_{f16}} \cdot s_{w_{f16}}} \cdot \boldsymbol{C}_{i32}$$

Dettmers et al. 2022, “LLM.int8()”

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from previous slide

Dettmers et al. 2022, “LLM.int8()”

LLM.int8()

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$$\approx \frac{1}{s_{x_{f16}} s_{w_{f16}}} \cdot \boldsymbol{X}_{i8}\boldsymbol{W}_{i8}$$

Dettmers et al. 2022, “LLM.int8()”

LLM.int8()

int8 matmul with fp16 inputs and outputs

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$$\approx \frac{1}{s_{x_{f16}} s_{w_{f16}}} \cdot \boldsymbol{X}_{i8}\boldsymbol{W}_{i8}$$

Why int32?

Dettmers et al. 2022, "LLM.int8()"

LLM.int8()

int8 matmul with fp16 inputs and outputs

- Given inputs $\boldsymbol{X}_{f16} \in \mathbb{R}^{s \times h}$ and $\boldsymbol{W}_{f16} \in \mathbb{R}^{h \times o}$, we compute $\boldsymbol{C}_{f16} \in \mathbb{R}^{s \times o}$

$$
\begin{aligned}
\boldsymbol{X}_{f16}\boldsymbol{W}_{f16} = \boldsymbol{C}_{f16} &\approx \frac{1}{s_{x_{f16}} s_{w_{f16}}} \cdot \boldsymbol{C}_{i32} \\
&\approx \frac{1}{s_{x_{f16}} s_{w_{f16}}} \cdot \boldsymbol{X}_{i8}\boldsymbol{W}_{i8} \\
&= \frac{1}{s_{x_{f16}} s_{w_{f16}}} \cdot Q(\boldsymbol{X}_{f16})\, Q(\boldsymbol{W}_{f16})
\end{aligned}
$$

Dettmers et al. 2022, “LLM.int8()”

LLM.int8()

Vector-wise Quantization

LLM.int8()

Vector-wise Quantization

- More scaling constants ⇒ improved quantization error

LLM.int8()

Vector-wise Quantization

- More scaling constants ⇒ improved quantization error
- Matrix multiplication is a series of dot products

LLM.int8()

Vector-wise Quantization

- More scaling constants ⇒ improved quantization error
- Matrix multiplication is a series of dot products
- Idea: one scaling factor for each row of $\boldsymbol{X}$ and each column of $\boldsymbol{W}$

LLM.int8()

Vector-wise Quantization

LLM.int8()

Vector-wise Quantization

- Given inputs $\boldsymbol{X}_{f16} \in \mathbb{R}^{s \times h}$ and $\boldsymbol{W}_{f16} \in \mathbb{R}^{h \times o}$, we compute $\boldsymbol{C}_{f16} \in \mathbb{R}^{s \times o}$

LLM.int8()

Vector-wise Quantization

- Given inputs $\boldsymbol{X}_{f16} \in \mathbb{R}^{s \times h}$ and $\boldsymbol{W}_{f16} \in \mathbb{R}^{h \times o}$, we compute $\boldsymbol{C}_{f16} \in \mathbb{R}^{s \times o}$
  - Assign a different scaling constant $s_{x_{f16}}$ to each row of $\boldsymbol{X}_{f16}$

LLM.int8()

Vector-wise Quantization

- Given inputs $\boldsymbol{X}_{f16} \in \mathbb{R}^{s \times h}$ and $\boldsymbol{W}_{f16} \in \mathbb{R}^{h \times o}$, we compute $\boldsymbol{C}_{f16} \in \mathbb{R}^{s \times o}$
  - Assign a different scaling constant $s_{x_{f16}}$ to each row of $\boldsymbol{X}_{f16}$
  - Assign a different scaling constant $s_{w_{f16}}$ to each col of $\boldsymbol{W}_{f16}$

LLM.int8()

Vector-wise Quantization

$$\begin{aligned}\text{first entry of } C &= (\text{first row of } X) \cdot (\text{first col of } W) \\ &\approx \frac{1}{s \text{ for first row of } X \cdot s \text{ for first col of } W} \cdot Q\left(\text{first row of } X\right) \cdot Q\left(\text{first col of } W\right)\end{aligned}$$

LLM.int8()

Vector-wise Quantization

$$\text{first entry of } C = (\text{first row of } X) \cdot (\text{first col of } W)$$

$$\approx \frac{1}{s \text{ for first row of } X \cdot s \text{ for first col of } W} \cdot Q\,(\text{first row of } X) \cdot Q\,(\text{first col of } W)$$

$$\text{first entry of } C = \begin{bmatrix} 2 & 6 & -15 \end{bmatrix} \bullet \begin{bmatrix} 7 \\ 2 \\ -4 \end{bmatrix}$$

LLM.int8()

Vector-wise Quantization

$$\text{first entry of } C = (\text{first row of } X) \cdot (\text{first col of } W)$$

$$\approx \frac{1}{s \text{ for first row of } X \cdot s \text{ for first col of } W} \cdot Q\,(\text{first row of } X) \cdot Q\,(\text{first col of } W)$$

$$\text{first entry of } C = \begin{bmatrix} 2 & 6 & -15 \end{bmatrix} \bullet \begin{bmatrix} 7 \\ 2 \\ -4 \end{bmatrix}$$

$$\approx \frac{1}{8.46\bar{6} \cdot 18.143} \bullet Q\left(\begin{bmatrix} 2 & 6 & -15 \end{bmatrix}\right) \bullet Q\left(\begin{bmatrix} 7 \\ 2 \\ -4 \end{bmatrix}\right)$$

derived via $\frac{127}{absmax}$ formula

LLM.int8()

Vector-wise Quantization

$$= \frac{1}{8.46\bar{6} \cdot 18.143} \cdot Q(\begin{array}{|c|c|c|}\hline 2 & 6 & -15 \\ \hline \end{array}) \cdot Q(\begin{array}{|c|}\hline 7 \\ \hline 2 \\ \hline -4 \\ \hline \end{array})$$

LLM.int8()

Vector-wise Quantization

$$= \frac{1}{8.46\bar{6} \cdot 18.143} \cdot Q\left(\begin{array}{|c|c|c|}\hline 2 & 6 & -15 \\ \hline\end{array}\right) \cdot Q\left(\begin{array}{|c|}\hline 7 \\ \hline 2 \\ \hline -4 \\ \hline\end{array}\right)$$

$$= \frac{1}{8.46\bar{6} \cdot 18.143} \cdot \begin{array}{|c|c|c|}\hline \lfloor 2 \cdot 8.46\bar{6} \rceil & \lfloor 6 \cdot 8.46\bar{6} \rceil & \lfloor -15 \cdot 8.46\bar{6} \rceil \\ \hline\end{array} \cdot \begin{array}{|c|}\hline \lfloor 18.143 \cdot 7 \rceil \\ \hline \lfloor 18.143 \cdot 2 \rceil \\ \hline \lfloor 18.143 \cdot -4 \rceil \\ \hline\end{array}$$

LLM.int8()

Vector-wise Quantization

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$$= \frac{1}{8.46\bar{6} \cdot 18.143} \cdot \begin{array}{|c|c|c|}\hline \lfloor 2 \cdot 8.46\bar{6} \rceil & \lfloor 6 \cdot 8.46\bar{6} \rceil & \lfloor -15 \cdot 8.46\bar{6} \rceil \\ \hline\end{array} \cdot \begin{array}{|c|}\hline \lfloor 18.143 \cdot 7 \rceil \\ \hline \lfloor 18.143 \cdot 2 \rceil \\ \hline \lfloor 18.143 \cdot -4 \rceil \\ \hline\end{array}$$

$$= \frac{1}{153.61} \cdot \begin{array}{|c|c|c|}\hline 17 & 51 & -127 \\ \hline\end{array} \cdot \begin{array}{|c|}\hline 127 \\ \hline 36 \\ \hline -73 \\ \hline\end{array}$$

LLM.int8()

Vector-wise Quantization

$$= \frac{1}{8.46\bar{6} \cdot 18.143} \bullet Q\left(\begin{array}{|c|c|c|}\hline 2 & 6 & -15 \\ \hline\end{array}\right) \bullet Q\left(\begin{array}{|c|}\hline 7 \\ \hline 2 \\ \hline -4 \\ \hline\end{array}\right)$$

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$$= \frac{1}{153.61} \cdot \Big(17(127) + 51(36) - 127(-73)\Big) = \frac{1}{153.61} \cdot \Big(13266\Big) = 86.36$$

LLM.int8()

Vector-wise Quantization

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for reference,
the correct result is 86

LLM.int8()

Vector-wise Quantization

X_{f16}

2	6	-15
-7	9	12
3	-10	8
8	1	3
-1	11	9

W_{f16}

7	-3
2	8
-4	0

LLM.int8()

Vector-wise Quantization

real → quantized

$$s_{f16} = \frac{127}{r_{absmax}}$$

X_{f16}

2	6	-15
-7	9	12
3	-10	8
8	1	3
-1	11	9

W_{f16}

7	-3
2	-8
-4	0

7	8

18.143	15.875

LLM.int8()

Vector-wise Quantization

real → quantized

$$s_{f16} = \frac{127}{r_{absmax}}$$

X_{f16}

2	6	-15
-7	9	12
3	-10	8
8	1	3
-1	11	9

row-wise

absmax

W_{f16}

7	-3
2	-8
-4	0

col-wise

absmax

7	8

18.143	15.875

LLM.int8()

Vector-wise Quantization

real → quantized

$$s_{f16} = \frac{127}{r_{absmax}}$$

X_{f16}

2	6	-15
-7	9	12
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row-wise

absmax

15
12
10
8
11

W_{f16}

7	-3
2	-8
-4	0

col-wise

absmax

7	8

18.143	15.875

LLM.int8()

Vector-wise Quantization

real → quantized

$$s_{f16} = \frac{127}{r_{absmax}}$$

X_{f16}

2	6	-15
-7	9	12
3	-10	8
8	1	3
-1	11	9

row-wise

absmax

15
12
10
8
11

compute s

W_{f16}

7	-3
2	-8
-4	0

col-wise

absmax

7	8

compute s

18.143	15.875

LLM.int8()

Vector-wise Quantization

real → quantized

$$s_{f16} = \frac{127}{r_{absmax}}$$

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row-wise

absmax

15
12
10
8
11

compute s

8.466
10.583
12.7
15.875
11.545

W_{f16}

7	-3
2	-8
-4	0

col-wise

absmax

7	8

compute s

18.143	15.875

LLM.int8()

Vector-wise Quantization

X_{f16}

2	6	-15
-7	9	12
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-1	11	9

s_{f16}

8.466
10.583
12.7
15.875
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W_{f16}

7	-3
2	-8
-4	0

s_{f16}

18.143	15.875

LLM.int8()

Vector-wise Quantization

X_{f16}

2	6	-15
-7	9	12
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8	1	3
-1	11	9

s_{f16}

8.466
10.583
12.7
15.875
11.545

$$Q(X_{f16}) = \left\lfloor s_{f16} X_{f16} \right\rceil$$

W_{f16}

7	-3
2	-8
-4	0

s_{f16}

18.143	15.875

$$Q(W_{f16}) = \left\lfloor s_{f16} W_{f16} \right\rceil$$

LLM.int8()

Vector-wise Quantization

X_{f16}

2	6	-15
-7	9	12
3	-10	8
8	1	3
-1	11	9

s_{f16}

8.466
10.583
12.7
15.875
11.545

$$Q(X_{f16}) = \left\lfloor s_{f16} X_{f16} \right\rceil$$

17	51	-127
-74	95	127
38	-127	102
127	16	3
-12	127	104

W_{f16}

7	-3
2	-8
-4	0

s_{f16}

18.143	15.875

$$Q(W_{f16}) = \left\lfloor s_{f16} W_{f16} \right\rceil$$

127	-48
36	-127
-73	0

LLM.int8()

Vector-wise Quantization

- Recall our dequantization equation

$$\boldsymbol{C}_{f16} \approx \frac{1}{\boldsymbol{s}_{x_{f16}} \boldsymbol{s}_{w_{f16}}} \cdot Q(\boldsymbol{X}_{f16})\, Q(\boldsymbol{W}_{f16})$$

LLM.int8()

Vector-wise Quantization

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- We now have multiple s
- How can we efficiently dequantize?

LLM.int8()

Vector-wise Quantization

- How can we efficiently dequantize?
  - ij-th entry of $\boldsymbol{C}$ is the dot product of the i-th row of $\boldsymbol{X}$ & the j-th col of $\boldsymbol{W}$
  - i-th row of $\boldsymbol{X}$ was quantized with i-th entry of $\boldsymbol{s}_{f16}$
  - j-th col of $\boldsymbol{W}$ was quantized with j-th entry of $\boldsymbol{s}_{f16}$

LLM.int8()

Vector-wise Quantization

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What should the ij-th entry of $\boldsymbol{C}$ be dequantized by?

LLM.int8()

Vector-wise Quantization

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LLM.int8()

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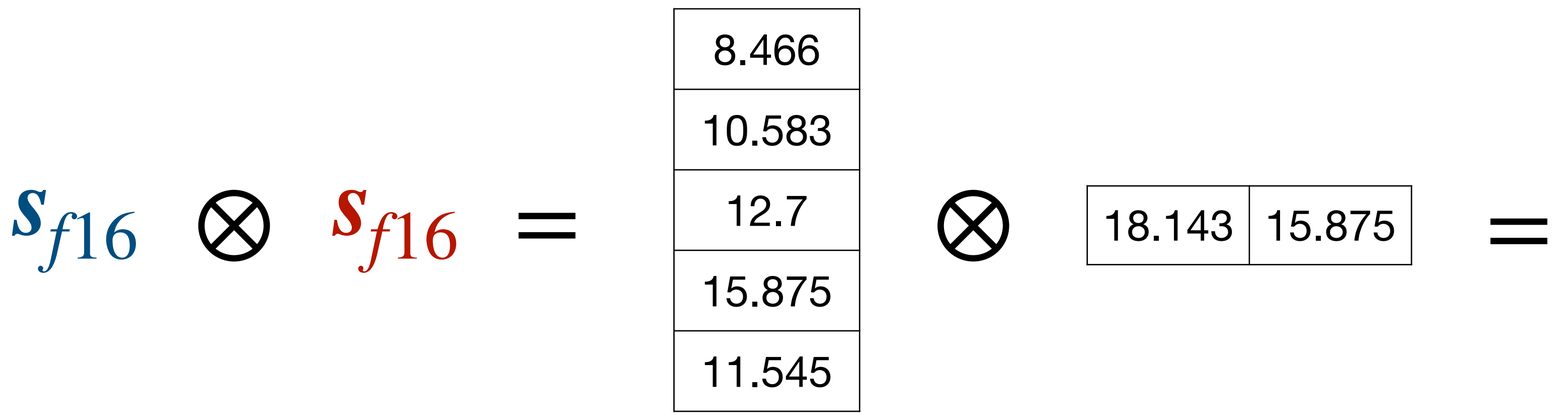
What should C be dequantized by?

$$\boldsymbol{s}_{f16} \otimes \boldsymbol{s}_{f16}$$

LLM.int8()

Vector-wise Quantization



LLM.int8()

Vector-wise Quantization

$$\boldsymbol{s}_{f16} \otimes \boldsymbol{s}_{f16} =$$

8.466
10.583
12.7
15.875
11.545

$\otimes$

18.143	15.875

$=$

153.610	134.408
192.012	168.010
230.414	201.613
288.018	252.016
209.468	183.284

LLM.int8()

Vector-wise Quantization

- Given inputs $\boldsymbol{X}_{f16} \in \mathbb{R}^{b \times h}$ and $\boldsymbol{W}_{f16} \in \mathbb{R}^{h \times o}$, we compute $\boldsymbol{C}_{f16} \in \mathbb{R}^{s \times o}$

$$\boldsymbol{C}_{f16} = \frac{1}{\boldsymbol{s}_{f16} \otimes \boldsymbol{s}_{f16}} \cdot Q\left(\boldsymbol{X}_{f16}\right) Q\left(\boldsymbol{W}_{f16}\right)$$

LLM.int8()

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$b \times o$ **matrix** $b \times o$ **matrix** $b \times o$ **matrix**

LLM.int8()

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$b \times o$ **matrix** $b \times o$ **matrix** $b \times o$ **matrix**

elementwise multiplication

LLM.int8()

Vector-wise Quantization

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$$\boldsymbol{C}_{f16} = \frac{1}{\boldsymbol{s}_{f16} \otimes \boldsymbol{s}_{f16}} \cdot Q\left(\underbrace{\lfloor \boldsymbol{s}_{f16} \boldsymbol{X}_{f16} \rceil}_{=}\right) Q\left(\underbrace{\lfloor \boldsymbol{s}_{f16} \boldsymbol{W}_{f16} \rceil}_{=}\right)$$

LLM.int8()

Vector-wise Quantization

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LLM.int8()

Vector-wise Quantization

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$$\boldsymbol{C}_{f16} = \boldsymbol{S} \cdot Q\left(\boldsymbol{X}_{f16}\right) Q\left(\boldsymbol{W}_{f16}\right)$$

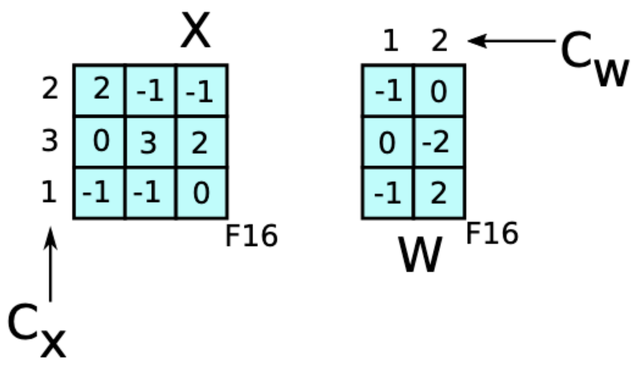
$$\left\lfloor s_{f16} \boldsymbol{X}_{f16} \right\rceil \quad \left\lfloor s_{f16} \boldsymbol{W}_{f16} \right\rceil$$

$$\| \qquad \|$$

LLM.int8()

Vector-wise Quantization

(1) Find vector-wise constants: C_w & C_x



(2) Quantize

$$X_{F16} * (127/C_x) = X_{I8}$$

$$W_{F16} * (127/C_w) = W_{I8}$$

(3) Int8 Matmul

$$X_{I8} \; W_{I8} = Out_{I32}$$

(4) Dequantize

$$\frac{Out_{I32} * (C_x \otimes C_w)}{127*127} = Out_{F16}$$

Dettmers et al. 2022, “LLM.int8()”

LLM.int8()

Outliers

X

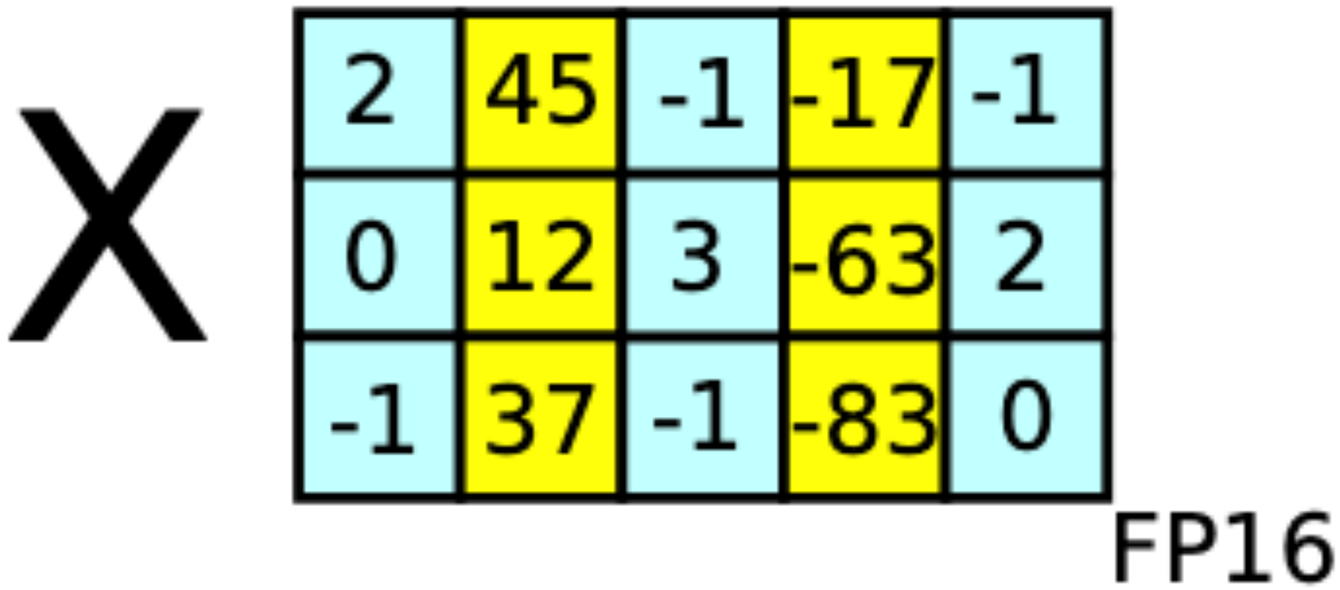
2	45	-1	-17	-1
0	12	3	-63	2
-1	37	-1	-83	0

FP16

Dettmers et al. 2022, “LLM.int8()”

LLM.int8()

Outliers



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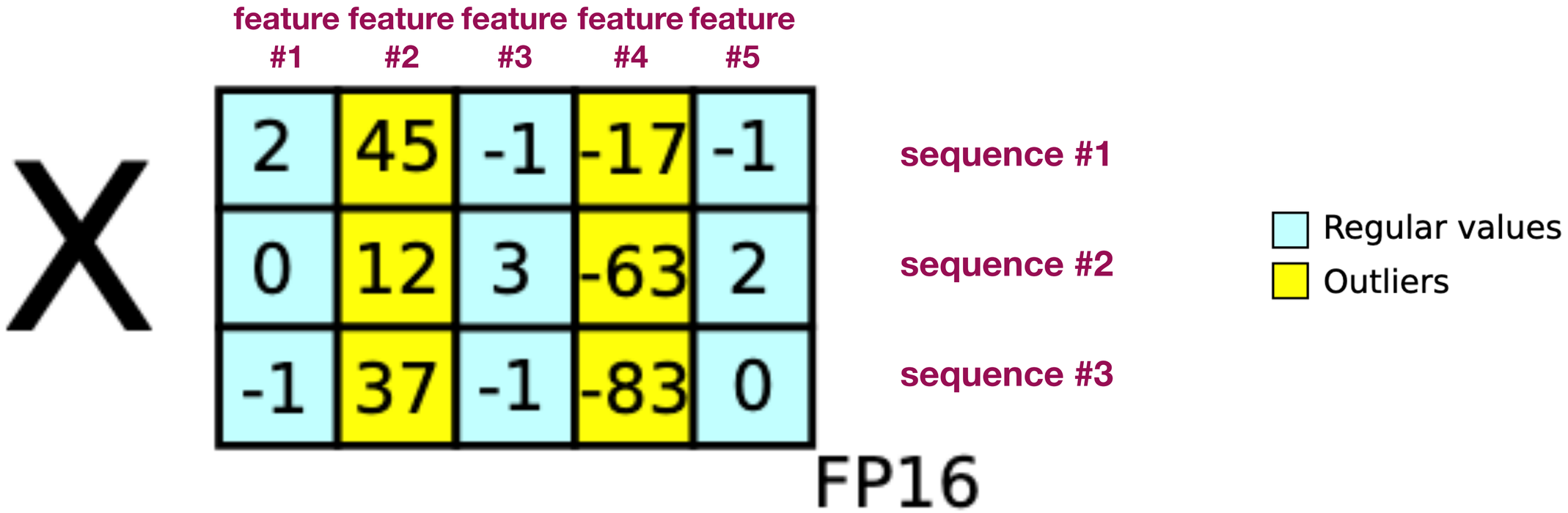
Outliers



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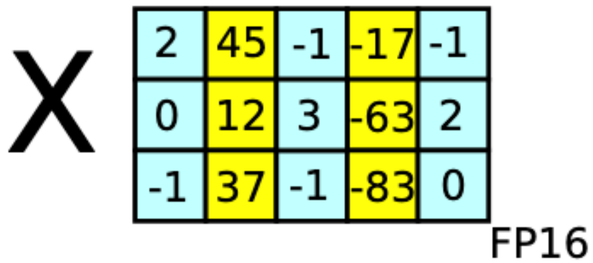
Outliers



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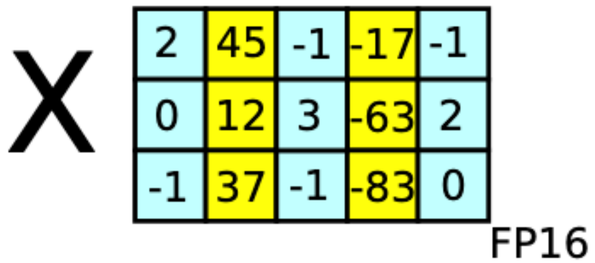
LLM.int8()

Outliers

- Empirically, large (>6B) transformer models have **outliers**:
  - Large magnitude features (columns)



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- Empirically, large (>6B) transformer models have **outliers**:
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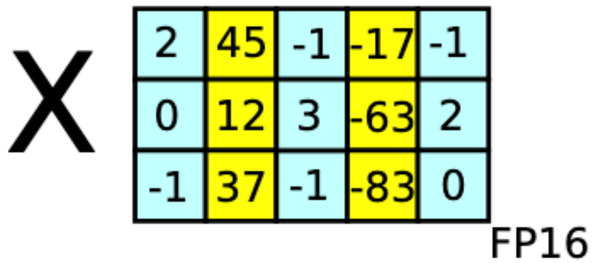
  - Extremely important for performance



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Outliers

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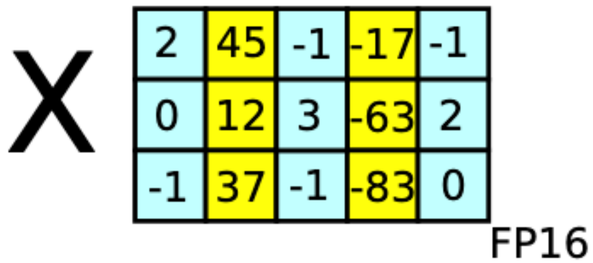
  - Extremely important for performance
  - Require high quantization precision



Dettmers et al. 2022, “LLM.int8()”

LLM.int8()

Outliers

- Empirically, large transformer models have…
  - 99.9% regular features — medium quantization precision OK
  - 0.01% outlier features — require high quantization precision

Dettmers et al. 2022, “LLM.int8()”

LLM.int8()

Outliers

- Empirically, large transformer models have…
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Great, we already employ vector-wise quantization, right?

Dettmers et al. 2022, “LLM.int8()”

LLM.int8()

Outliers

- Vector-wise quantization assigns different scaling factors for rows in X
- Outliers occur in columns in X

LLM.int8()

Outliers

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Vector-wise quantization doesn’t help

LLM.int8()

Mixed-precision Decomposition

- What if we handled the normal features and outlier features separately?

Dettmers et al. 2022, “LLM.int8()”

LLM.int8()

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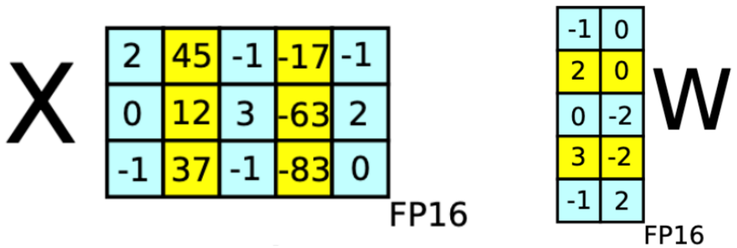

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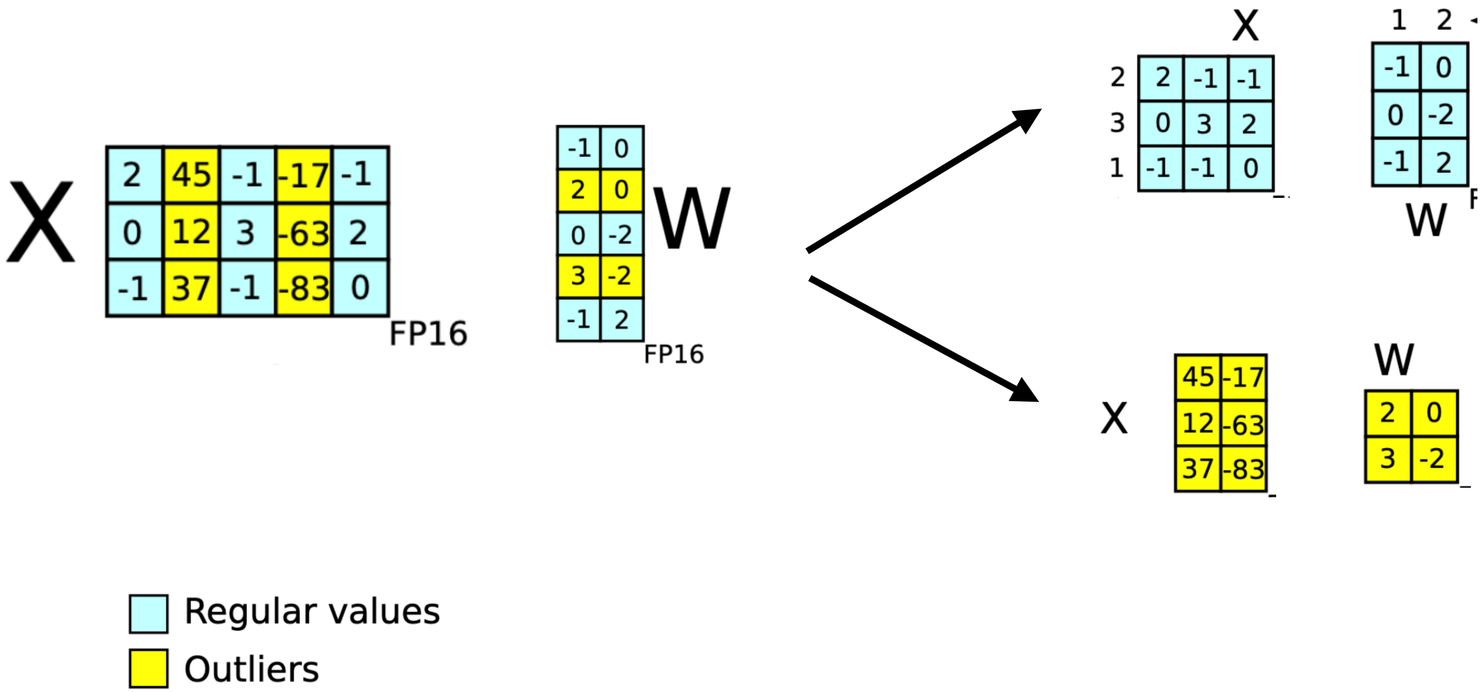

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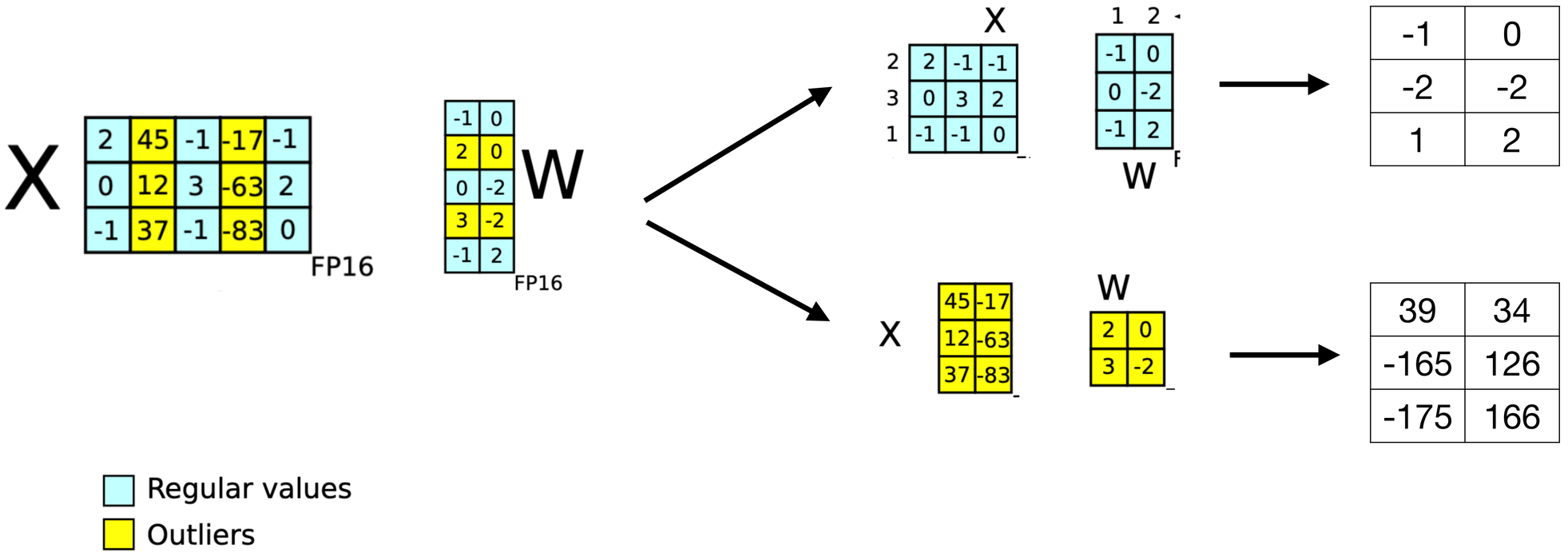

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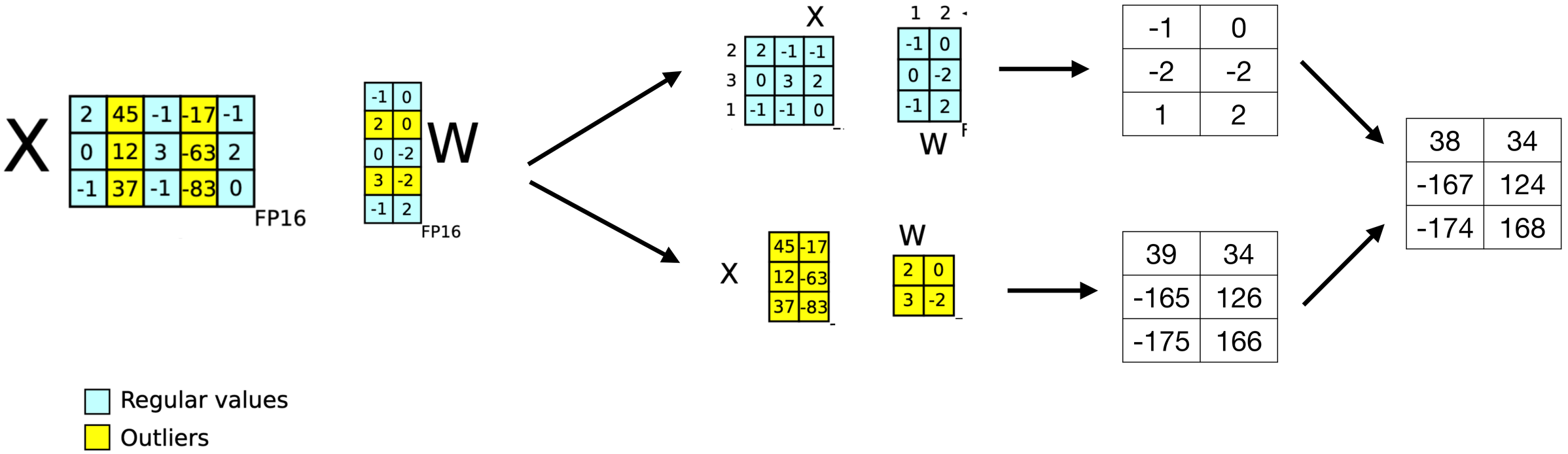

Dettmers et al. 2022, "LLM.int8()"

Aside: Einstein Notation

This is non-standard

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This is non-standard

$$X^2W^2 + X^4W^4$$

LLM.int8()

Mixed-precision Decomposition

- Notation for handling outliers O “separately” (still with normal fp16 matmul)

$$\boldsymbol{C}_{f16} = \sum_{h \in O} \boldsymbol{X}^h_{f16} \boldsymbol{W}^h_{f16} + \sum_{h \notin O} \boldsymbol{X}^h_{f16} \boldsymbol{W}^h_{f16}$$

LLM.int8()

Mixed-precision Decomposition

$$C_{f16} = s \cdot Q\left(\boldsymbol{X}_{f16}\right) Q\left(\boldsymbol{W}_{f16}\right)$$

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Dettmers et al. 2022, “LLM.int8()”

LLM.int8()

Mixed-precision Decomposition

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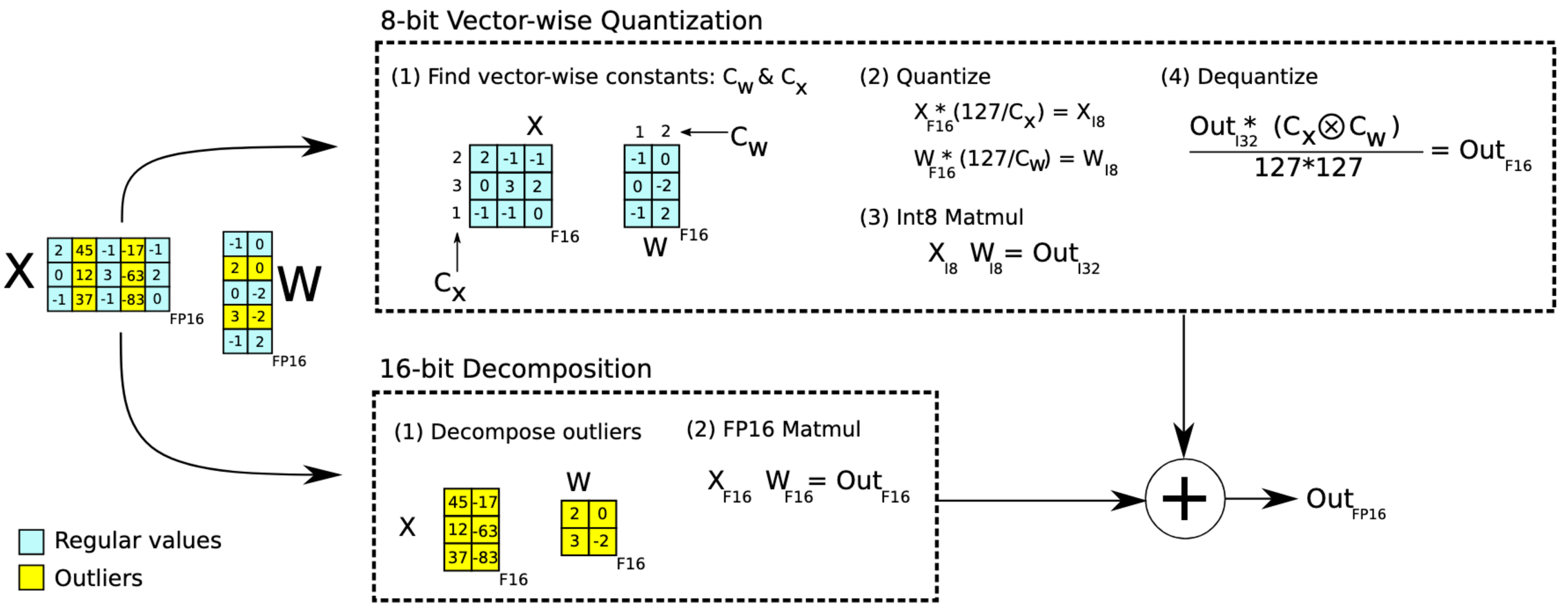
$$\boldsymbol{C}_{f16} = \sum_{h \in O} \boldsymbol{X}^{h}_{f16} \boldsymbol{W}^{h}_{f16} + \boldsymbol{S} \cdot \sum_{h \notin O} \boldsymbol{X}^{h}_{i8} \boldsymbol{W}^{h}_{i8}$$

Dettmers et al. 2022, “LLM.int8()”

LLM.int8()



Dettmers et al. 2022, "LLM.int8()"

LLM.int8()

Outliers are critical to transformer performance

Dettmers et al. 2022, “LLM.int8()”

LLM.int8()

Outliers are critical to transformer performance

- Removing **<7 outlier features** causes…
  - top-1 softmax probability cut in half
  - validation perplexity increases 6-10x

Dettmers et al. 2022, “LLM.int8()”

LLM.int8()

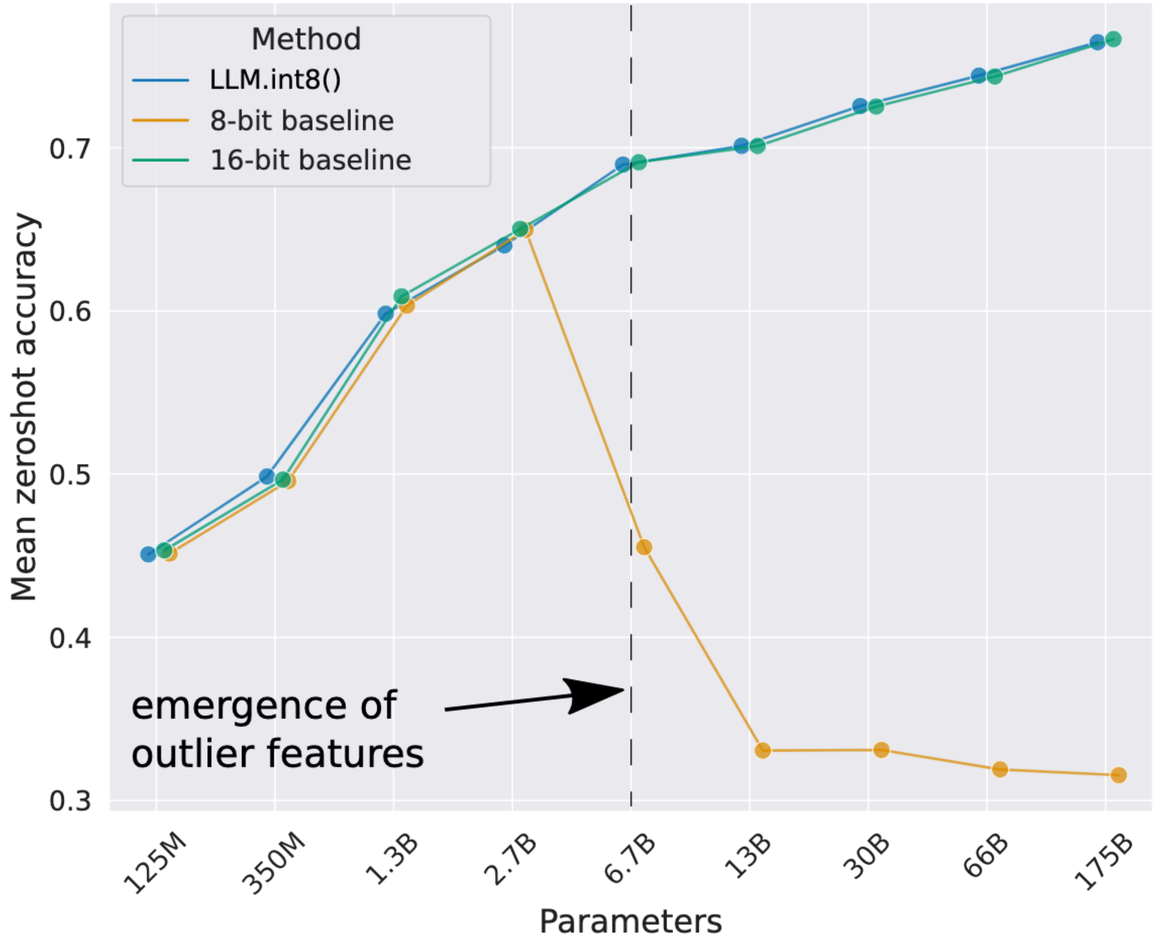
Outliers are critical to transformer performance

- Removing **<7 outlier features** causes…
  - top-1 softmax probability cut in half
  - validation perplexity increases 6-10x
- Removing **7 random features** causes…
  - top-1 softmax probability decreases by 0.3%
  - validation perplexity increases by 0.1%

Dettmers et al. 2022, “LLM.int8()”

LLM.int8()

Phase Shift



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LLM.int8()

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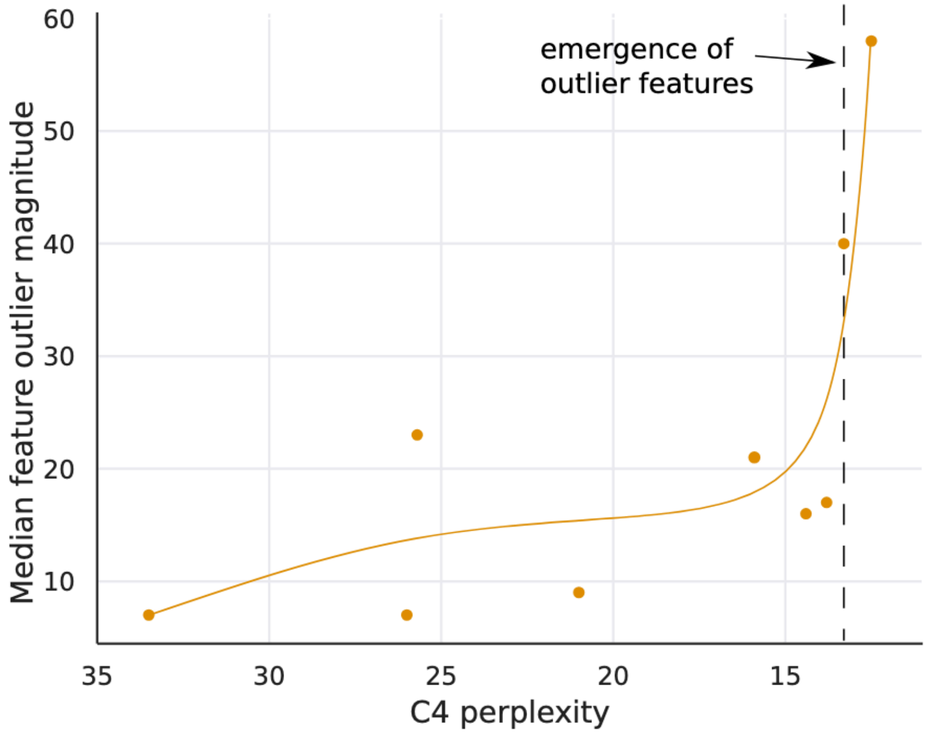
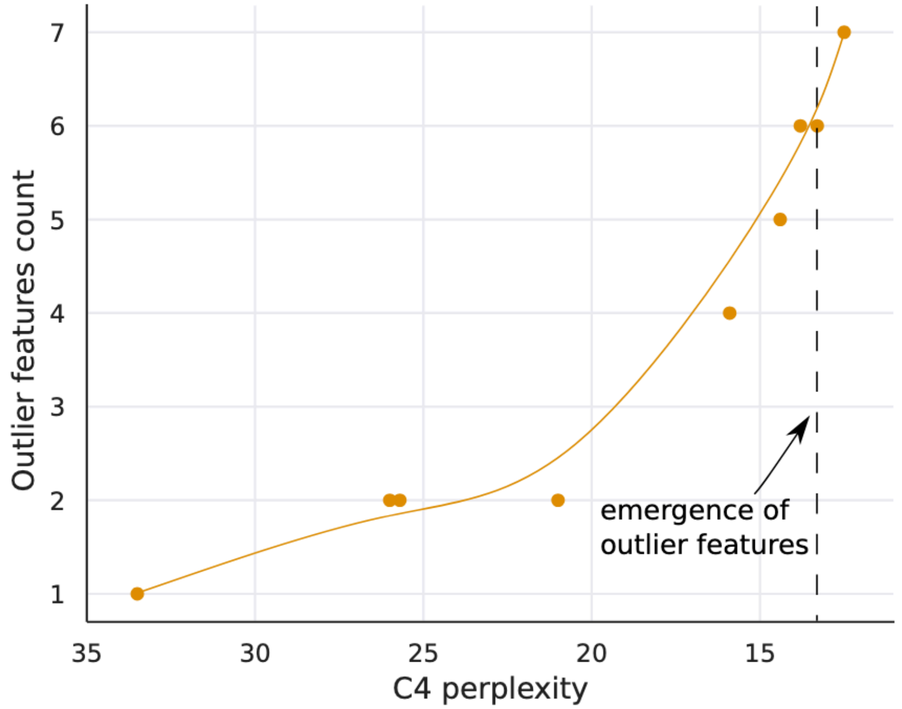


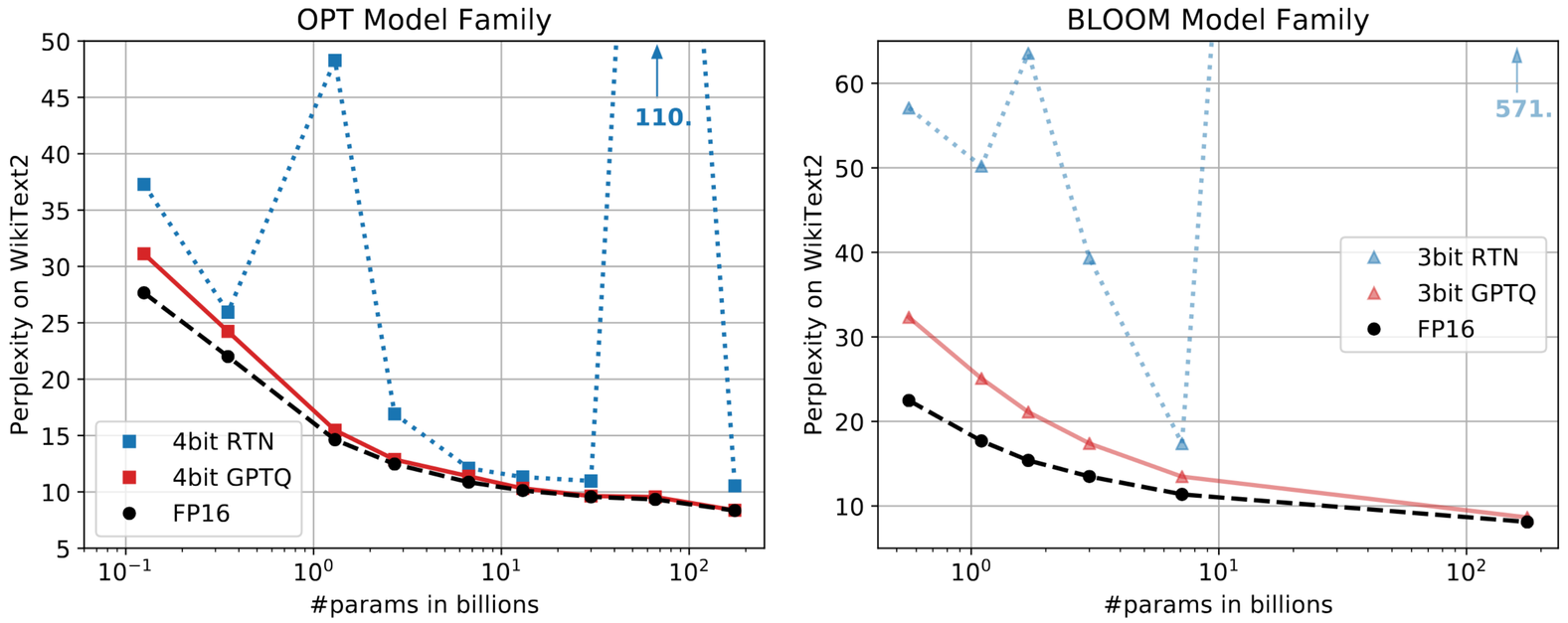


Dettmers et al. 2022, “LLM.int8()”

GPTQ

4 Bit Quantization

- Integrated with 🤗 Transformers
- First paper to break the 8 bit quantization barrier



Frantar et al. 2022, “GPTQ”

AWQ

4 Bit Quantization… but simpler :)

Lin et al. 2024, “AWQ”

AWQ

4 Bit Quantization… but simpler :)

- 1% of weights are “salient weights”
  - Similar insight to LLM.int8()

Lin et al. 2024, “AWQ”

AWQ

4 Bit Quantization… but simpler :)

- 1% of weights are “salient weights”
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- Can be found via activation distribution instead of weights
  - Intuition: input features with large magnitude are important

Lin et al. 2024, “AWQ”

AWQ

4 Bit Quantization… but simpler :)

- 1% of weights are “salient weights”
  - Similar insight to LLM.int8()
- Can be found via activation distribution instead of weights
  - Intuition: input features with large magnitude are important
- Neat trick: multiply weight by s, input by s^{-1}

Lin et al. 2024, “AWQ”

Conclusion

Summary

Quantization is like printers.

Summary

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- Powerful tool to reduce memory footprint of models with many use-cases
  - Frontier labs use quantization to train very large models
  - Academics & home users use quantization to fit large models on their GPUs

Summary

Quantization is like printers.

- Powerful tool to reduce memory footprint of models with many use-cases
  - Frontier labs use quantization to train very large models
  - Academics & home users use quantization to fit large models on their GPUs
- Awe-inspiring results

References

Recommended further reading is bolded

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