Lecture 6:

Activation Functions & Normalization Layers

Administrative: EdStem

Please make sure to check and read all pinned EdStem posts.

Administrative: Assignment 2

Due 4/25 11:59pm

- Multi-layer Neural Networks,
- Image Features,
- Optimizers

Administrative: Fridays

This Friday

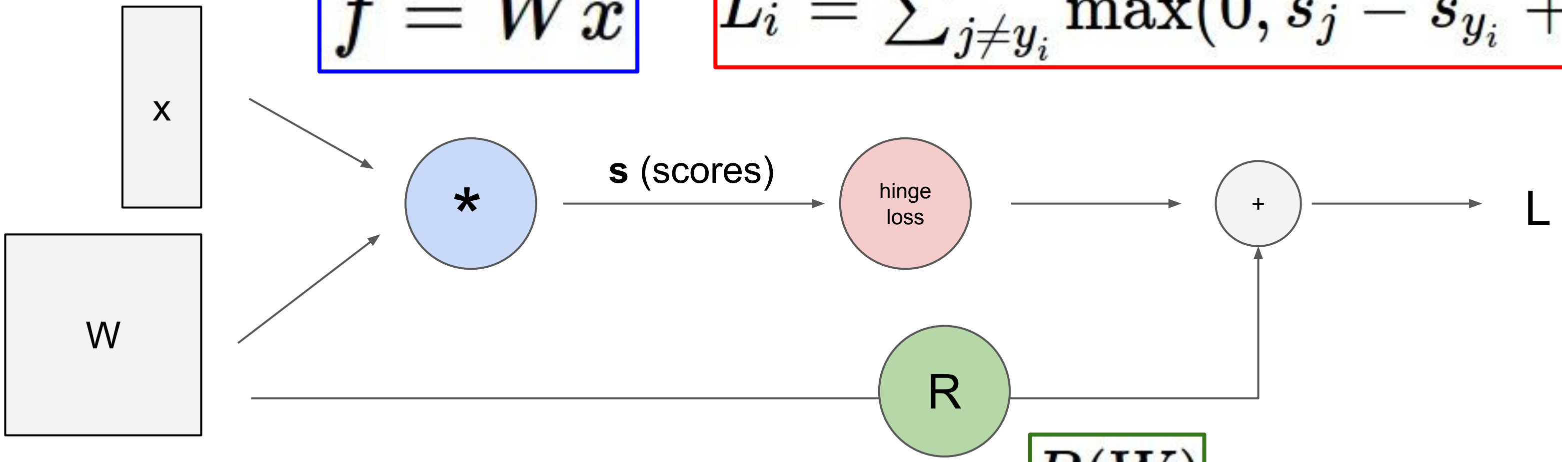
Convolutions & Vectorization

Where we are now...

Computational graphs

$$f = Wx$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$



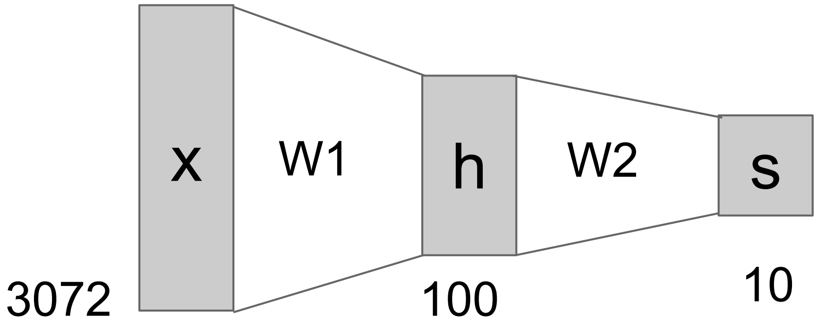
$$R(W)$$

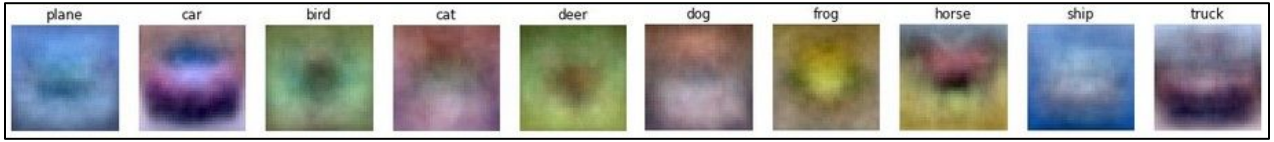
Where we are now...

Neural Networks

Linear score function: $f = Wx$

2-layer Neural Network $f = W_2 \max(0, W_1 x)$





Where we are now...

Convolutional Neural Networks

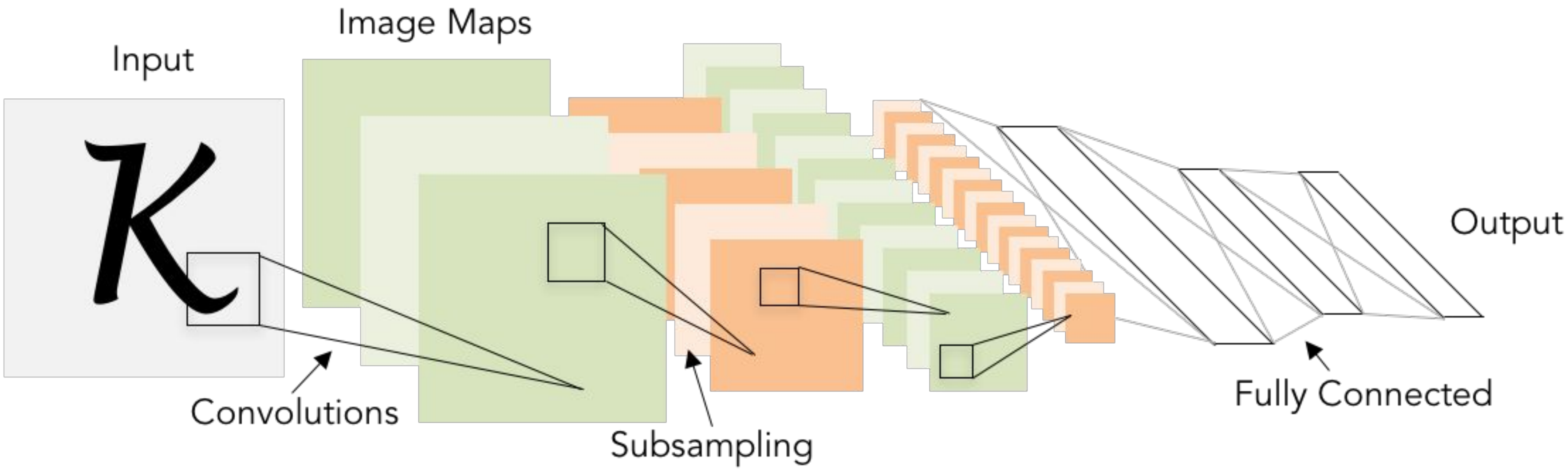
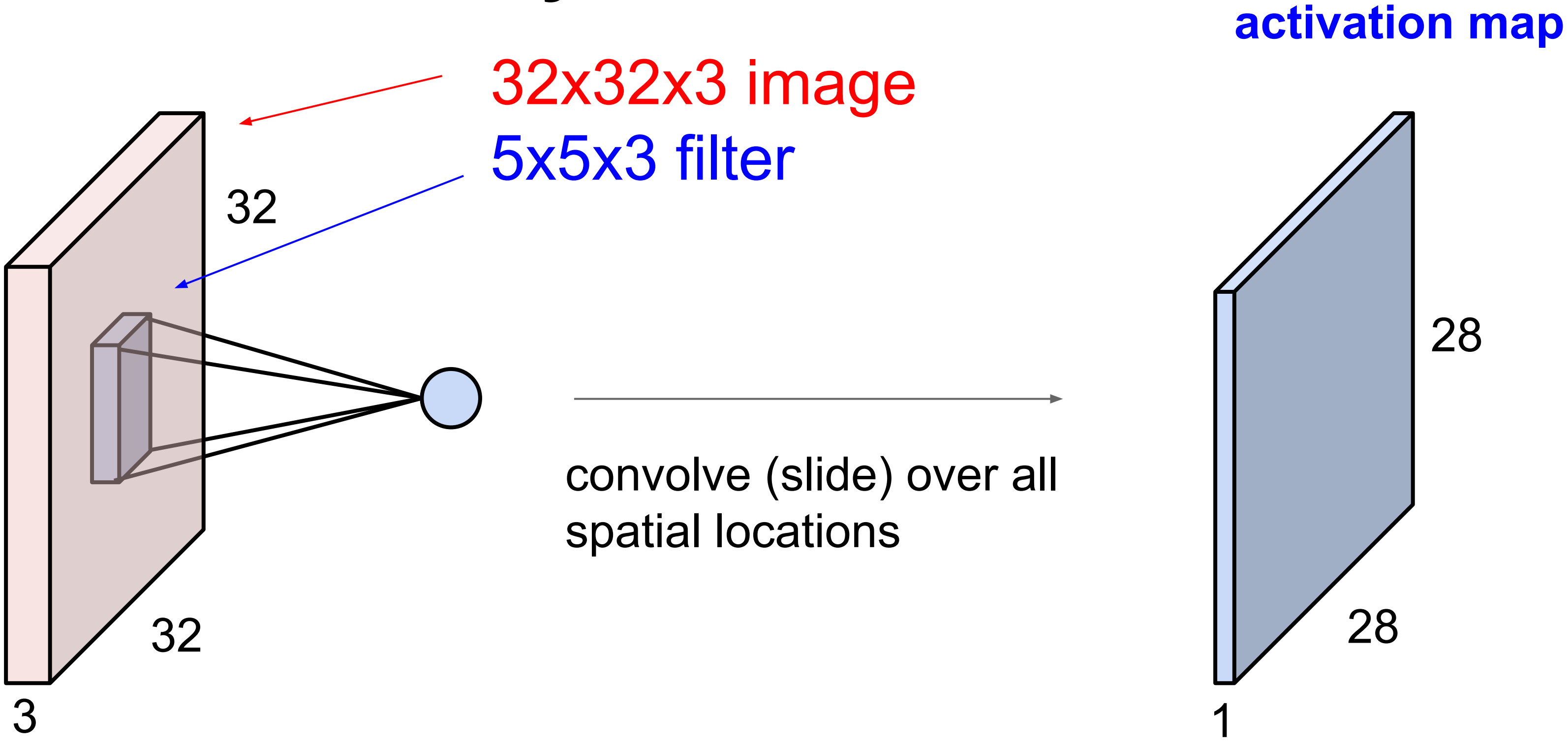


Illustration of LeCun et al. 1998 from CS231n 2017 Lecture 1

Where we are now...

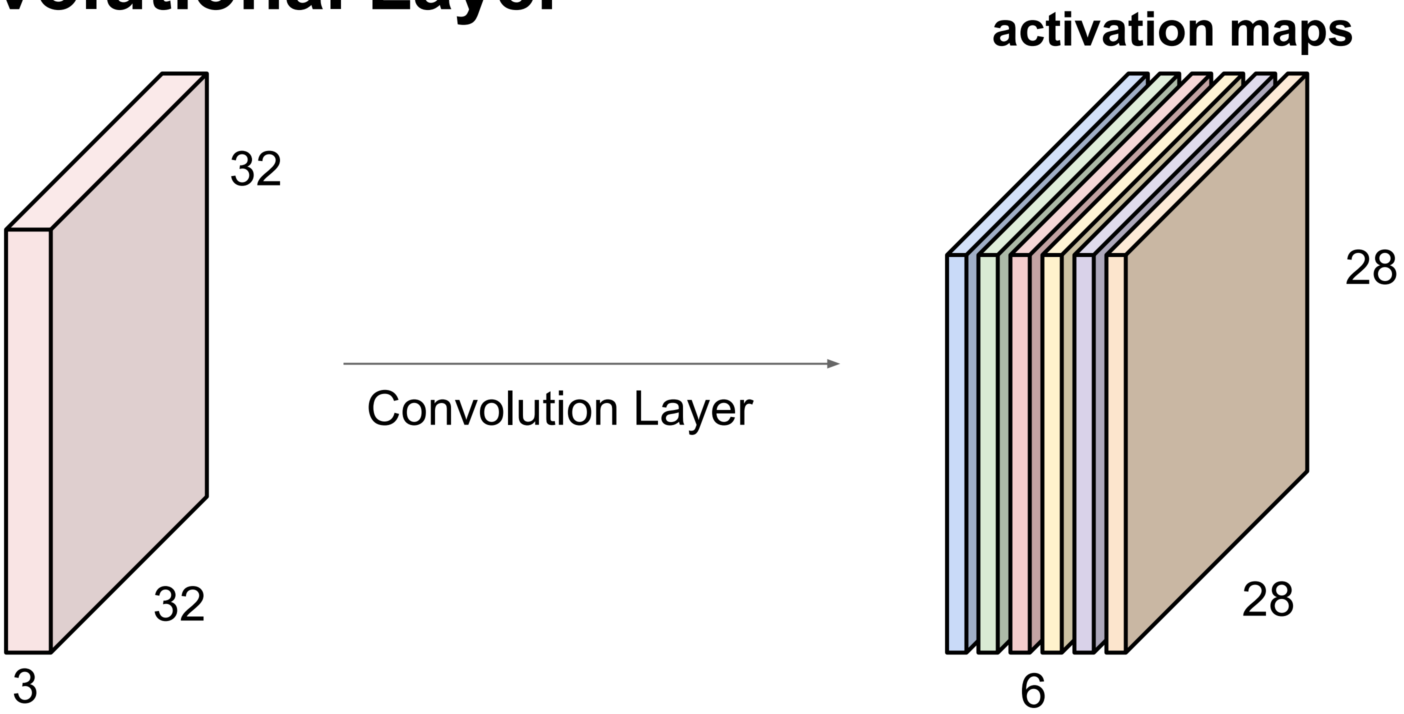
Convolutional Layer



Where we are now...

Convolutional Layer

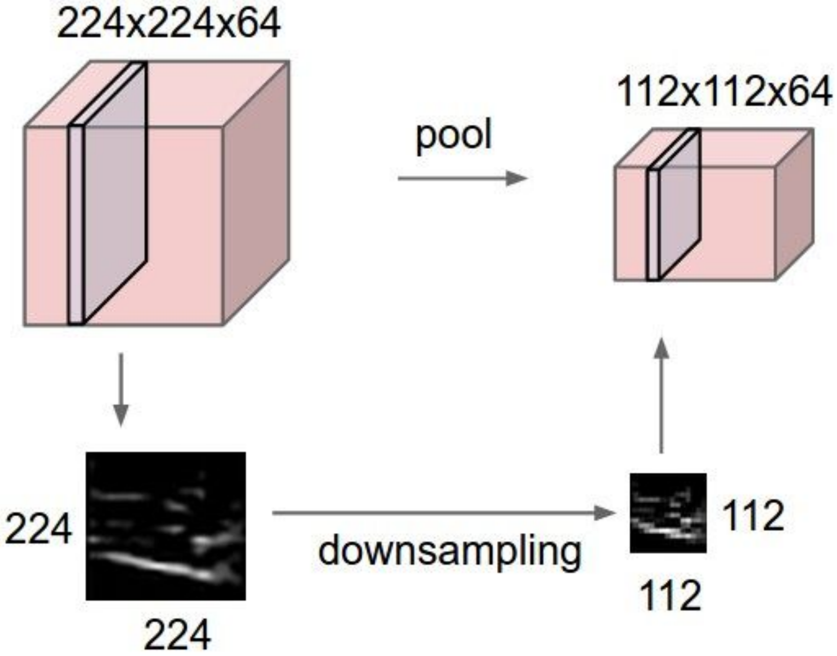
For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:



We stack these up to get a "new image" of size 28x28x6!

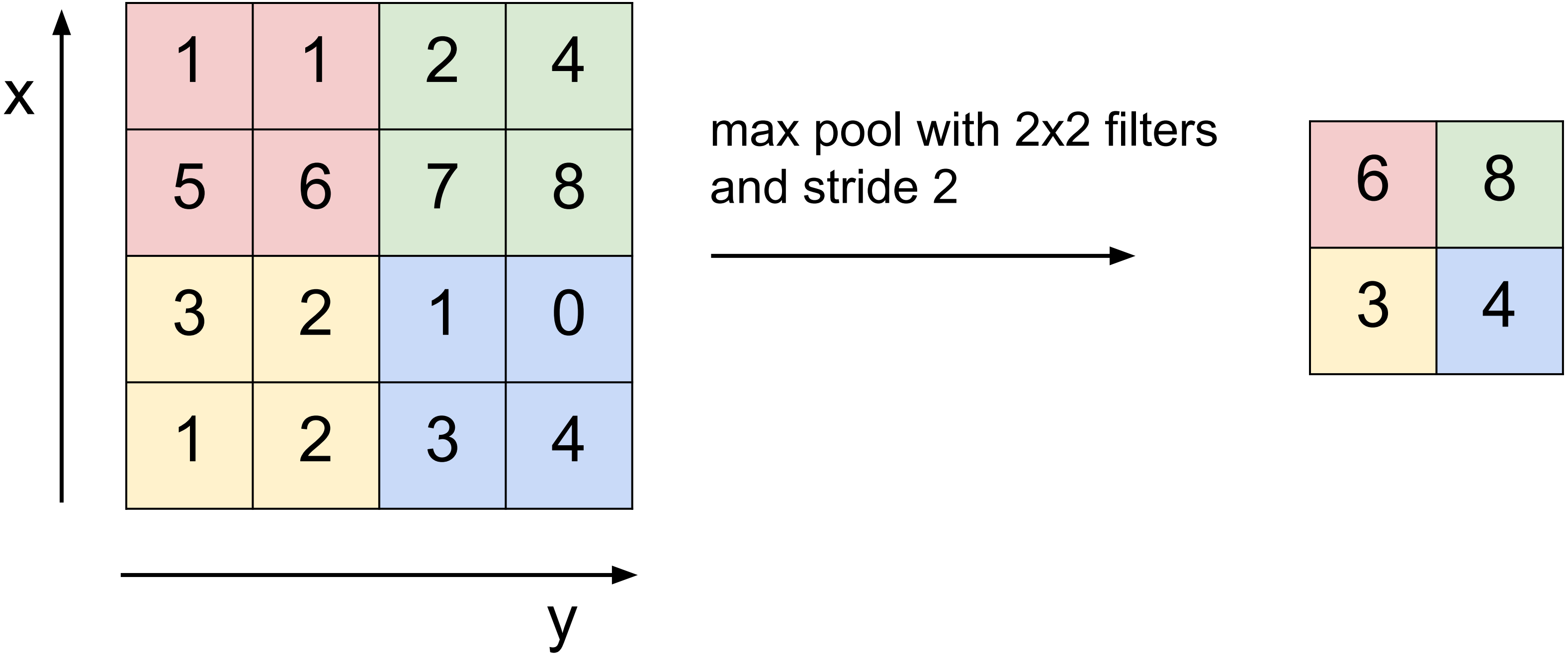
Pooling layer

- makes the representations smaller and more manageable
- operates over each activation map independently:



MAX POOLING

Single depth slice



Pooling layer: summary

Let's assume input is W_1 x H_1 x C
Conv layer needs 2 hyperparameters:

- The spatial extent **F**
- The stride **S**

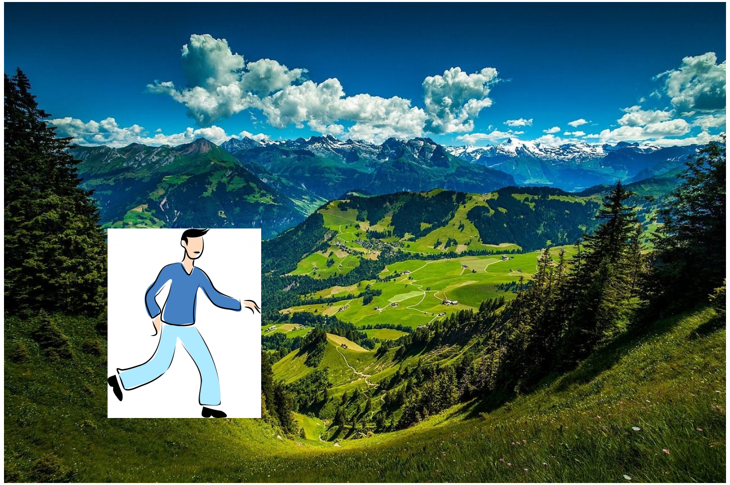
This will produce an output of W_2 x H_2 x C where:

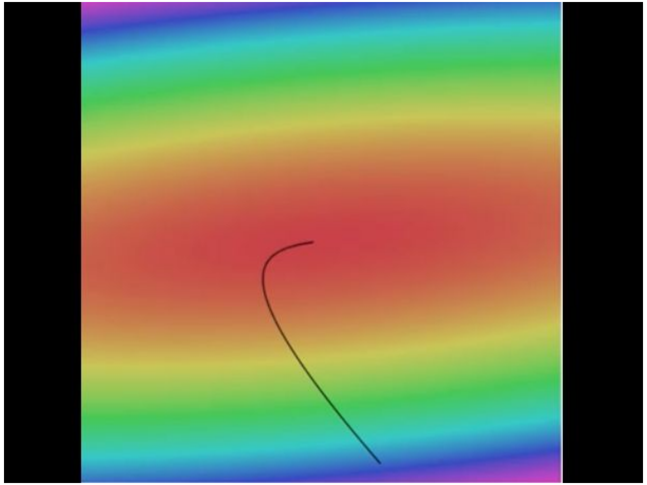
- $W_2 = (W_1 - F)/S + 1$
- $H_2 = (H_1 - F)/S + 1$

Number of parameters: 0

Where we are now...

Learning network parameters through optimization





```
# Vanilla Gradient Descent

while True:
  weights_grad = evaluate_gradient(loss_fun, data, weights)
  weights += - step_size * weights_grad # perform parameter update
```

Mini-batch SGD

Loop:

1. **Sample** a batch of data
2. **Forward** prop it through the graph (network), get loss
3. **Backprop** to calculate the gradients
4. **Update** the parameters using the gradient

Today:

- Activation Function
- Normalization layers

There are a few more steps before we start training

1. **One time setup**
   activation functions, preprocessing, weight initialization, regularization, gradient checking
2. **Training dynamics**
   babysitting the learning process,
   parameter updates, hyperparameter optimization
3. **Evaluation**
   model ensembles, test-time augmentation, transfer learning

Today's agenda

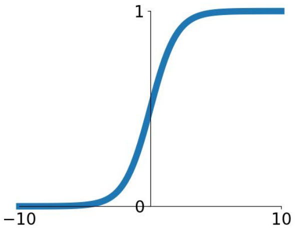
- Activation Functions
- Data Preprocessing
- Weight Initialization
- Normalization Layers

Activation Functions

Activation Functions

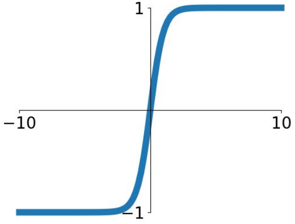
Sigmoid

$\sigma(x) = \frac{1}{1+e^{-x}}$



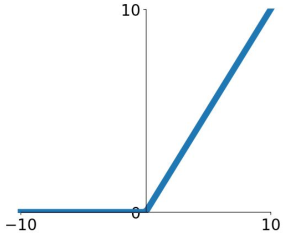
tanh

$\tanh(x)$



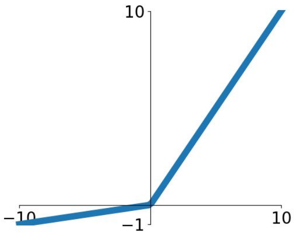
ReLU

$\max(0, x)$



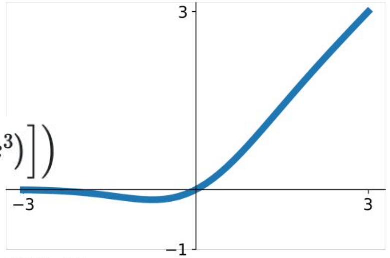
Leaky ReLU

$\max(0.1x, x)$



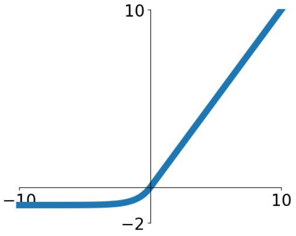
GeLU

$0.5x\left(1 + \tanh\left[\sqrt{2/\pi}(x + 0.044715x^3)\right]\right)$



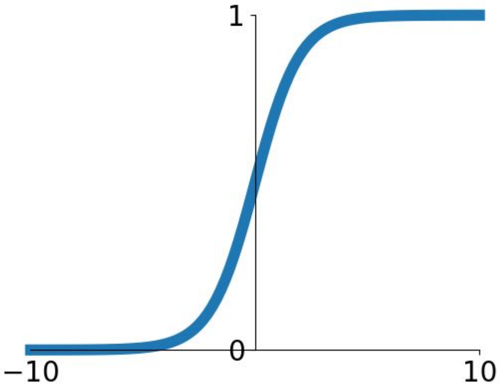
ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$

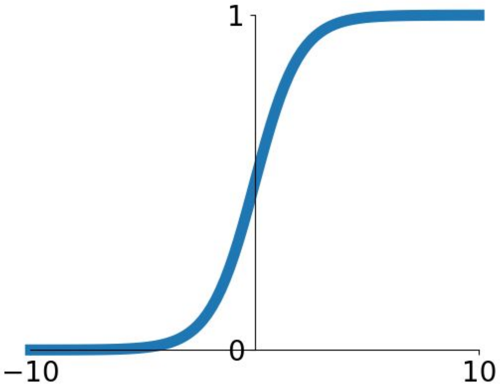


Sigmoid

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$

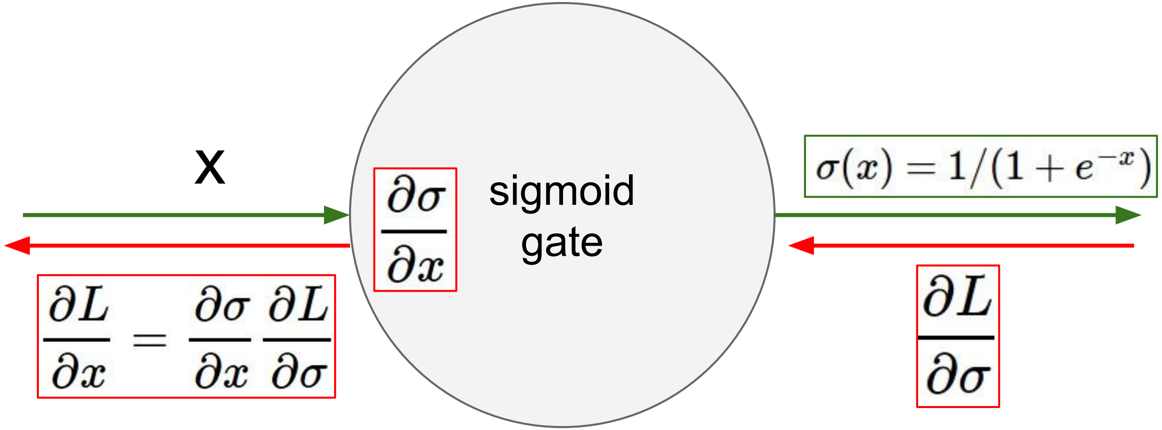


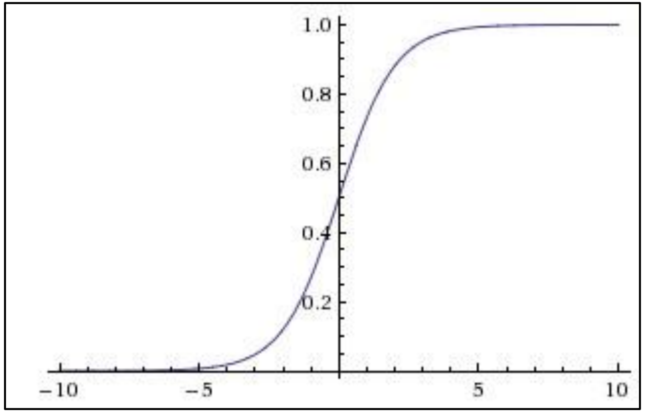
Sigmoid

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating "firing rate" of a neuron

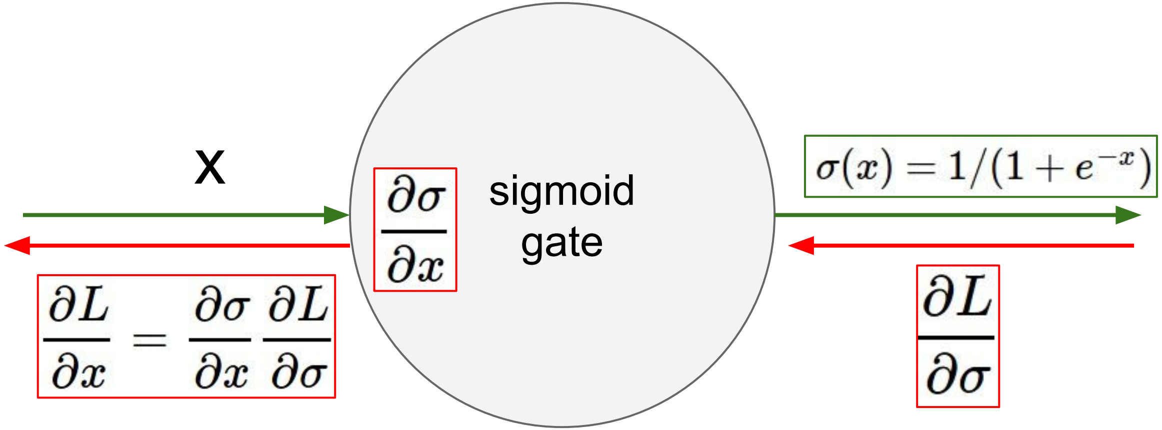
3 problems:

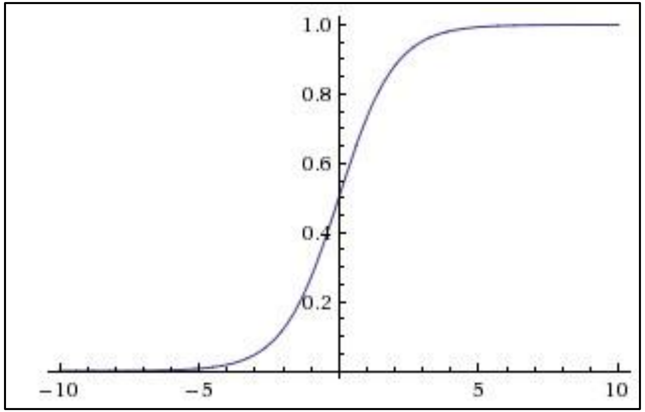
1. Saturated neurons "kill" the gradients





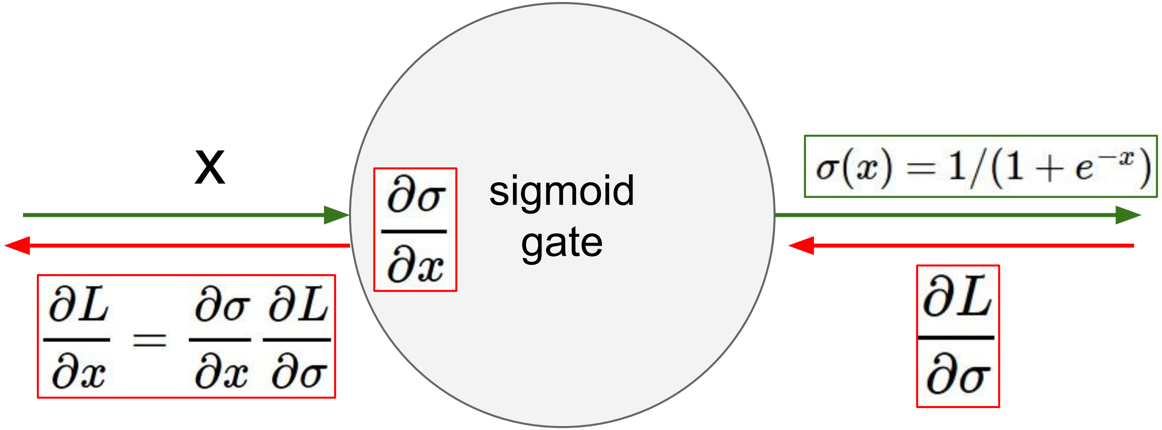
$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x)\,(1 - \sigma(x)\,)$$

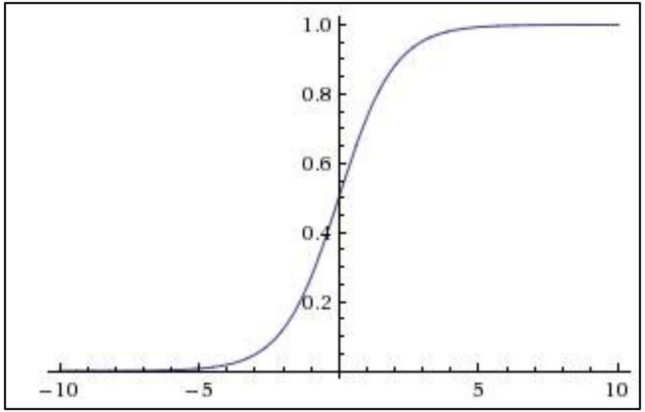




What happens when x = -10?

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x)\,(1 - \sigma(x)\,)$$



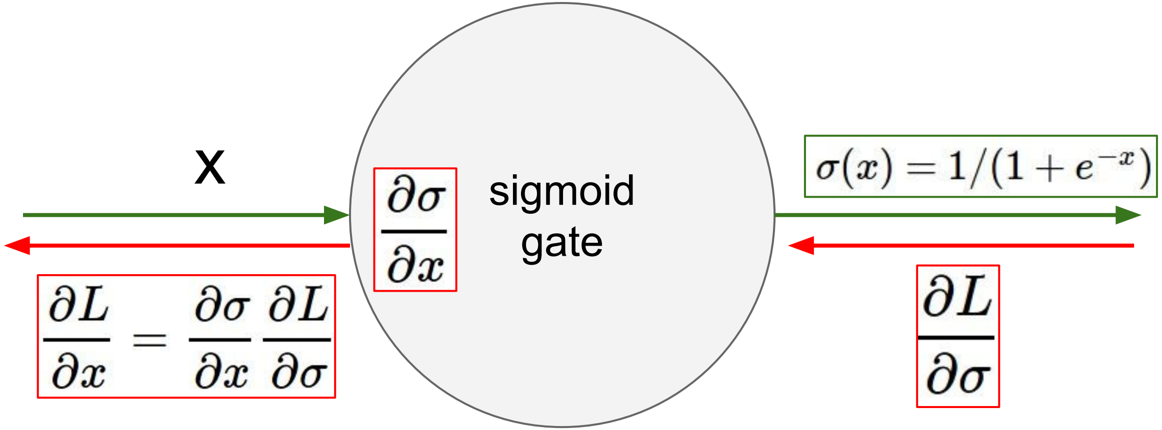


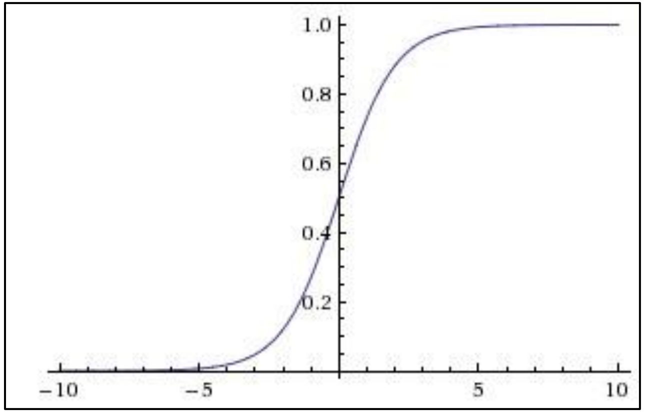
What happens when x = -10?

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x)\,(1 - \sigma(x)\,)$$

$$\sigma(x) \;= \sim 0$$

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x)\,(1 - \sigma(x)\,) = 0(1-0) \;=\; 0$$

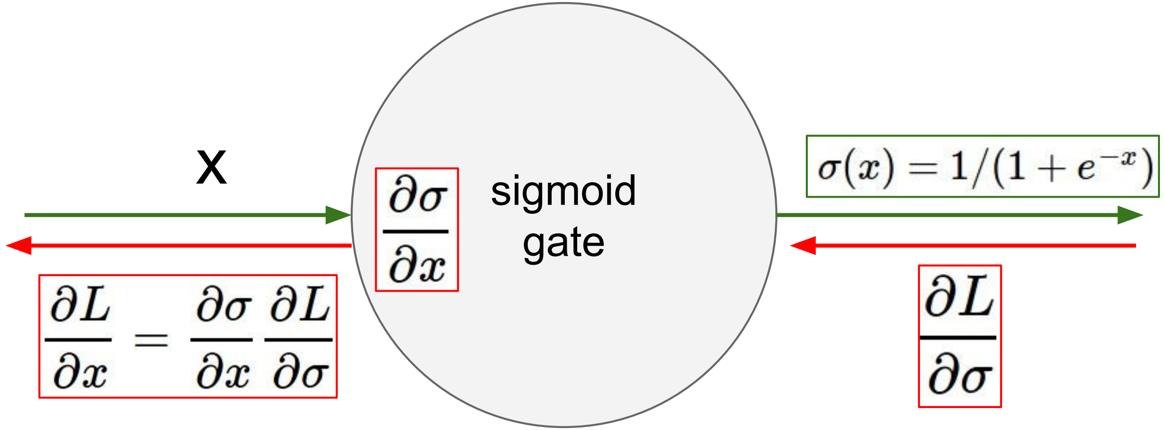


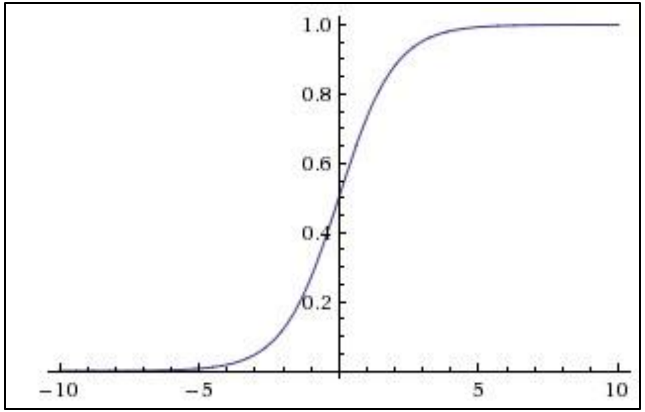


What happens when x = -10?

What happens when x = 0?

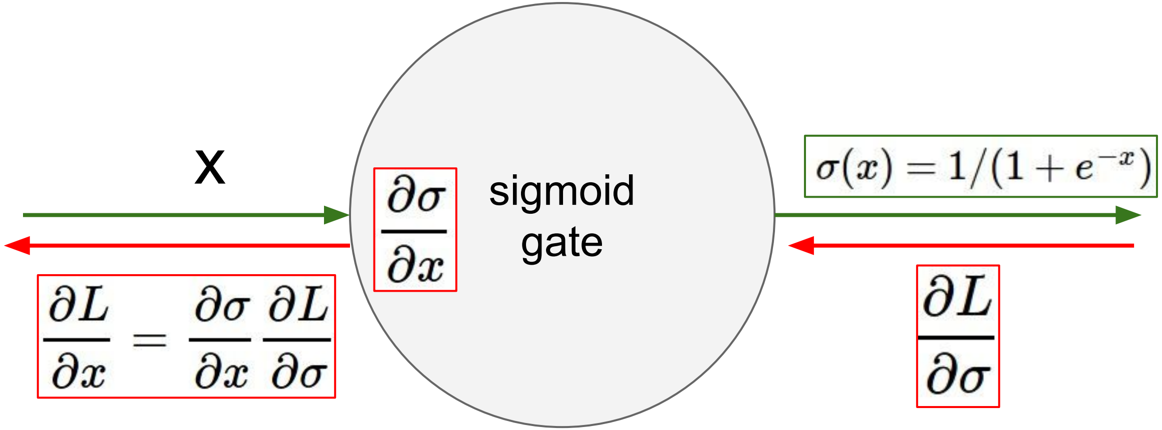
$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x)\,(1 - \sigma(x))$$

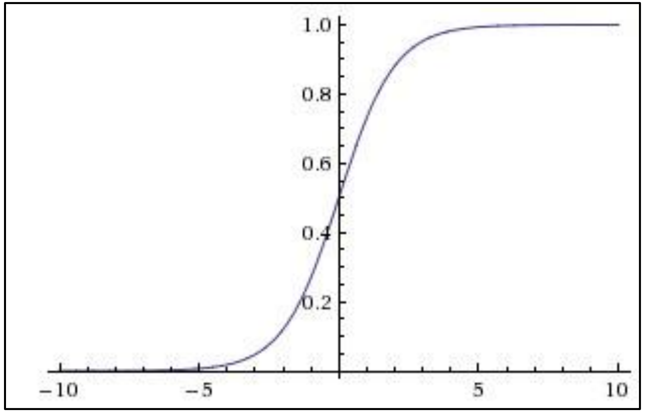




What happens when x = -10?
What happens when x = 0?
What happens when x = 10?

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x)\,(1 - \sigma(x))$$





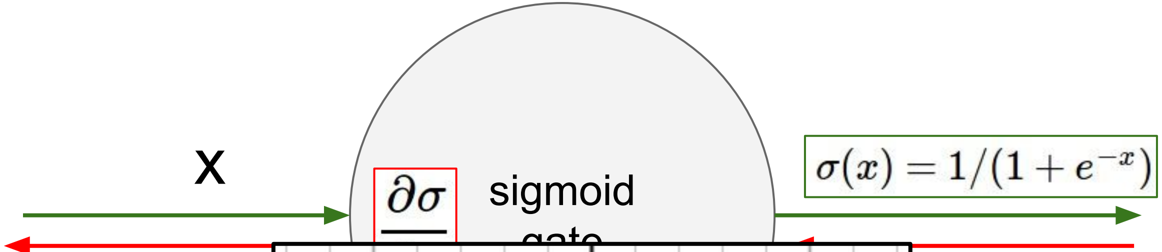
What happens when x = -10?

What happens when x = 0?

What happens when x = 10?

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x)\left(1 - \sigma(x)\right)$$

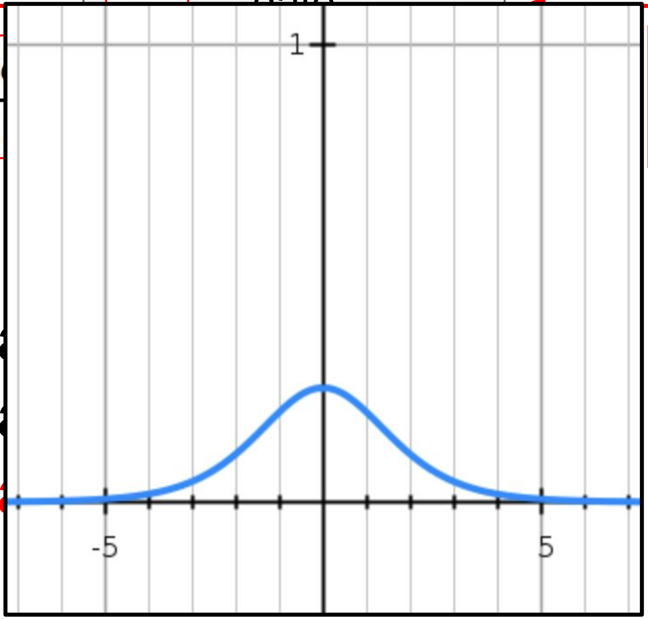
$$\sigma(x) = \sim 1 \qquad \frac{\partial \sigma(x)}{\partial x} = \sigma(x)\left(1 - \sigma(x)\right) = 1(1-1) = 0$$

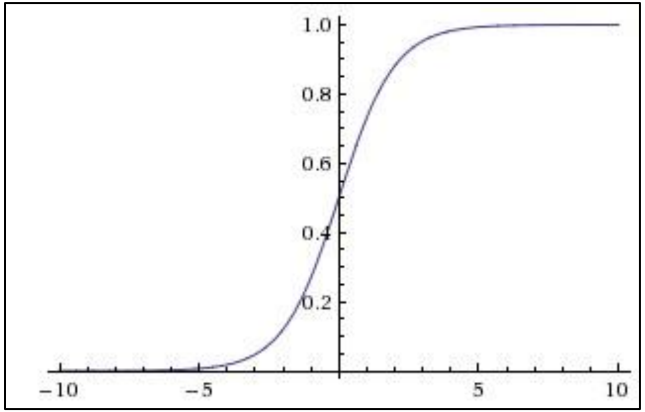


$$\sigma(x) = 1/(1+e^{-x})$$

$$\frac{\partial \sigma}{}$$

$$\frac{\partial L}{\partial x} = \frac{\partial \sigma}{\partial x} \quad \frac{\partial L}{\partial \sigma}$$



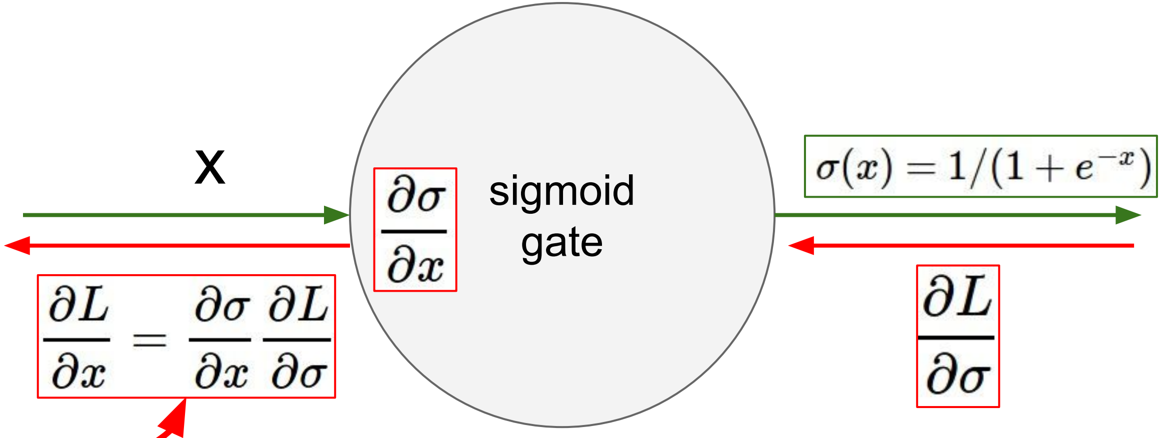


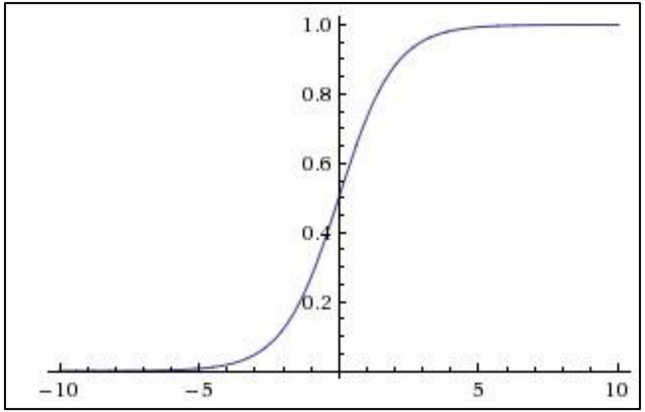
What ha

What ha

What ha

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x)\,(1 - \sigma(x)\,)$$





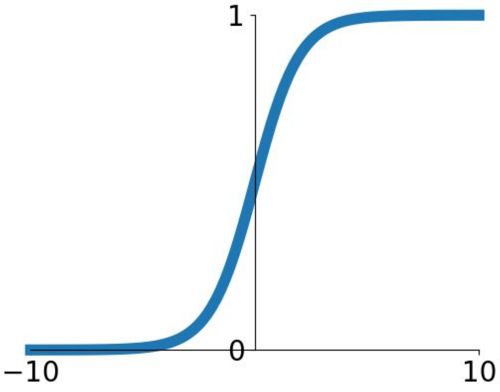
$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x)\,(1 - \sigma(x))$$

Why is this a problem?

If all the gradients flowing back will be zero and weights will never change

Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$



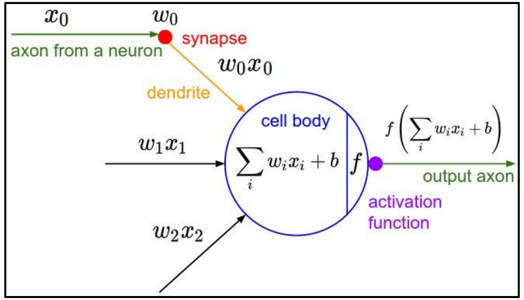
Sigmoid

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

3 problems:

1. Saturated neurons “kill” the gradients
2. Sigmoid outputs are not zero-centered

Consider what happens when the input to a neuron is always positive...

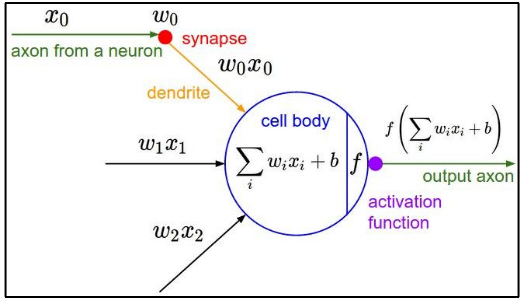


$$f\left(\sum_i w_i x_i + b\right)$$

What can we say about the gradients on **w**?

Consider what happens when the input to a neuron is always positive...

$$f\left(\sum_i w_i x_i + b\right)$$

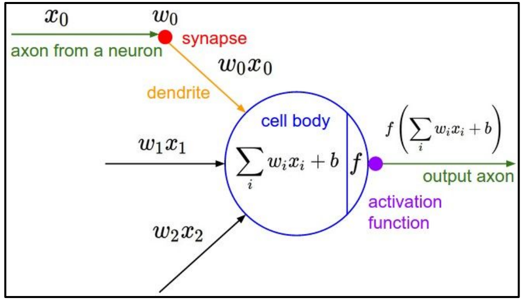


What can we say about the gradients on **w**?

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x)\,(1 - \sigma(x)\,) \qquad \frac{\partial L}{\partial x} = \frac{\partial \sigma}{\partial x}\frac{\partial L}{\partial \sigma}$$

Consider what happens when the input to a neuron is always positive...

$$f\left(\sum_i w_i x_i + b\right)$$



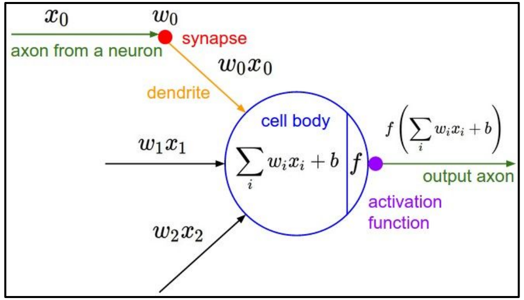
What can we say about the gradients on **w**?

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x)\,(1 - \sigma(x)\,) \qquad \frac{\partial L}{\partial x} = \frac{\partial \sigma}{\partial x}\frac{\partial L}{\partial \sigma}$$

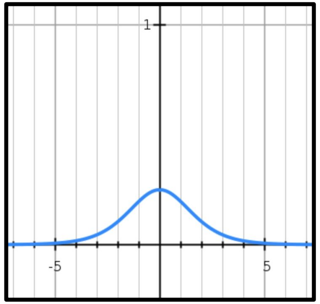
$$\frac{\partial L}{\partial w} = \sigma(\textstyle\sum_i w_i x_i + b)(1 - \sigma(\textstyle\sum_i w_i x_i + b))x \times upstream\_gradient$$

Consider what happens when the input to a neuron is always positive...

$$f\left(\sum_i w_i x_i + b\right)$$



What can we say about the gradients on **w**?

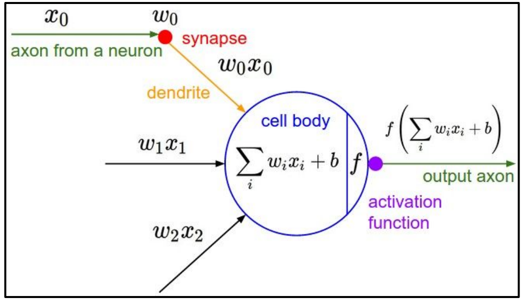


We know that local gradient of sigmoid is always positive

$$\frac{\partial L}{\partial w} = \boxed{\sigma(\textstyle\sum_i w_i x_i + b)(1 - \sigma(\sum_i w_i x_i + b))} x \times upstream\_gradient$$

Consider what happens when the input to a neuron is always positive...

$$f\left(\sum_i w_i x_i + b\right)$$



What can we say about the gradients on **w**?

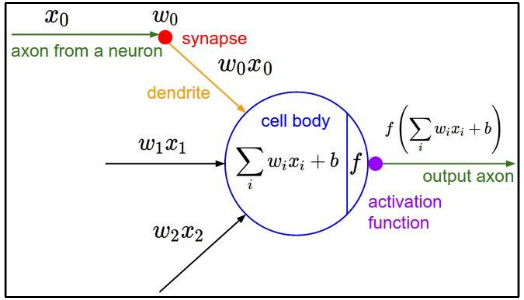
We know that local gradient of sigmoid is always positive

We are assuming x is always positive

$$\frac{\partial L}{\partial w} = \boxed{\sigma(\textstyle\sum_i w_i x_i + b)(1 - \sigma(\sum_i w_i x_i + b))}\boxed{x} \times upstream\_gradient$$

Consider what happens when the input to a neuron is always positive...

$$f\left(\sum_i w_i x_i + b\right)$$



What can we say about the gradients on **w**?

We know that local gradient of sigmoid is always positive

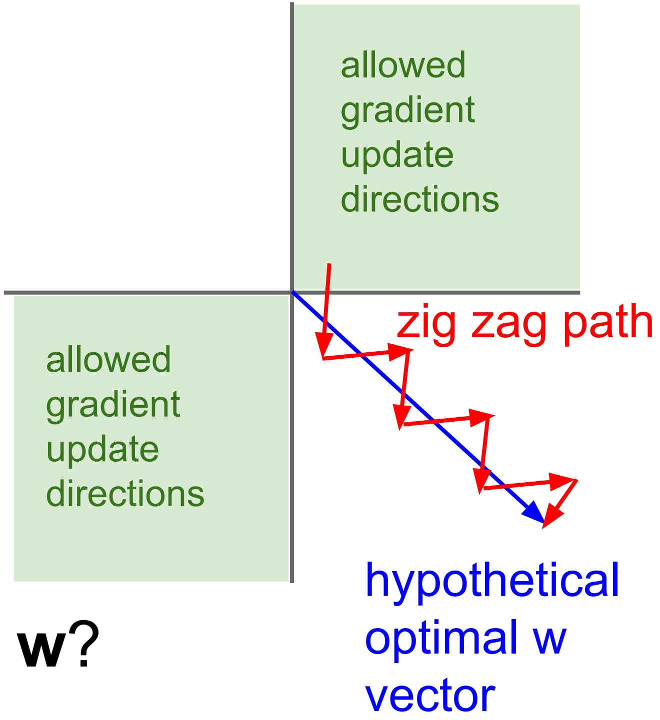
We are assuming x is always positive

So!! Sign of gradient **for all w_i** is the same as the sign of upstream scalar gradient!

$$\boxed{\frac{\partial L}{\partial w}} = \boxed{\sigma(\sum_i w_i x_i + b)(1 - \sigma(\sum_i w_i x_i + b))}\boxed{x} \times \boxed{upstream\_gradient}$$

Consider what happens when the input to a neuron is always positive...

$$f\left(\sum_i w_i x_i + b\right)$$

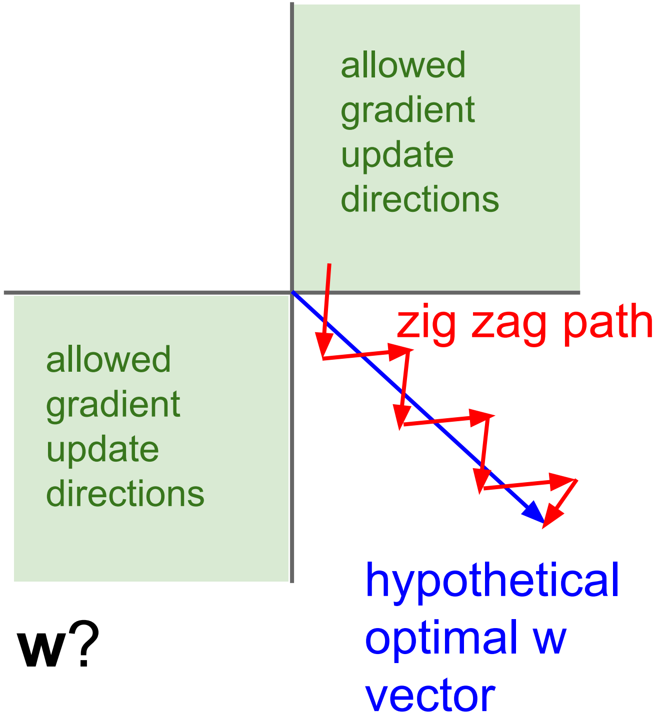


What can we say about the gradients on **w**?

Always all positive or all negative :(

Consider what happens when the input to a neuron is always positive...

$$f\left(\sum_i w_i x_i + b\right)$$

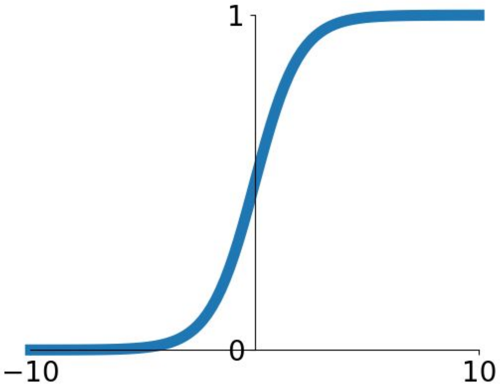


What can we say about the gradients on **w**?

Always all positive or all negative :(

(For a single element! Minibatches help)

Sigmoid



Sigmoid

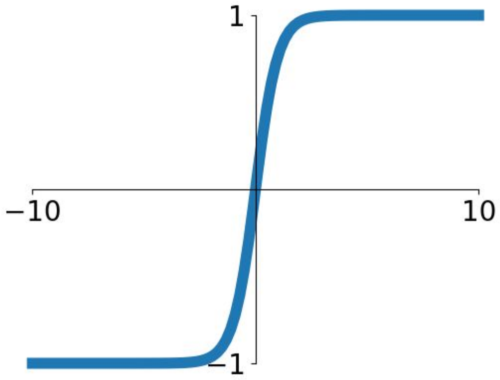
$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating "firing rate" of a neuron

3 problems:

1. Saturated neurons "kill" the gradients
2. Sigmoid outputs are not zero-centered
3. exp() is a bit compute expensive

Tanh

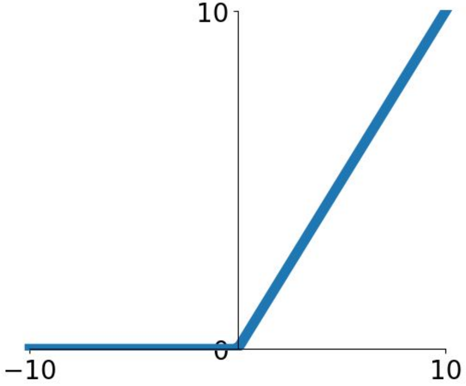


tanh(x)

- Squashes numbers to range [-1,1]
- zero centered (nice)
- still kills gradients when saturated :(

[LeCun et al., 1991]

ReLU



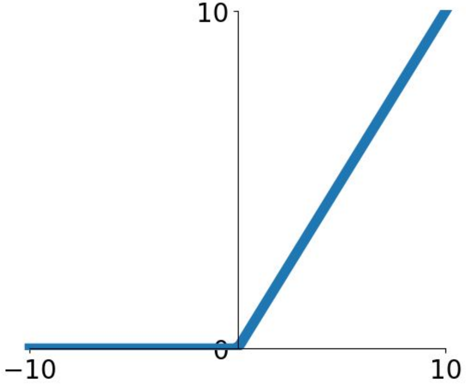
ReLU
(Rectified Linear Unit)

- Computes **f(x) = max(0,x)**

- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)

[Krizhevsky et al., 2012]

ReLU



ReLU
(Rectified Linear Unit)

- Computes **f(x) = max(0,x)**

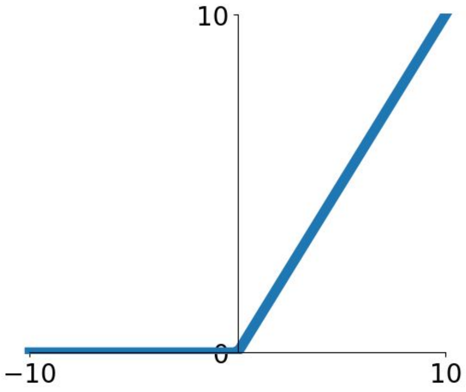
- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)

- Not zero-centered output

ReLU

- Computes **f(x) = max(0,x)**

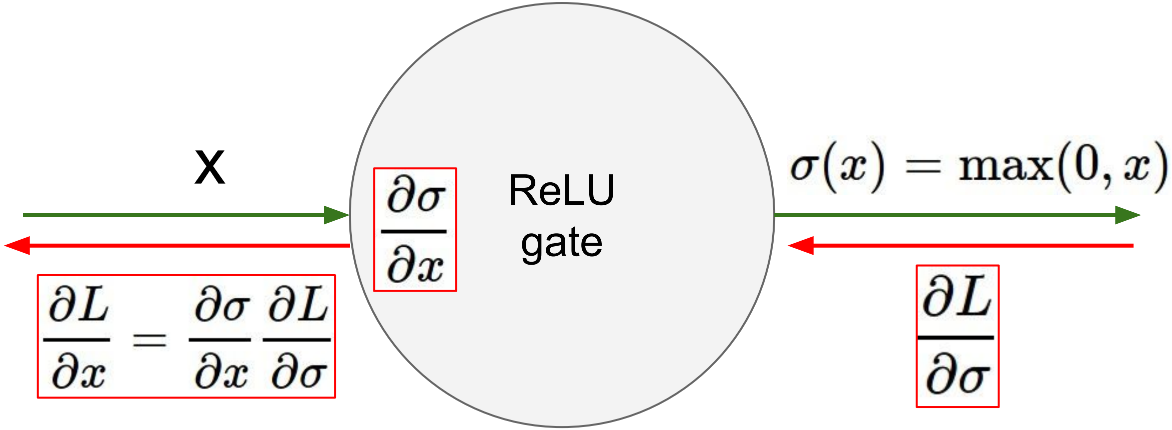
- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)

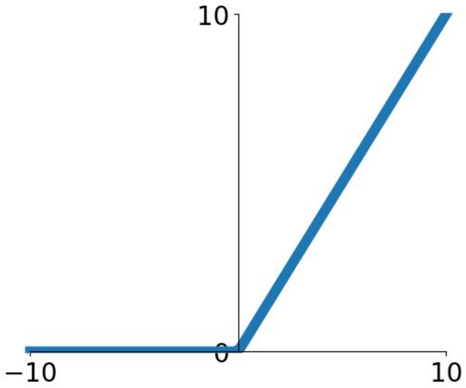


- Not zero-centered output
- An annoyance:

ReLU
(Rectified Linear Unit)

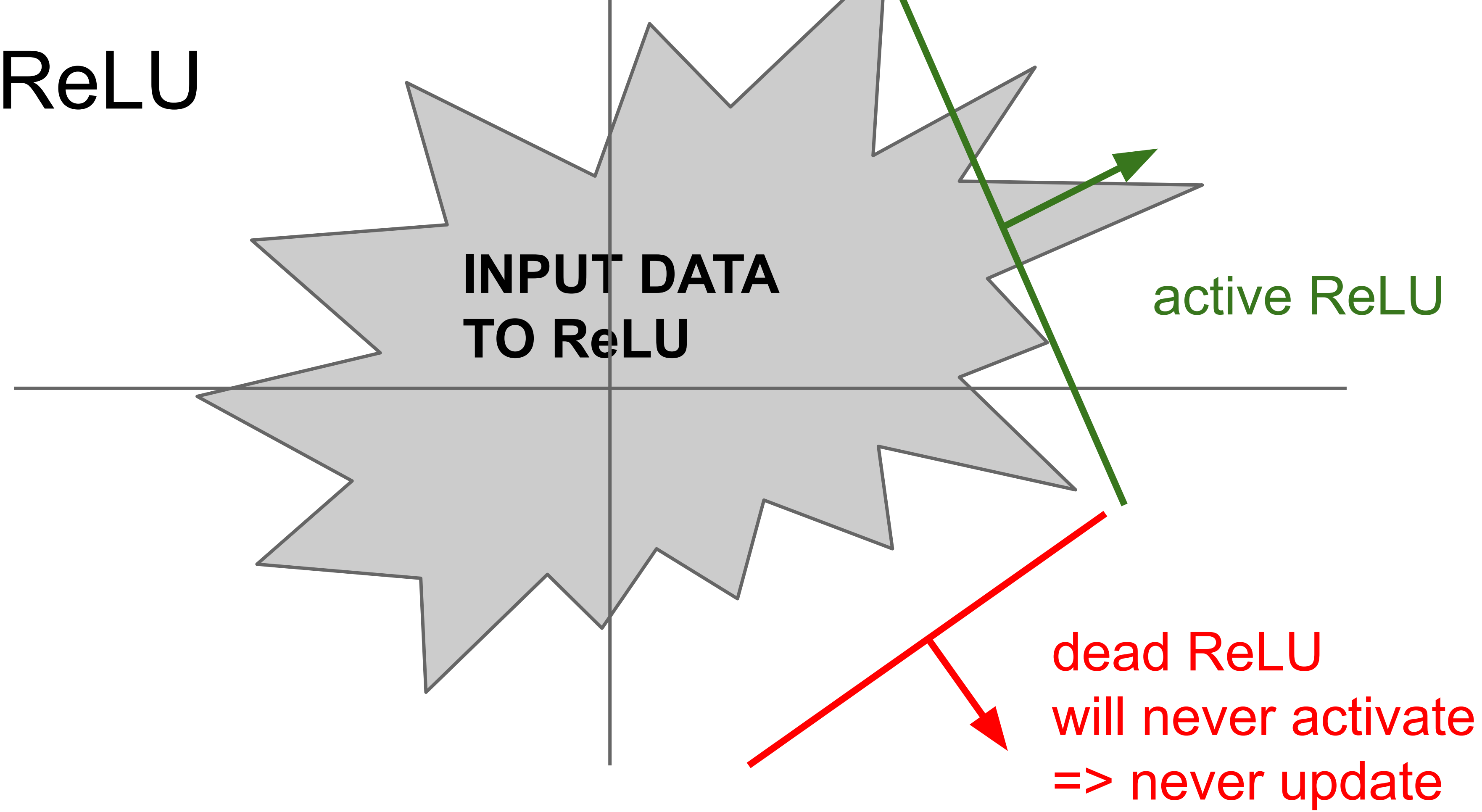
hint: what is the gradient when x < 0?





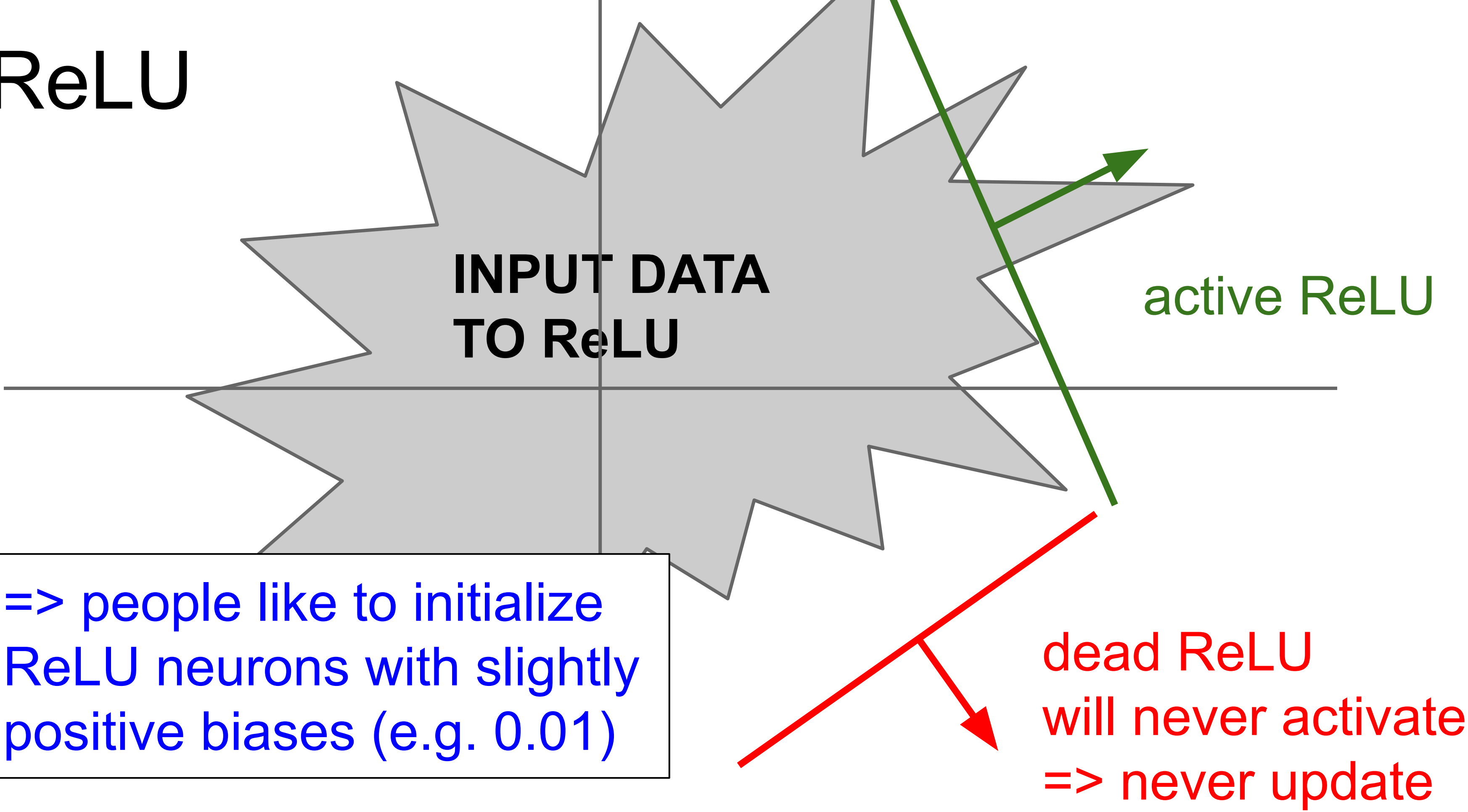
What happens when x = -10?
What happens when x = 0?
What happens when x = 10?

ReLU



INPUT DATA
TO ReLU
active ReLU
dead ReLU
will never activate
=> never update

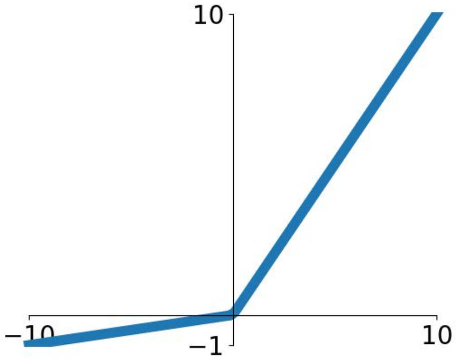
ReLU



=> people like to initialize ReLU neurons with slightly positive biases (e.g. 0.01)

Leaky ReLU

[Mass et al., 2013]
[He et al., 2015]



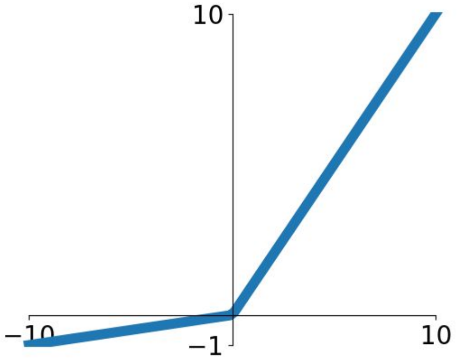
- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not "die".**

Leaky ReLU

$$f(x) = \max(0.01x, x)$$

Leaky ReLU

[Mass et al., 2013]
[He et al., 2015]



- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not "die".**

Leaky ReLU

$$f(x) = \max(0.01x, x)$$

Parametric Rectifier (PReLU)

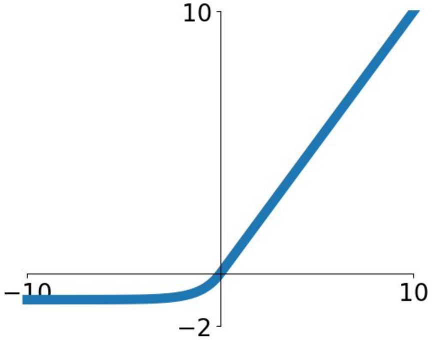
$$f(x) = \max(\alpha x, x)$$

backprop into \alpha (parameter)

ELU

[Clevert et al., 2015]

Exponential Linear Units (ELU)



$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha\,(\exp(x) - 1) & \text{if } x \le 0 \end{cases}$$

(Alpha default = 1)

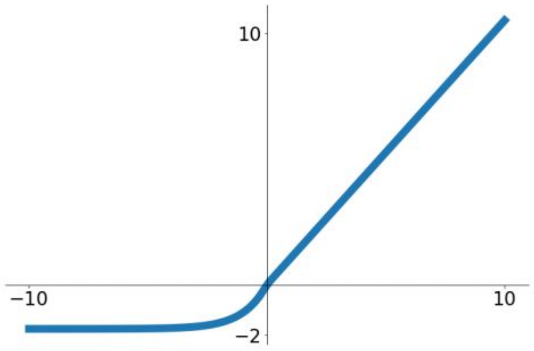
- All benefits of ReLU
- Closer to zero mean outputs

- Computation requires exp()
- Negative saturation can kill gradients for large negative

SELU

[Klambauer et al. ICLR 2017]

Scaled Exponential Linear Units (SELU)



$$f(x) = \begin{cases} \lambda x & \text{if } x > 0 \\ \lambda\alpha(e^x - 1) & \text{otherwise} \end{cases}$$

$\alpha = 1.6733$, $\lambda = 1.0507$

- Scaled versionof ELU that works better for deep networks
- "Self-normalizing" property;
- Can train deep SELU networks without BatchNorm
  - (will discuss more later)

Maxout "Neuron"

[Goodfellow et al., 2013]

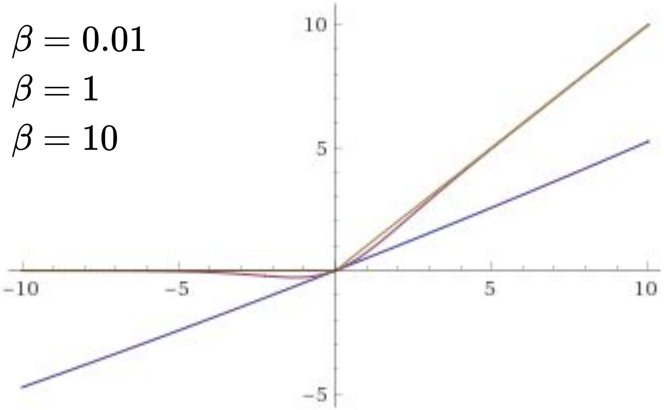
- Does not have the basic form of dot product -> nonlinearity
- Generalizes ReLU and Leaky ReLU
- Linear Regime! Does not saturate! Does not die!

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

Problem: doubles the number of parameters/weights :(

Swish

[Ramachandran et al. 2018]



- They trained a neural network to generate and test out different non-linearities.
- Swish outperformed all other options for CIFAR-10 accuracy

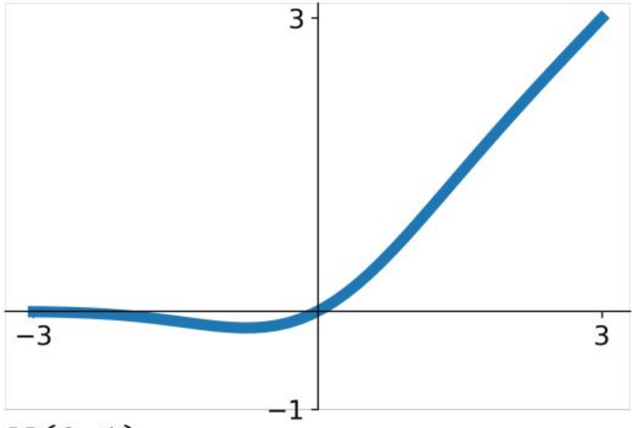
Swish

$$f(x) = x\sigma(\beta x)$$

GeLU

[Hendrycks and Gimpel, Gaussian Error Linear Units (GELUs), 2016]

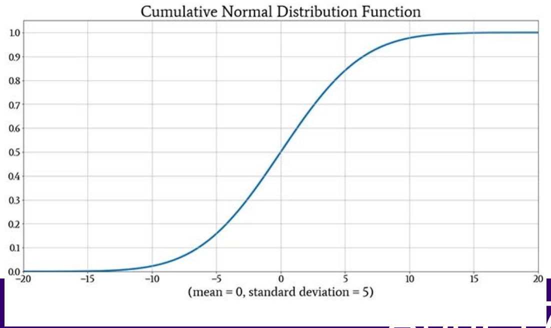
Gaussian Error Linear Units



$$X \sim N(0,1)$$

$$gelu(x) = xP(X \leq x) = \frac{x}{2}\left(1 + \text{erf}(x/\sqrt{2})\right)$$
$$\approx x\sigma(1.702x)$$

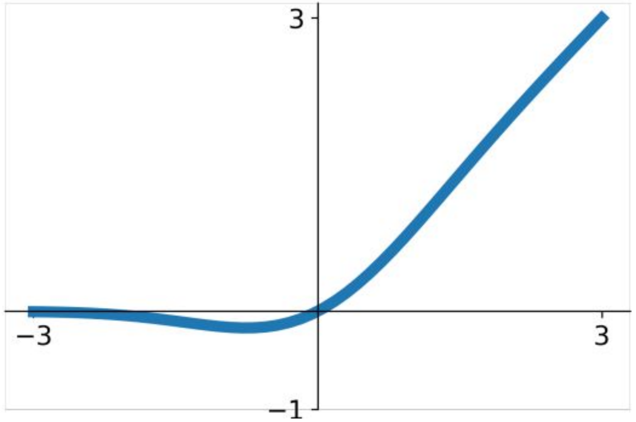
- Idea: combine the best parts of sigmoid and relu.
- GELU transitions more smoothly
- It weights each input according to the Gaussian (Normal) CDF



GeLU

[Hendrycks and Gimpel, Gaussian Error Linear Units (GELUs), 2016]

Mathematically, $\mathrm{GELU}(x) = x \times \Phi(x)$



where $\Phi(x)$ is the cumulative distribution function (CDF) of a standard normal distribution N (0,1)

$$\Phi(x) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right)\right]$$

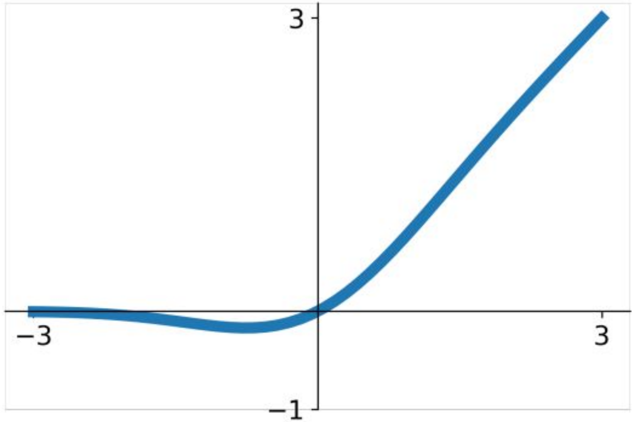
It is approximated as

$$\mathrm{GELU}(x) \approx 0.5\, x \left[1 + \tanh\left(\sqrt{\frac{2}{\pi}}\,(x + 0.044715\, x^3)\right)\right]$$

Activation Functions

[Hendrycks and Gimpel, Gaussian Error Linear Units (GELUs), 2016]

GeLU



$X \sim N(0,1)$

$$gelu(x) = xP(X \leq x) = \frac{x}{2}\left(1 + \operatorname{erf}(x/\sqrt{2})\right)$$
$$\approx x\sigma(1.702x)$$

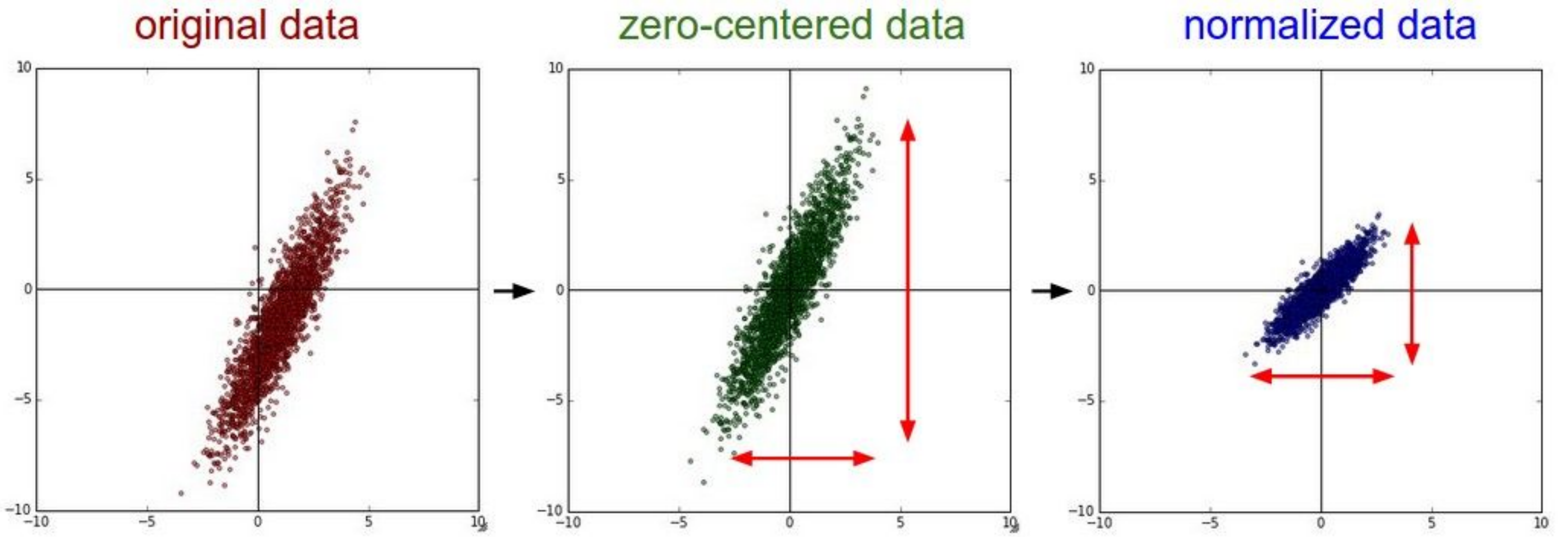
- More continuous version of ReLU. It scales the input x by a probability factor dependent on x
- Avoid dead neurons
- Empirically stable training
- Very common in Transformers (BERT, GPT, ViT)

TLDR: In practice:

- Use ReLU. Be careful with your learning rates
- Use GeLU when using transformers
- Try out Leaky ReLU / Maxout / ELU / SELU
  - To squeeze out some marginal gains
- Don't use sigmoid or tanh

Data Preprocessing

Data Preprocessing

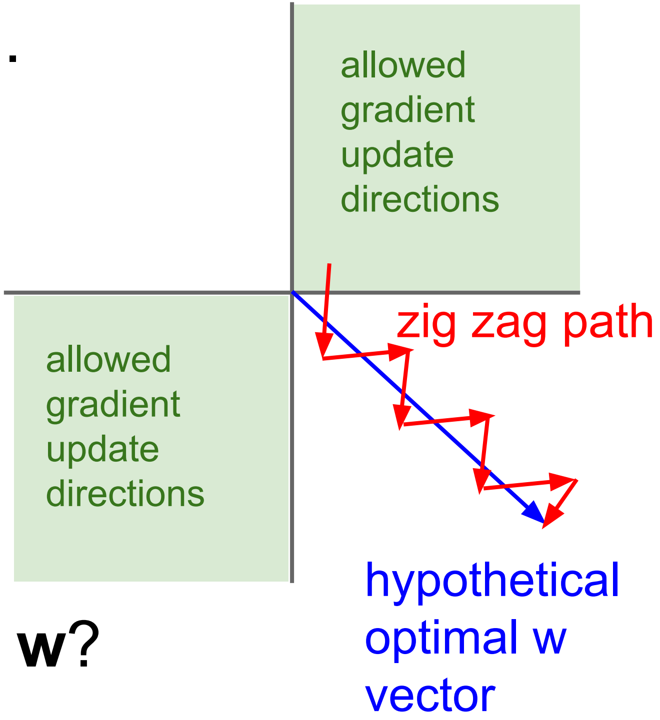



```
X -= np.mean(X, axis = 0)
X /= np.std(X, axis = 0)
```

(Assume X [NxD] is data matrix, each example in a row)

Remember: Consider what happens when the input to a neuron is always positive...

$$f\left(\sum_i w_i x_i + b\right)$$



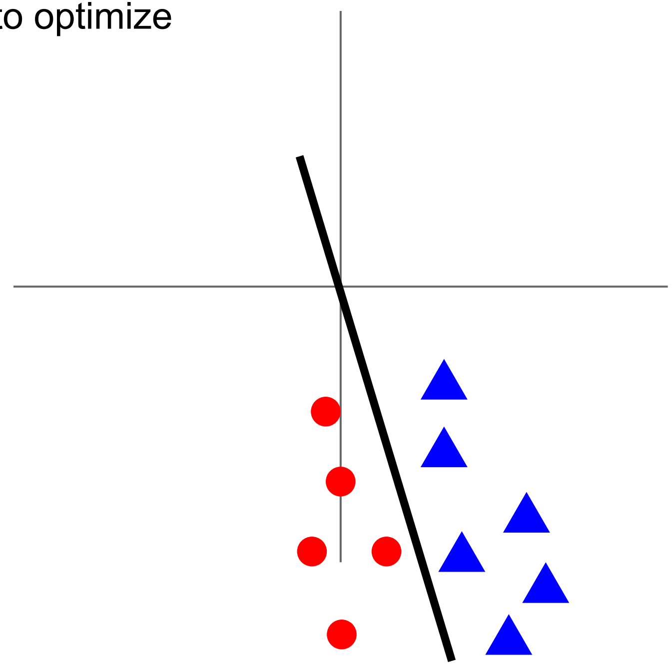
What can we say about the gradients on **w**?

Always all positive or all negative :(

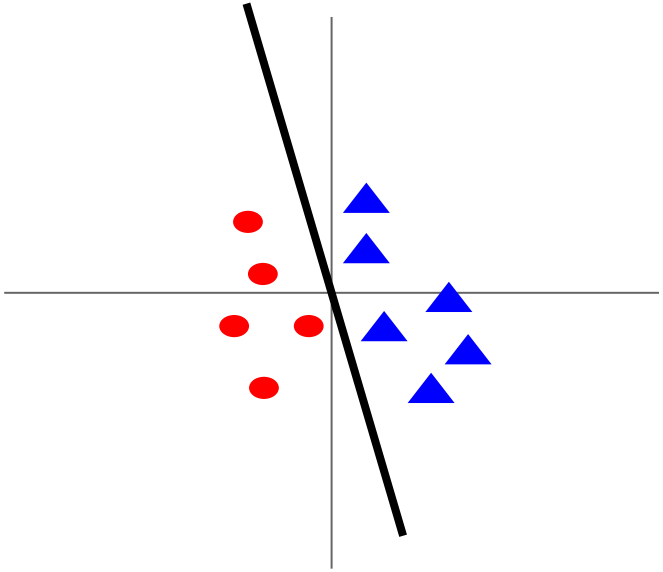
(this is also why you want zero-mean data!)

Data Preprocessing

Before normalization: classification loss very sensitive to changes in weight matrix; hard to optimize

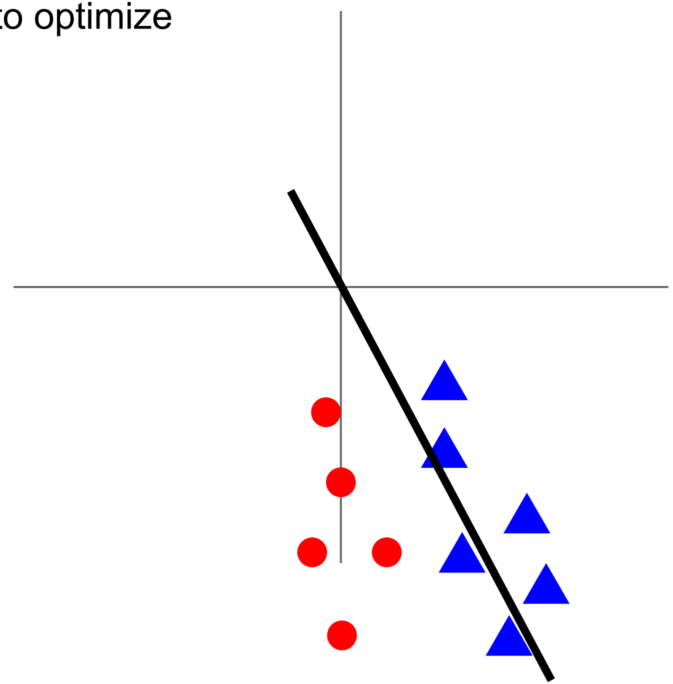


After normalization: less sensitive to small changes in weights; easier to optimize

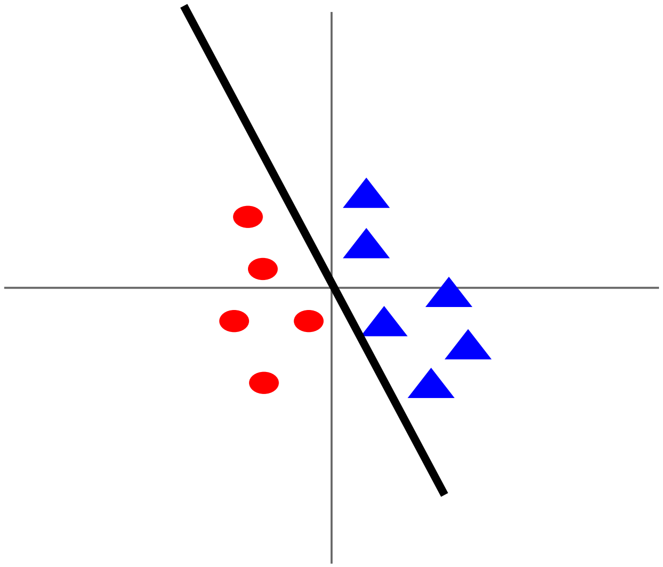


Data Preprocessing

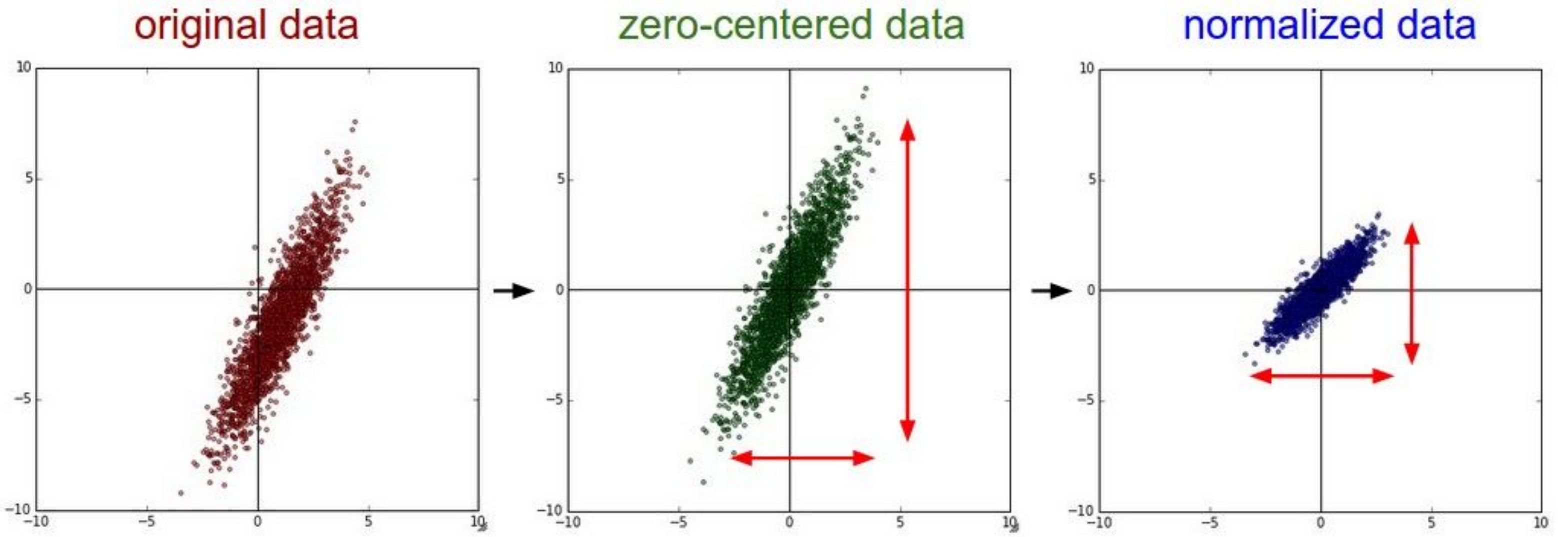
Before normalization: classification loss very sensitive to changes in weight matrix; hard to optimize



After normalization: less sensitive to small changes in weights; easier to optimize



Data Preprocessing



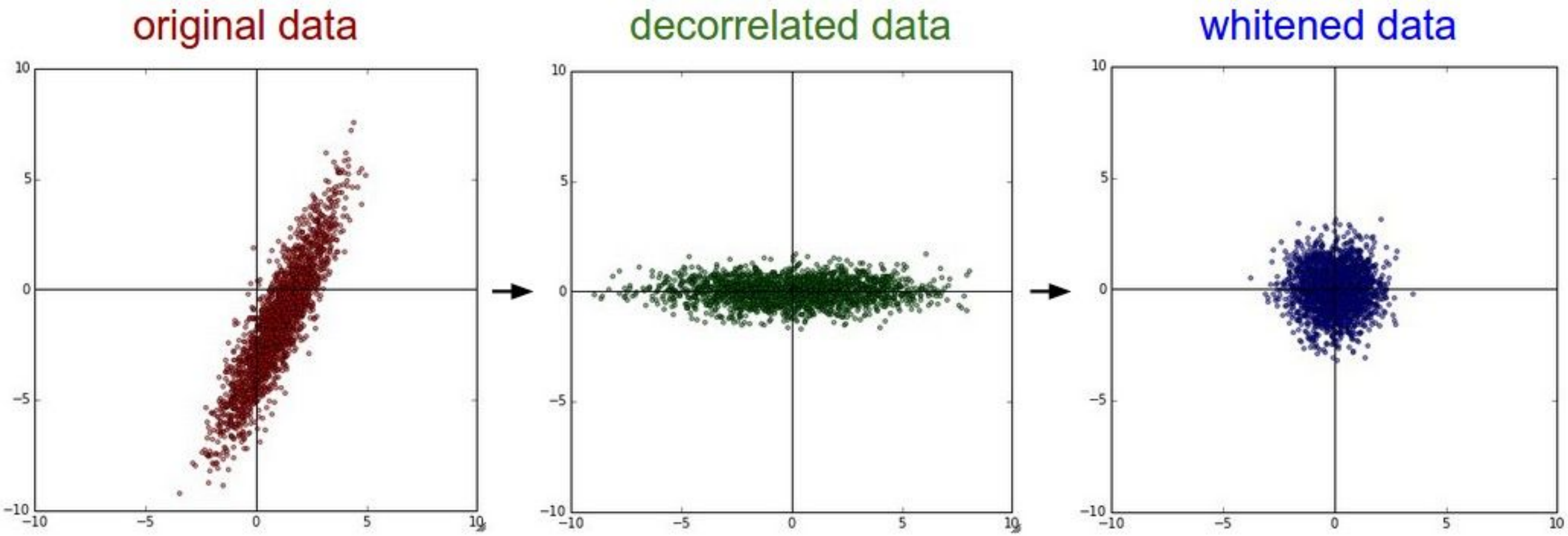

```
X -= np.mean(X, axis = 0)
```

```
X /= np.std(X, axis = 0)
```

(Assume X [NxD] is data matrix, each example in a row)

Data Preprocessing

In practice, you may also see **PCA** and **Whitening** of the data



(data has diagonal covariance matrix)

(covariance matrix is the identity matrix)

TLDR: In practice for Images: center only

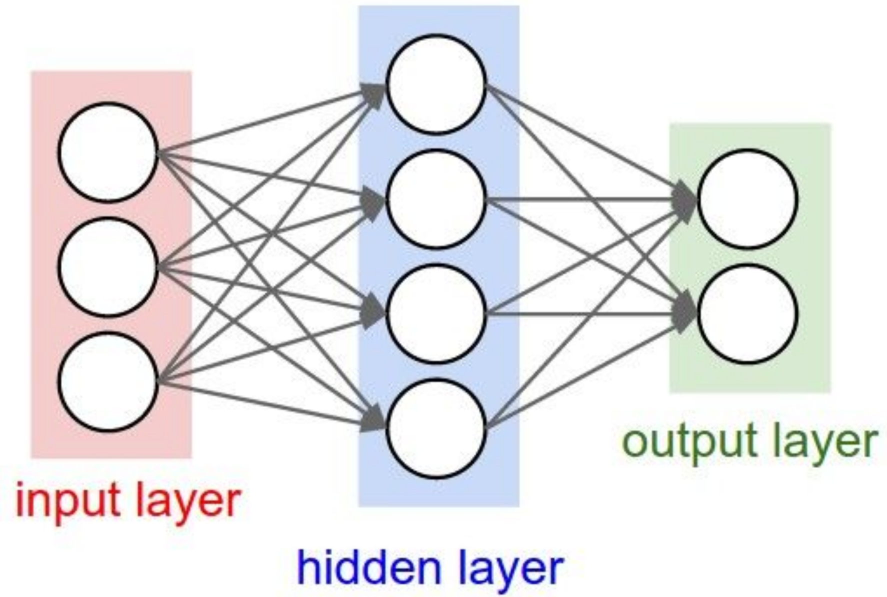
e.g. consider CIFAR-10 example with [32,32,3] images

- Subtract the mean image (e.g. AlexNet)
  (mean image = [32,32,3] array)
- Subtract per-channel mean (e.g. VGGNet)
  (mean along each channel = 3 numbers)
- Subtract per-channel mean and
  Divide by per-channel std (e.g. ResNet)
  (mean along each channel = 3 numbers)

Not common to do PCA or whitening

Weight Initialization

- Q: what happens when W=constant init is used?



- First idea: **Small random numbers**
(gaussian with zero mean and 1e-2 standard deviation)

```
W = 0.01 * np.random.randn(Din, Dout)
```

- First idea: **Small random numbers**

(gaussian with zero mean and 1e-2 standard deviation)

```
W = 0.01 * np.random.randn(Din, Dout)
```

Works ~okay for small networks, but problems with deeper networks.

Weight Initialization: Activation statistics

```
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = 0.01 * np.random.randn(Din, Dout)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

Forward pass for a 6-layer net with hidden size 4096

What will happen to the activations for the last layer?

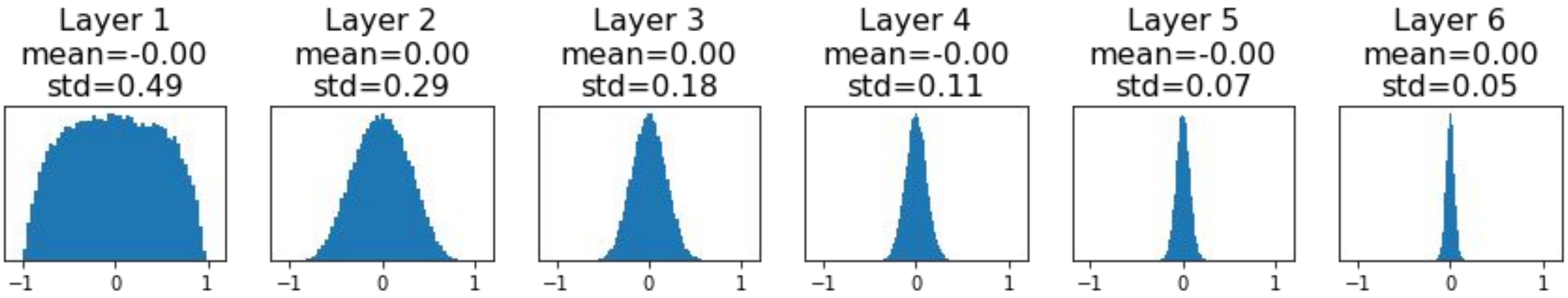
Weight Initialization: Activation statistics

```
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = 0.01 * np.random.randn(Din, Dout)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

Forward pass for a 6-layer net with hidden size 4096

All activations tend to zero for deeper network layers

Q: What do the gradients dL/dW look like?



Weight Initialization: Activation statistics

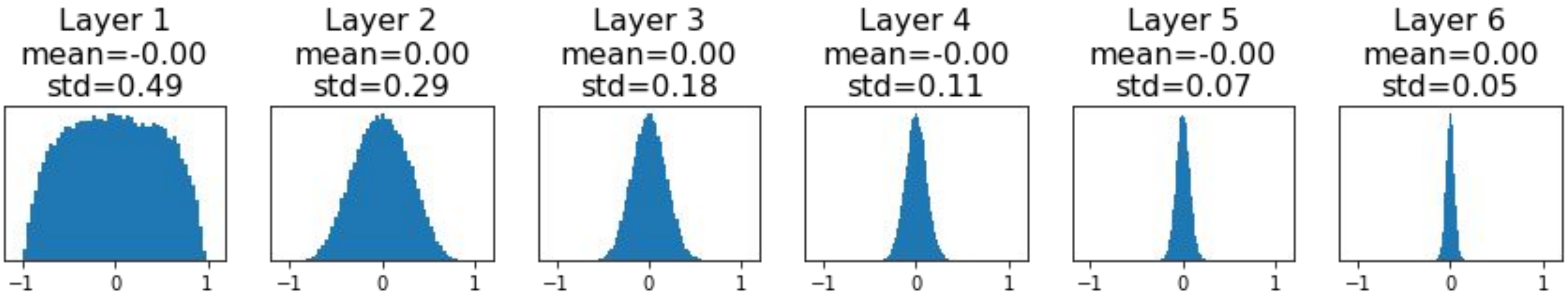
```
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = 0.01 * np.random.randn(Din, Dout)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

Forward pass for a 6-layer net with hidden size 4096

All activations tend to zero for deeper network layers

Q: What do the gradients dL/dW look like?

A: All zero, no learning =(



Weight Initialization: Activation statistics

```
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = 0.05 * np.random.randn(Din, Dout)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

Increase std of initial weights from 0.01 to 0.05

What will happen to the activations when the weights are initialized with larger values?

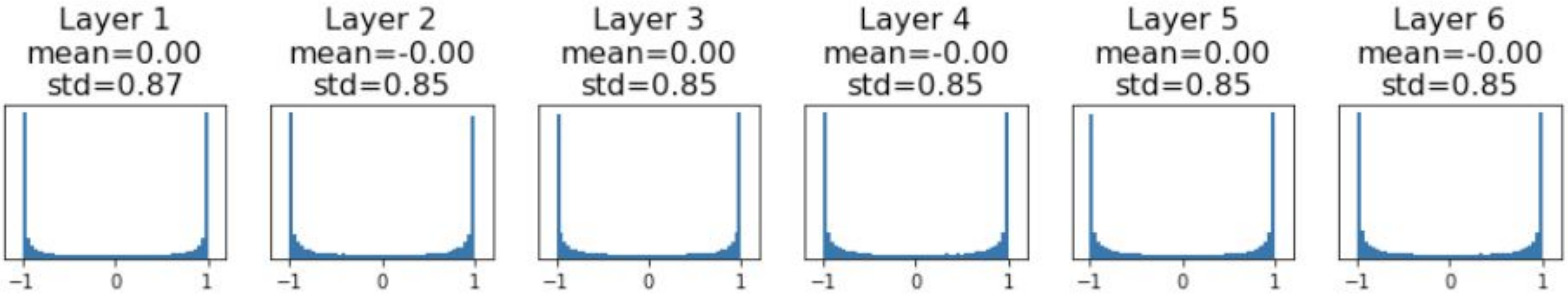
Weight Initialization: Activation statistics

```
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = 0.05 * np.random.randn(Din, Dout)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

Increase std of initial weights from 0.01 to 0.05

All activations saturate

Q: What do the gradients look like?



Weight Initialization: Activation statistics

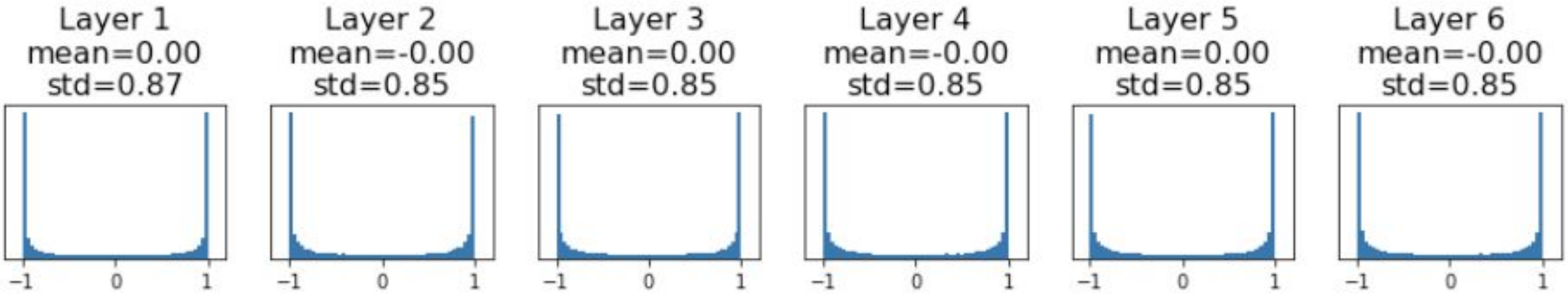
```
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = 0.05 * np.random.randn(Din, Dout)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

Increase std of initial weights from 0.01 to 0.05

All activations saturate

Q: What do the gradients look like?

A: Local gradients all zero, no learning =(



Weight Initialization: "Xavier" Initialization

```
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

"Xavier" initialization:
std = 1/sqrt(Din)

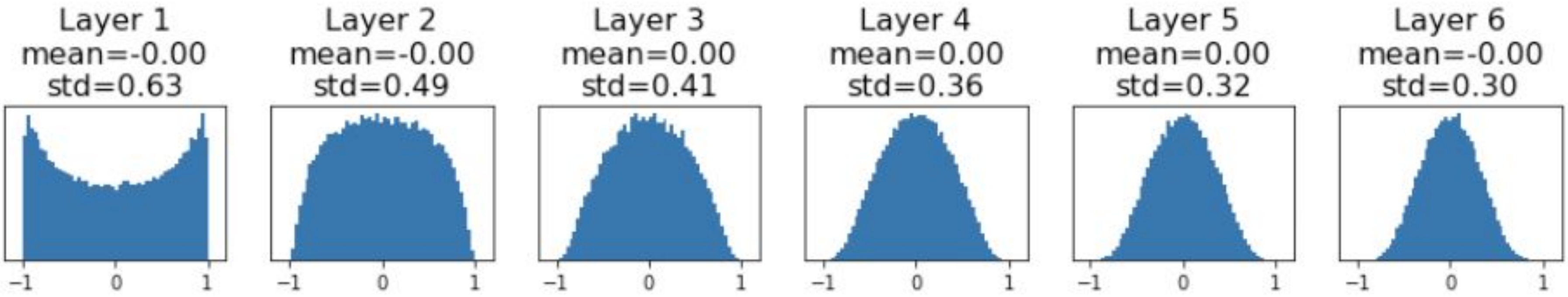
Glorot and Bengio, "Understanding the difficulty of training deep feedforward neural networks", AISTAT 2010

Weight Initialization: "Xavier" Initialization

```
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

"Xavier" initialization:
std = 1/sqrt(Din)

"Just right": Activations are nicely scaled for all layers!



Glorot and Bengio, "Understanding the difficulty of training deep feedforward neural networks", AISTAT 2010

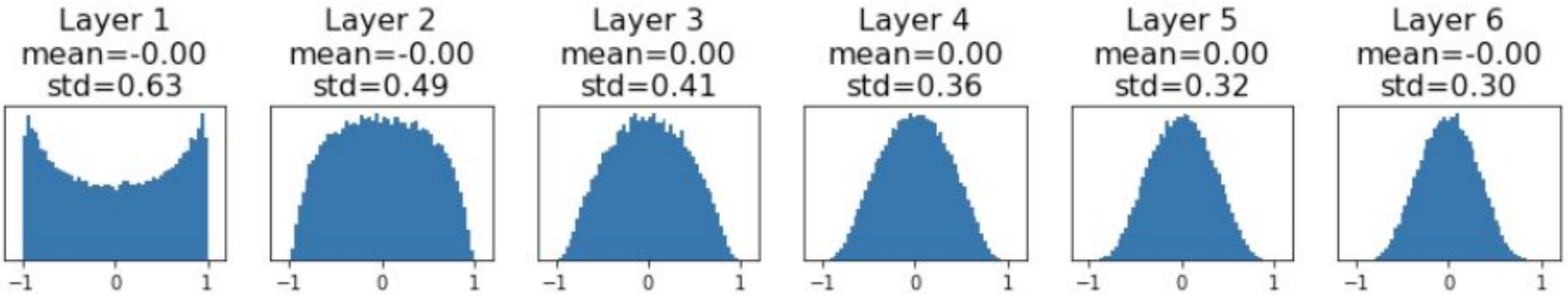
Weight Initialization: “Xavier” Initialization

```
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

“Xavier” initialization:
std = 1/sqrt(Din)

“Just right”: Activations are nicely scaled for all layers!

For conv layers, Din is filter_size^2 * input_channels



Glorot and Bengio, “Understanding the difficulty of training deep feedforward neural networks”, AISTAT 2010

Weight Initialization: "Xavier" Initialization

```
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

"Xavier" initialization:
std = 1/sqrt(Din)

"Just right": Activations are nicely scaled for all layers!

For conv layers, Din is filter_size^2 * input_channels

Let: $y = x_1w_1+x_2w_2+...+x_{Din}w_{Din}$

Glorot and Bengio, "Understanding the difficulty of training deep feedforward neural networks", AISTAT 2010

Weight Initialization: "Xavier" Initialization

```
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

"Xavier" initialization:
std = 1/sqrt(Din)

"Just right": Activations are nicely scaled for all layers!

For conv layers, Din is filter_size^2 * input_channels

Let: $y = x_1w_1+x_2w_2+\ldots+x_{Din}w_{Din}$

Assume: $Var(x_1) = Var(x_2) = \ldots = Var(x_{Din})$

Glorot and Bengio, "Understanding the difficulty of training deep feedforward neural networks", AISTAT 2010

Weight Initialization: “Xavier” Initialization

```
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

“Xavier” initialization:
std = 1/sqrt(Din)

“Just right”: Activations are nicely scaled for all layers!

For conv layers, Din is $\text{filter\_size}^2 * \text{input\_channels}$

Let: $y = x_1w_1+x_2w_2+\ldots+x_{Din}w_{Din}$

Assume: $Var(x_1) = Var(x_2)= \ldots=Var(x_{Din})$

We want: $Var(y) = Var(x_i)$

Glorot and Bengio, “Understanding the difficulty of training deep feedforward neural networks”, AISTAT 2010

Weight Initialization: “Xavier” Initialization

```python
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

“Xavier” initialization:
std = 1/sqrt(Din)

“Just right”: Activations are nicely scaled for all layers!

For conv layers, Din is filter_size^2 * input_channels

Let: $y = x_1w_1+x_2w_2+...+x_{Din}w_{Din}$

Assume: $Var(x_1) = Var(x_2)= \ldots =Var(x_{Din})$

We want: $Var(y) = Var(x_i)$

$Var(y) = Var(x_1w_1+x_2w_2+...+x_{Din}w_{Din})$
[substituting value of y]

Glorot and Bengio, “Understanding the difficulty of training deep feedforward neural networks”, AISTAT 2010

Weight Initialization: "Xavier" Initialization

```python
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

"Xavier" initialization:
std = 1/sqrt(Din)

"Just right": Activations are nicely scaled for all layers!

For conv layers, Din is $filter\_size^2$ * input_channels

Let: $y = x_1w_1+x_2w_2+...+x_{Din}w_{Din}$

Assume: $Var(x_1) = Var(x_2)= \ldots =Var(x_{Din})$

We want: $Var(y) = Var(x_i)$

$Var(y) = Var(x_1w_1+x_2w_2+...+x_{Din}w_{Din})$
$= Din\ Var(x_iw_i)$
[Assume all x_i, w_i are iid]

Glorot and Bengio, "Understanding the difficulty of training deep feedforward neural networks", AISTAT 2010

Weight Initialization: "Xavier" Initialization

```python
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

"Xavier" initialization:
std = 1/sqrt(Din)

"Just right": Activations are nicely scaled for all layers!

For conv layers, Din is filter_size^2 * input_channels

Let: $y = x_1w_1+x_2w_2+...+x_{Din}w_{Din}$

Assume: $Var(x_1) = Var(x_2) = ... = Var(x_{Din})$

We want: $Var(y) = Var(x_i)$

$Var(y) = Var(x_1w_1+x_2w_2+...+x_{Din}w_{Din})$
$= Din\ Var(x_iw_i)$
$= Din\ Var(x_i)\ Var(w_i)$

[Assume all x_i, w_i are zero mean]

Glorot and Bengio, "Understanding the difficulty of training deep feedforward neural networks", AISTAT 2010

Weight Initialization: "Xavier" Initialization

```python
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

"Xavier" initialization:
std = 1/sqrt(Din)

"Just right": Activations are nicely scaled for all layers!

For conv layers, Din is filter_size^2 * input_channels

Let: $y = x_1w_1+x_2w_2+...+x_{Din}w_{Din}$

Assume: $Var(x_1) = Var(x_2)= ...=Var(x_{Din})$

We want: $Var(y) = Var(x_i)$

$Var(y) = Var(x_1w_1+x_2w_2+...+x_{Din}w_{Din})$
$= Din\ Var(x_iw_i)$
$= Din\ Var(x_i)\ Var(w_i)$
[Assume all x_i, w_i are iid]

So, $Var(y) = Var(x_i)$ only when $Var(w_i) = 1/Din$

Glorot and Bengio, "Understanding the difficulty of training deep feedforward neural networks", AISTAT 2010

Weight Initialization: What about ReLU?

```
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.maximum(0, x.dot(W))
    hs.append(x)
```

Change from tanh to ReLU

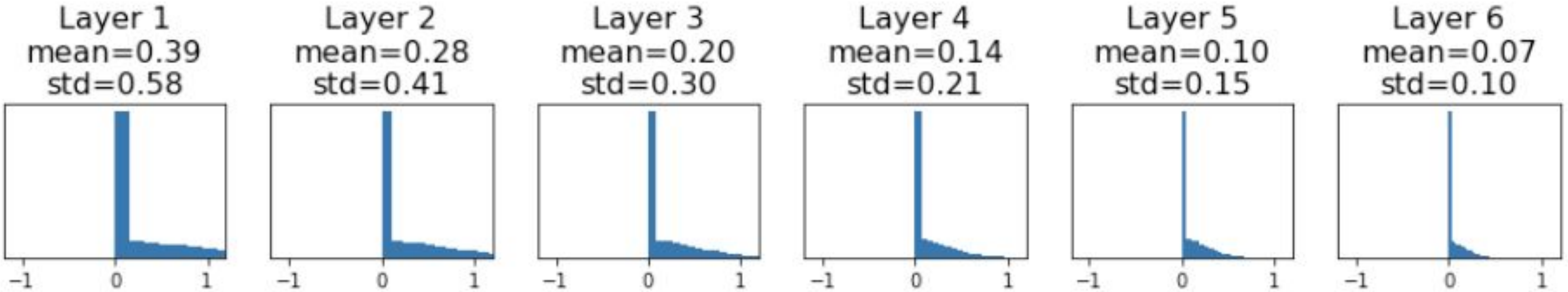
Weight Initialization: What about ReLU?

```
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.maximum(0, x.dot(W))
    hs.append(x)
```

Change from tanh to ReLU

Xavier assumes zero centered activation function

Activations collapse to zero again, no learning =(

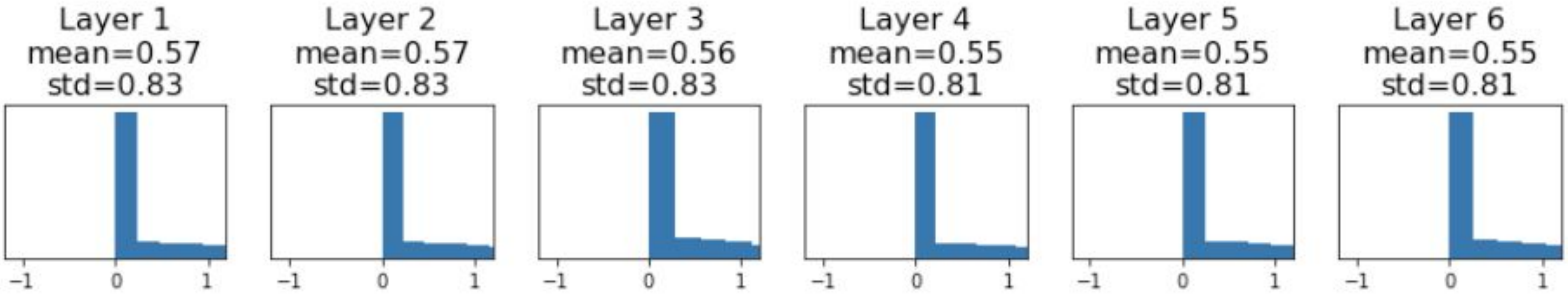


Weight Initialization: Kaiming / MSRA Initialization

```
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) * np.sqrt(2/Din)
    x = np.maximum(0, x.dot(W))
    hs.append(x)
```

ReLU correction: std = sqrt(2 / Din)

"Just right": Activations are nicely scaled for all layers!



He et al, "Delving Deep into Rectifiers: Surpassing Human-Level Performance on ImageNet Classification", ICCV 2015

Proper initialization is an active area of research…

Understanding the difficulty of training deep feedforward neural networks
by Glorot and Bengio, 2010

Exact solutions to the nonlinear dynamics of learning in deep linear neural networks by Saxe et al, 2013

Random walk initialization for training very deep feedforward networks by Sussillo and Abbott, 2014

Delving deep into rectifiers: Surpassing human-level performance on ImageNet classification by He et al., 2015

Data-dependent Initializations of Convolutional Neural Networks by Krähenbühl et al., 2015

All you need is a good init, Mishkin and Matas, 2015

Fixup Initialization: Residual Learning Without Normalization, Zhang et al, 2019

The Lottery Ticket Hypothesis: Finding Sparse, Trainable Neural Networks, Frankle and Carbin, 2019

Batch Normalization

Batch Normalization

[Ioffe and Szegedy, 2015]

"you want zero-mean unit-variance activations? just make them so."

consider a batch of activations at some layer. To make each dimension zero-mean unit-variance, apply:

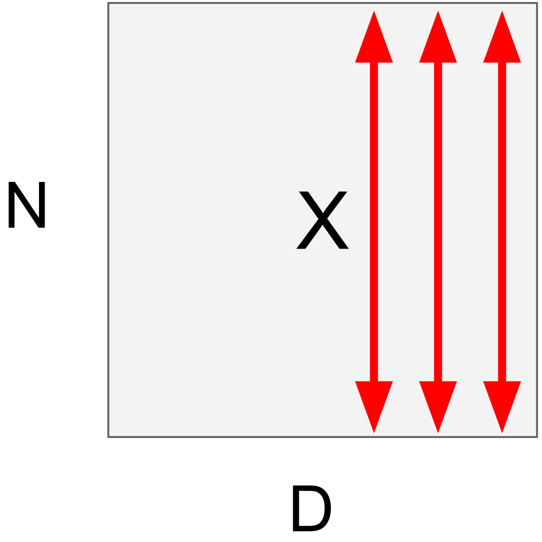
$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

this is a vanilla differentiable function...

Batch Normalization

[Ioffe and Szegedy, 2015]

Input: $x : N \times D$



$$\mu_j = \frac{1}{N} \sum_{i=1}^{N} x_{i,j}$$

Per-channel mean, shape is D

$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^{N} (x_{i,j} - \mu_j)^2$$

Per-channel var, shape is D

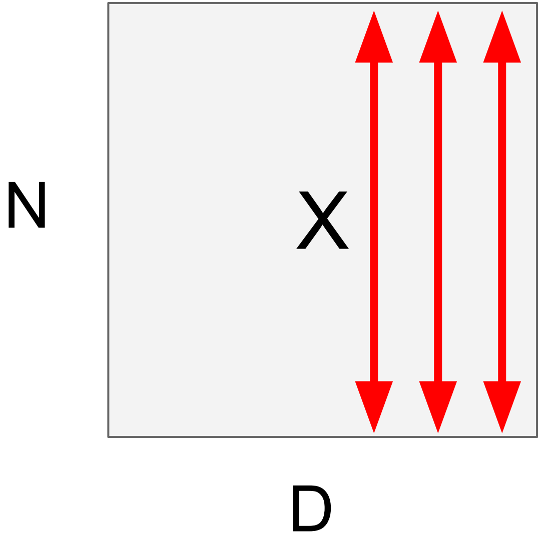
$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

Batch Normalization

[Ioffe and Szegedy, 2015]

Input: $x : N \times D$



$$\mu_j = \frac{1}{N} \sum_{i=1}^{N} x_{i,j}$$

Per-channel mean, shape is D

$$\sigma_j^2 = \frac{1}{N} \sum_{i=1}^{N} (x_{i,j} - \mu_j)^2$$

Per-channel var, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

Problem: What if zero-mean, unit variance is too hard of a constraint?

Batch Normalization

[Ioffe and Szegedy, 2015]

Input: $x : N \times D$

Learnable scale and shift parameters:

$$\gamma, \beta : D$$

Learning $\gamma = \sigma$, $\beta = \mu$ will recover the identity function!

$$\mu_j = \frac{1}{N}\sum_{i=1}^{N} x_{i,j}$$

Per-channel mean, shape is D

$$\sigma_j^2 = \frac{1}{N}\sum_{i=1}^{N}(x_{i,j} - \mu_j)^2$$

Per-channel var, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Batch Normalization: Test-Time

Estimates depend on minibatch; can't do this at test-time!

Input: $x : N \times D$

Learnable scale and shift parameters:

$$\gamma, \beta : D$$

Learning $\gamma = \sigma$, $\beta = \mu$ will recover the identity function!

$$\mu_j = \frac{1}{N}\sum_{i=1}^{N} x_{i,j}$$

Per-channel mean, shape is D

$$\sigma_j^2 = \frac{1}{N}\sum_{i=1}^{N}(x_{i,j} - \mu_j)^2$$

Per-channel var, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Batch Normalization: Test-Time

Input: $x : N \times D$

Learnable scale and shift parameters:

$$\gamma, \beta : D$$

During testing batchnorm becomes a linear operator! Can be fused with the previous fully-connected or conv layer

$$\mu_j =$$ (Running) average of values seen during training

Per-channel mean, shape is D

$$\sigma_j^2 =$$ (Running) average of values seen during training

Per-channel var, shape is D

$$\hat{x}_{i,j} = \frac{x_{i,j} - \mu_j}{\sqrt{\sigma_j^2 + \varepsilon}}$$

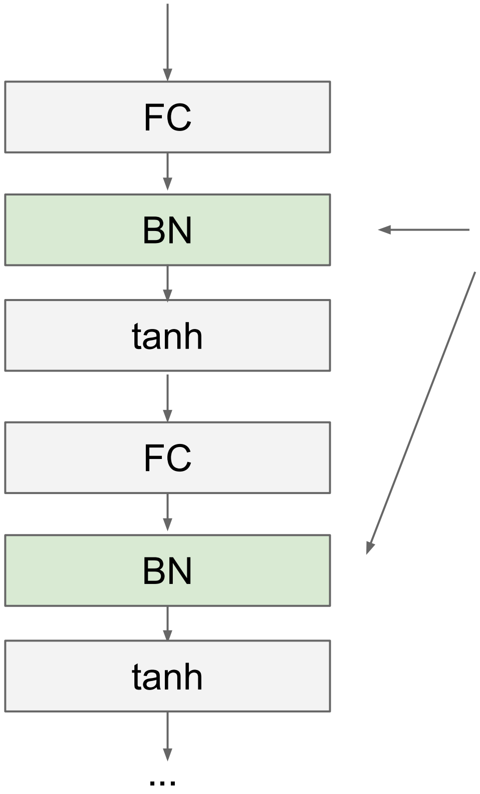
Normalized x, Shape is N x D

$$y_{i,j} = \gamma_j \hat{x}_{i,j} + \beta_j$$

Output, Shape is N x D

Batch Normalization

[Ioffe and Szegedy, 2015]

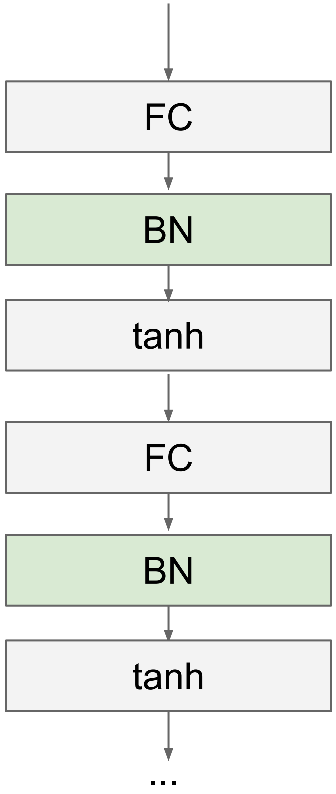


Usually inserted after Fully Connected or Convolutional layers, and before nonlinearity.

$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

Batch Normalization

[Ioffe and Szegedy, 2015]



- Makes deep networks **much** easier to train!
- Improves gradient flow
- Allows higher learning rates, faster convergence
- Networks become more robust to initialization
- Acts as regularization during training
- Zero overhead at test-time: can be fused with conv!
- Behaves differently during training and testing: this is a very common source of debugging!

Batch Normalization for convolutions

Batch Normalization for **fully-connected** networks

$$x:\ N \times D$$

Normalize $\downarrow$ (over N)

$$\mu, \sigma:\ 1 \times D$$

$$\gamma, \beta:\ 1 \times D$$

$$y = \gamma(x-\mu)/\sigma+\beta$$

Batch Normalization for **convolutional** networks (Spatial Batchnorm, BatchNorm2D)

$$x:\ N\times C\times H\times W$$

Normalize

$$\mu, \sigma:\ 1\times C\times 1\times 1$$

$$\gamma, \beta:\ 1\times C\times 1\times 1$$

$$y = \gamma(x-\mu)/\sigma+\beta$$

Layer Normalization

Layer Normalization for MLPs

Batch Normalization for fully-connected networks

$\mathbf{x}: \mathbf{N} \times \mathbf{D}$

Normalize

$\boldsymbol{\mu}, \boldsymbol{\sigma}: 1 \times \mathbf{D}$

$\boldsymbol{\gamma}, \boldsymbol{\beta}: 1 \times \mathbf{D}$

$\mathbf{y} = \boldsymbol{\gamma}(\mathbf{x}-\boldsymbol{\mu})/\boldsymbol{\sigma}+\boldsymbol{\beta}$

Layer Normalization for fully-connected networks
Same behavior at train and test!
Often used in transformers

$\mathbf{x}: \mathbf{N} \times \mathbf{D}$

Normalize

$\boldsymbol{\mu}, \boldsymbol{\sigma}: \mathbf{N} \times 1$

$\boldsymbol{\gamma}, \boldsymbol{\beta}: 1 \times \mathbf{D}$

$\mathbf{y} = \boldsymbol{\gamma}(\mathbf{x}-\boldsymbol{\mu})/\boldsymbol{\sigma}+\boldsymbol{\beta}$

Ba, Kiros, and Hinton, “Layer Normalization”, arXiv 2016

Instance Normalization

Instance Normalization for Convolutions

Batch Normalization for convolutional networks

$$x:\ N \times C \times H \times W$$

Normalize

$$\mu, \sigma:\ 1 \times C \times 1 \times 1$$

$$\gamma, \beta:\ 1 \times C \times 1 \times 1$$

$$y = \gamma(x-\mu)/\sigma+\beta$$

Instance Normalization for convolutional networks
Same behavior at train / test!

$$x:\ N \times C \times H \times W$$

Normalize

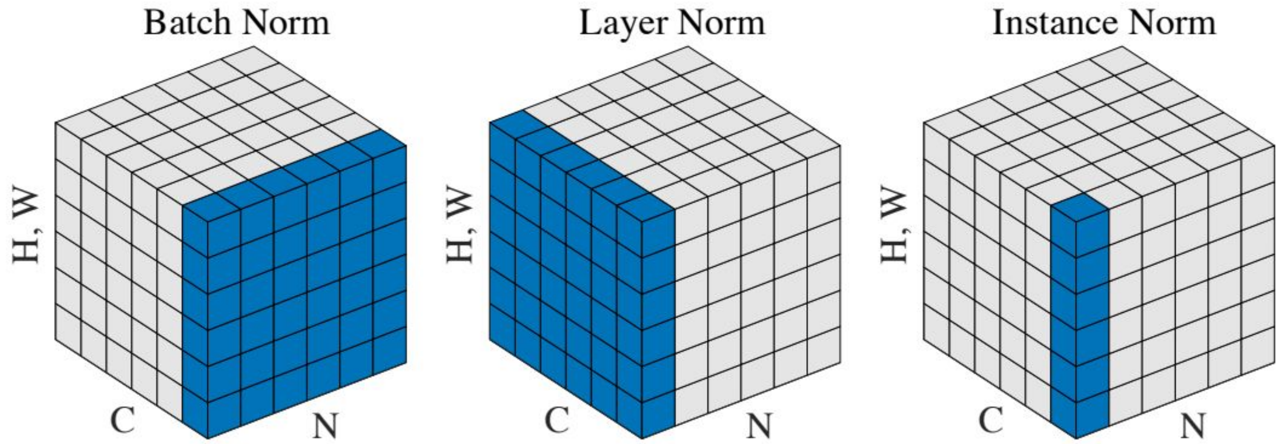
$$\mu, \sigma:\ N \times C \times 1 \times 1$$

$$\gamma, \beta:\ 1 \times C \times 1 \times 1$$

$$y = \gamma(x-\mu)/\sigma+\beta$$

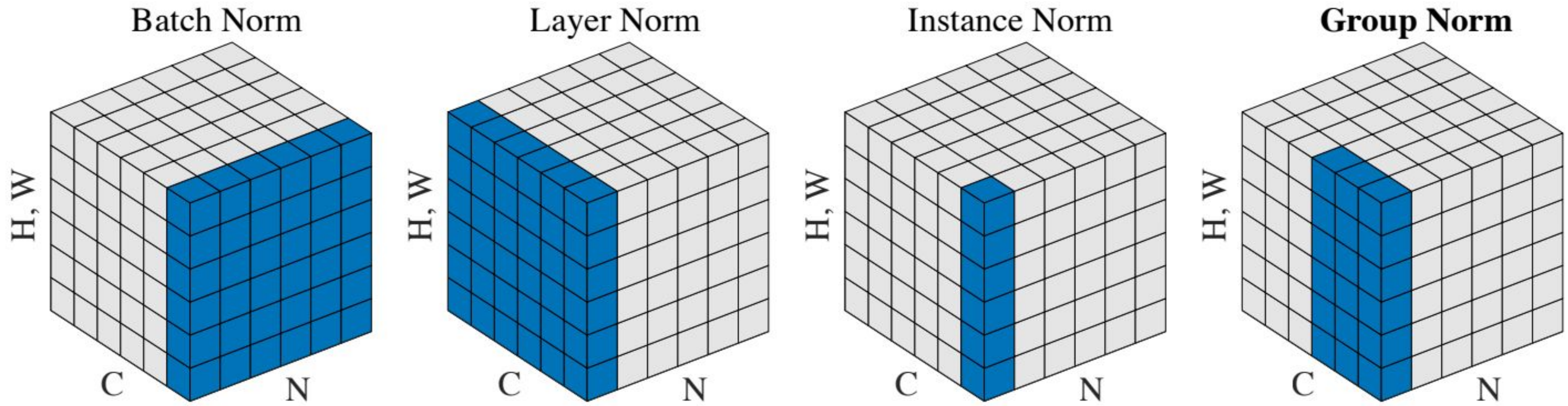
Ulyanov et al, Improved Texture Networks: Maximizing Quality and Diversity in Feed-forward Stylization and Texture Synthesis, CVPR 2017

Comparison of Normalization Layers



Wu and He, “Group Normalization”, ECCV 2018

Group Normalization



Wu and He, "Group Normalization", ECCV 2018

Summary

TLDRs

We looked in detail at:

- Activation Functions (use ReLU or GeLU)
- Data Preprocessing (images: subtract mean)
- Weight Initialization (use Xavier/He init)
- Batch Normalization (use this!)
- Layer Normalization (used in transformers!)

Next time: Optimizers

- Parameter update schemes
- Learning rate schedules
- Gradient checking
- Regularization (Dropout etc.)
- Babysitting learning
- Evaluation (Ensembles etc.)
- Hyperparameter Optimization