Lecture 5:
Convolutional Neural Networks

Administrative: EdStem

Please make sure to check and read all pinned EdStem posts.

Administrative: Assignment 1

Due 1/21 11:59pm

- K-Nearest Neighbor
- Linear classifiers: SVM, Softmax

Pushed back deadline by a few days.

Administrative: Assignment 2

Will be released this weekend

Due 1/30 11:59pm

- Multi-layer Neural Networks,
- Image Features,
- Optimizers

Administrative: Fridays

This Friday

Quiz 1: 6% of your grade

Backpropagation part 1 - the main algorithm for training neural networks

Presenter: Tanush Tadav

Administrative: Course Project

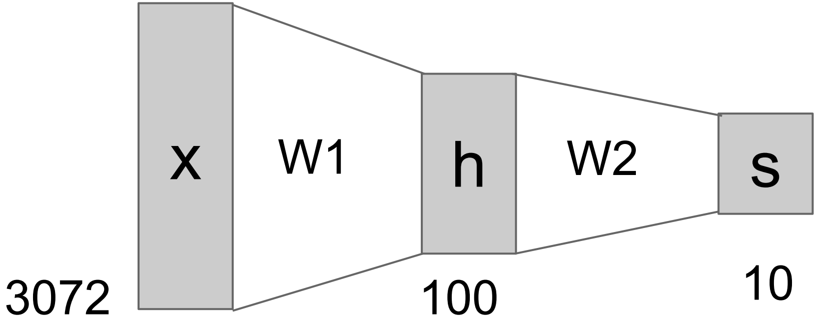
Project proposal due 2/06 11:59pm

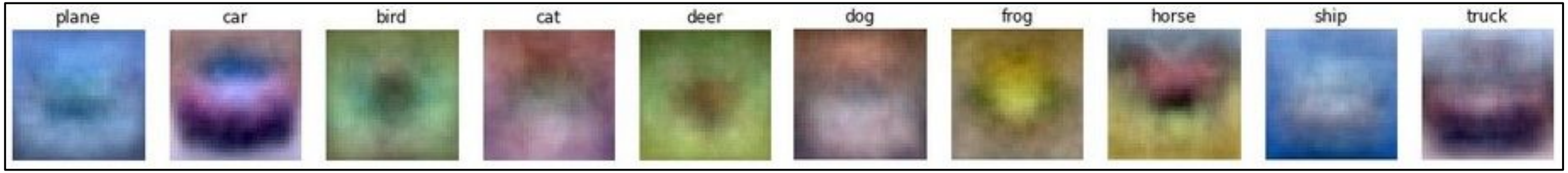
Come to office hours to talk about your ideas

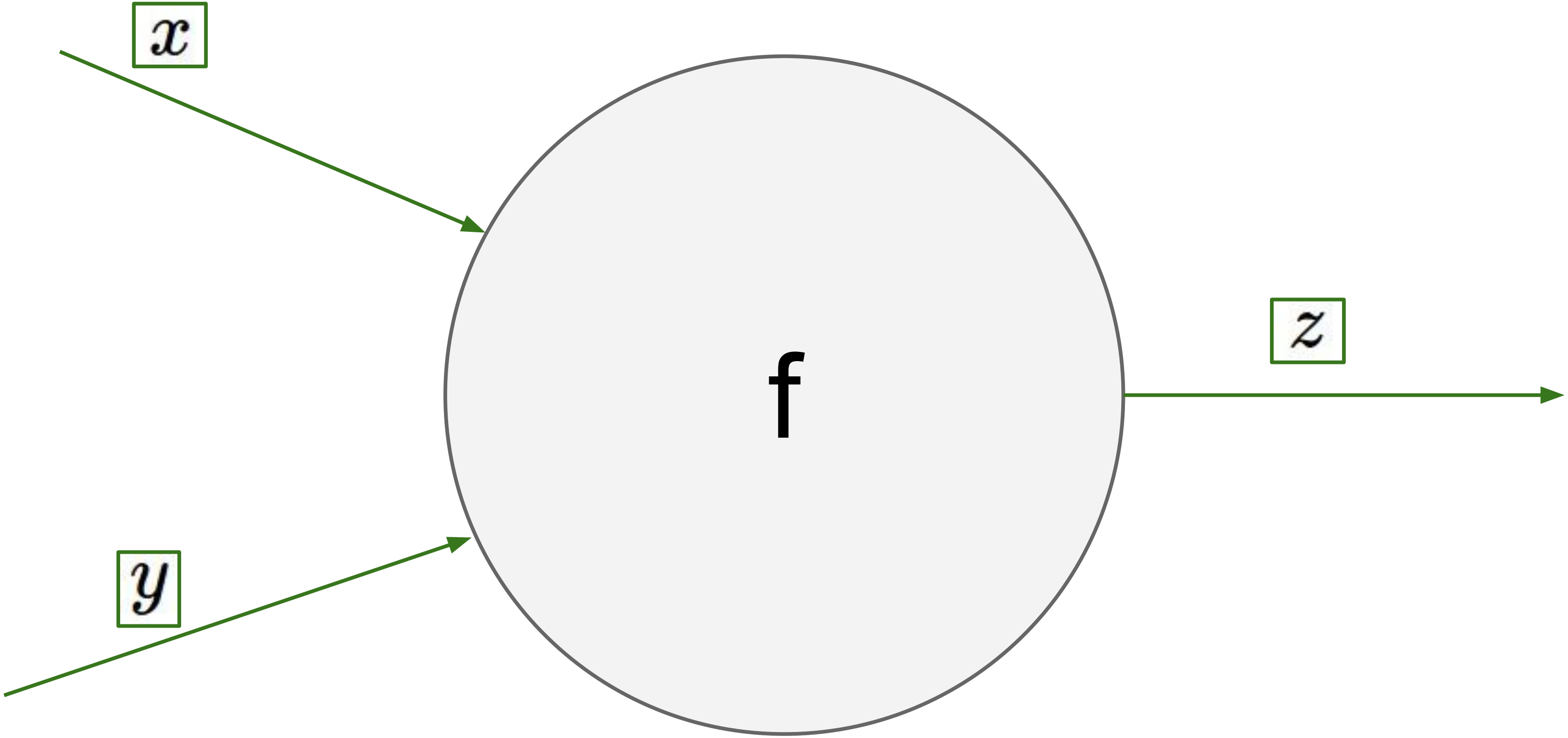
Last time: Neural Networks

Linear score function: $f = Wx$

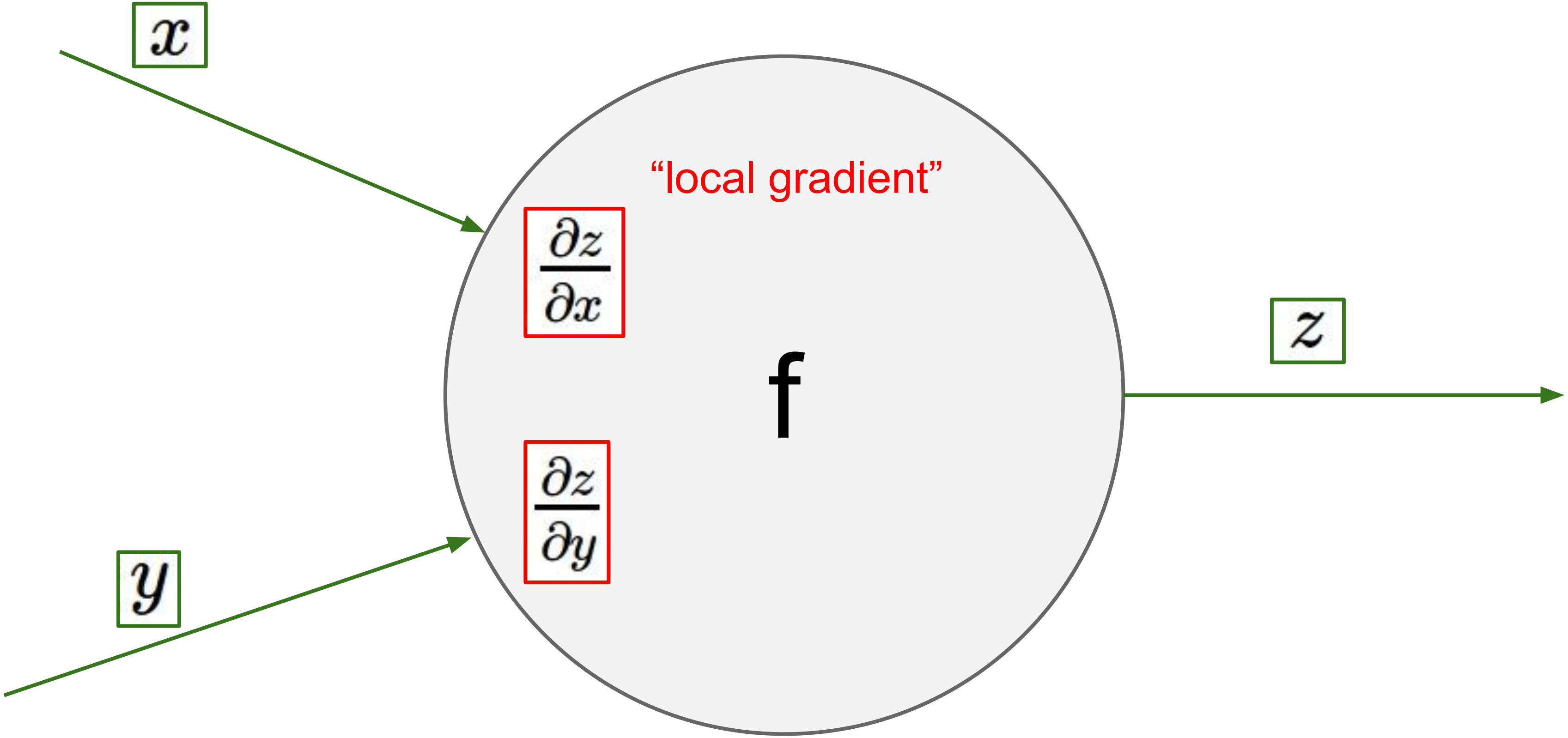
2-layer Neural Network $f = W_2 \max(0, W_1 x)$



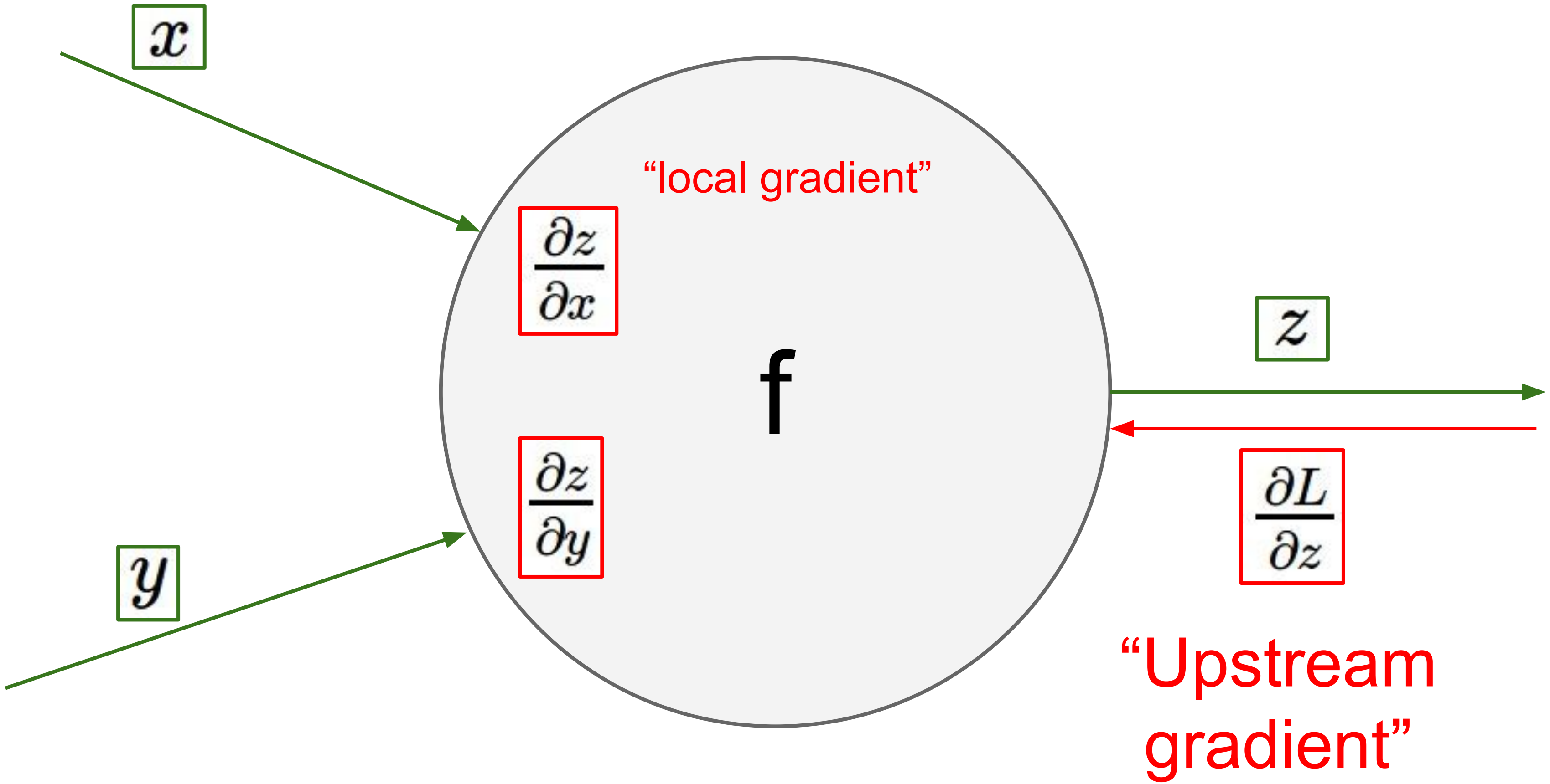


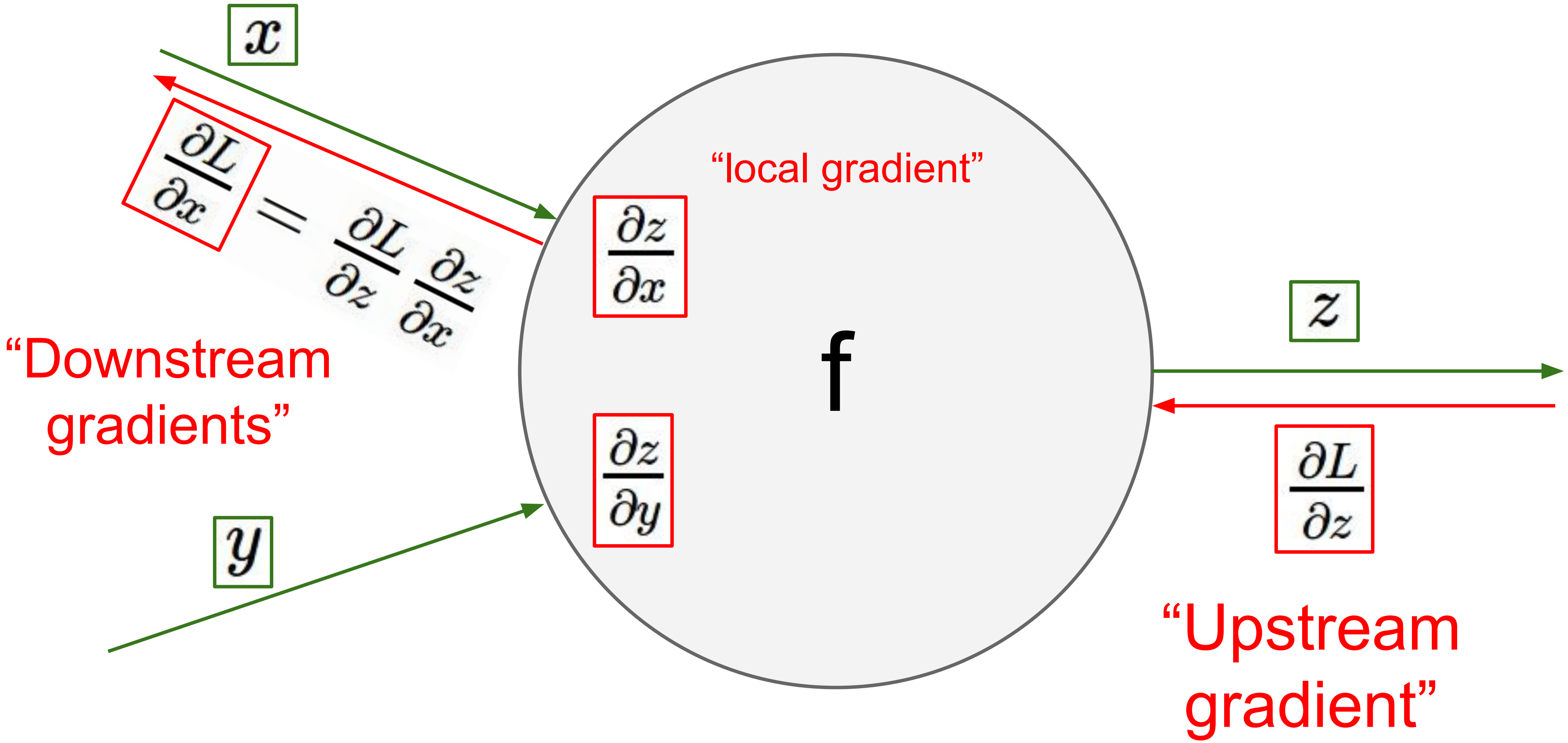


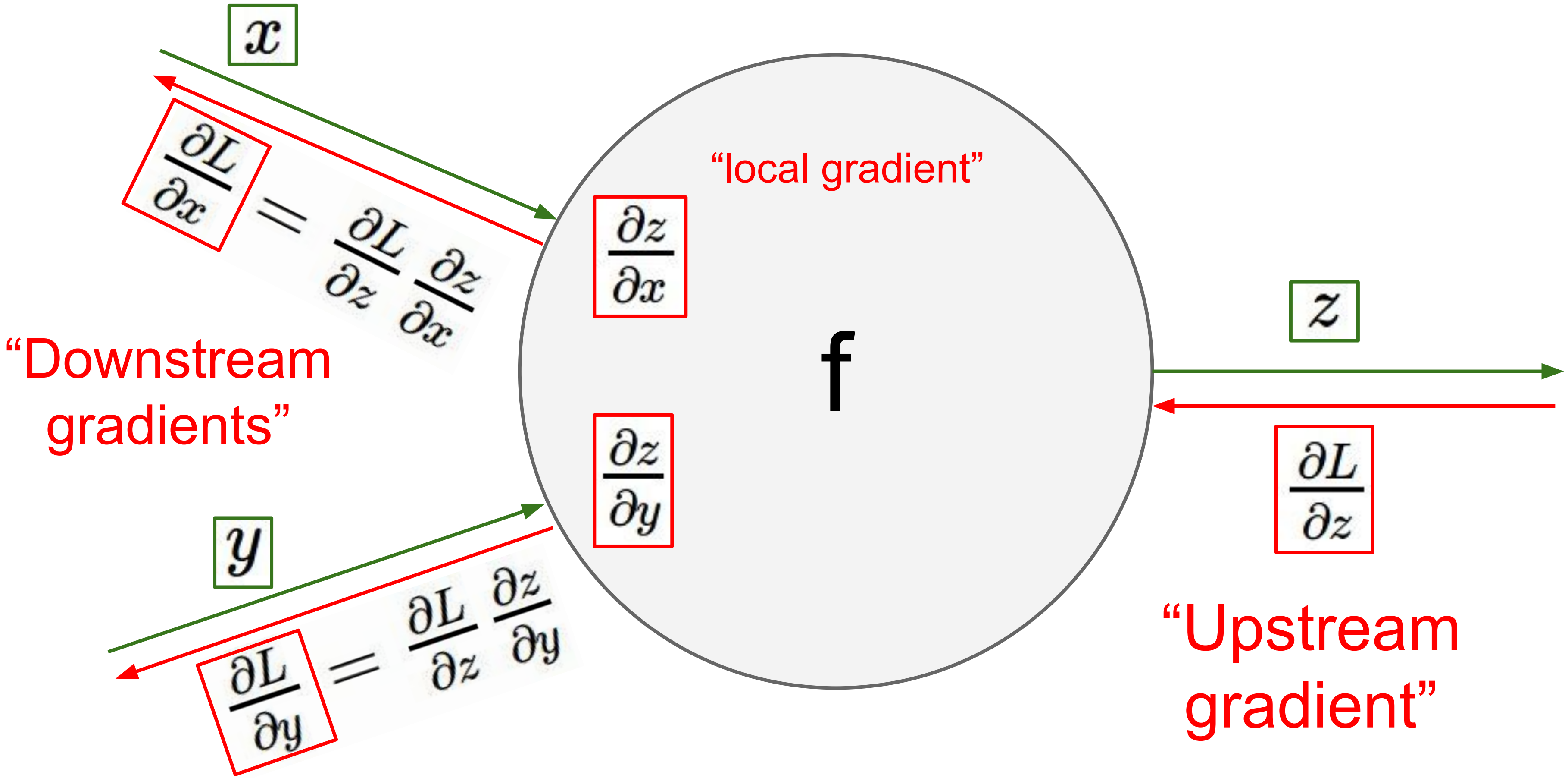
x
y
f
z



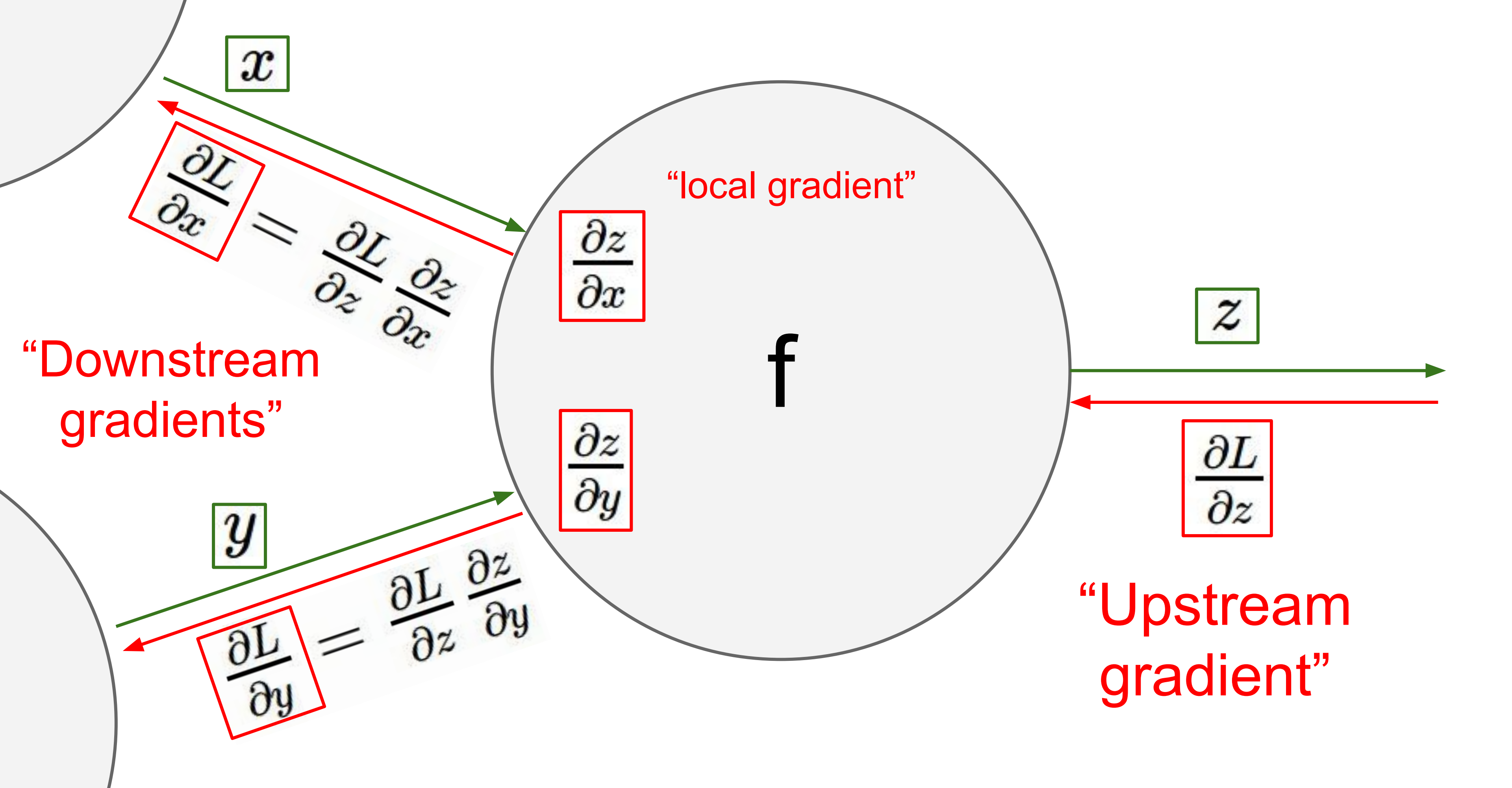
x
"local gradient"
$\frac{\partial z}{\partial x}$
f
z
$\frac{\partial z}{\partial y}$
y


x
"local gradient"
$\frac{\partial z}{\partial x}$
$\frac{\partial z}{\partial y}$
f
y
z
$\frac{\partial L}{\partial z}$
"Upstream gradient"

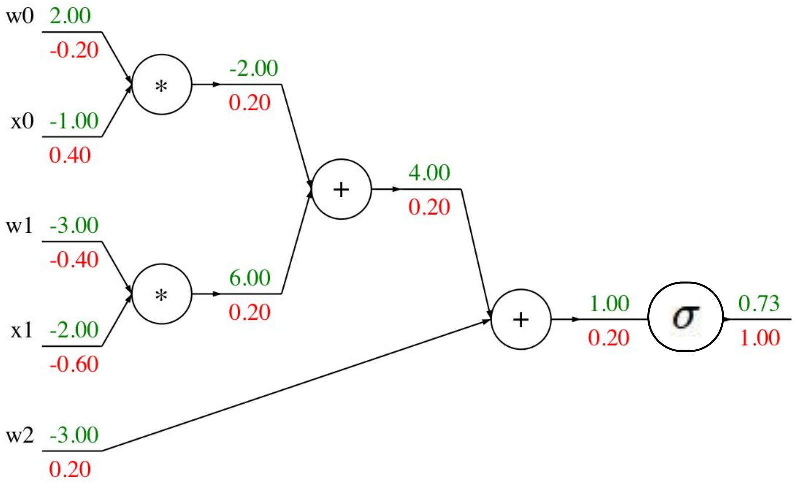

x
$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial x}$
"Downstream gradients"
"local gradient"
$\frac{\partial z}{\partial x}$
f
$\frac{\partial z}{\partial y}$
y
z
$\frac{\partial L}{\partial z}$
"Upstream gradient"



x
$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial x}$
"Downstream gradients"
y
$\frac{\partial L}{\partial y} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial y}$
"local gradient"
$\frac{\partial z}{\partial x}$
$\frac{\partial z}{\partial y}$
f
z
$\frac{\partial L}{\partial z}$
"Upstream gradient"


x
$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial x}$
"Downstream gradients"
y
$\frac{\partial L}{\partial y} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial y}$
"local gradient"
$\frac{\partial z}{\partial x}$
$\frac{\partial z}{\partial y}$
f
z
$\frac{\partial L}{\partial z}$
"Upstream gradient"

Backprop Implementation: "Flat" code



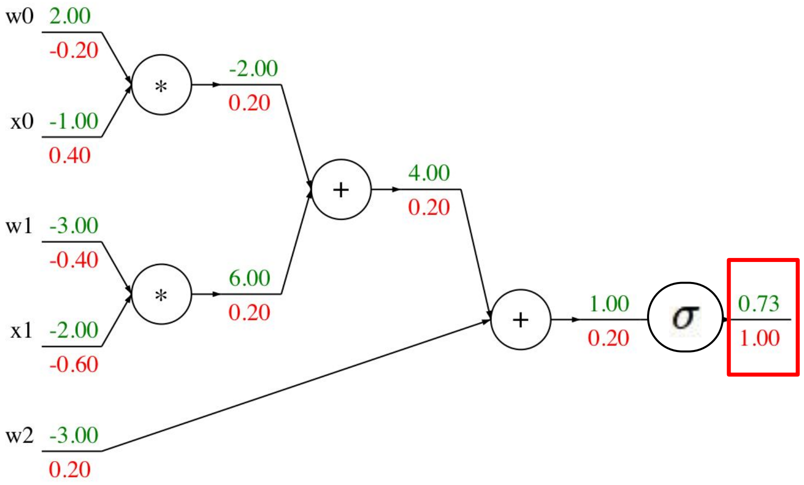
Forward pass:
Compute output

```
def f(w0, x0, w1, x1, w2):
  s0 = w0 * x0
  s1 = w1 * x1
  s2 = s0 + s1
  s3 = s2 + w2
  L = sigmoid(s3)
```

Backward pass:
Compute grads

```
  grad_L = 1.0
  grad_s3 = grad_L * (1 - L) * L
  grad_w2 = grad_s3
  grad_s2 = grad_s3
  grad_s0 = grad_s2
  grad_s1 = grad_s2
  grad_w1 = grad_s1 * x1
  grad_x1 = grad_s1 * w1
  grad_w0 = grad_s0 * x0
  grad_x0 = grad_s0 * w0
```

Backprop Implementation: "Flat" code



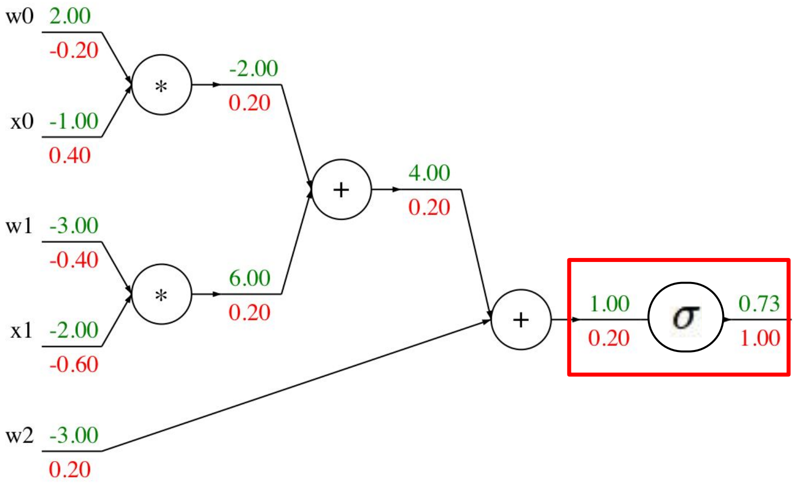
Forward pass: Compute output

```
def f(w0, x0, w1, x1, w2):
    s0 = w0 * x0
    s1 = w1 * x1
    s2 = s0 + s1
    s3 = s2 + w2
    L = sigmoid(s3)
```

Base case

```
    grad_L = 1.0
    grad_s3 = grad_L * (1 - L) * L
    grad_w2 = grad_s3
    grad_s2 = grad_s3
    grad_s0 = grad_s2
    grad_s1 = grad_s2
    grad_w1 = grad_s1 * x1
    grad_x1 = grad_s1 * w1
    grad_w0 = grad_s0 * x0
    grad_x0 = grad_s0 * w0
```

Backprop Implementation: "Flat" code



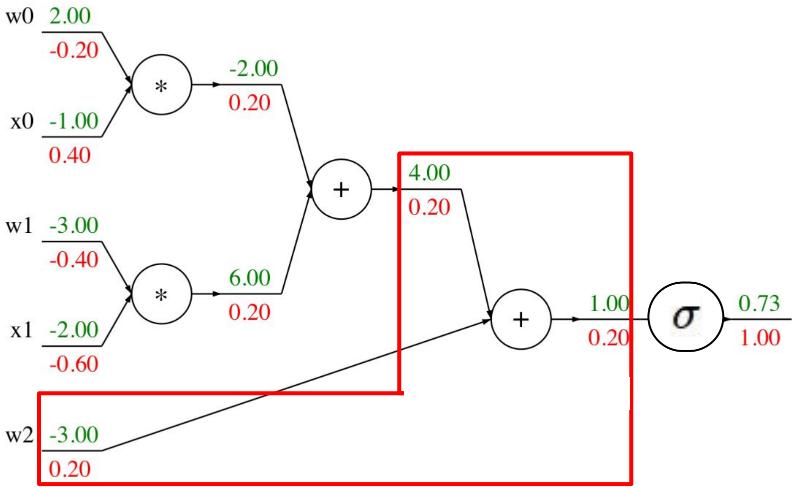
Forward pass:
Compute output

```
def f(w0, x0, w1, x1, w2):
  s0 = w0 * x0
  s1 = w1 * x1
  s2 = s0 + s1
  s3 = s2 + w2
  L = sigmoid(s3)
```

Sigmoid

```
  grad_L = 1.0
  grad_s3 = grad_L * (1 - L) * L
  grad_w2 = grad_s3
  grad_s2 = grad_s3
  grad_s0 = grad_s2
  grad_s1 = grad_s2
  grad_w1 = grad_s1 * x1
  grad_x1 = grad_s1 * w1
  grad_w0 = grad_s0 * x0
  grad_x0 = grad_s0 * w0
```

Backprop Implementation: "Flat" code



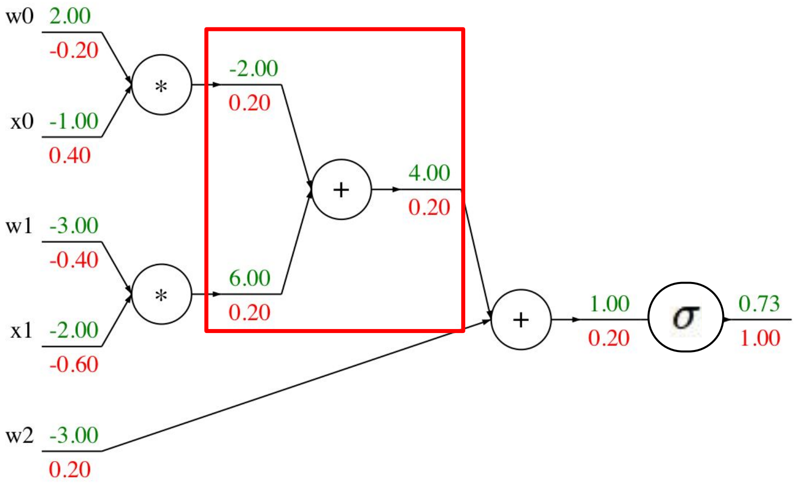
Forward pass: Compute output

```
def f(w0, x0, w1, x1, w2):
  s0 = w0 * x0
  s1 = w1 * x1
  s2 = s0 + s1
  s3 = s2 + w2
  L = sigmoid(s3)

  grad_L = 1.0
  grad_s3 = grad_L * (1 - L) * L
  grad_w2 = grad_s3
  grad_s2 = grad_s3
  grad_s0 = grad_s2
  grad_s1 = grad_s2
  grad_w1 = grad_s1 * x1
  grad_x1 = grad_s1 * w1
  grad_w0 = grad_s0 * x0
  grad_x0 = grad_s0 * w0
```

Add gate

Backprop Implementation: “Flat” code



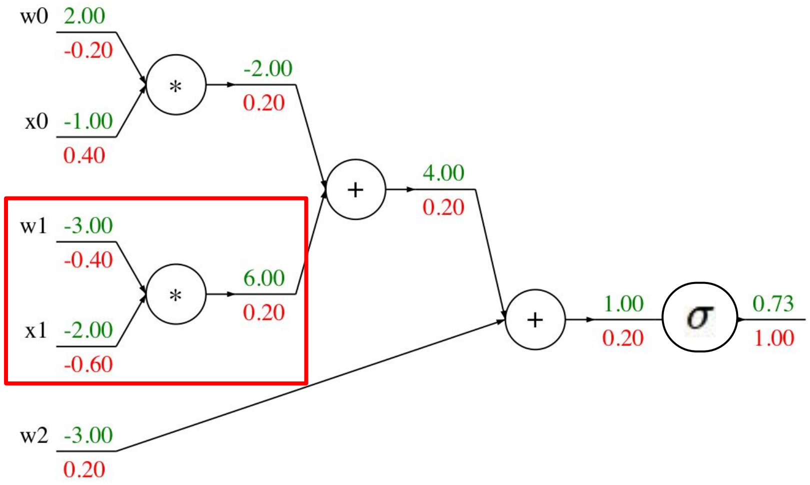
Forward pass:
Compute output

```
def f(w0, x0, w1, x1, w2):
    s0 = w0 * x0
    s1 = w1 * x1
    s2 = s0 + s1
    s3 = s2 + w2
    L = sigmoid(s3)

    grad_L = 1.0
    grad_s3 = grad_L * (1 - L) * L
    grad_w2 = grad_s3
    grad_s2 = grad_s3
    grad_s0 = grad_s2
    grad_s1 = grad_s2
    grad_w1 = grad_s1 * x1
    grad_x1 = grad_s1 * w1
    grad_w0 = grad_s0 * x0
    grad_x0 = grad_s0 * w0
```

Add gate

Backprop Implementation: "Flat" code



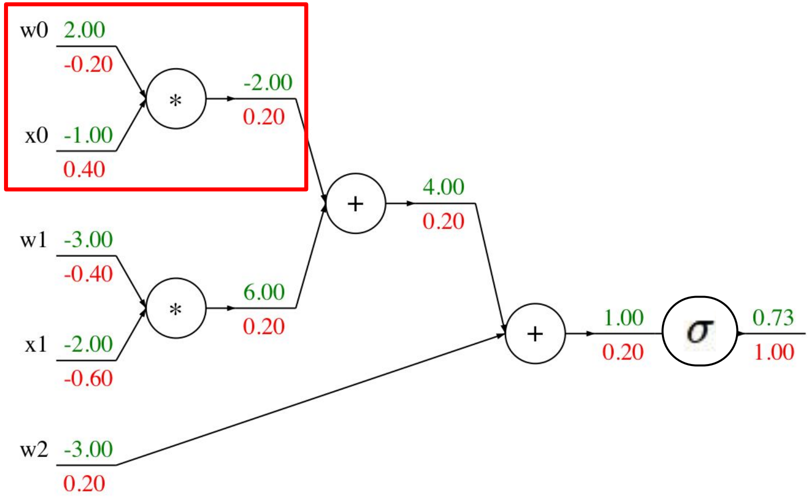
Forward pass: Compute output

```
def f(w0, x0, w1, x1, w2):
  s0 = w0 * x0
  s1 = w1 * x1
  s2 = s0 + s1
  s3 = s2 + w2
  L = sigmoid(s3)

  grad_L = 1.0
  grad_s3 = grad_L * (1 - L) * L
  grad_w2 = grad_s3
  grad_s2 = grad_s3
  grad_s0 = grad_s2
  grad_s1 = grad_s2
  grad_w1 = grad_s1 * x1
  grad_x1 = grad_s1 * w1
  grad_w0 = grad_s0 * x0
  grad_x0 = grad_s0 * w0
```

Multiply gate

Backprop Implementation: "Flat" code



Forward pass:
Compute output

Multiply gate

```
def f(w0, x0, w1, x1, w2):
  s0 = w0 * x0
  s1 = w1 * x1
  s2 = s0 + s1
  s3 = s2 + w2
  L = sigmoid(s3)

  grad_L = 1.0
  grad_s3 = grad_L * (1 - L) * L
  grad_w2 = grad_s3
  grad_s2 = grad_s3
  grad_s0 = grad_s2
  grad_s1 = grad_s2
  grad_w1 = grad_s1 * x1
  grad_x1 = grad_s1 * w1
  grad_w0 = grad_s0 * x0
  grad_x0 = grad_s0 * w0
```

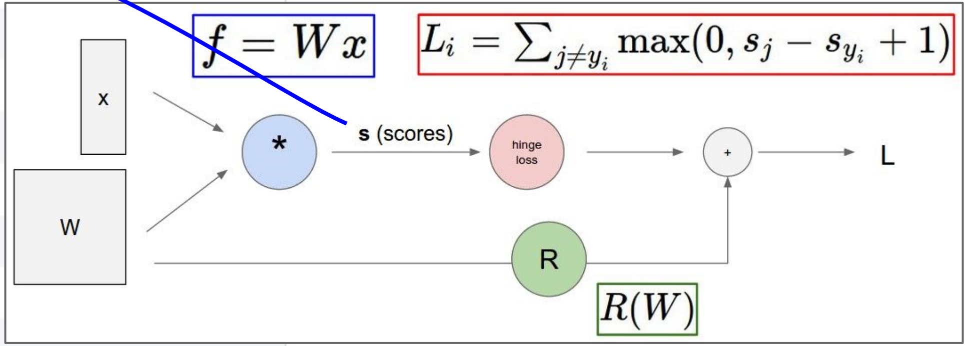
“Flat” Backprop: Do this for assignment 2!

Stage your forward/backward computation!

E.g. for the SVM:

```
# receive W (weights), X (data)
# forward pass (we have 8 lines)
scores = #...
margins = #...
data_loss = #...
reg_loss = #...
loss = data_loss + reg_loss
# backward pass (we have 5 lines)
dmargins = # ... (optionally, we go direct to dscores)
dscores = #...
dW = #...
```

margins

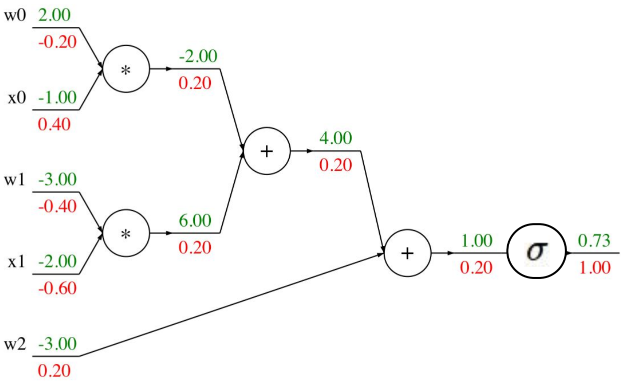


“Flat” Backprop: Do this for assignment 1!

E.g. for two-layer neural net:

```
# receive W1,W2,b1,b2 (weights/biases), X (data)
# forward pass:
h1 = #... function of X,W1,b1
scores = #... function of h1,W2,b2
loss = #... (several lines of code to evaluate Softmax loss)
# backward pass:
dscores = #...
dh1,dW2,db2 = #...
dW1,db1 = #...
```

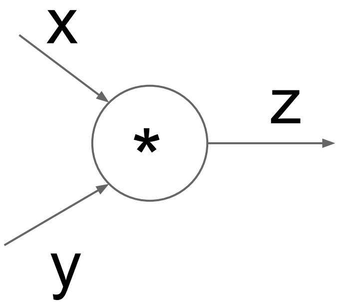
Backprop Implementation: Modularized API



Graph (or Net) object *(rough pseudo code)*

```
class ComputationalGraph(object):
    #...
    def forward(inputs):
        # 1. [pass inputs to input gates...]
        # 2. forward the computational graph:
        for gate in self.graph.nodes_topologically_sorted():
            gate.forward()
        return loss # the final gate in the graph outputs the loss
    def backward():
        for gate in reversed(self.graph.nodes_topologically_sorted()):
            gate.backward() # little piece of backprop (chain rule applied)
        return inputs_gradients
```

Modularized implementation: forward / backward API



(x,y,z are scalars)

Gate / Node / Function object: Actual PyTorch code

```
class Multiply(torch.autograd.Function):
  @staticmethod
  def forward(ctx, x, y):
    ctx.save_for_backward(x, y)
    z = x * y
    return z
  @staticmethod
  def backward(ctx, grad_z):
    x, y = ctx.saved_tensors
    grad_x = y * grad_z   # dz/dx * dL/dz
    grad_y = x * grad_z   # dz/dy * dL/dz
    return grad_x, grad_y
```

Need to stash some values for use in backward

Upstream gradient

Multiply upstream and local gradients

Example: PyTorch operators

pytorch / pytorch

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Tree: 517c7c9861 | pytorch / aten / src / THNN / generic /

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ezyang and facebook-github-bot Canonicalize all includes in PyTorch. (#14849) ... Latest commit 517c7c9 on Dec 8, 2018

..		
AbsCriterion.c	Canonicalize all includes in PyTorch. (#14849)	4 months ago
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linear_upsampling.h	Implement nn.functional.interpolate based on upsample. (#8591)	9 months ago
pooling_shape.h	Use integer math to compute output size of pooling operations (#14405)	4 months ago
unfold.c	Canonicalize all includes in PyTorch. (#14849)	4 months ago

PyTorch sigmoid layer

```
#ifndef TH_GENERIC_FILE
#define TH_GENERIC_FILE "THNN/generic/Sigmoid.c"
#else

void THNN_(Sigmoid_updateOutput)(
          THNNState *state,
          THTensor *input,
          THTensor *output)
{
  THTensor_(sigmoid)(output, input);
}

void THNN_(Sigmoid_updateGradInput)(
          THNNState *state,
          THTensor *gradOutput,
          THTensor *gradInput,
          THTensor *output)
{
  THNN_CHECK_NELEMENT(output, gradOutput);
  THTensor_(resizeAs)(gradInput, output);
  TH_TENSOR_APPLY3(scalar_t, gradInput, scalar_t, gradOutput, scalar_t, output,
    scalar_t z = *output_data;
    *gradInput_data = *gradOutput_data * (1. - z) * z;
  );
}

#endif
```

Forward

$$\boxed{\sigma(x) = \frac{1}{1+e^{-x}}}$$

Source

PyTorch sigmoid layer

```
#ifndef TH_GENERIC_FILE
#define TH_GENERIC_FILE "THNN/generic/Sigmoid.c"
#else

void THNN_(Sigmoid_updateOutput)(
          THNNState *state,
          THTensor *input,
          THTensor *output)
{
  THTensor_(sigmoid)(output, input);
}

void THNN_(Sigmoid_updateGradInput)(
          THNNState *state,
          THTensor *gradOutput,
          THTensor *gradInput,
          THTensor *output)
{
  THNN_CHECK_NELEMENT(output, gradOutput);
  THTensor_(resizeAs)(gradInput, output);
  TH_TENSOR_APPLY3(scalar_t, gradInput, scalar_t, gradOutput, scalar_t, output,
    scalar_t z = *output_data;
    *gradInput_data = *gradOutput_data * (1. - z) * z;
  );
}

#endif
```

Forward

$$\boxed{\sigma(x) = \frac{1}{1+e^{-x}}}$$

```
static void sigmoid_kernel(TensorIterator& iter) {
  AT_DISPATCH_FLOATING_TYPES(iter.dtype(), "sigmoid_cpu", [&]() {
    unary_kernel_vec(
        iter,
        [=](scalar_t a) -> scalar_t { return (1 / (1 + std::exp((-a)))); },
        [=](Vec256<scalar_t> a) {
          a = Vec256<scalar_t>((scalar_t)(0)) - a;
          a = a.exp();
          a = Vec256<scalar_t>((scalar_t)(1)) + a;
          a = a.reciprocal();
          return a;
        });
  });
}
```

Forward actually defined elsewhere...

```
return (1 / (1 + std::exp((-a))));
```

Source

PyTorch sigmoid layer

```c
#ifndef TH_GENERIC_FILE
#define TH_GENERIC_FILE "THNN/generic/Sigmoid.c"
#else

void THNN_(Sigmoid_updateOutput)(
          THNNState *state,
          THTensor *input,
          THTensor *output)
{
  THTensor_(sigmoid)(output, input);
}

void THNN_(Sigmoid_updateGradInput)(
          THNNState *state,
          THTensor *gradOutput,
          THTensor *gradInput,
          THTensor *output)
{
  THNN_CHECK_NELEMENT(output, gradOutput);
  THTensor_(resizeAs)(gradInput, output);
  TH_TENSOR_APPLY3(scalar_t, gradInput, scalar_t, gradOutput, scalar_t, output,
    scalar_t z = *output_data;
    *gradInput_data = *gradOutput_data * (1. - z) * z;
  );
}

#endif
```

Forward

$$\boxed{\sigma(x) = \frac{1}{1+e^{-x}}}$$

```cpp
static void sigmoid_kernel(TensorIterator& iter) {
  AT_DISPATCH_FLOATING_TYPES(iter.dtype(), "sigmoid_cpu", [&]() {
    unary_kernel_vec(
        iter,
        [=](scalar_t a) -> scalar_t { return (1 / (1 + std::exp((-a)))); },
        [=](Vec256<scalar_t> a) {
          a = Vec256<scalar_t>((scalar_t)(0)) - a;
          a = a.exp();
          a = Vec256<scalar_t>((scalar_t)(1)) + a;
          a = a.reciprocal();
          return a;
        });
  });
}
```

Forward actually defined elsewhere...

Backward

$$\boxed{(1-\sigma(x))\,\sigma(x)}$$

Source

So far: backprop with scalars

What about vector-valued functions?

Recap: Vector derivatives

Scalar to Scalar

$$x \in \mathbb{R}, y \in \mathbb{R}$$

Regular derivative:

$$\frac{\partial y}{\partial x} \in \mathbb{R}$$

If x changes by a small amount, how much will y change?

Recap: Vector derivatives

Scalar to Scalar

$$x \in \mathbb{R}, y \in \mathbb{R}$$

Regular derivative:

$$\frac{\partial y}{\partial x} \in \mathbb{R}$$

If x changes by a small amount, how much will y change?

Vector to Scalar

$$x \in \mathbb{R}^N, y \in \mathbb{R}$$

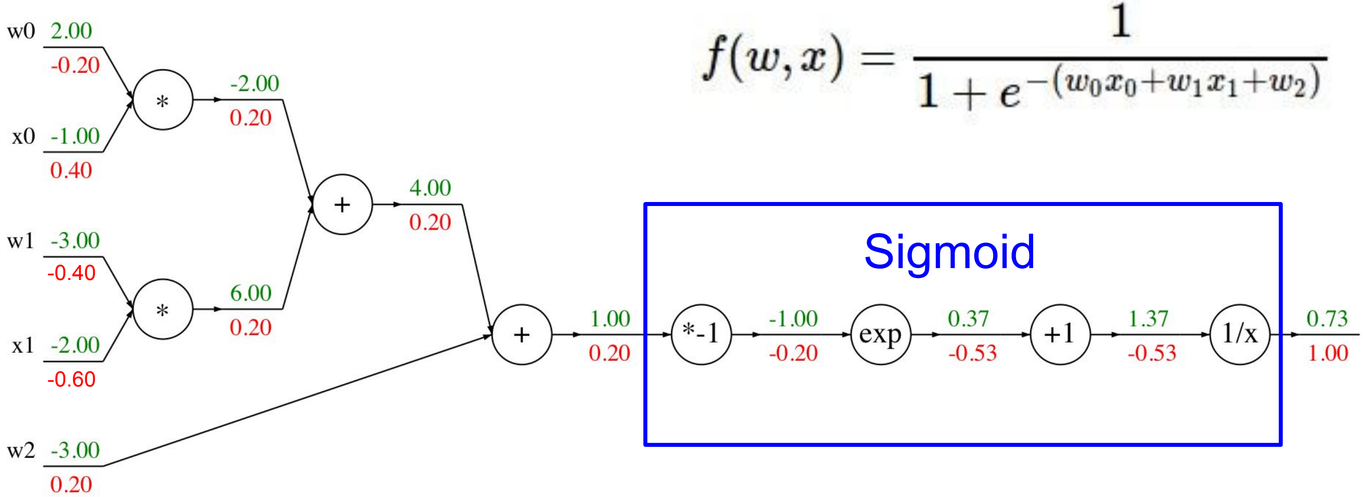
Derivative is **Gradient**:

$$\frac{\partial y}{\partial x} \in \mathbb{R}^N \quad \left(\frac{\partial y}{\partial x}\right)_n = \frac{\partial y}{\partial x_n}$$

For each element of x, if it changes by a small amount then how much will y change?

Remember this example from last lecture?

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



Vector to Scalar

$\begin{bmatrix} -1.00 \\ -2.00 \end{bmatrix}$ $x \in \mathbb{R}^N, y \in \mathbb{R}$ 0.73

Derivative is **Gradient**:

$$\frac{\partial y}{\partial x} \in \mathbb{R}^N \quad \left(\frac{\partial y}{\partial x}\right)_n = \frac{\partial y}{\partial x_n} \quad \begin{bmatrix} 0.40 \\ -0.60 \end{bmatrix}$$

Recap: Vector derivatives

Scalar to Scalar

$$x \in \mathbb{R}, y \in \mathbb{R}$$

Regular derivative:

$$\frac{\partial y}{\partial x} \in \mathbb{R}$$

If x changes by a small amount, how much will y change?

Vector to Scalar

$$x \in \mathbb{R}^N, y \in \mathbb{R}$$

Derivative is **Gradient**:

$$\frac{\partial y}{\partial x} \in \mathbb{R}^N \quad \left(\frac{\partial y}{\partial x}\right)_n = \frac{\partial y}{\partial x_n}$$

For each element of x, if it changes by a small amount then how much will y change?

Vector to Vector

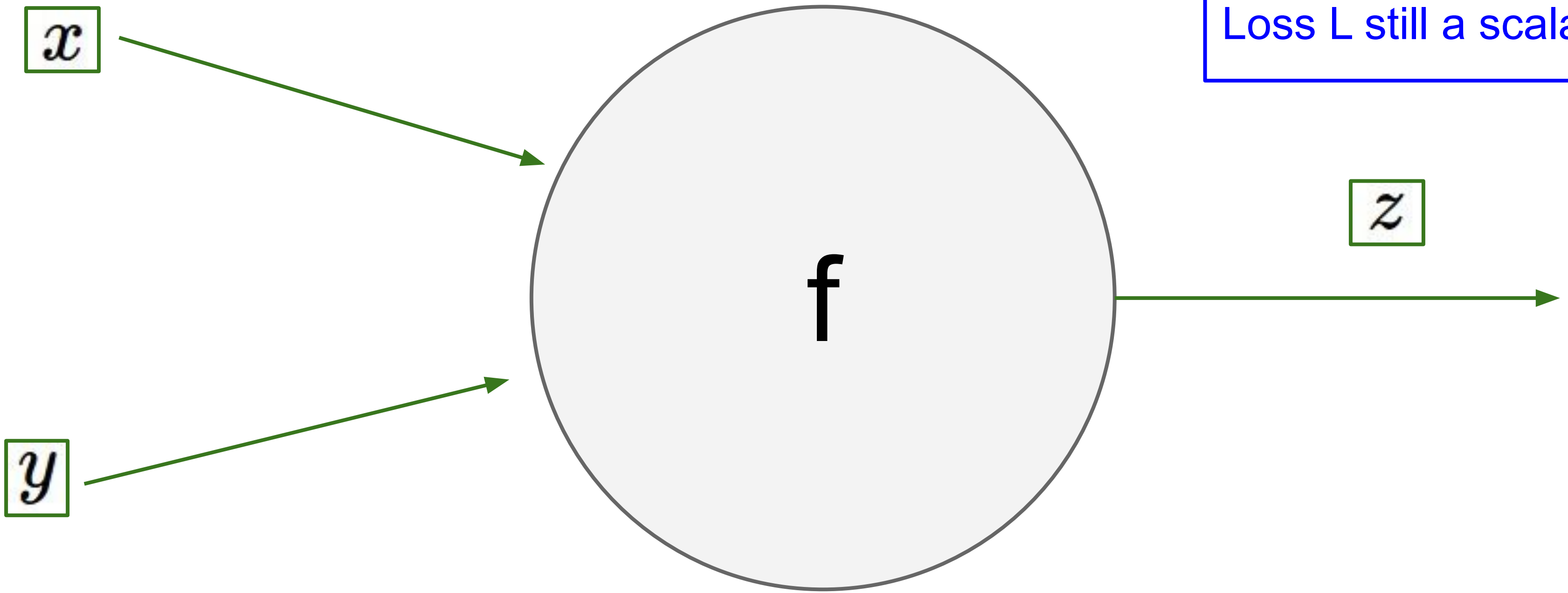
$$x \in \mathbb{R}^N, y \in \mathbb{R}^M$$

Derivative is **Jacobian**:

$$\frac{\partial y}{\partial x} \in \mathbb{R}^{N \times M} \quad \left(\frac{\partial y}{\partial x}\right)_{n,m} = \frac{\partial y_m}{\partial x_n}$$

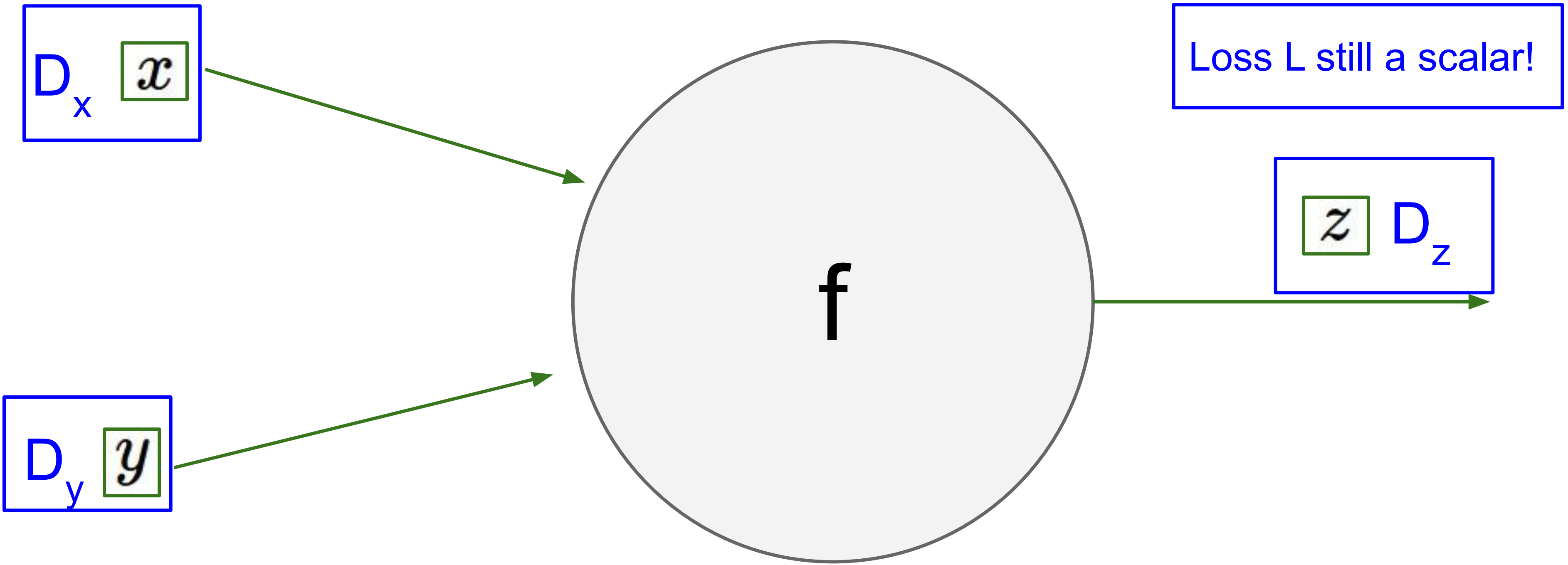
For each element of x, if it changes by a small amount then how much will each element of y change?

Backprop with Vectors

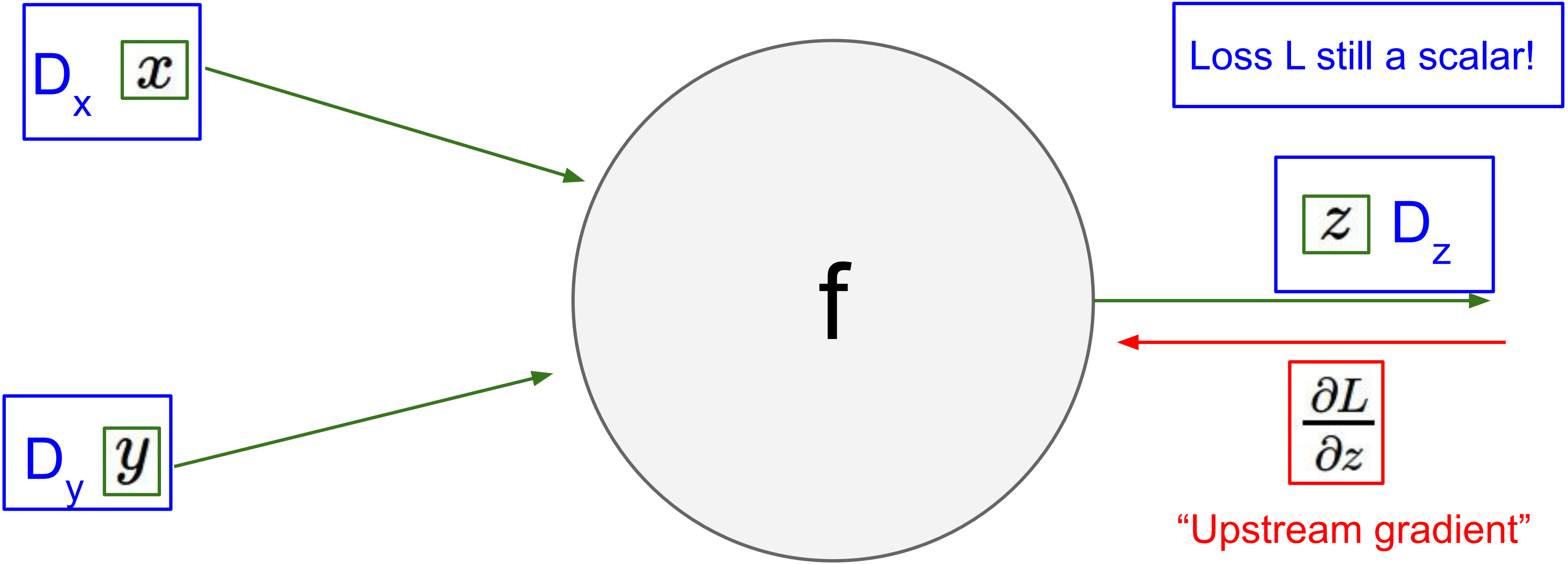


Loss L still a scalar!

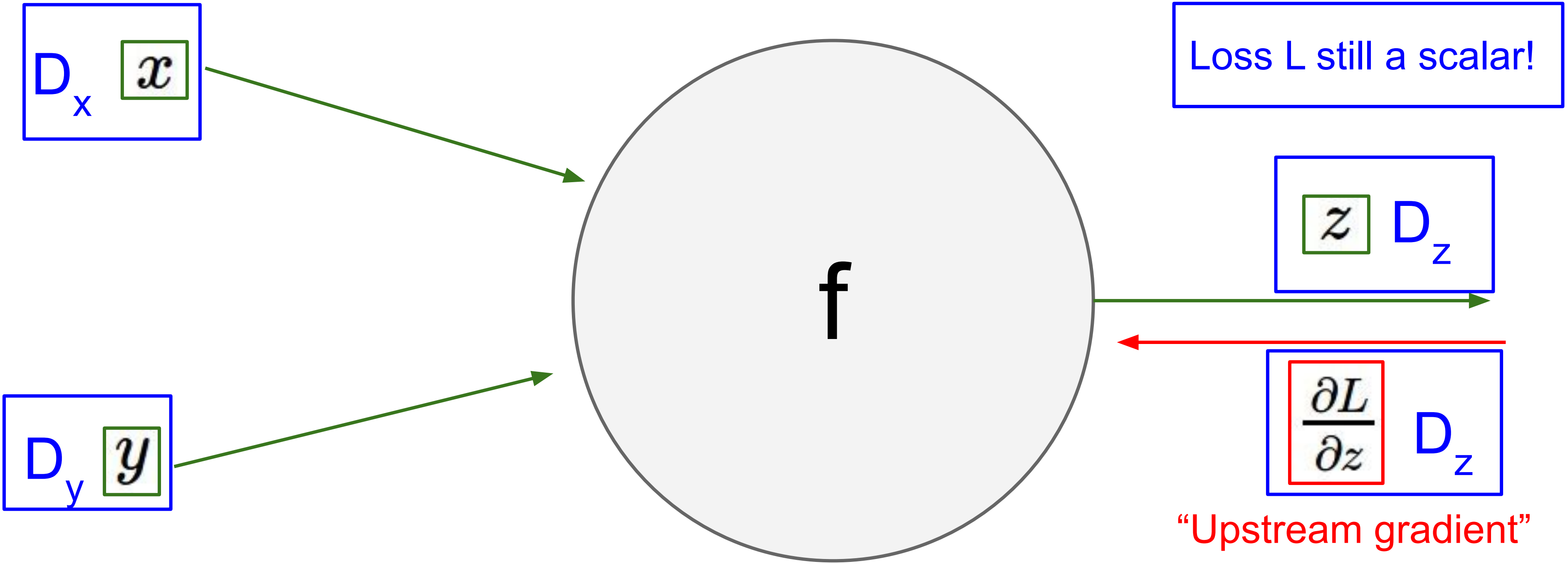
Backprop with Vectors



Backprop with Vectors

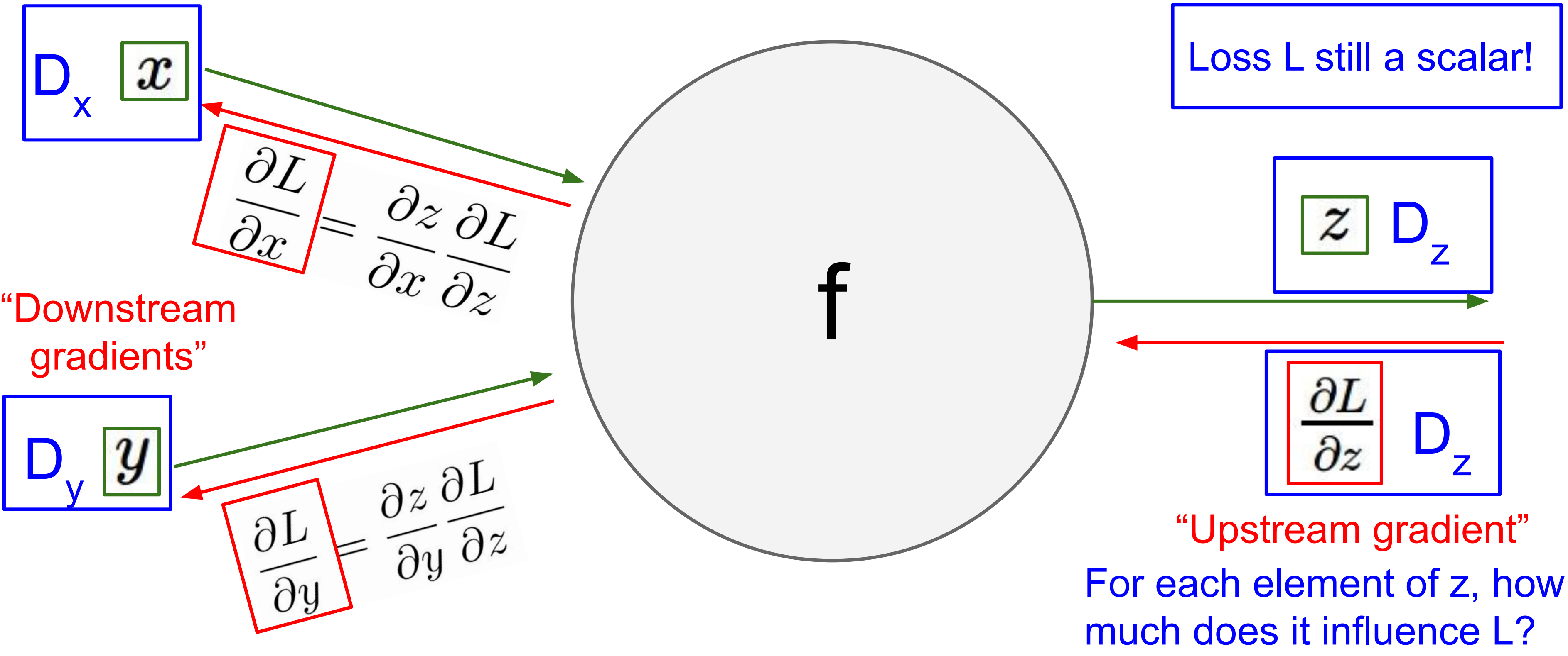


Backprop with Vectors

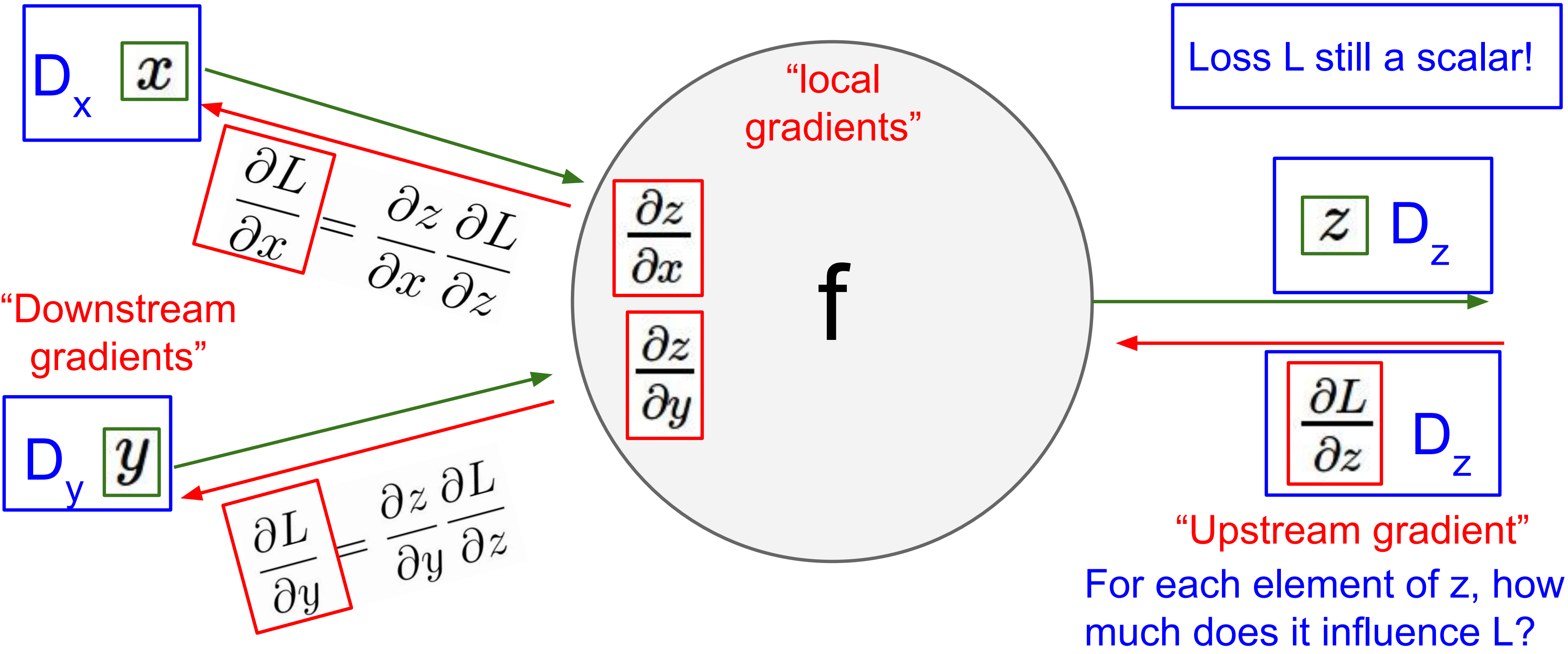


For each element of z, how much does it influence L?

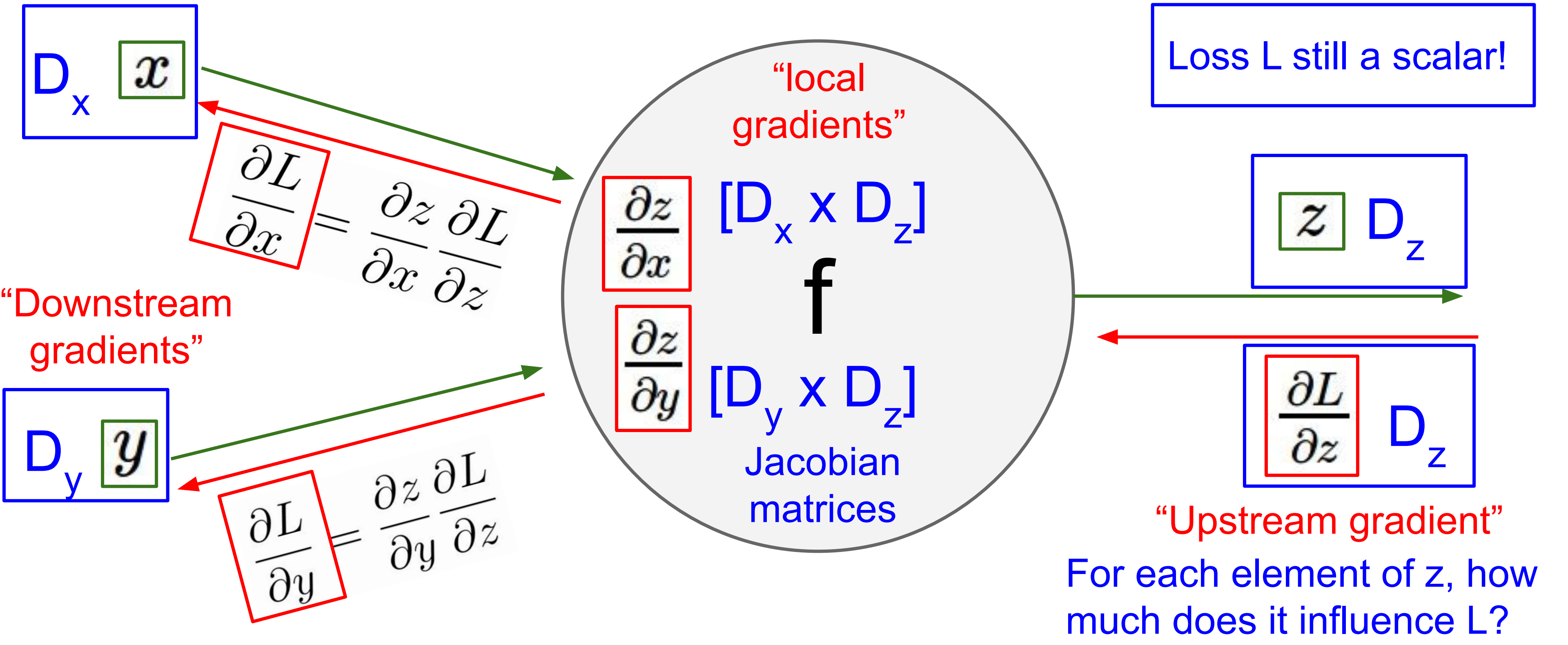
Backprop with Vectors



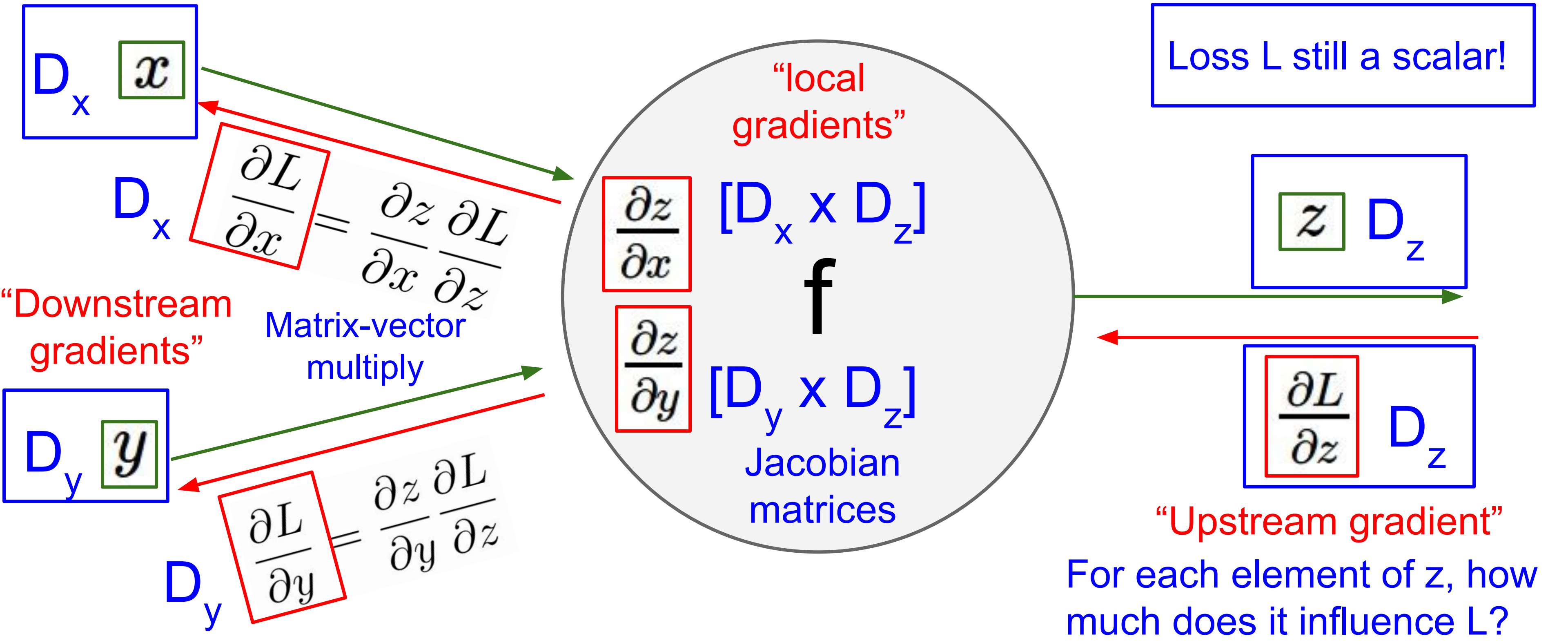
Backprop with Vectors



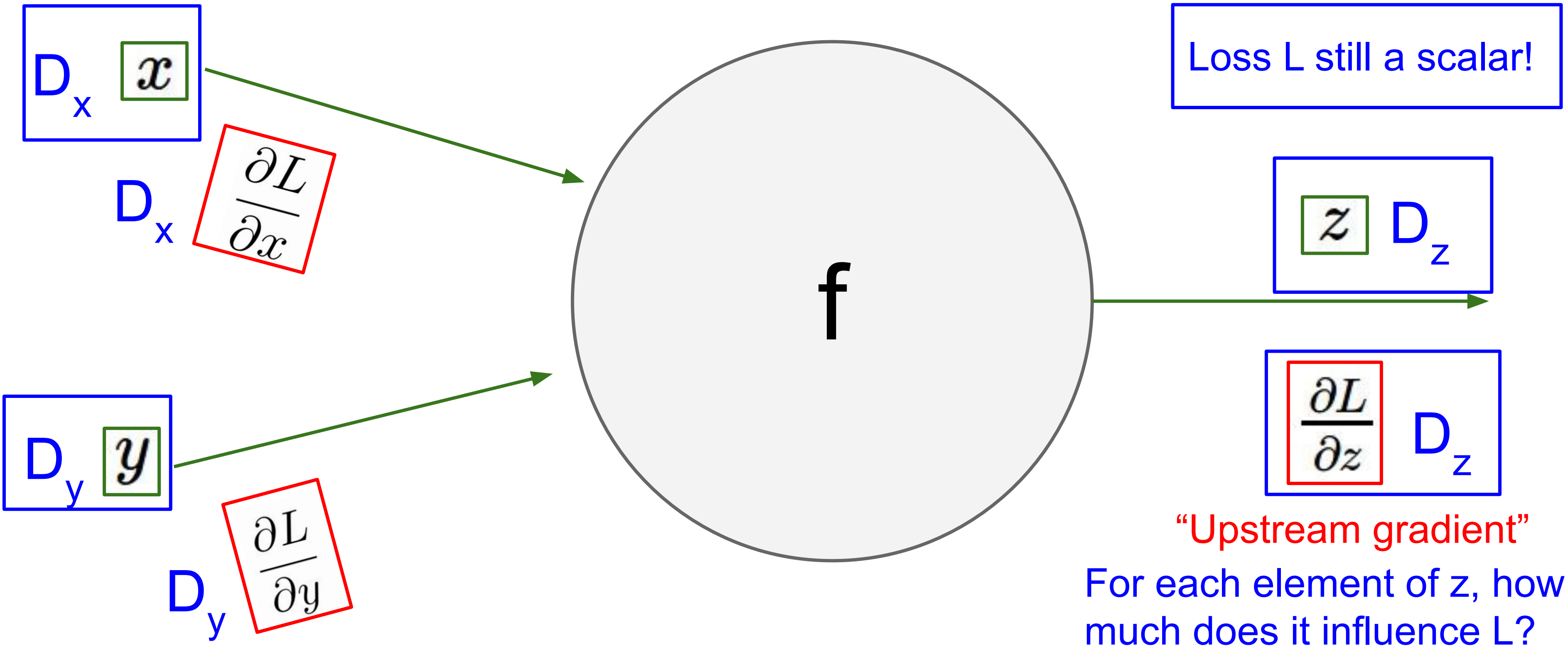
Backprop with Vectors



Backprop with Vectors



Gradients of variables wrt loss have same dims as the original variable



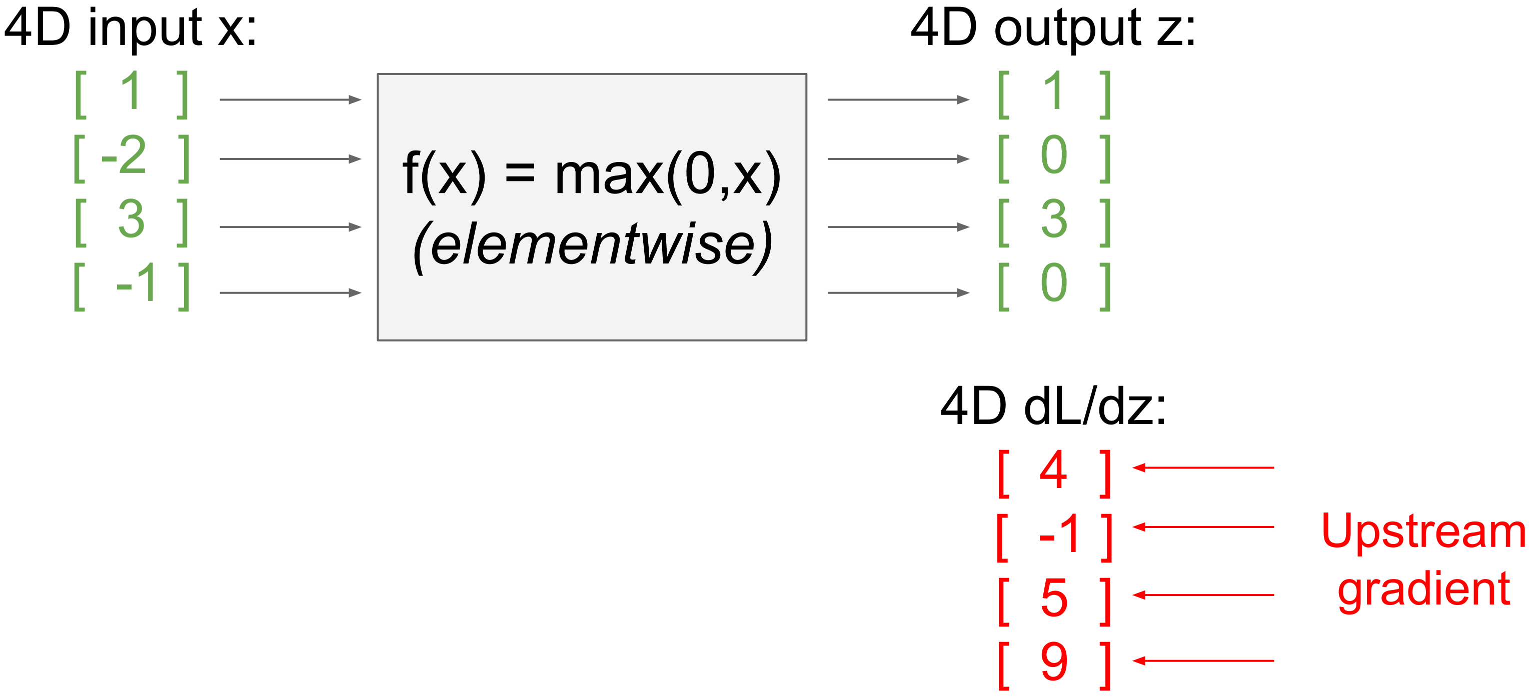
Backprop with Vectors

4D input x:

$$\begin{bmatrix} 1 \\ -2 \\ 3 \\ -1 \end{bmatrix} \longrightarrow \boxed{\begin{array}{c} f(x) = \max(0,x) \\ \textit{(elementwise)} \end{array}} \longrightarrow \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$

4D output z:

Backprop with Vectors



Backprop with Vectors

4D input x:

$$\begin{bmatrix} 1 \\ -2 \\ 3 \\ -1 \end{bmatrix}$$

$f(x) = \max(0,x)$
(elementwise)

4D output z:

$$\begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$

Jacobian dz/dx

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

4D dL/dz:

$$\begin{bmatrix} 4 \\ -1 \\ 5 \\ 9 \end{bmatrix}$$

Upstream gradient

Backprop with Vectors

4D input x:

$$\begin{bmatrix} 1 \\ -2 \\ 3 \\ -1 \end{bmatrix}$$

f(x) = max(0,x)
(elementwise)

4D output z:

$$\begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$

[dz/dx] [dL/dz]

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 5 \\ 9 \end{bmatrix}$$

4D dL/dz:

$$\begin{bmatrix} 4 \\ -1 \\ 5 \\ 9 \end{bmatrix}$$

Upstream gradient

Backprop with Vectors

4D input x:

$$\begin{bmatrix} 1 \\ -2 \\ 3 \\ -1 \end{bmatrix}$$

$f(x) = \max(0,x)$ *(elementwise)*

4D output z:

$$\begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix}$$

4D dL/dx:

$$\begin{bmatrix} 4 \\ 0 \\ 5 \\ 0 \end{bmatrix}$$

[dz/dx] [dL/dz]

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 5 \\ 9 \end{bmatrix}$$

4D dL/dz:

$$\begin{bmatrix} 4 \\ -1 \\ 5 \\ 9 \end{bmatrix}$$

Upstream gradient

Backprop with Vectors

4D input x:

[1]
[-2]
[3]
[-1]

f(x) = max(0,x)
(elementwise)

4D output z:

[1]
[0]
[3]
[0]

Jacobian is **sparse**: off-diagonal entries always zero! Never **explicitly** form Jacobian -- instead use **implicit** multiplication

4D dL/dx:

[4]
[0]
[5]
[0]

[dz/dx] [dL/dz]

[1 0 0 0] [4]
[0 0 0 0] [-1]
[0 0 1 0] [5]
[0 0 0 0] [9]

4D dL/dz:

[4]
[-1]
[5]
[9]

Upstream gradient

Backprop with Vectors

Jacobian is **sparse**: off-diagonal entries always zero! Never **explicitly** form Jacobian -- instead use **implicit** multiplication

4D input x:

[1]
[-2]
[3]
[-1]

f(x) = max(0,x)
(elementwise)

4D output z:

[1]
[0]
[3]
[0]

4D dL/dx:

[4]
[0]
[5]
[0]

[dz/dx] [dL/dz]

$$\left(\frac{\partial L}{\partial x}\right)_i = \begin{cases} \left(\frac{\partial L}{\partial z}\right)_i & \text{if } x_i > 0 \\ 0 & \text{otherwise} \end{cases}$$

4D dL/dz:

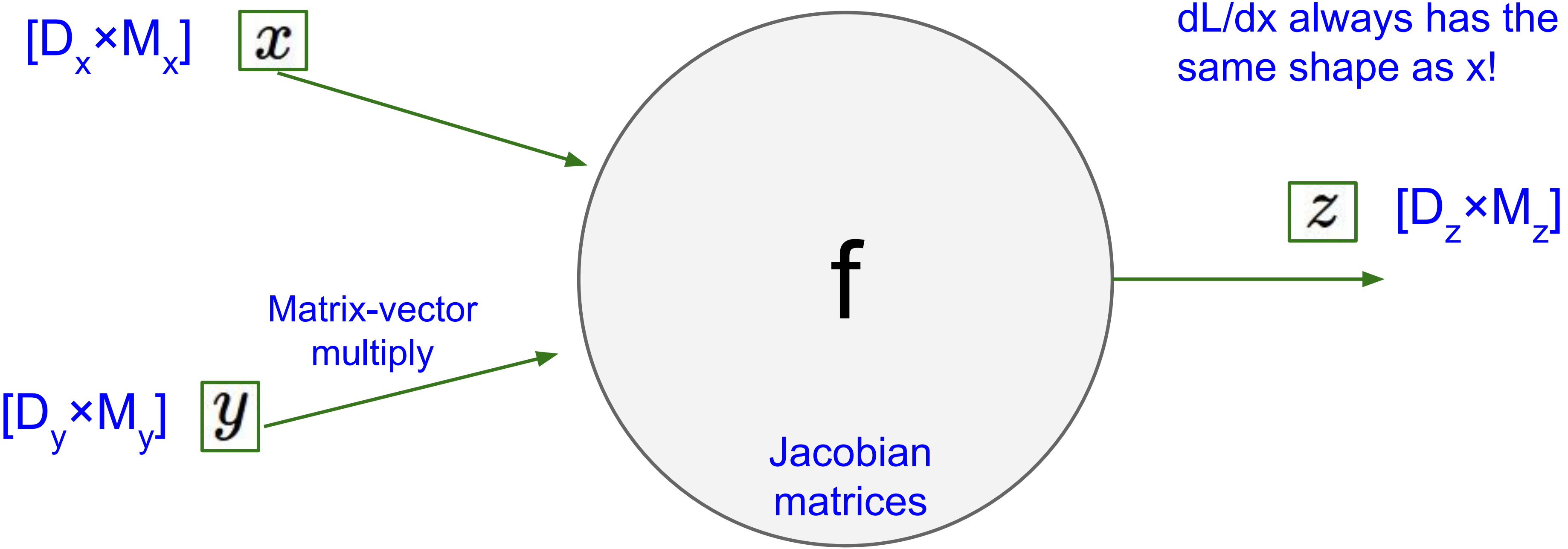
[4]
[-1]
[5]
[9]

Upstream gradient

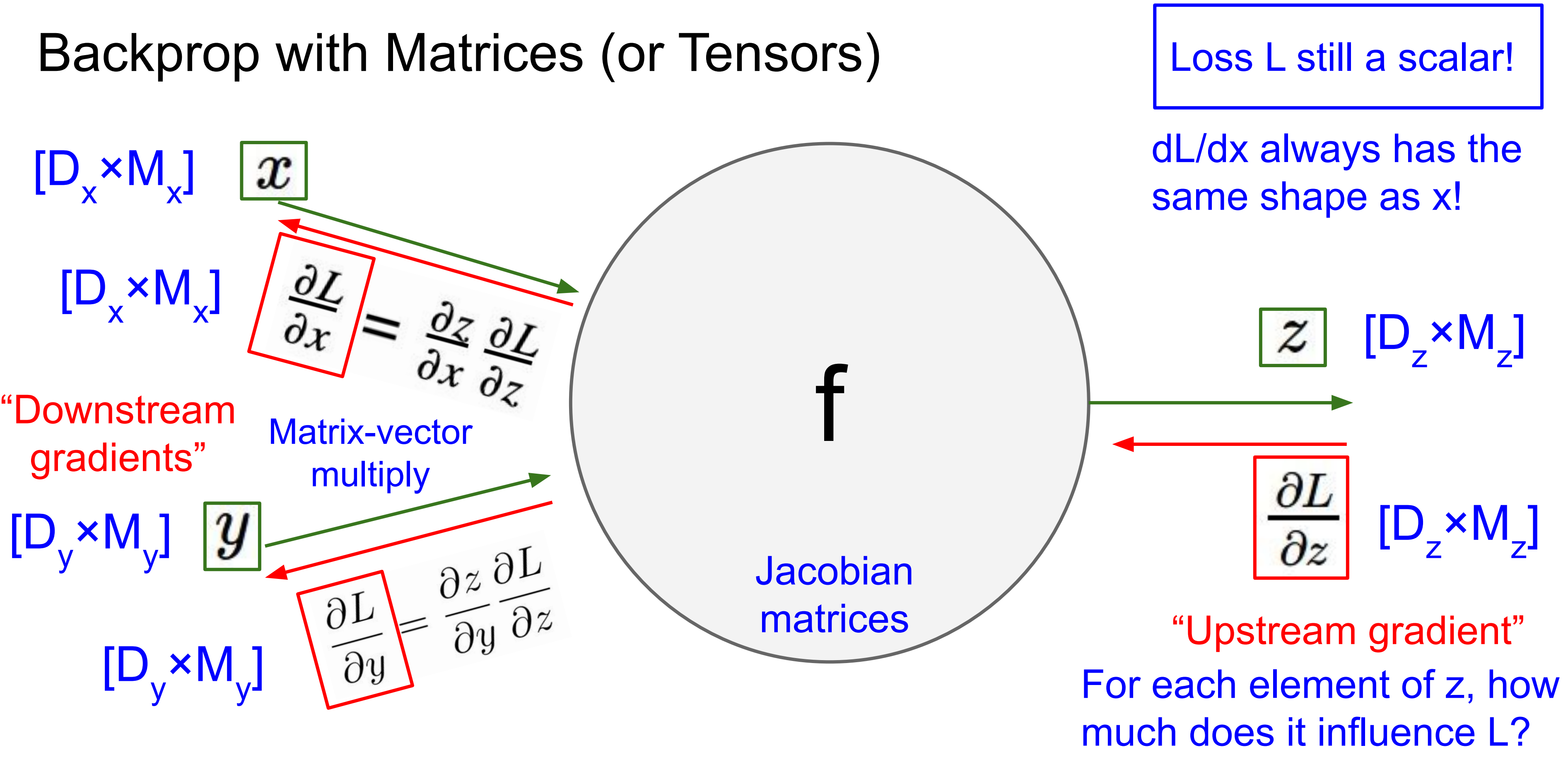
Backprop with Matrices (or Tensors)

Loss L still a scalar!

dL/dx always has the same shape as x!

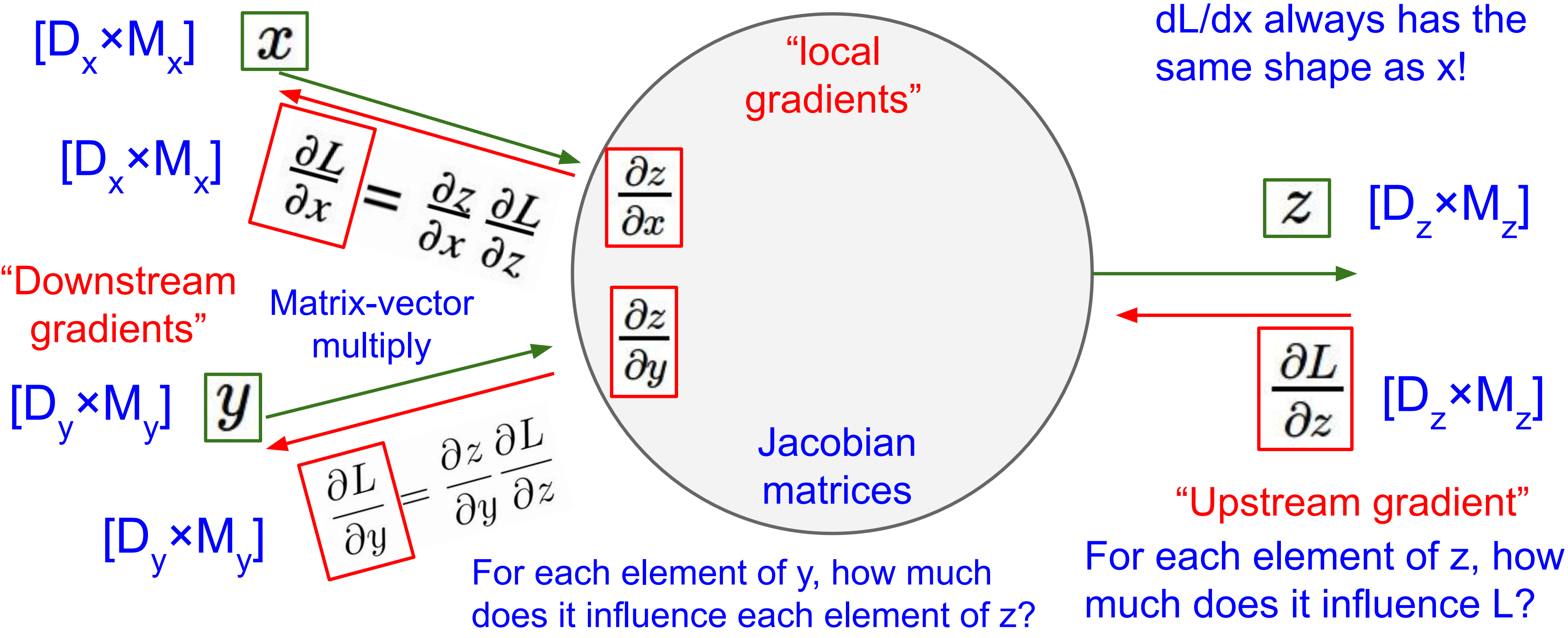


Backprop with Matrices (or Tensors)



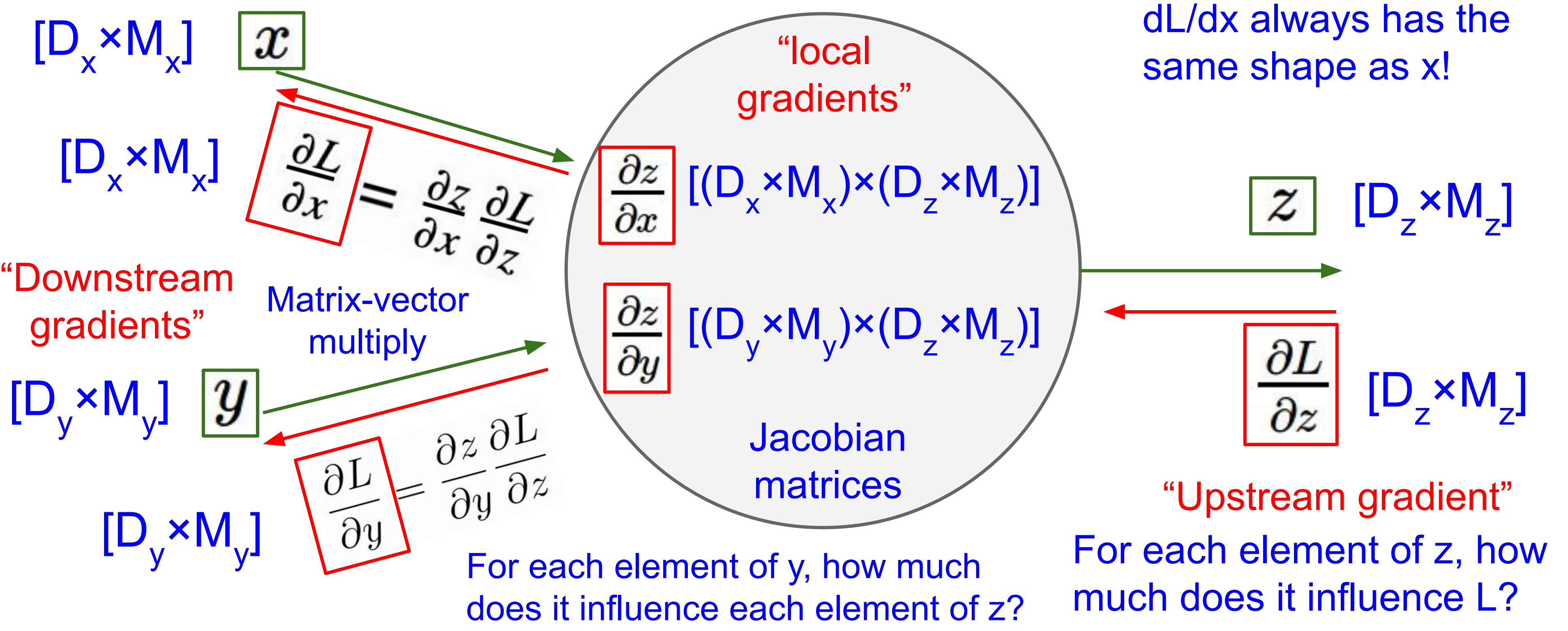
Backprop with Matrices (or Tensors)

Loss L still a scalar!



Backprop with Matrices (or Tensors)

Loss L still a scalar!



Backprop with Matrices

x: [N×D]

$$\begin{bmatrix} 2 & 1 & -3 \\ -3 & 4 & 2 \end{bmatrix}$$

w: [D×M]

$$\begin{bmatrix} 3 & 2 & 1 & -1 \\ 2 & 1 & 3 & 2 \\ 3 & 2 & 1 & -2 \end{bmatrix}$$

Matrix Multiply

$$y_{n,m} = \sum_d x_{n,d} w_{d,m}$$

y: [N×M]

$$\begin{bmatrix} 13 & 9 & -2 & -6 \\ 5 & 2 & 17 & 1 \end{bmatrix}$$

dL/dy: [N×M]

$$\begin{bmatrix} 2 & 3 & -3 & 9 \\ -8 & 1 & 4 & 6 \end{bmatrix}$$

Also see derivation by Prof. Justin Johnson:
https://courses.cs.washington.edu/courses/cse493g1/23sp/resources/linear-backprop.pdf

Backprop with Matrices

x: [N×D]
[2 1 -3]
[-3 4 2]

w: [D×M]
[3 2 1 -1]
[2 1 3 2]
[3 2 1 -2]

Matrix Multiply

$$y_{n,m} = \sum_d x_{n,d} w_{d,m}$$

y: [N×M]
[13 9 -2 -6]
[5 2 17 1]

dL/dy: [N×M]
[2 3 -3 9]
[-8 1 4 6]

Jacobians:
dy/dx: [(N×D)×(N×M)]
dy/dw: [(D×M)×(N×M)]

For a neural net we may have
N=64, D=M=4096
Each Jacobian takes ~256 GB of memory! Must work with them implicitly!

Backprop with Matrices

x: [N×D]

$$\begin{bmatrix} 2 & \boxed{1} & -3 \\ -3 & 4 & 2 \end{bmatrix}$$

w: [D×M]

$$\begin{bmatrix} 3 & 2 & 1 & -1 \\ 2 & 1 & 3 & 2 \\ 3 & 2 & 1 & -2 \end{bmatrix}$$

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dL/dy: [N×M]

$$\begin{bmatrix} 2 & 3 & -3 & 9 \\ -8 & 1 & 4 & 6 \end{bmatrix}$$

Q: What parts of y are affected by one element of x?

Backprop with Matrices

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$$\begin{bmatrix} 2 & \boxed{1} & -3 \\ -3 & 4 & 2 \end{bmatrix}$$

w: [D×M]

$$\begin{bmatrix} 3 & 2 & 1 & -1 \\ 2 & 1 & 3 & 2 \\ 3 & 2 & 1 & -2 \end{bmatrix}$$

Matrix Multiply

$$y_{n,m} = \sum_d x_{n,d} w_{d,m}$$

y: [N×M]

$$\begin{bmatrix} \boxed{13 \quad 9 \quad -2 \quad -6} \\ 5 \quad 2 \quad 17 \quad 1 \end{bmatrix}$$

dL/dy: [N×M]

$$\begin{bmatrix} \boxed{2 \quad 3 \quad -3 \quad 9} \\ -8 \quad 1 \quad 4 \quad 6 \end{bmatrix}$$

Q: What parts of y are affected by one element of x?

A: $\boxed{x_{n,d}}$ affects the whole row $y_{n,\cdot}$

$$\frac{\partial L}{\partial x_{n,d}} = \sum_m \frac{\partial L}{\partial y_{n,m}} \frac{\partial y_{n,m}}{\partial x_{n,d}}$$

Backprop with Matrices

x: [N×D]

$$\begin{bmatrix} 2 & \boxed{1} & -3 \\ -3 & 4 & 2 \end{bmatrix}$$

w: [D×M]

$$\begin{bmatrix} 3 & 2 & 1 & -1 \\ 2 & 1 & 3 & 2 \\ 3 & 2 & 1 & -2 \end{bmatrix}$$

Matrix Multiply

$$y_{n,m} = \sum_d x_{n,d} w_{d,m}$$

y: [N×M]

$$\begin{bmatrix} \mathbf{13} & \mathbf{9} & \boxed{\mathbf{-2}} & \mathbf{-6} \\ 5 & 2 & 17 & 1 \end{bmatrix}$$

dL/dy: [N×M]

$$\begin{bmatrix} 2 & 3 & -3 & 9 \\ -8 & 1 & 4 & 6 \end{bmatrix}$$

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A: $\boxed{x_{n,d}}$ affects the whole row $y_{n,\cdot}$

$$\frac{\partial L}{\partial x_{n,d}} = \sum_m \frac{\partial L}{\partial y_{n,m}} \frac{\partial y_{n,m}}{\partial x_{n,d}}$$

Q: How much does $\boxed{x_{n,d}}$ affect $\boxed{y_{n,m}}$?

Backprop with Matrices

x: [N×D]

[2 $\boxed{1}$ -3]
[-3 4 2]

w: [D×M]

[3 2 1 -1]
[2 1 $\boxed{3}$ 2]
[3 2 1 -2]

Matrix Multiply

$$y_{n,m} = \sum_d x_{n,d} w_{d,m}$$

y: [N×M]

[13 9 -2 -6]
[5 2 17 1]

dL/dy: [N×M]

[2 3 -3 9]
[-8 1 4 6]

Q: What parts of y are affected by one element of x?

A: $\boxed{x_{n,d}}$ affects the whole row $y_{n,\cdot}$

Q: How much does $\boxed{x_{n,d}}$ affect $\boxed{y_{n,m}}$?

A: $\boxed{w_{d,m}}$

$$\frac{\partial L}{\partial x_{n,d}} = \sum_m \frac{\partial L}{\partial y_{n,m}} \frac{\partial y_{n,m}}{\partial x_{n,d}} = \sum_m \frac{\partial L}{\partial y_{n,m}} w_{d,m}$$

Backprop with Matrices

x: [N×D]

[2 $\boxed{1}$ -3]
[-3 4 2]

w: [D×M]

[3 2 1 -1]
[2 1 $\boxed{3}$ 2]
[3 2 1 -2]

Matrix Multiply

$$y_{n,m} = \sum_d x_{n,d} w_{d,m}$$

y: [N×M]

[13 9 $\boxed{-2}$ -6]
[5 2 17 1]

dL/dy: [N×M]

[2 3 -3 9]
[-8 1 4 6]

Q: What parts of y are affected by one element of x?

A: $\boxed{x_{n,d}}$ affects the whole row $y_{n,\cdot}$

Q: How much does $\boxed{x_{n,d}}$ affect $\boxed{y_{n,m}}$?

A: $\boxed{w_{d,m}}$

[N×D] [N×M] [M×D]

$$\boxed{\frac{\partial L}{\partial x} = \left(\frac{\partial L}{\partial y}\right) w^T}$$

$$\frac{\partial L}{\partial x_{n,d}} = \sum_m \frac{\partial L}{\partial y_{n,m}} \frac{\partial y_{n,m}}{\partial x_{n,d}} = \sum_m \frac{\partial L}{\partial y_{n,m}} w_{d,m}$$

Backprop with Matrices

x: [N×D]

$$\begin{bmatrix} 2 & \boxed{1} & -3 \\ -3 & 4 & 2 \end{bmatrix}$$

w: [D×M]

$$\begin{bmatrix} 3 & 2 & 1 & -1 \\ 2 & 1 & \boxed{3} & 2 \\ 3 & 2 & 1 & -2 \end{bmatrix}$$

Matrix Multiply

$$y_{n,m} = \sum_d x_{n,d} w_{d,m}$$

y: [N×M]

$$\begin{bmatrix} 13 & 9 & \boxed{-2} & -6 \\ 5 & 2 & 17 & 1 \end{bmatrix}$$

dL/dy: [N×M]

$$\begin{bmatrix} 2 & 3 & -3 & 9 \\ -8 & 1 & 4 & 6 \end{bmatrix}$$

By similar logic:

[N×D] [N×M] [M×D]

$$\frac{\partial L}{\partial x} = \left(\frac{\partial L}{\partial y}\right) w^T$$

[D×M] [D×N] [N×M]

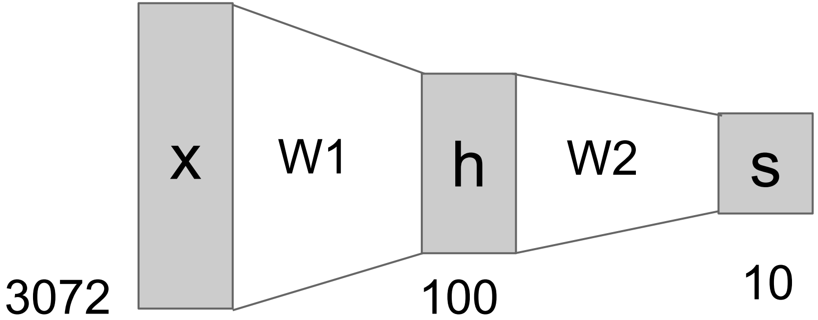
$$\frac{\partial L}{\partial w} = x^T \left(\frac{\partial L}{\partial y}\right)$$

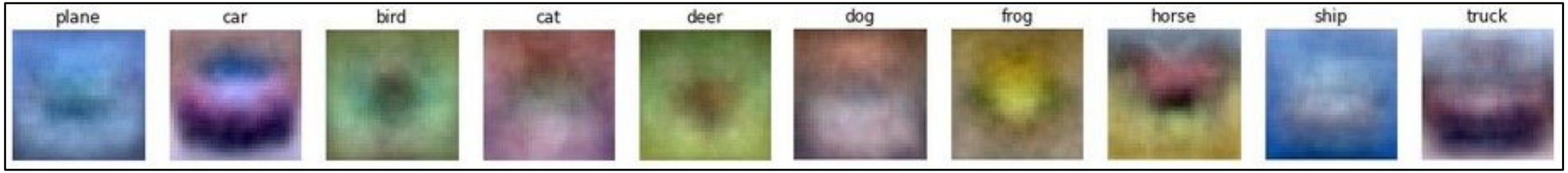
These formulas are easy to remember: they are the only way to make shapes match up!

Wrapping up: Neural Networks

Linear score function: $f = Wx$

2-layer Neural Network $f = W_2 \max(0, W_1 x)$





Next: Convolutional Neural Networks

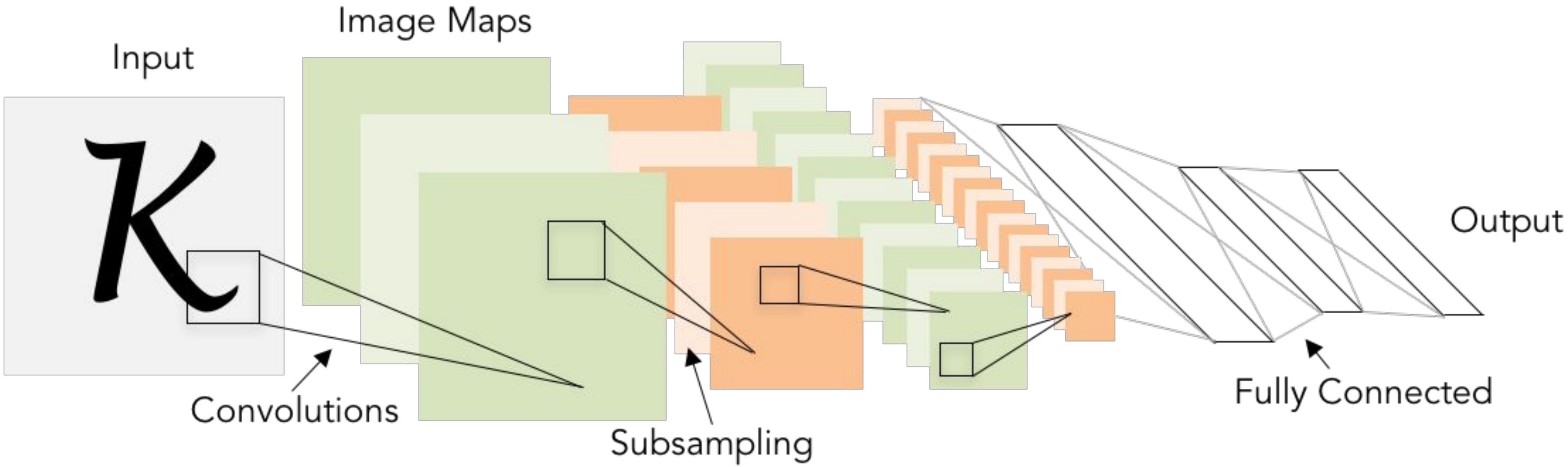
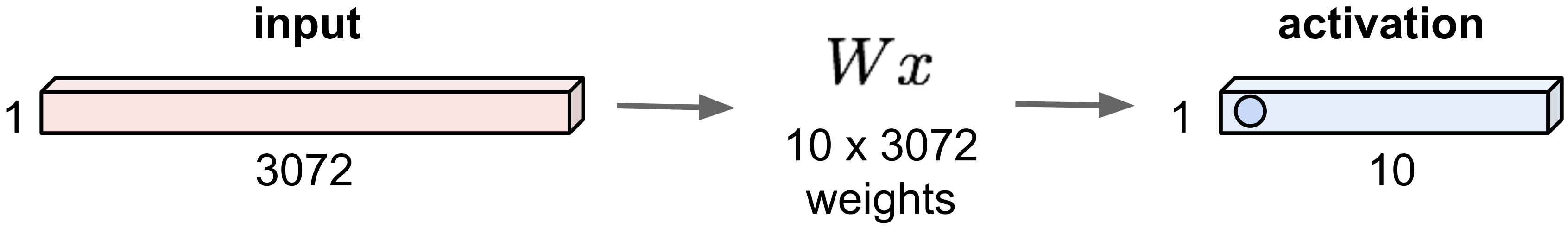


Illustration of LeCun et al. 1998

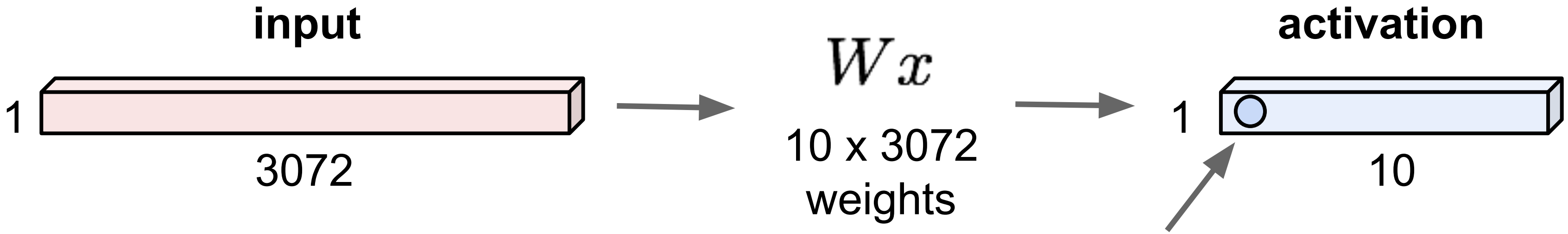
Recap: Fully Connected Layer

32x32x3 image -> stretch to 3072 x 1



Fully Connected Layer

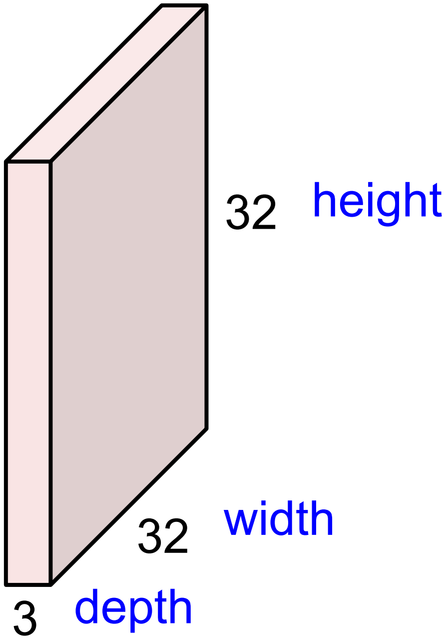
32x32x3 image -> stretch to 3072 x 1



1 number:
the result of taking a dot product between a row of W and the input (a 3072-dimensional dot product)

Convolution Layer

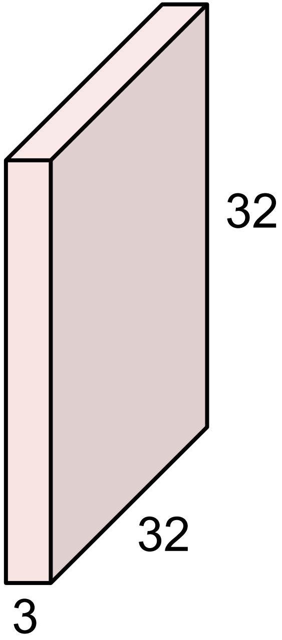
32x32x3 image -> preserve spatial structure



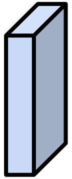
Main idea: only look at small patches of an image

Convolution Layer

32x32x3 image



5x5x3 filter

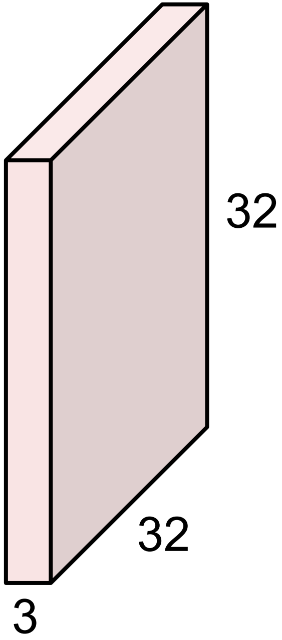


Convolve the filter with the image
i.e. “slide over the image spatially, computing dot products”

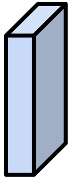
Convolution Layer

Filters always extend the full depth of the input volume

32x32x3 image

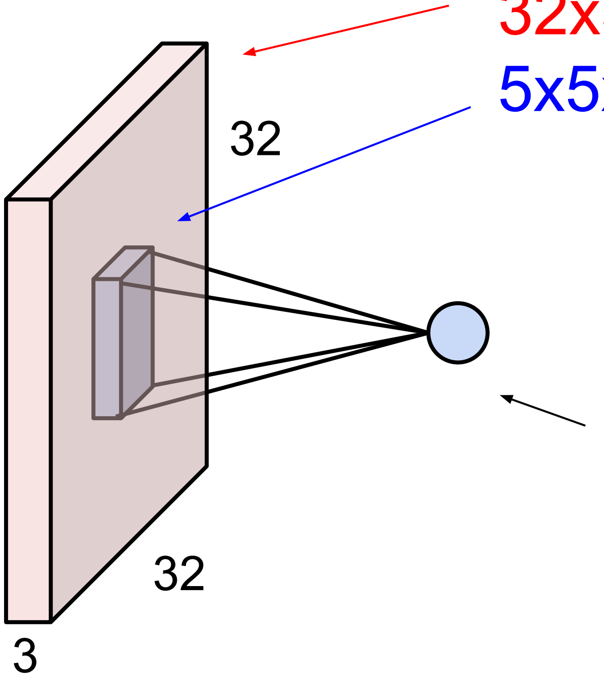


5x5x3 filter



Convolve the filter with the image
i.e. “slide over the image spatially, computing dot products”

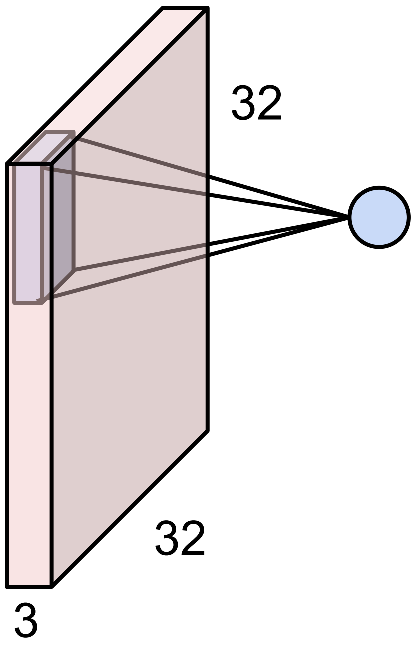
Convolution Layer



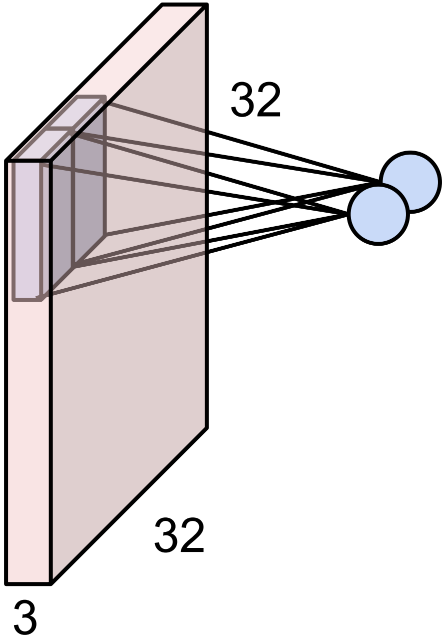
1 number:
the result of taking a dot product between the filter and a small 5x5x3 chunk of the image (i.e. 5*5*3 = 75-dimensional dot product + bias)

$$w^T x + b$$

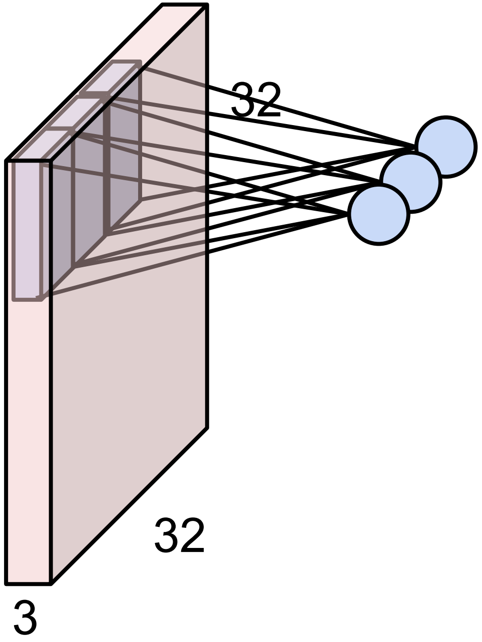
Convolution Layer



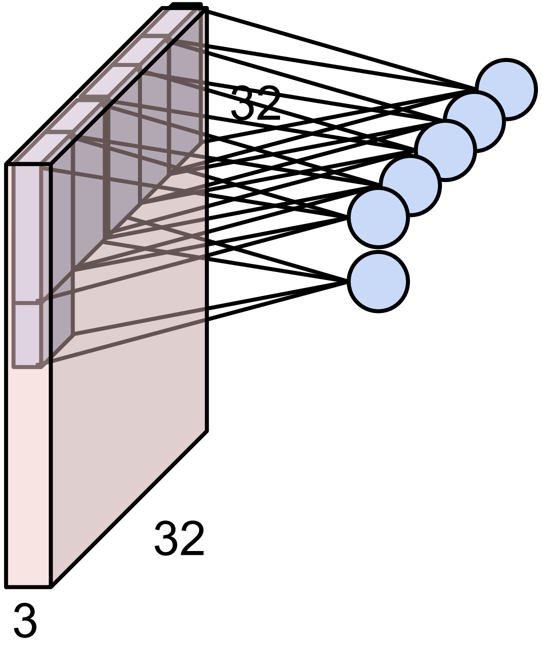
Convolution Layer



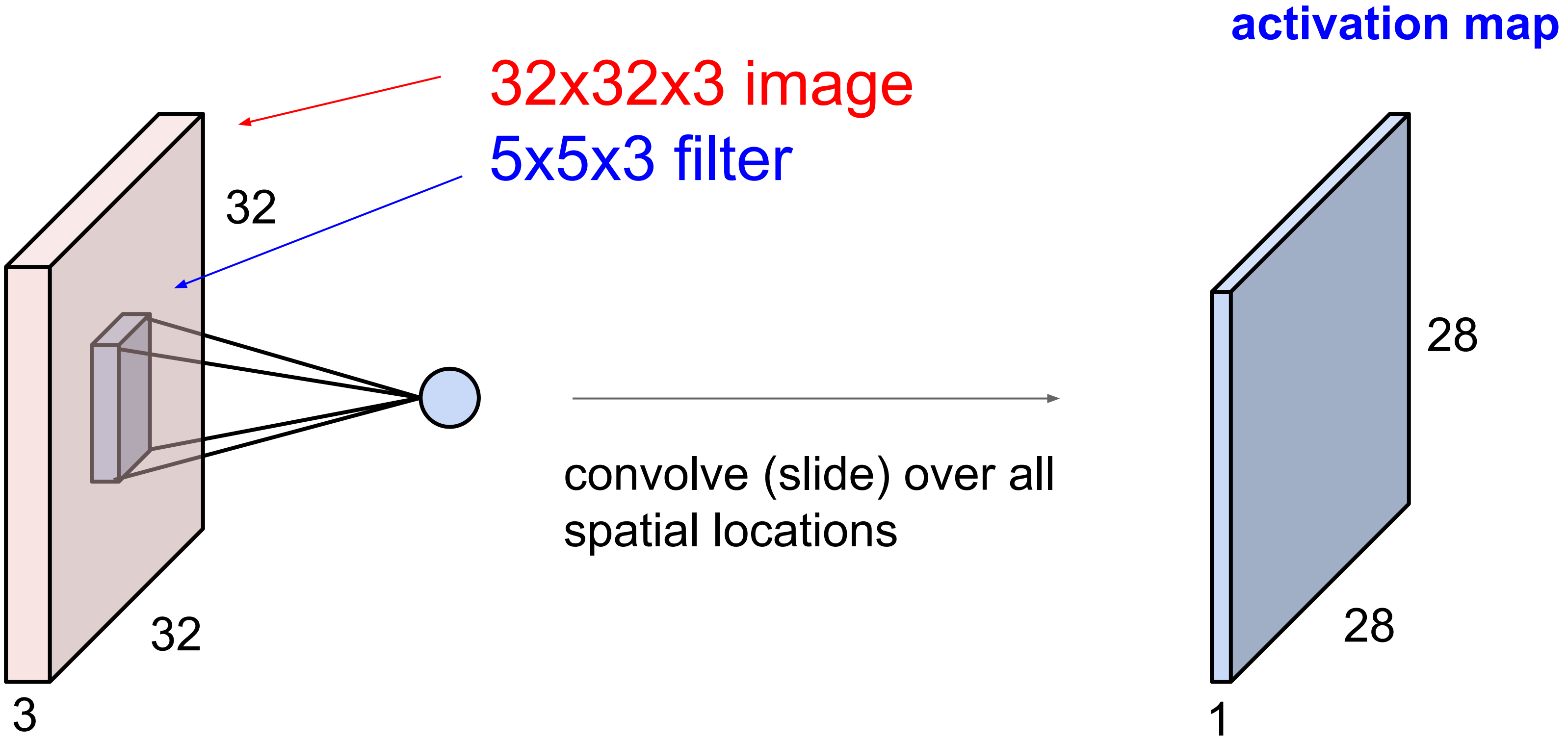
Convolution Layer



Convolution Layer

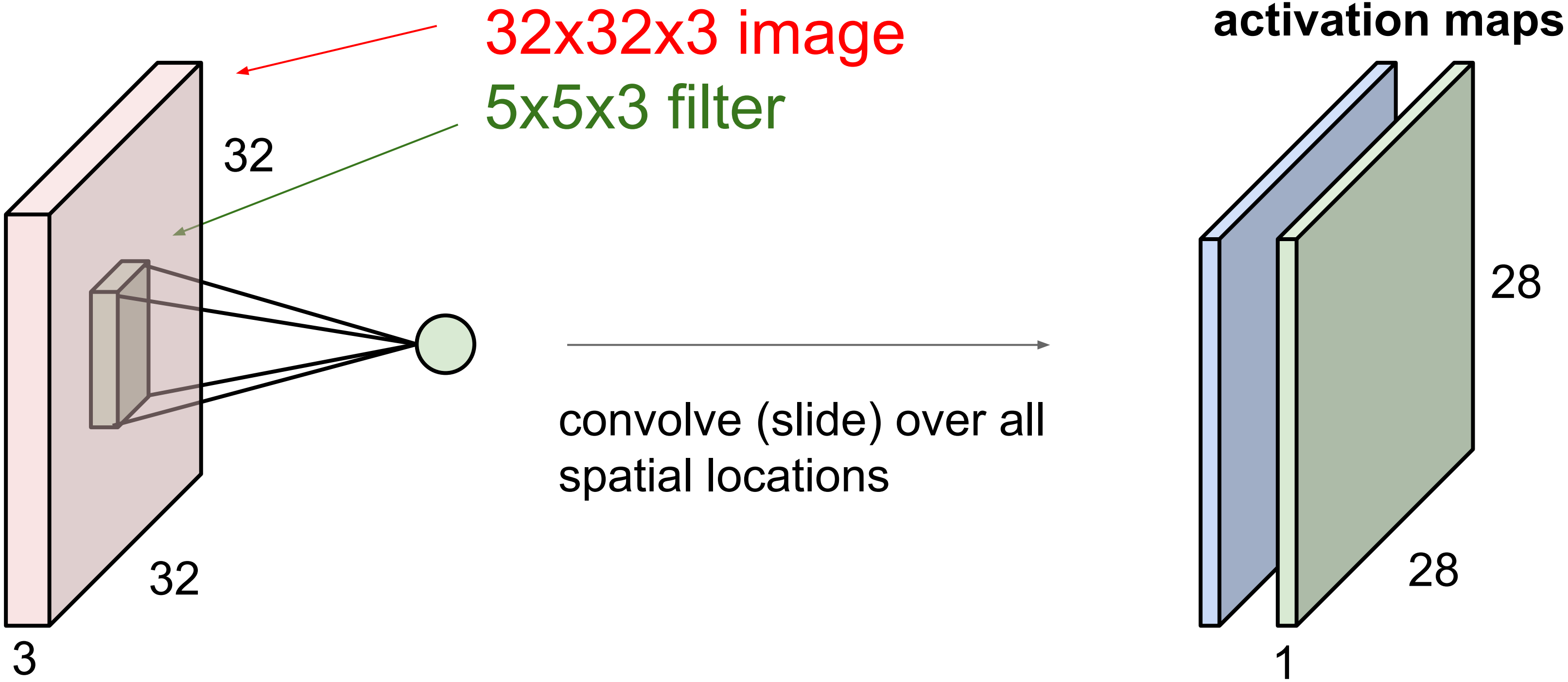


Convolution Layer

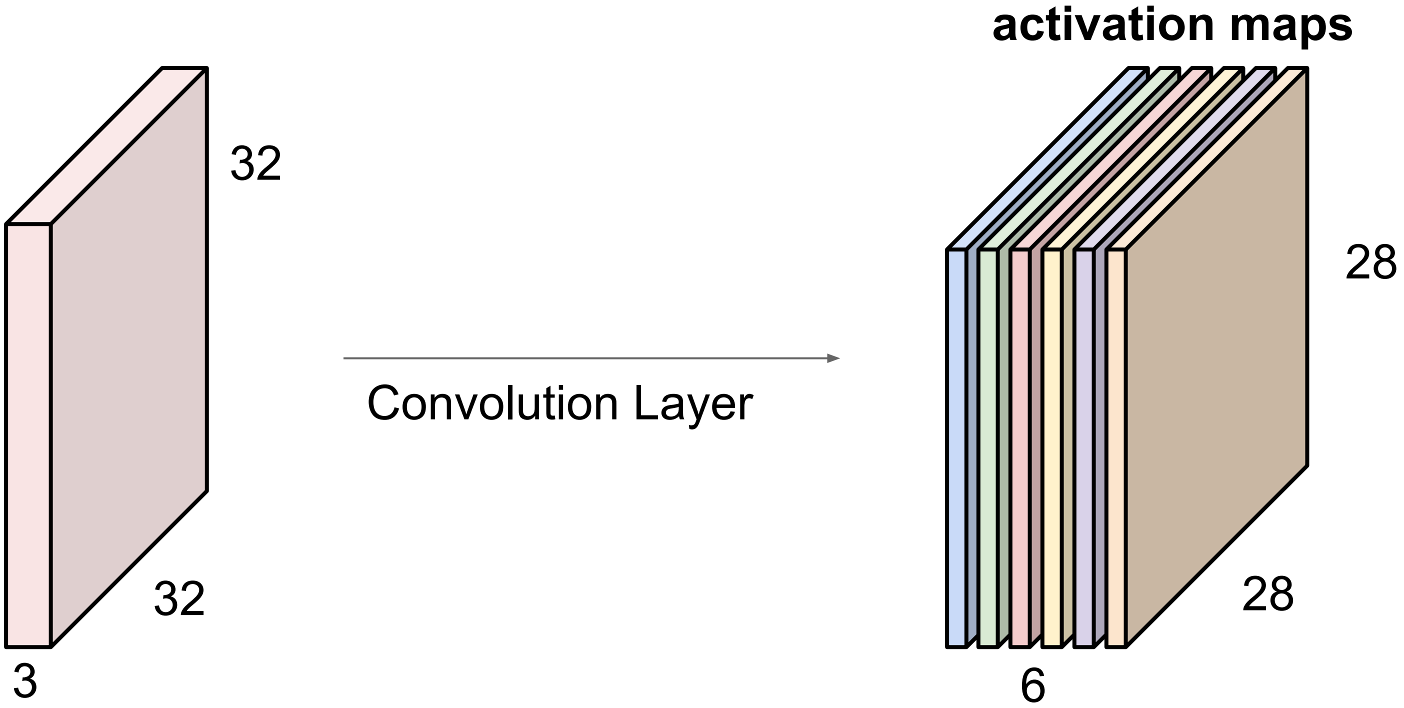


Convolution Layer

consider a second, green filter

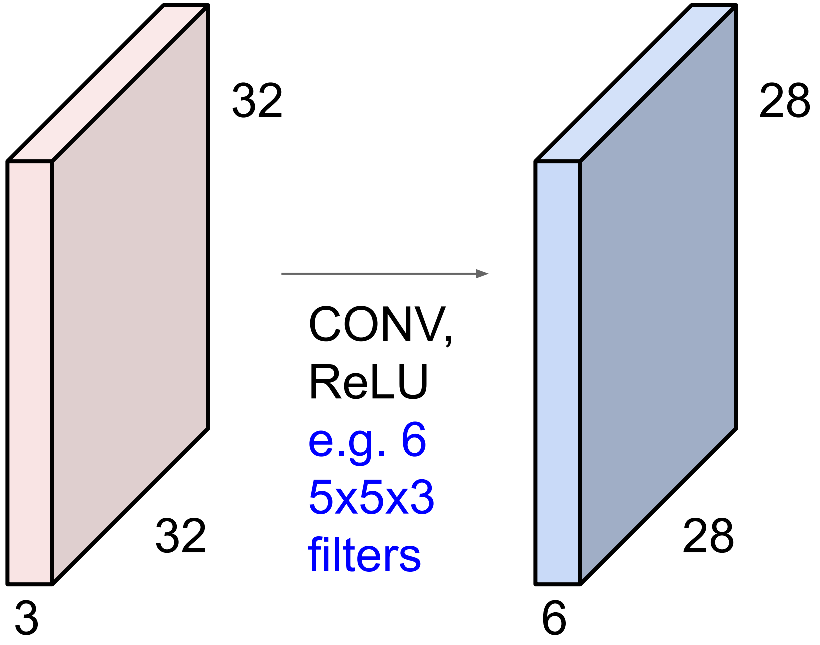


For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:

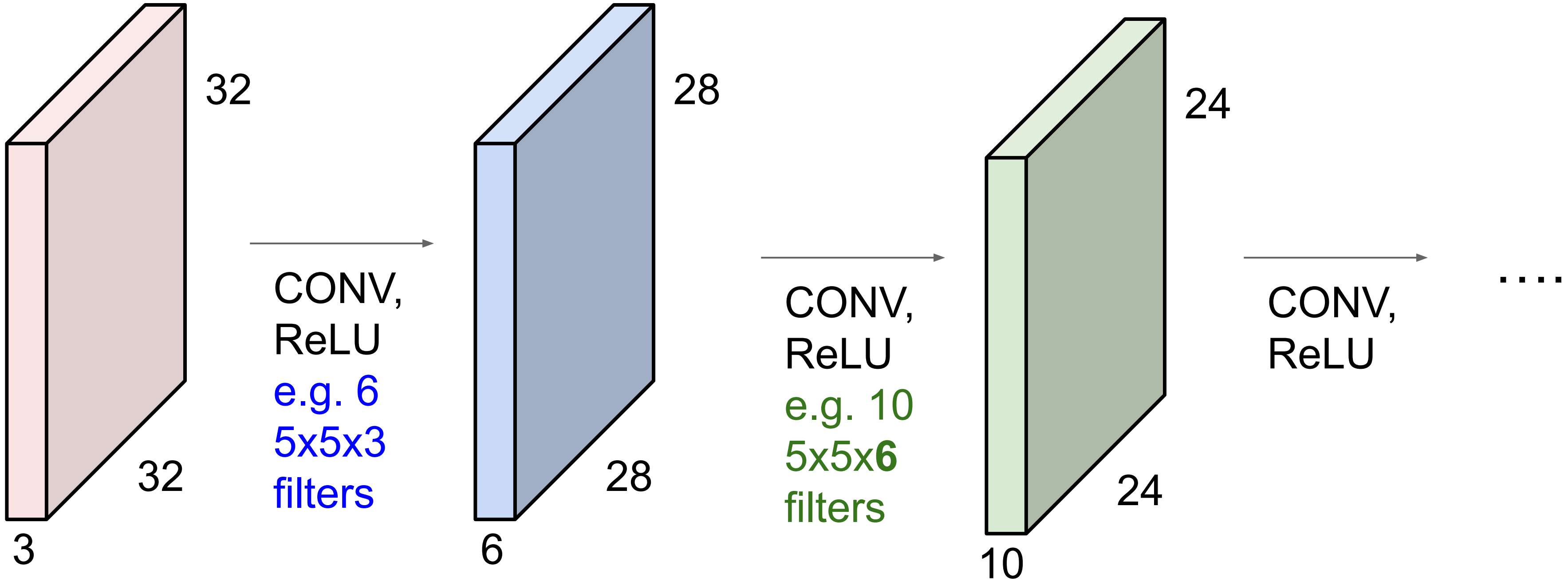


We stack these up to get a "new image" of size 28x28x6!

Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



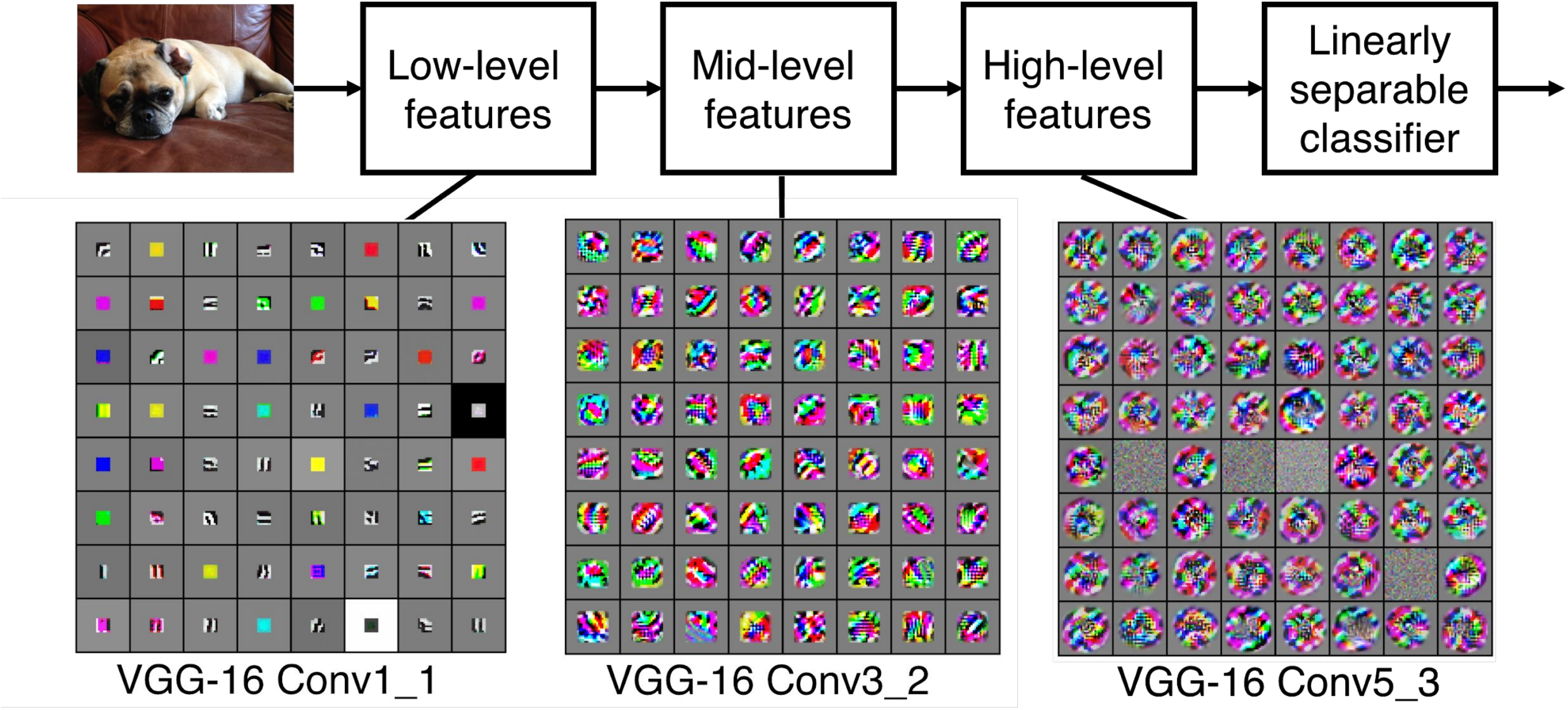
Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



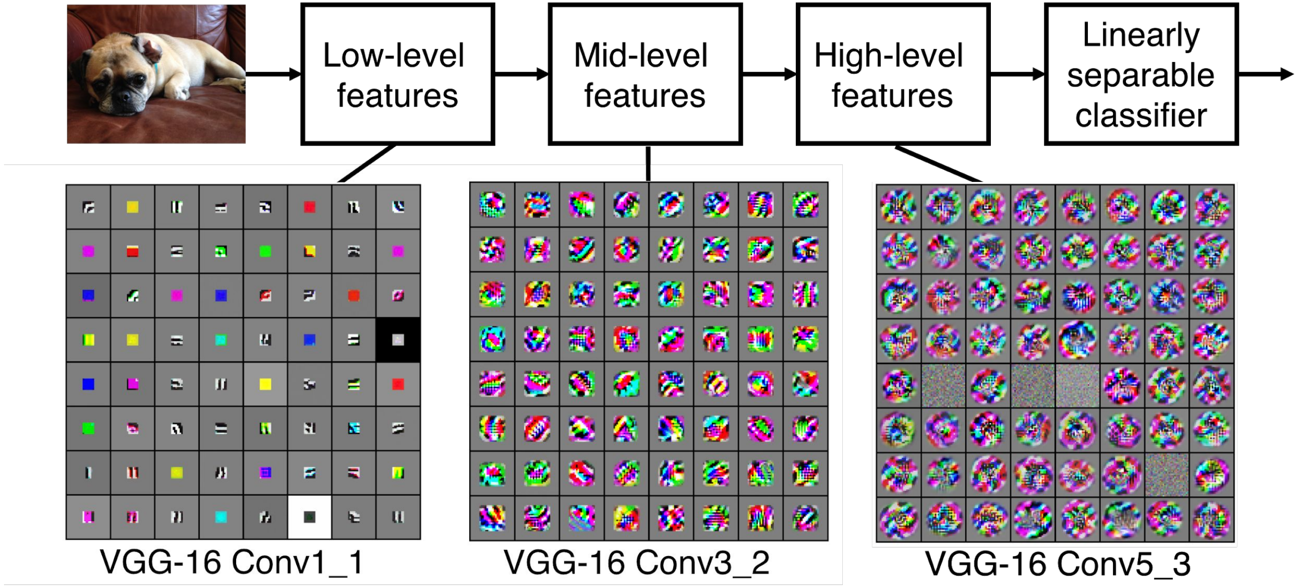
Preview

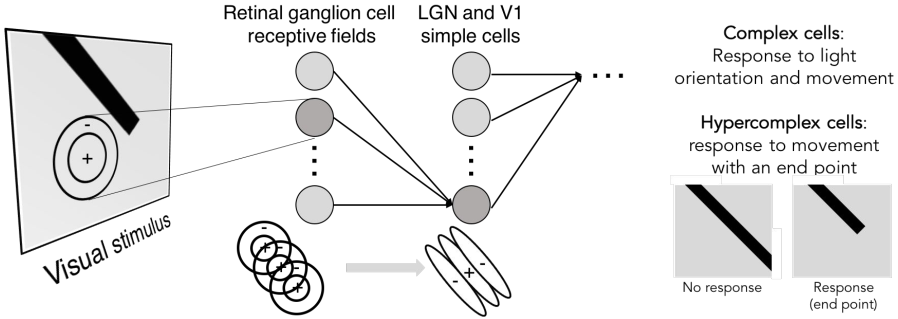
[Zeiler and Fergus 2013]

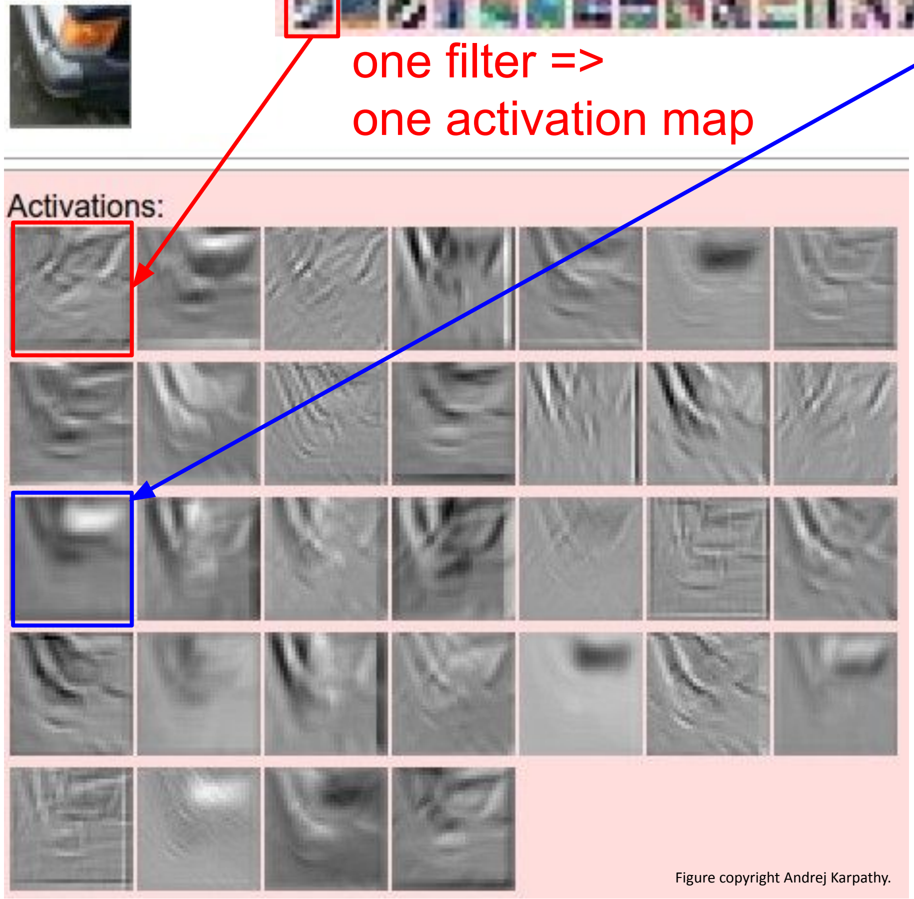
Visualization of VGG-16 by Lane McIntosh. VGG-16 architecture from [Simonyan and Zisserman 2014].



Preview


Low-level features
Mid-level features
High-level features
Linearly separable classifier
VGG-16 Conv1_1
VGG-16 Conv3_2
VGG-16 Conv5_3


Visual stimulus
Retinal ganglion cell receptive fields
LGN and V1 simple cells
Complex cells:
Response to light orientation and movement
Hypercomplex cells:
response to movement with an end point
No response
Response (end point)

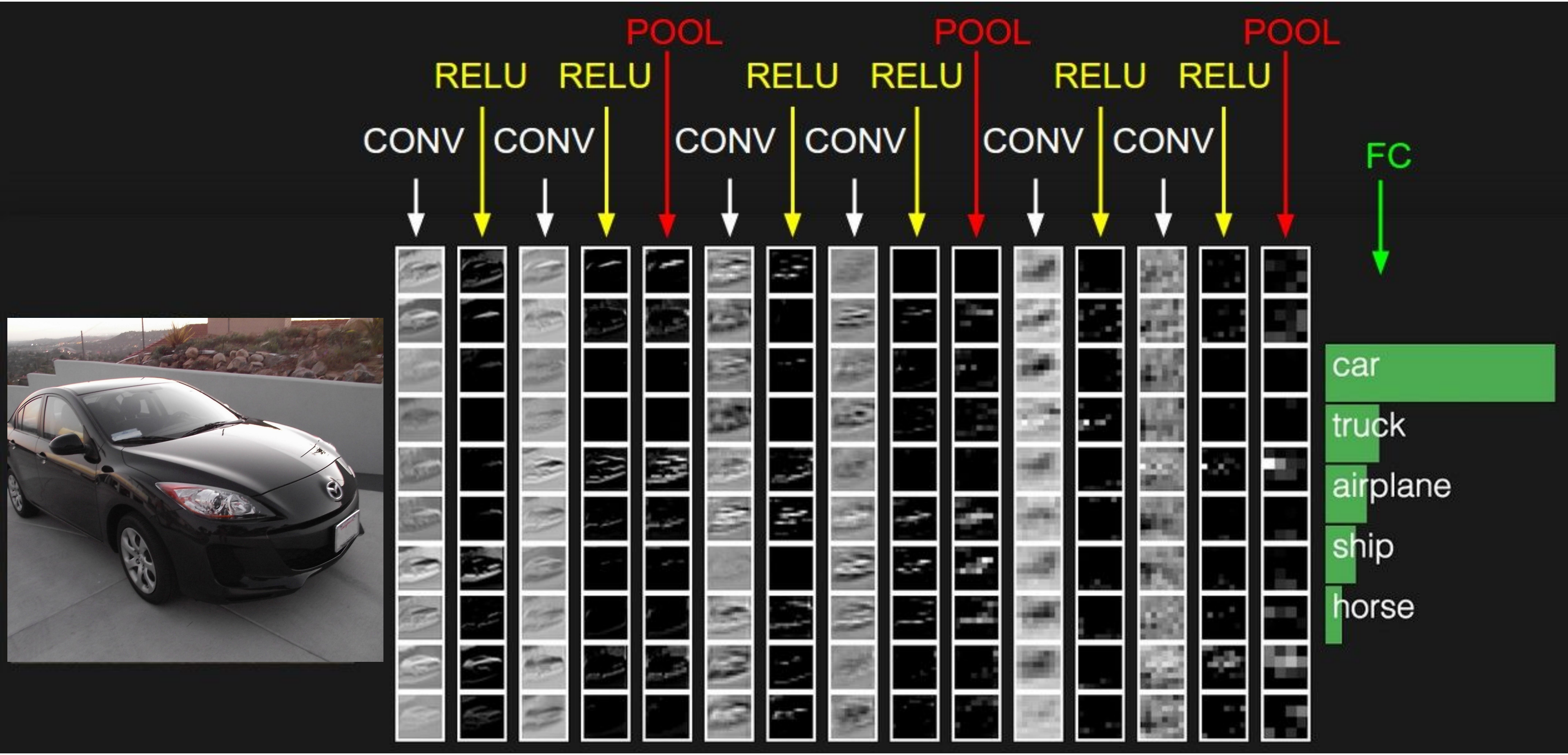


example 5x5 filters
(32 total)

We call the layer convolutional because it is related to convolution of two signals:

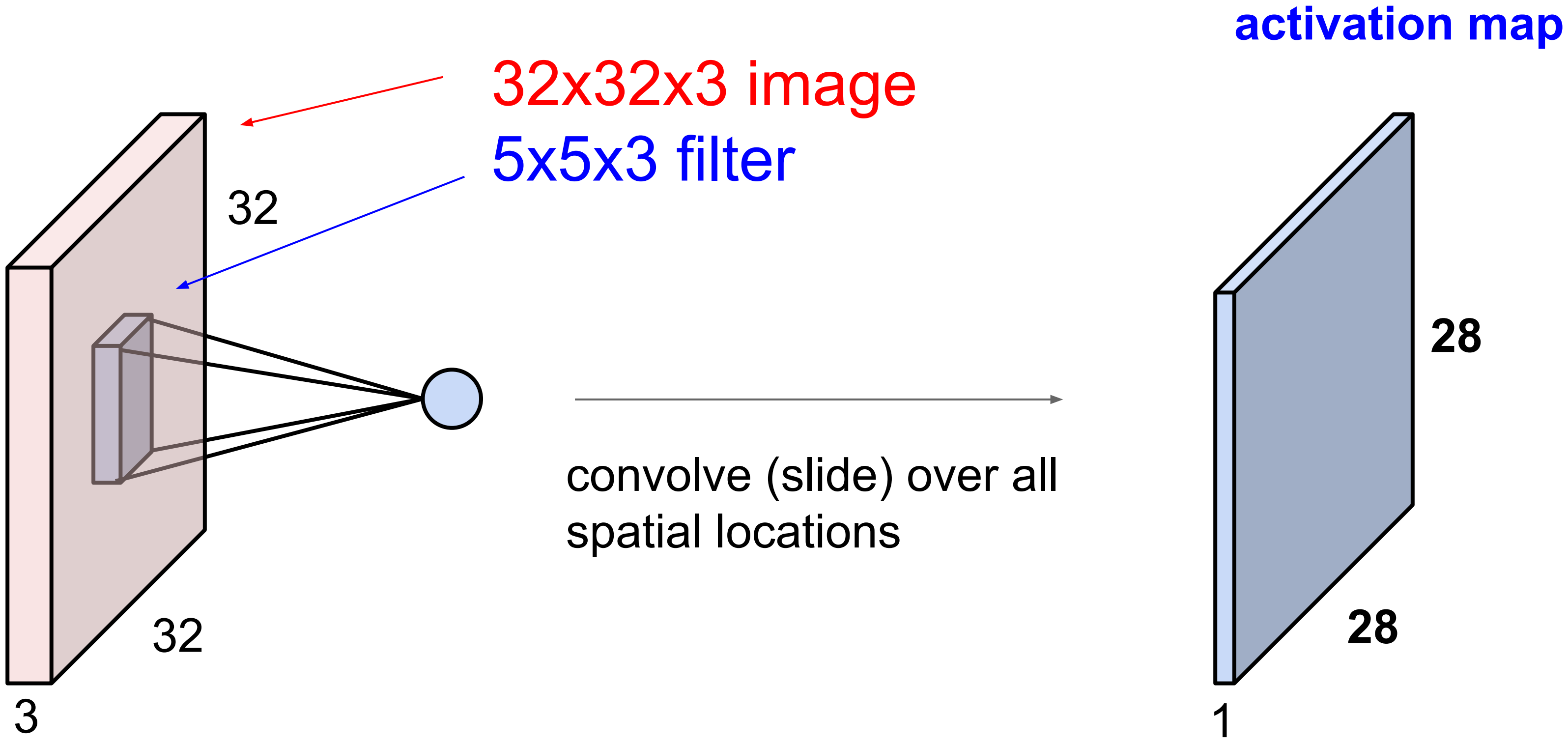
$$f[x,y] * g[x,y] = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} f[n_1,n_2] \cdot g[x-n_1, y-n_2]$$

elementwise multiplication and sum of a filter and the signal (image) preview:

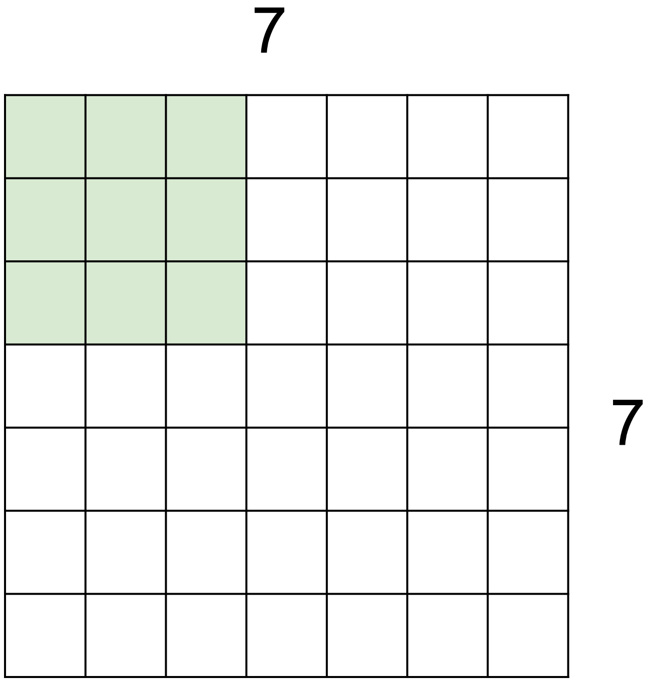


POOL
POOL
POOL
RELU
RELU
RELU
RELU
RELU
RELU
CONV
CONV
CONV
CONV
CONV
CONV
FC
car
truck
airplane
ship
horse

A closer look at spatial dimensions:

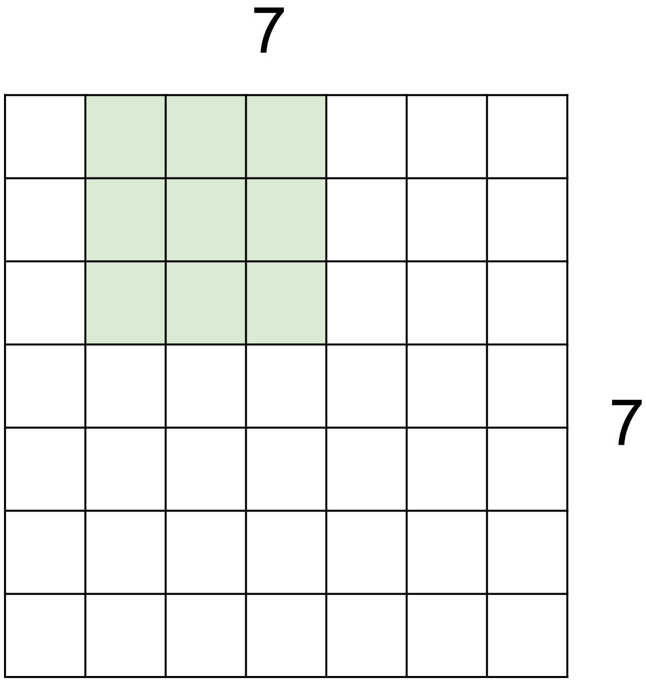


A closer look at spatial dimensions:



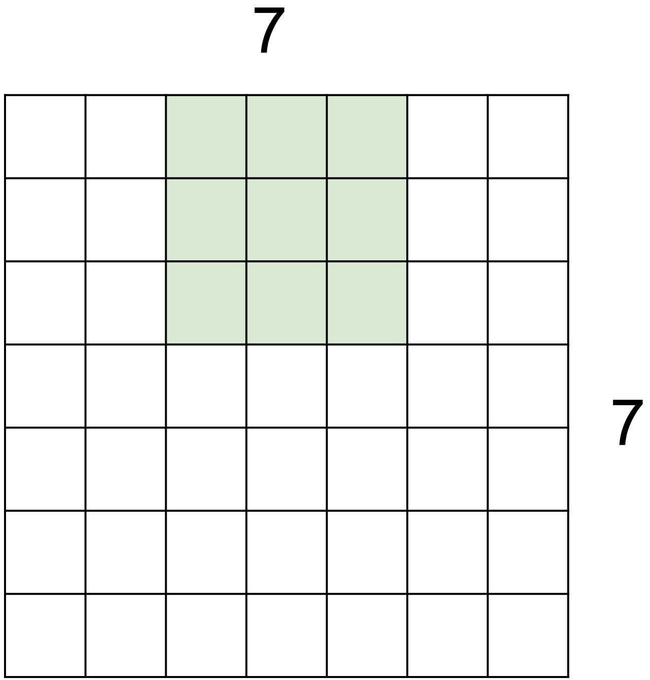
7x7 input (spatially)
assume 3x3 filter

A closer look at spatial dimensions:



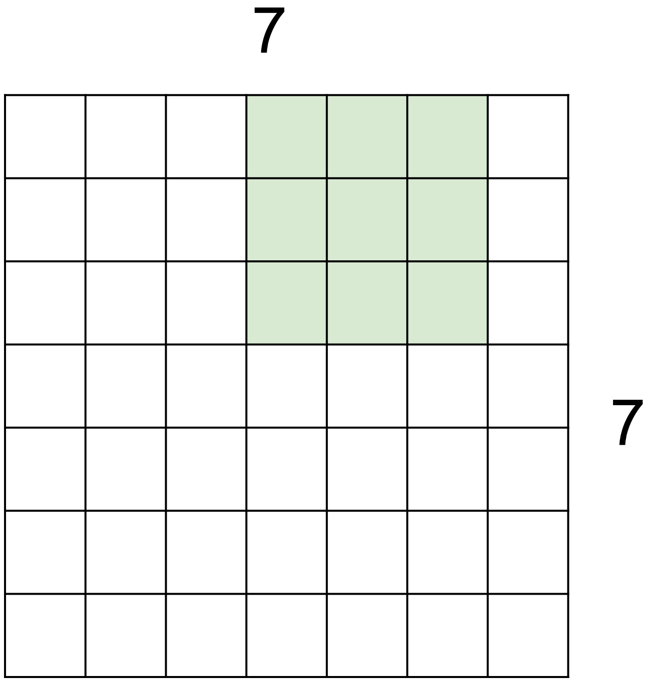
7x7 input (spatially)
assume 3x3 filter

A closer look at spatial dimensions:



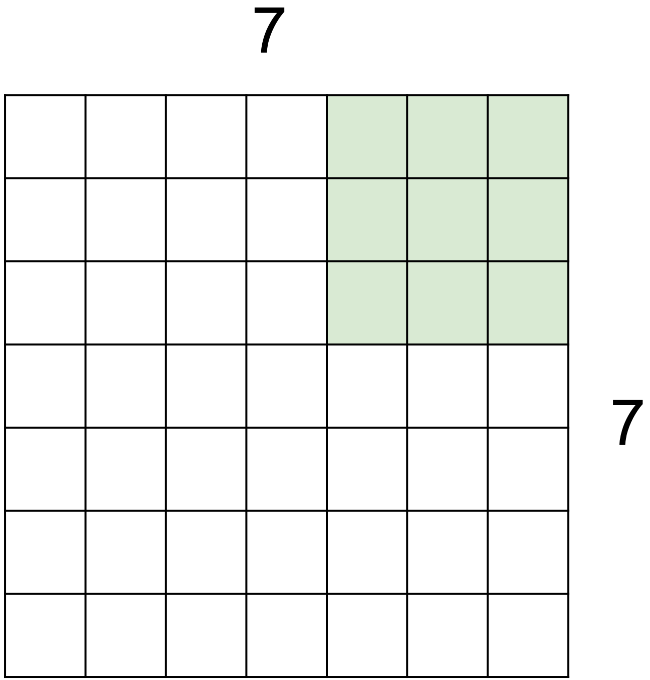
7x7 input (spatially)
assume 3x3 filter

A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter

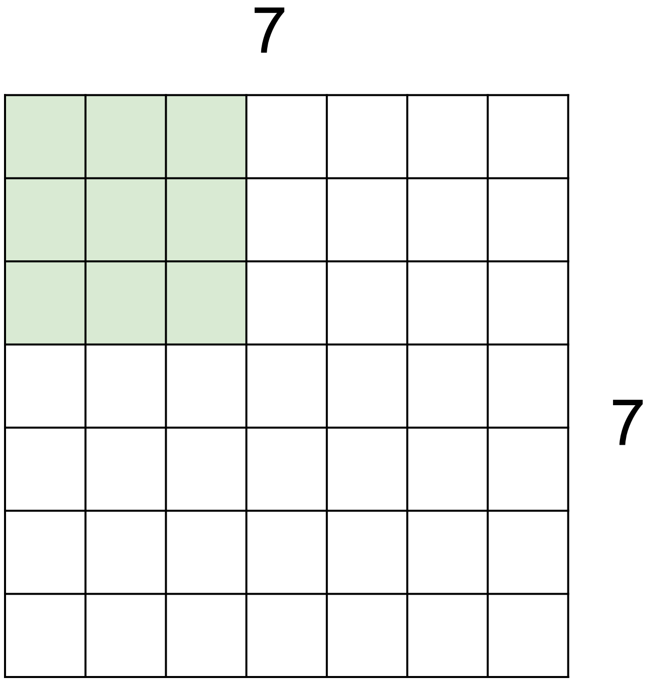
A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter

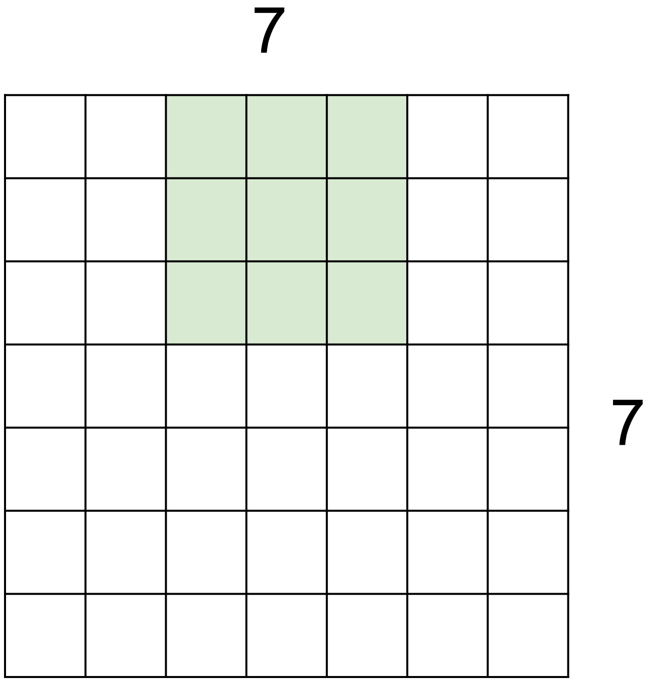
=> 5x5 output

A closer look at spatial dimensions:



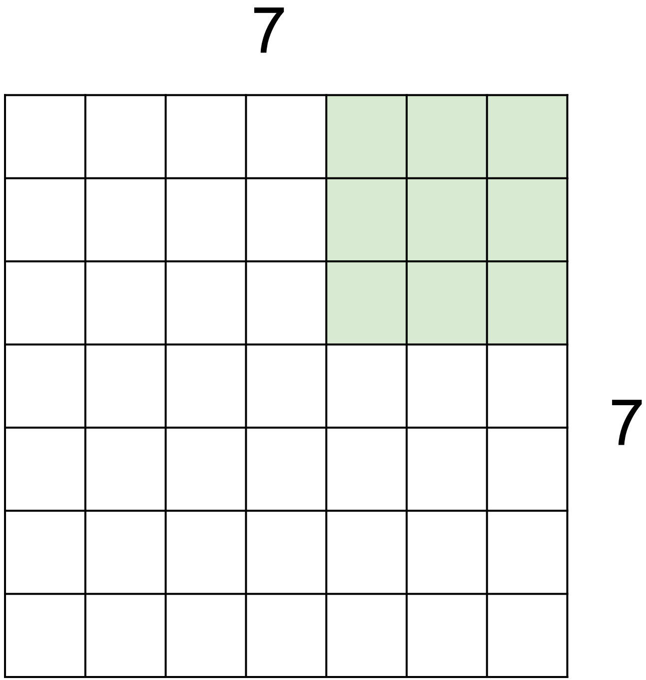
7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**

A closer look at spatial dimensions:



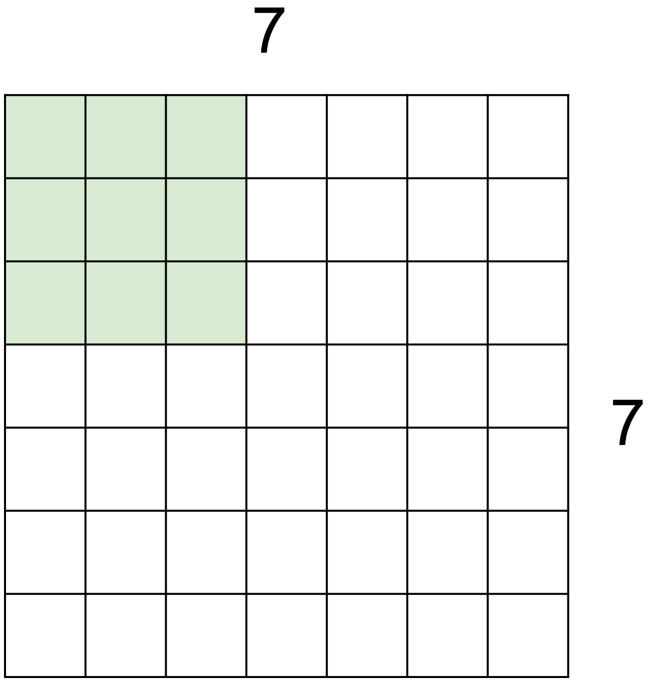
7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**

A closer look at spatial dimensions:



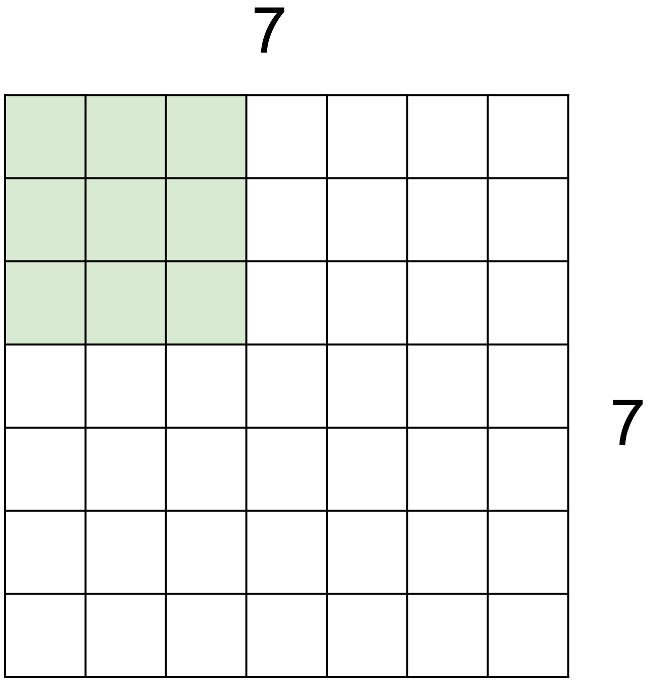
7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**
=> 3x3 output!

A closer look at spatial dimensions:



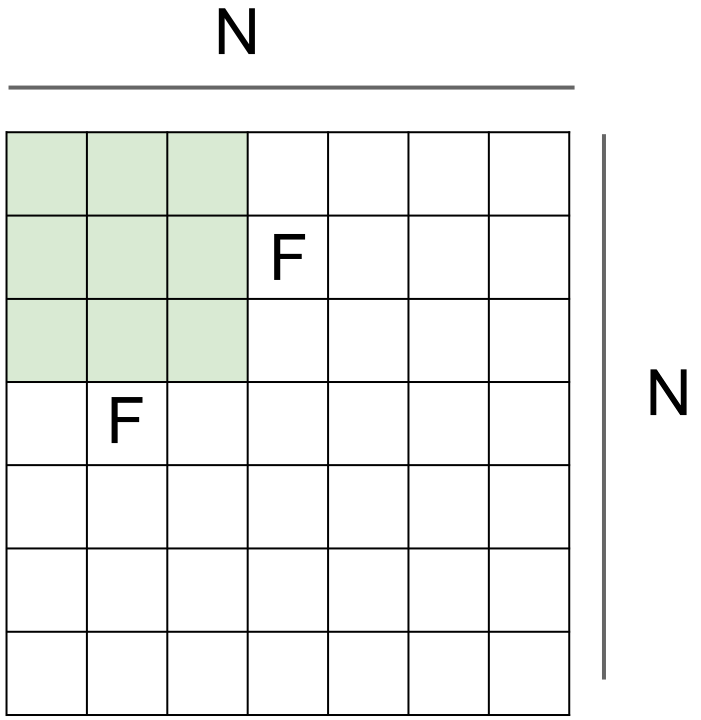
7x7 input (spatially)
assume 3x3 filter
applied **with stride 3?**

A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter
applied **with stride 3?**

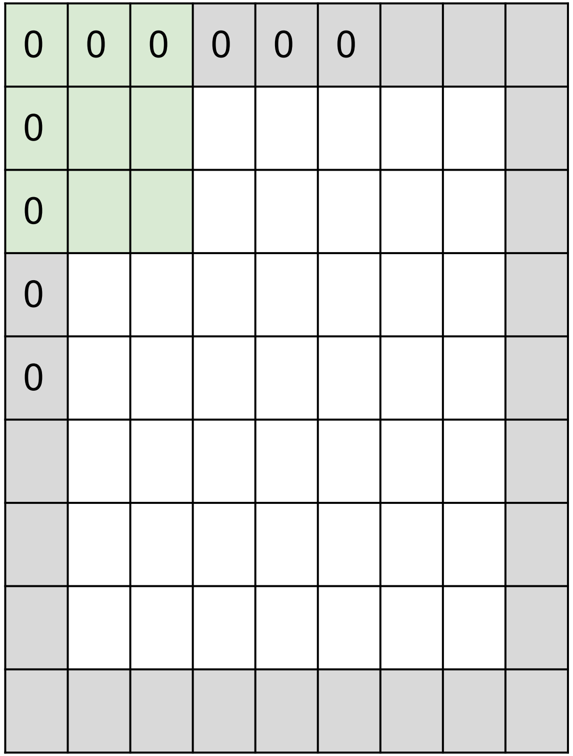
doesn't fit!
cannot apply 3x3 filter on
7x7 input with stride 3.



Output size:
(N - F) / stride + 1

e.g. N = 7, F = 3:
stride 1 => (7 - 3)/1 + 1 = 5
stride 2 => (7 - 3)/2 + 1 = 3
stride 3 => (7 - 3)/3 + 1 = 2.33 :\

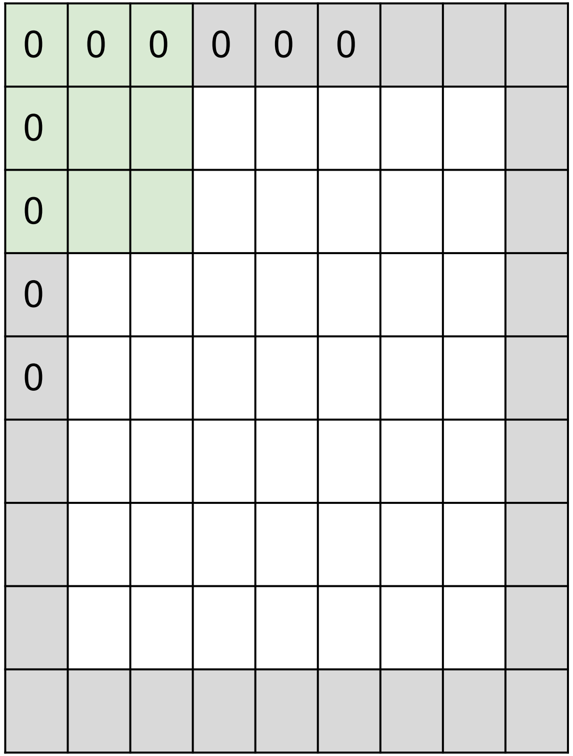
In practice: Common to zero pad the border



e.g. input 7x7
3x3 filter, applied with **stride 1**
pad with 1 pixel border => what is the output?

(recall:)
(N - F) / stride + 1

In practice: Common to zero pad the border

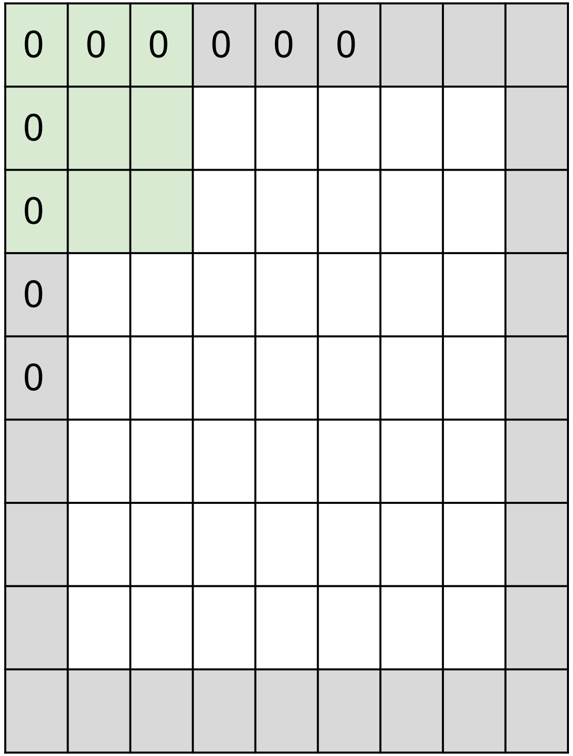


e.g. input 7x7
3x3 filter, applied with **stride 1**
pad with 1 pixel border => what is the output?

7x7 output!

(recall:)
(N + 2P - F) / stride + 1

In practice: Common to zero pad the border

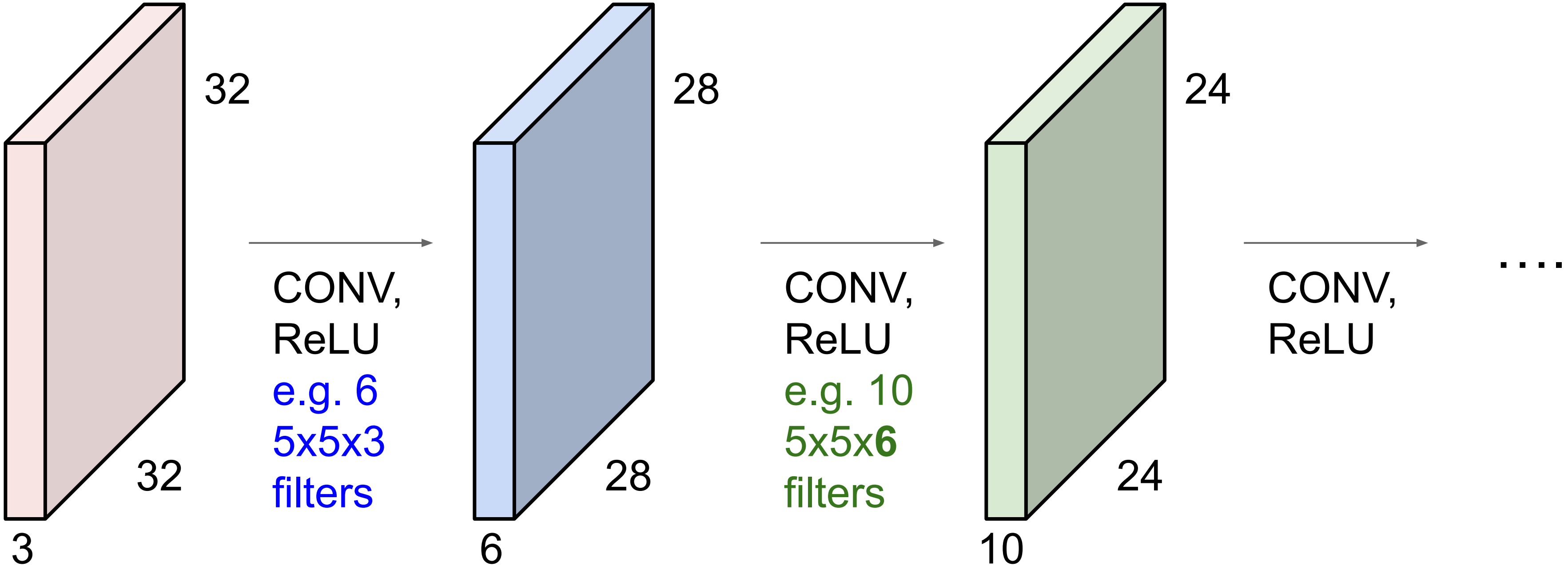


e.g. input 7x7
3x3 filter, applied with **stride 1**
pad with 1 pixel border => what is the output?

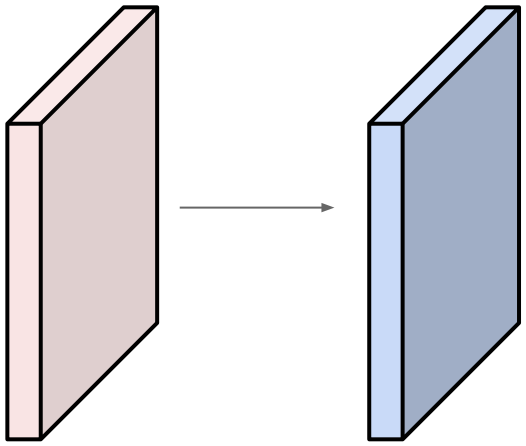
7x7 output!
in general, common to see CONV layers with stride 1, filters of size FxF, and zero-padding with (F-1)/2. (will preserve size spatially)
e.g. F = 3 => zero pad with 1
F = 5 => zero pad with 2
F = 7 => zero pad with 3

Remember back to…

E.g. 32x32 input convolved repeatedly with 5x5 filters shrinks volumes spatially! (32 -> 28 -> 24 ...). Shrinking too fast is not good, doesn’t work well.



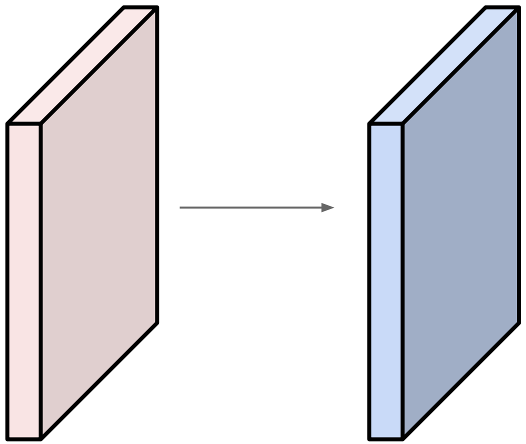
Examples time:



Input volume: **32x32x3**
10 5x5 filters with stride 1, pad 2

Let's assume output size is HxWxD.
What is D?

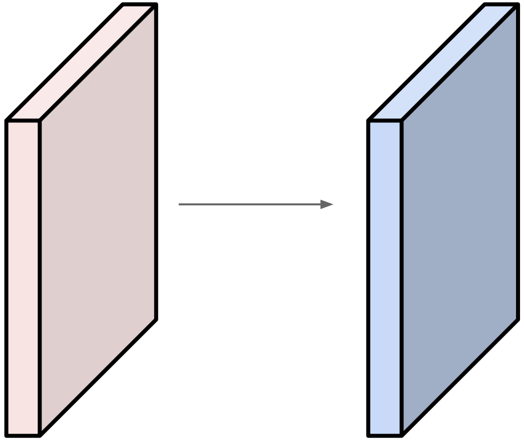
Examples time:



Input volume: **32x32x3**
10 5x5 filters with stride 1, pad 2

Let's assume output size is HxWxD.
What is D? **10**

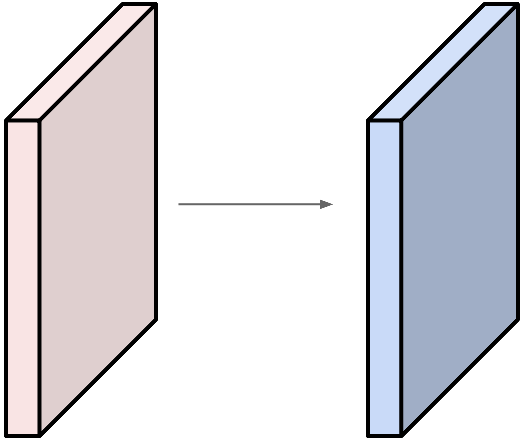
Examples time:



Input volume: **32x32x3**
10 5x5 filters with stride 1, pad 2

Let's assume output size is HxWxD.
What is D? **10**
What is H or W?

Examples time:



Input volume: **32x32x3**

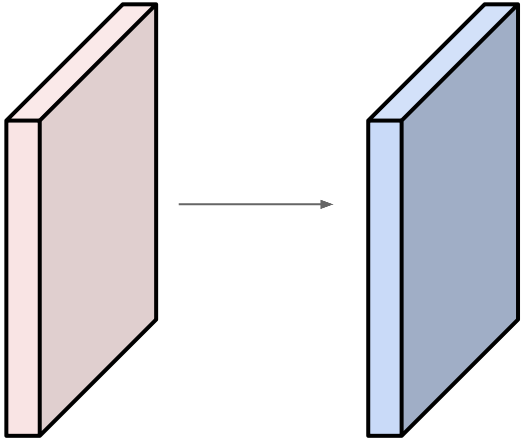
10 5x5 filters with stride 1, pad 2

Let's assume output size is HxWxD.

What is D? 10

What is H or W? (32+2*2-5)/1+1 = 32

Examples time:



Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2

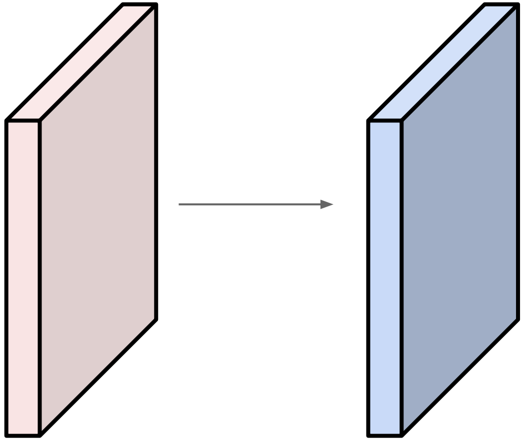
Let's assume output size is HxWxD.

What is D? 10

What is H or W? (32+2*2-5)/1+1 = 32

So the total output size is: **32x32x10**

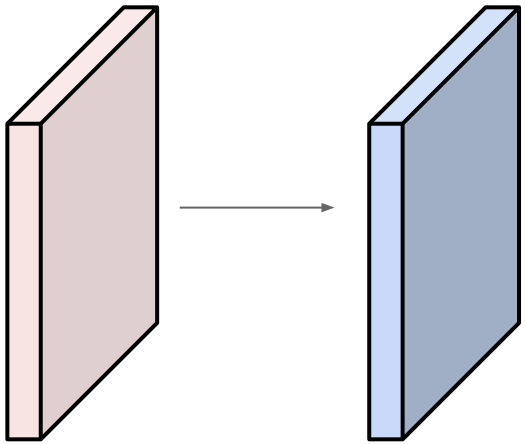
Examples time:



Input volume: **32x32x3**
10 5x5 filters with stride 1, pad 2

Number of parameters in this layer?

Examples time:



Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2

Number of parameters in this layer?

each filter has 5*5*3 + 1 = 76 params (+1 for bias)

=> 76*10 = **760**

Convolution layer: summary

Let's assume input is W_1 x H_1 x C
Conv layer needs 4 hyperparameters:

- Number of filters **K**
- The filter size **F**
- The stride **S**
- The zero padding **P**

This will produce an output of W_2 x H_2 x K
where:

- $W_2 = (W_1 - F + 2P)/S + 1$
- $H_2 = (H_1 - F + 2P)/S + 1$

Number of parameters: F^2CK and K biases

Convolution layer: summary

Let's assume input is $W_1 \times H_1 \times C$
Conv layer needs 4 hyperparameters:

- Number of filters **K**
- The filter size **F**
- The stride **S**
- The zero padding **P**

This will produce an output of $W_2 \times H_2 \times K$
where:

- $W_2 = (W_1 - F + 2P)/S + 1$
- $H_2 = (H_1 - F + 2P)/S + 1$

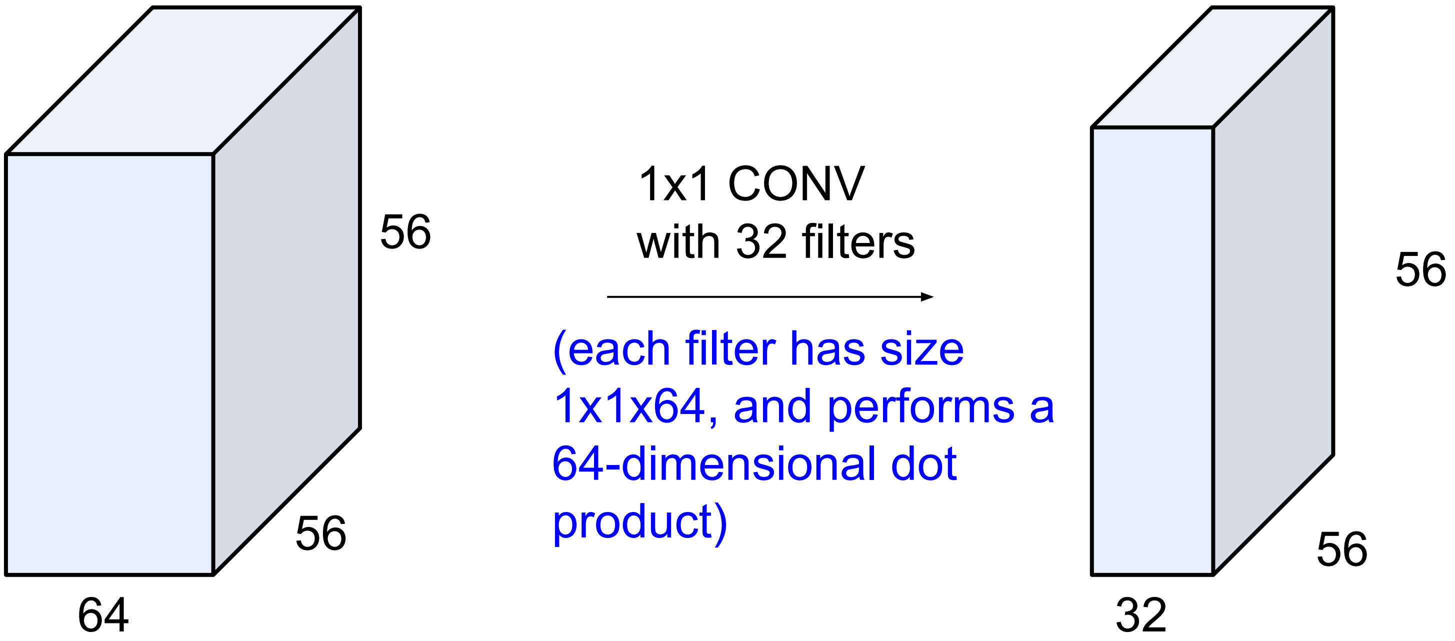
Number of parameters: F^2CK and K biases

Common settings:

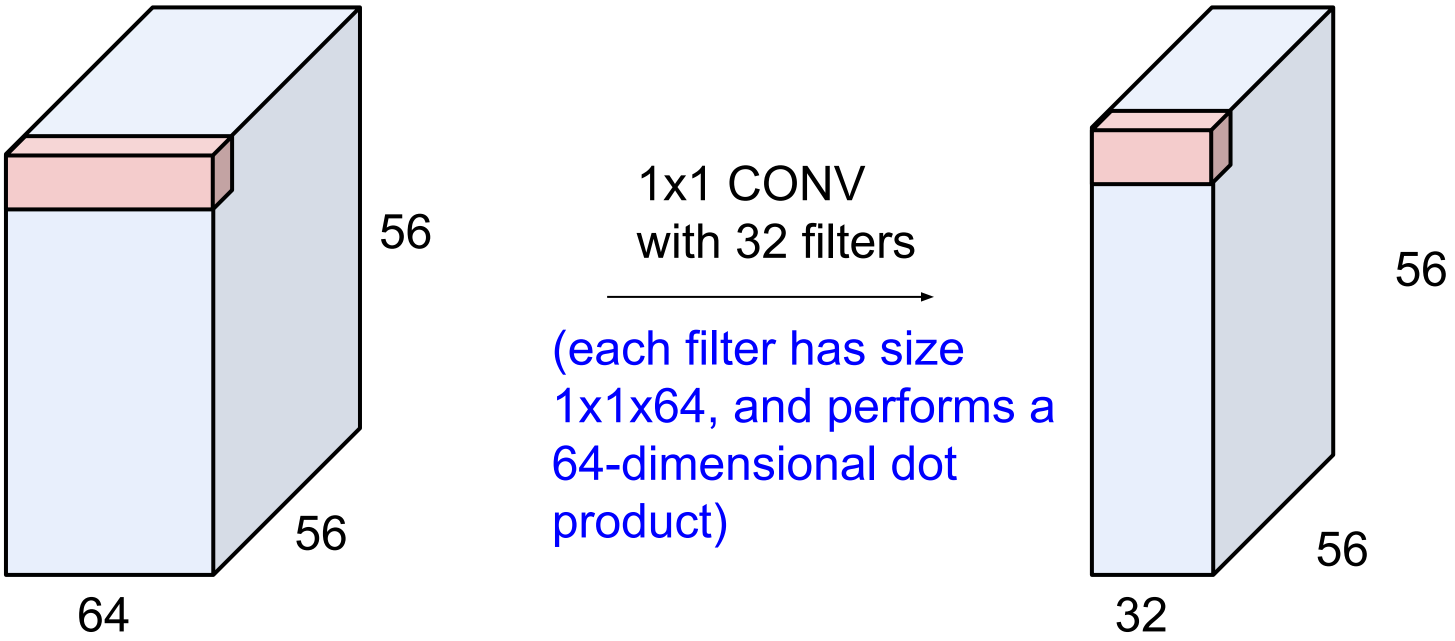
K = (powers of 2, e.g. 32, 64, 128, 512)

- F = 3, S = 1, P = 1
- F = 5, S = 1, P = 2
- F = 5, S = 2, P = ? (whatever fits)
- F = 1, S = 1, P = 0

(btw, 1x1 convolution layers make perfect sense)



(btw, 1x1 convolution layers make perfect sense)



Example: CONV layer in PyTorch

Conv layer needs 4 hyperparameters:
- Number of filters **K**
- The filter size **F**
- The stride **S**
- The zero padding **P**

Conv2d

CLASS `torch.nn.Conv2d(in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True)` [SOURCE]

Applies a 2D convolution over an input signal composed of several input planes.

In the simplest case, the output value of the layer with input size (N, C_{in}, H, W) and output $(N, C_{\text{out}}, H_{\text{out}}, W_{\text{out}})$ can be precisely described as:

$$\text{out}(N_i, C_{\text{out}_j}) = \text{bias}(C_{\text{out}_j}) + \sum_{k=0}^{C_{\text{in}}-1} \text{weight}(C_{\text{out}_j}, k) \star \text{input}(N_i, k)$$

where $\star$ is the valid 2D cross-correlation operator, N is a batch size, C denotes a number of channels, H is a height of input planes in pixels, and W is width in pixels.

- `stride` controls the stride for the cross-correlation, a single number or a tuple.
- `padding` controls the amount of implicit zero-paddings on both sides for `padding` number of points for each dimension.
- `dilation` controls the spacing between the kernel points; also known as the à trous algorithm. It is harder to describe, but this link has a nice visualization of what `dilation` does.
- `groups` controls the connections between inputs and outputs. `in_channels` and `out_channels` must both be divisible by `groups`. For example,
  - At groups=1, all inputs are convolved to all outputs.
  - At groups=2, the operation becomes equivalent to having two conv layers side by side, each seeing half the input channels, and producing half the output channels, and both subsequently concatenated.
  - At groups= `in_channels`, each input channel is convolved with its own set of filters, of size: $\left\lfloor \frac{C_{\text{out}}}{C_{\text{in}}} \right\rfloor$.

The parameters `kernel_size`, `stride`, `padding`, `dilation` can either be:

- a single `int` – in which case the same value is used for the height and width dimension
- a `tuple` of two ints – in which case, the first *int* is used for the height dimension, and the second *int* for the width dimension

Example: CONV layer in Keras

Conv layer needs 4 hyperparameters:

- Number of filters **K**
- The filter size **F**
- The stride **S**
- The zero padding **P**

Conv2D [source]

```
keras.layers.Conv2D(filters, kernel_size, strides=(1, 1), padding='valid', data_format=None, d:
```

2D convolution layer (e.g. spatial convolution over images).

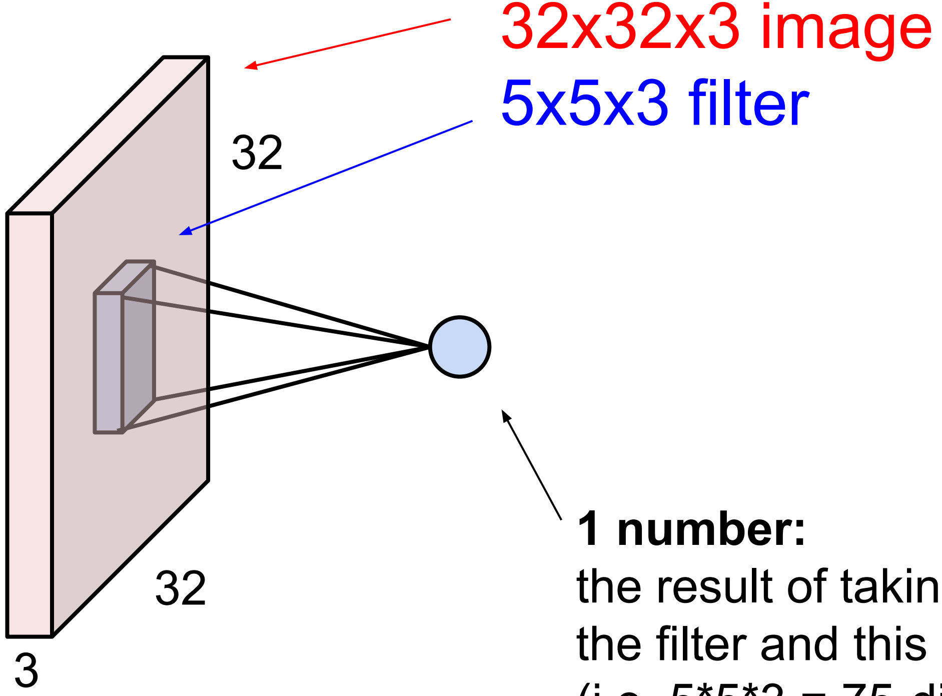
This layer creates a convolution kernel that is convolved with the layer input to produce a tensor of outputs. If `use_bias` is True, a bias vector is created and added to the outputs. Finally, if `activation` is not `None`, it is applied to the outputs as well.

When using this layer as the first layer in a model, provide the keyword argument `input_shape` (tuple of integers, does not include the batch axis), e.g. `input_shape=(128, 128, 3)` for 128x128 RGB pictures in `data_format="channels_last"`.

Arguments

- **filters**: Integer, the dimensionality of the output space (i.e. the number of output filters in the convolution).
- **kernel_size**: An integer or tuple/list of 2 integers, specifying the height and width of the 2D convolution window. Can be a single integer to specify the same value for all spatial dimensions.
- **strides**: An integer or tuple/list of 2 integers, specifying the strides of the convolution along the height and width. Can be a single integer to specify the same value for all spatial dimensions. Specifying any stride value != 1 is incompatible with specifying any `dilation_rate` value != 1.
- **padding**: one of `"valid"` or `"same"` (case-insensitive). Note that `"same"` is slightly inconsistent across backends with `strides` != 1, as described here
- **data_format**: A string, one of `"channels_last"` or `"channels_first"`. The ordering of the dimensions in the inputs. `"channels_last"` corresponds to inputs with shape `(batch, height, width, channels)` while `"channels_first"` corresponds to inputs with shape `(batch, channels, height, width)`. It defaults to the `image_data_format` value found in your Keras config file at `~/.keras/keras.json`. If you never set it, then it will be "channels_last".

The brain/neuron view of CONV Layer

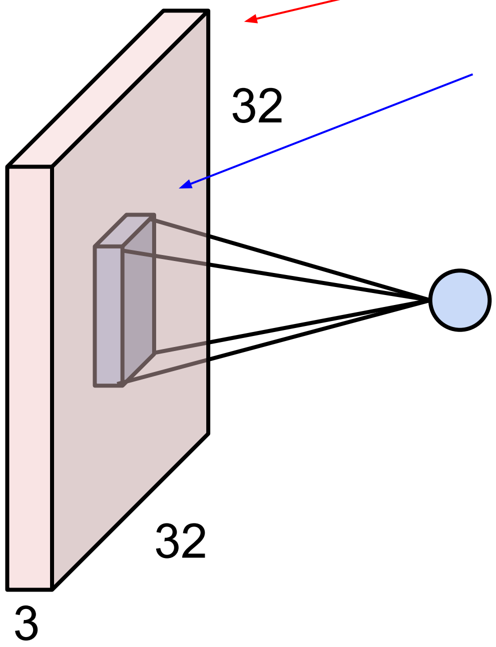


1 number:
the result of taking a dot product between the filter and this part of the image
(i.e. 5*5*3 = 75-dimensional dot product)

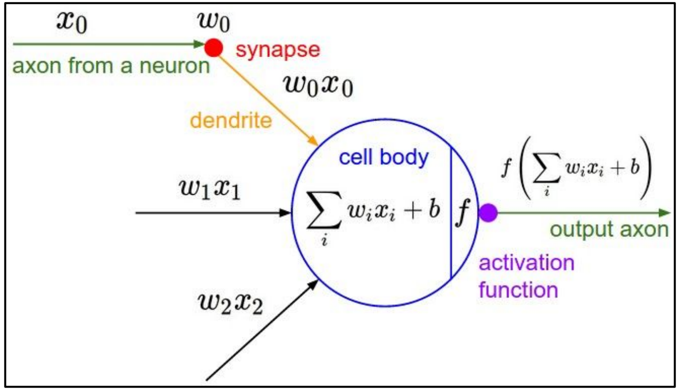
The brain/neuron view of CONV Layer

32x32x3 image

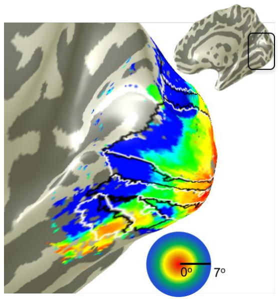
5x5x3 filter



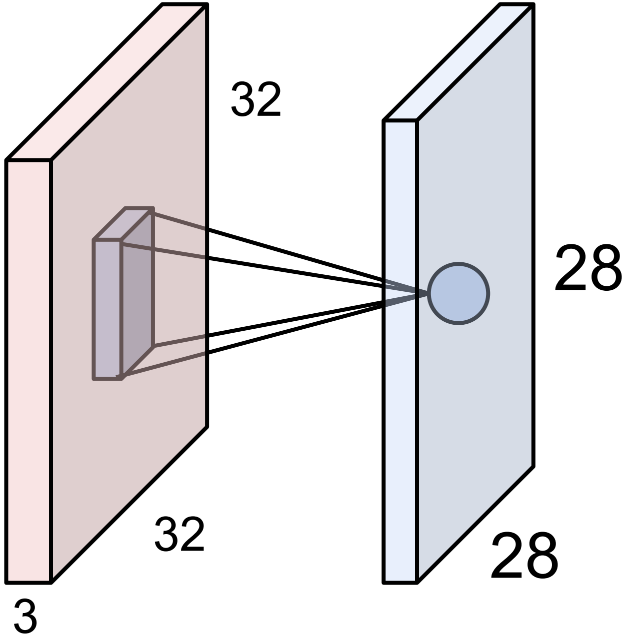
1 number:
the result of taking a dot product between the filter and this part of the image
(i.e. 5*5*3 = 75-dimensional dot product)

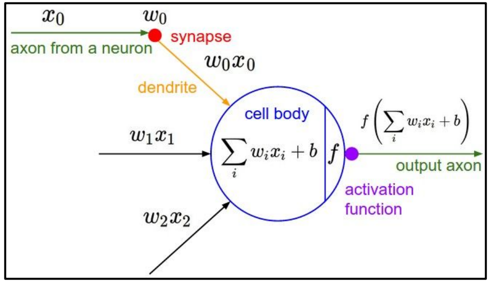


It's just a neuron with local connectivity...



Receptive field



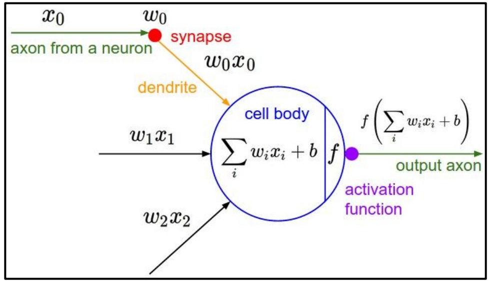


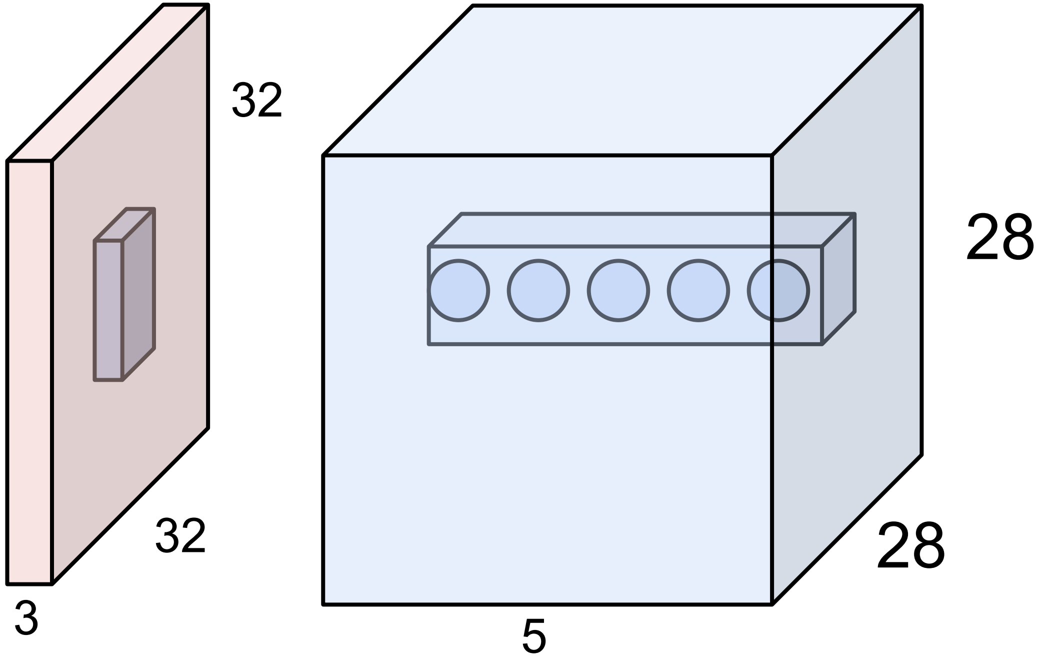
An activation map is a 28x28 sheet of neuron outputs:

1. Each is connected to a small region in the input
2. All of them share parameters

“5x5 filter” -> “5x5 receptive field for each neuron”

The brain/neuron view of CONV Layer





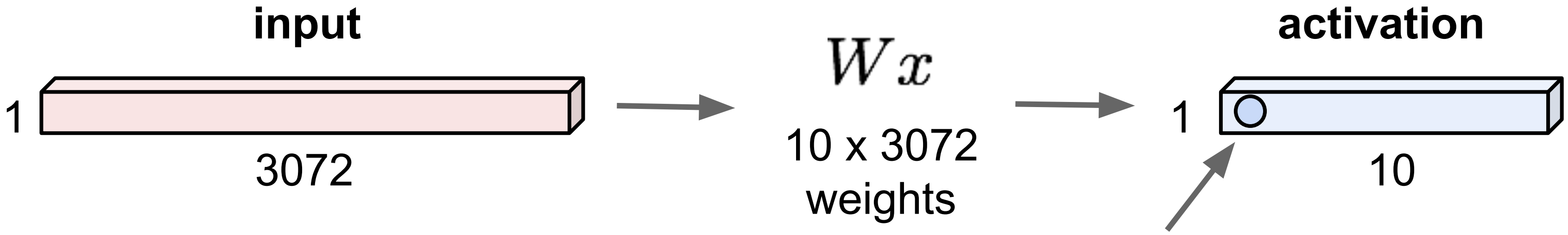
E.g. with 5 filters,
CONV layer consists of neurons arranged in a 3D grid (28x28x5)

There will be 5 different neurons all looking at the same region in the input volume

Reminder: Fully Connected Layer

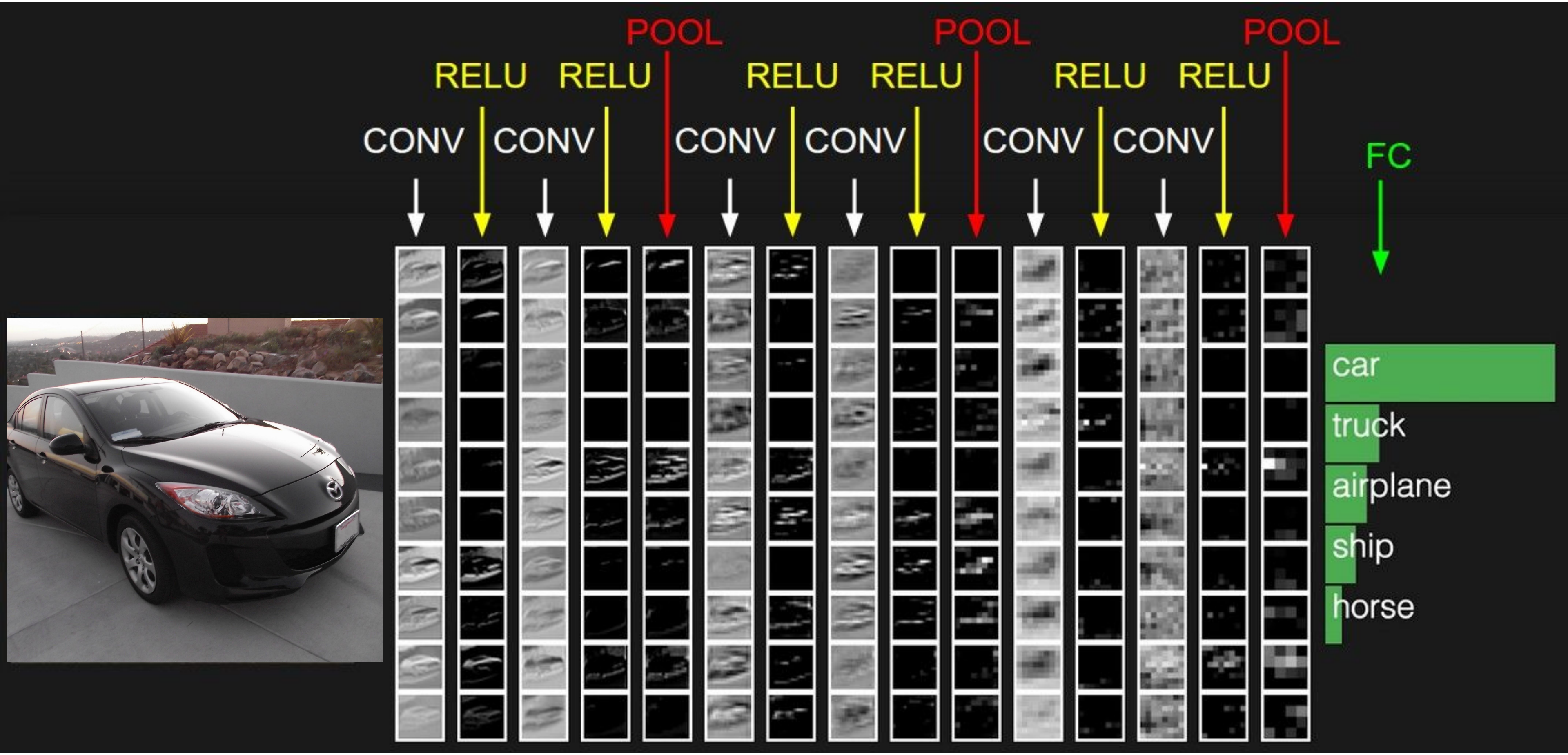
32x32x3 image -> stretch to 3072 x 1

Each neuron looks at the full input volume



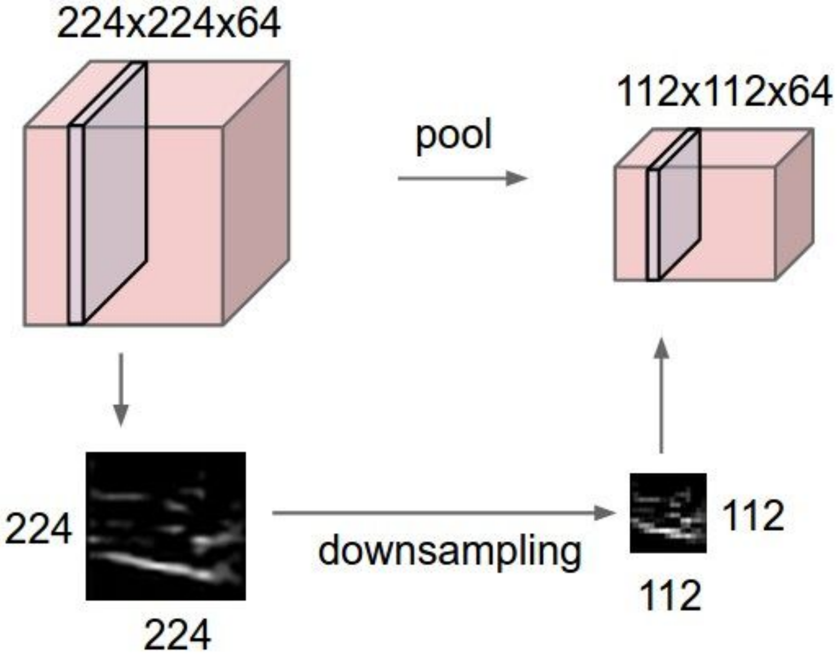
1 number:
the result of taking a dot product between a row of W and the input (a 3072-dimensional dot product)

FOUR layers in total: CONV/ReLU/POOL/FC

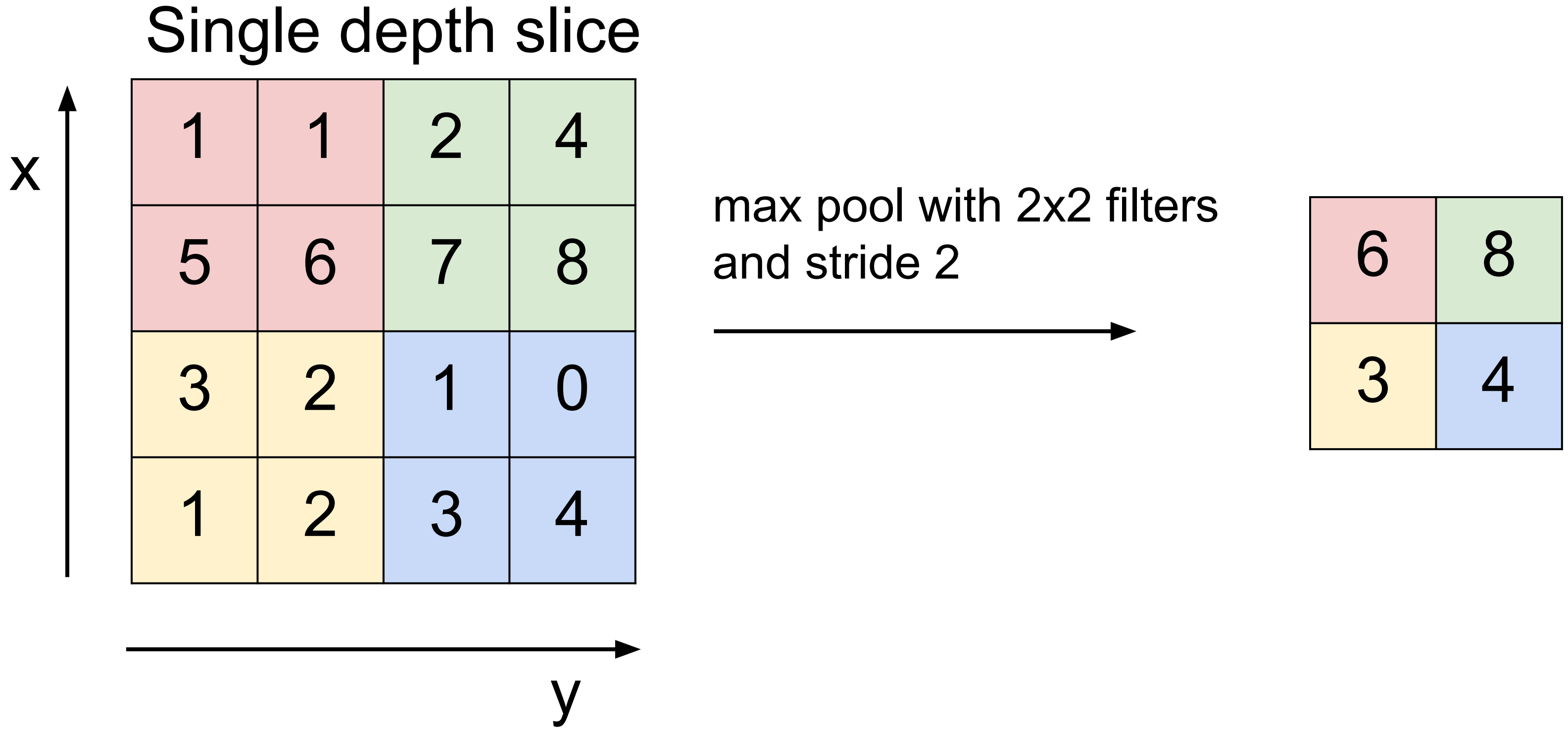


Pooling layer

- makes the representations smaller and more manageable
- operates over each activation map independently:



MAX POOLING



Single depth slice
x
1 1 2 4
5 6 7 8
3 2 1 0
1 2 3 4
y
max pool with 2x2 filters
and stride 2
6 8
3 4

Pooling layer: summary

Let's assume input is W_1 x H_1 x C
Conv layer needs 2 hyperparameters:

- The spatial extent **F**
- The stride **S**

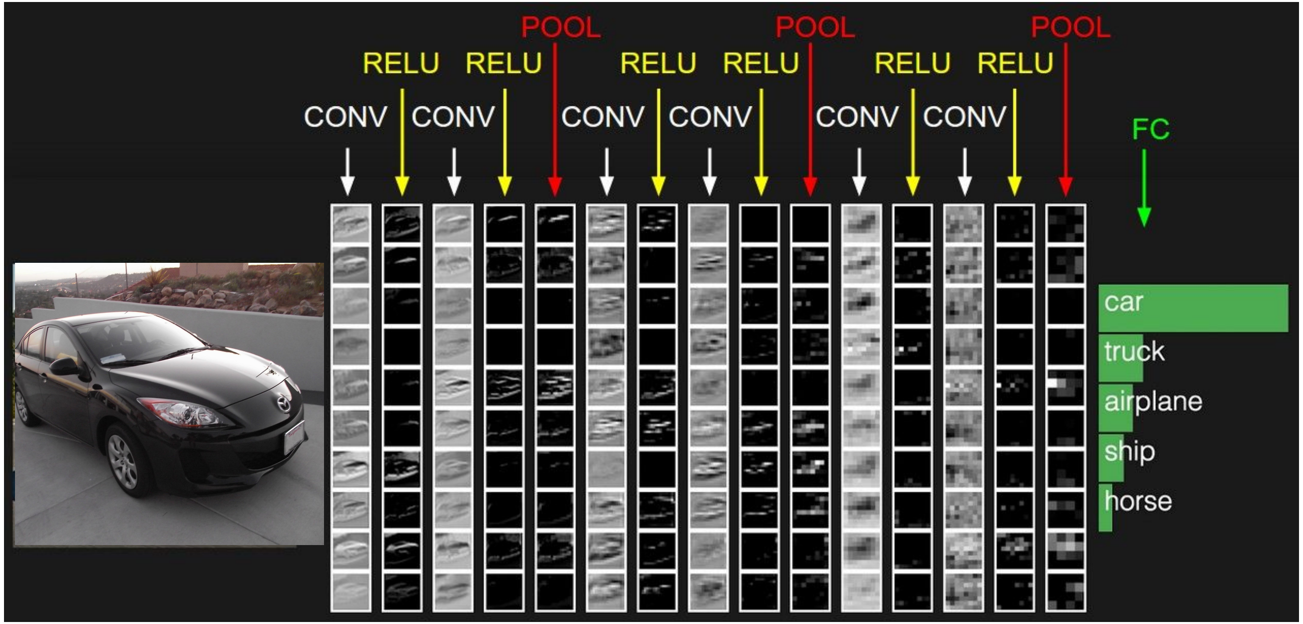
This will produce an output of W_2 x H_2 x C where:

- $W_2 = (W_1 - F)/S + 1$
- $H_2 = (H_1 - F)/S + 1$

Number of parameters: 0

Fully Connected Layer (FC layer)

- Contains neurons that connect to the entire input volume, as in ordinary Neural Networks



Summary

- ConvNets stack CONV,POOL,FC layers
- Trend towards smaller filters and deeper architectures
- Trend towards getting rid of POOL/FC layers (just CONV)
- Between 2012-2016 architectures looked like

  [(CONV-RELU)*N-POOL?]*M-(FC-RELU)*K,SOFTMAX

  where N is usually up to ~5, M is large, 0 <= K <= 2.

  - but recent advances such as ResNet/GoogLeNet have challenged this paradigm

A bit of history...

The **Mark I Perceptron** machine was the first implementation of the perceptron algorithm.

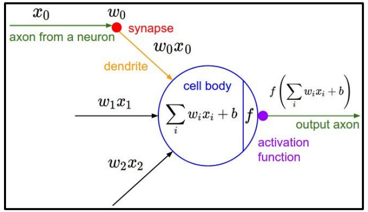
The machine was connected to a camera that used 20×20 cadmium sulfide photocells to produce a 400-pixel image.

recognized
letters of the alphabet

$$f(x) = \begin{cases} 1 & \text{if } w \cdot x + b > 0 \\ 0 & \text{otherwise} \end{cases}$$

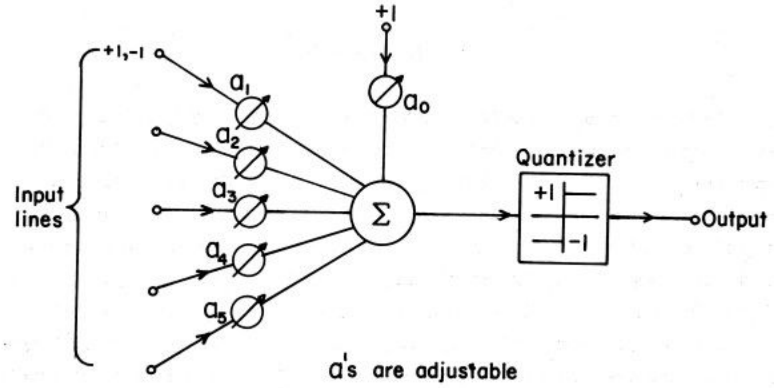
update rule:

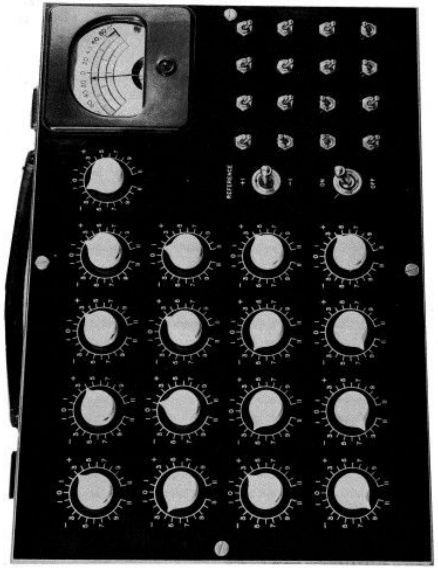
$$w_i(t+1) = w_i(t) + \alpha(d_j - y_j(t))x_{j,i},$$

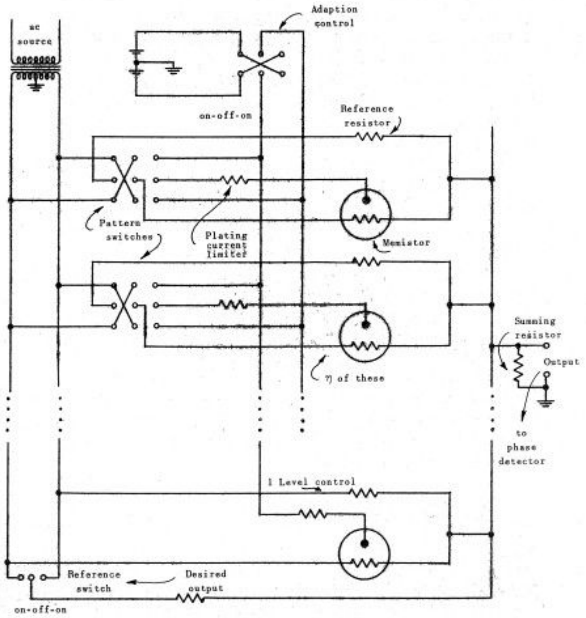


Frank Rosenblatt, ~1957: Perceptron

A bit of history...





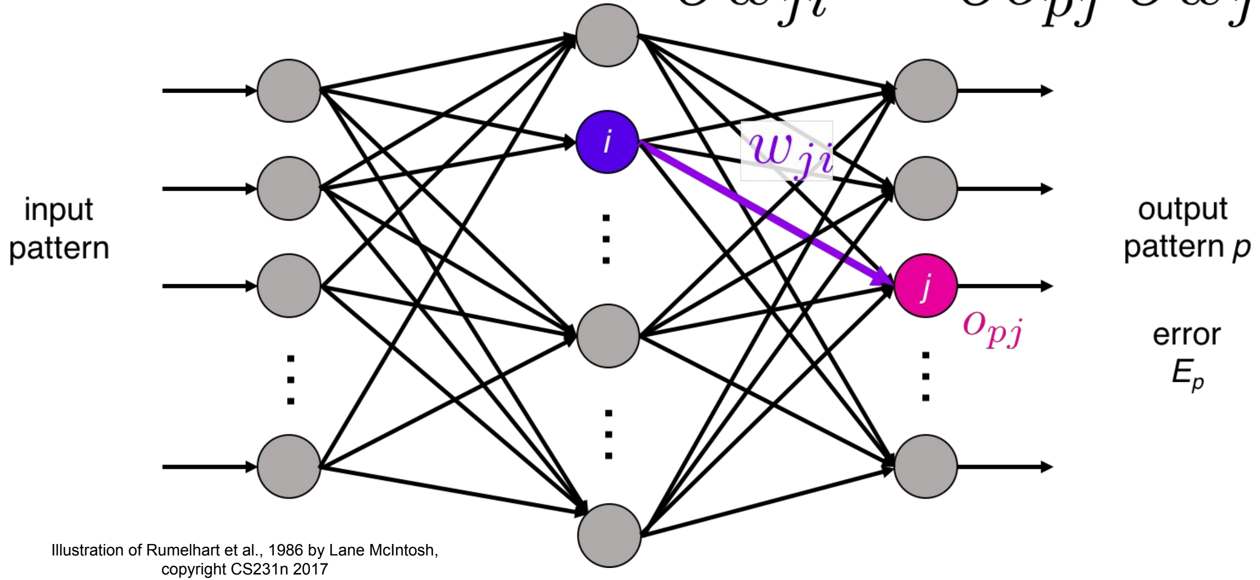


Widrow and Hoff, ~1960: Adaline/Madaline

These figures are reproduced from [Widrow 1960, Stanford Electronics Laboratories Technical Report] with permission from [Stanford University Special Collections].

A bit of history...

$$\frac{\partial E_p}{\partial w_{ji}} = \frac{\partial E_p}{\partial o_{pj}} \frac{\partial o_{pj}}{\partial w_{ji}}$$



recognizable math

Illustration of Rumelhart et al., 1986 by Lane McIntosh, copyright CS231n 2017

Rumelhart et al., 1986: First time back-propagation became popular

A bit of history...

[Hinton and Salakhutdinov 2006]

Reinvigorated research in Deep Learning

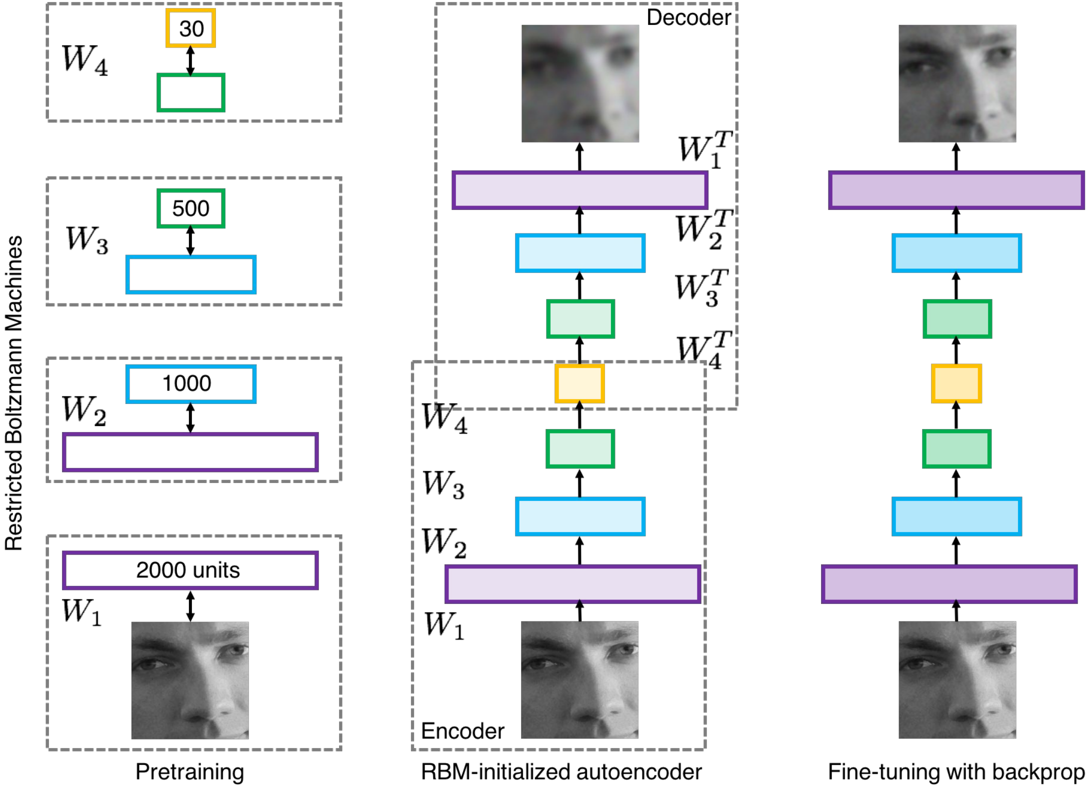


Illustration of Hinton and Salakhutdinov 2006 by Lane McIntosh, copyright CS231n 2017

First strong results

Acoustic Modeling using Deep Belief Networks
Abdel-rahman Mohamed, George Dahl, Geoffrey Hinton, 2010
Context-Dependent Pre-trained Deep Neural Networks for Large Vocabulary Speech Recognition
George Dahl, Dong Yu, Li Deng, Alex Acero, 2012

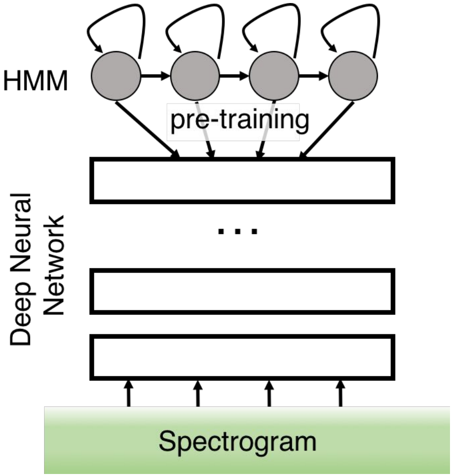
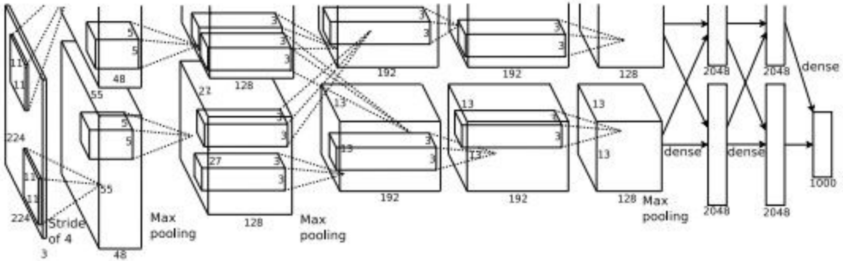
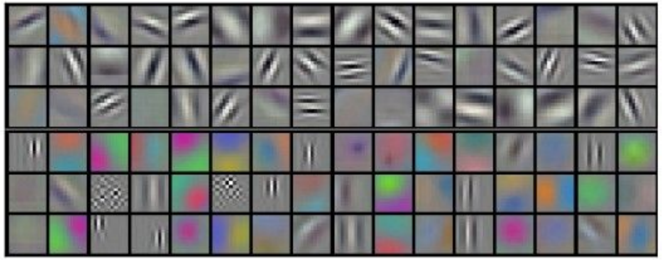


Illustration of Dahl et al. 2012 by Lane McIntosh, copyright CS231n 2017

Imagenet classification with deep convolutional neural networks
Alex Krizhevsky, Ilya Sutskever, Geoffrey E Hinton, 2012





Figures copyright Alex Krizhevsky, Ilya Sutskever, and Geoffrey Hinton, 2012. Reproduced with permission.

A bit of history:

Hubel & Wiesel,

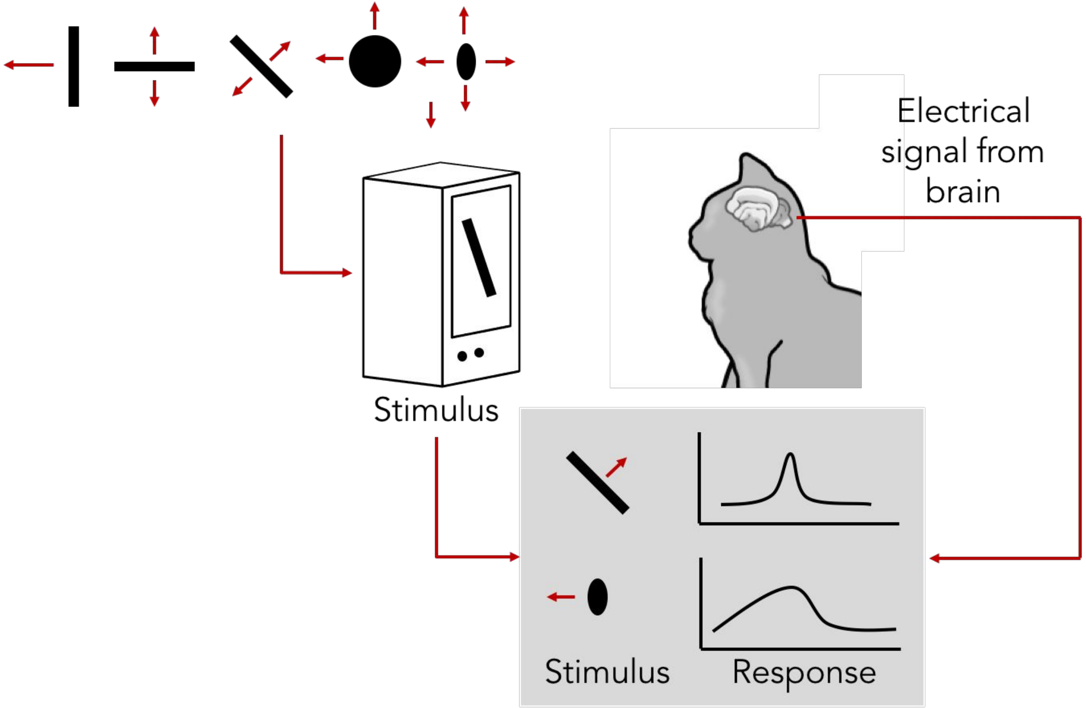
1959

RECEPTIVE FIELDS OF SINGLE NEURONES IN
THE CAT'S STRIATE CORTEX

1962

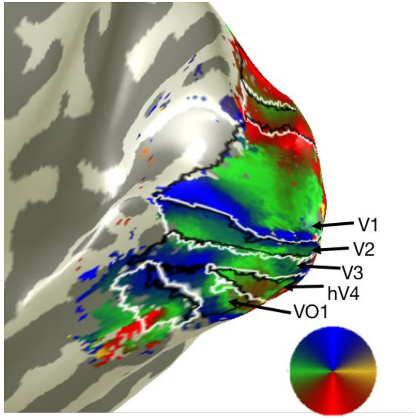
RECEPTIVE FIELDS, BINOCULAR INTERACTION
AND FUNCTIONAL ARCHITECTURE IN
THE CAT'S VISUAL CORTEX

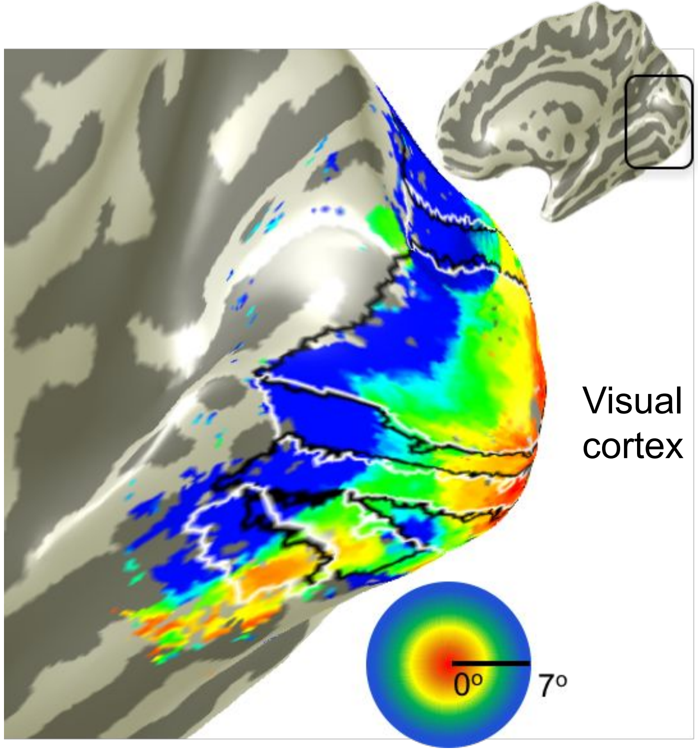
1968...



A bit of history

Topographical mapping in the cortex:
nearby cells in cortex represent
nearby regions in the visual field





Retinotopy images courtesy of Jesse Gomez in the Stanford Vision & Perception Neuroscience Lab.

Hierarchical organization

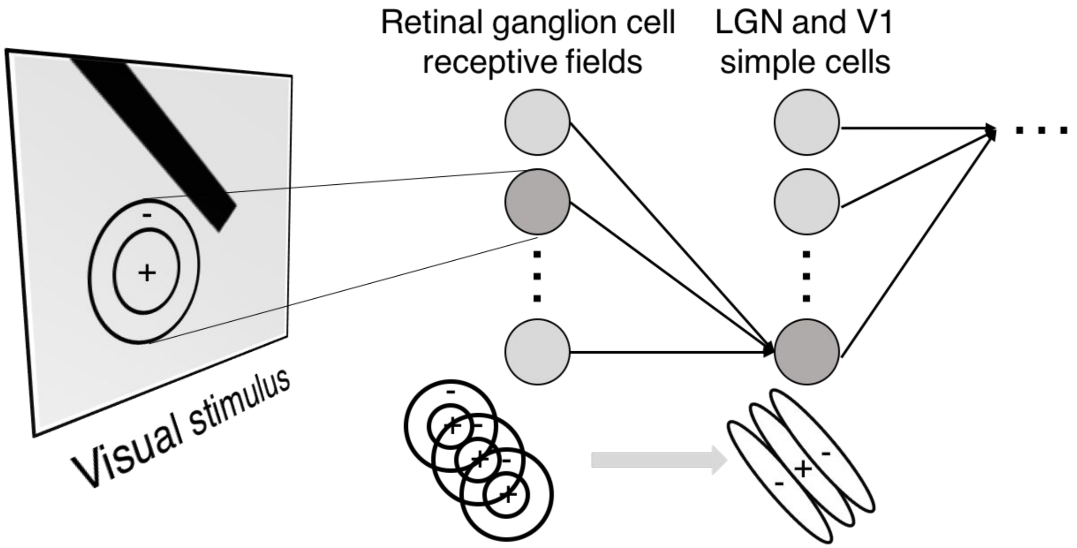
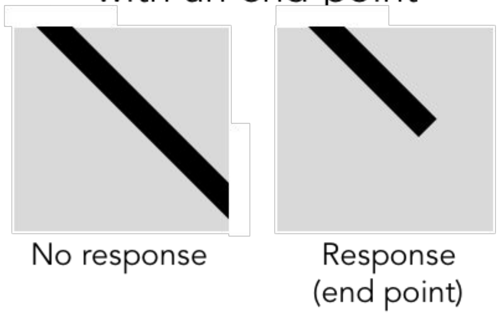


Illustration of hierarchical organization in early visual pathways by Lane McIntosh, copyright CS231n 2017

Simple cells: Response to light orientation

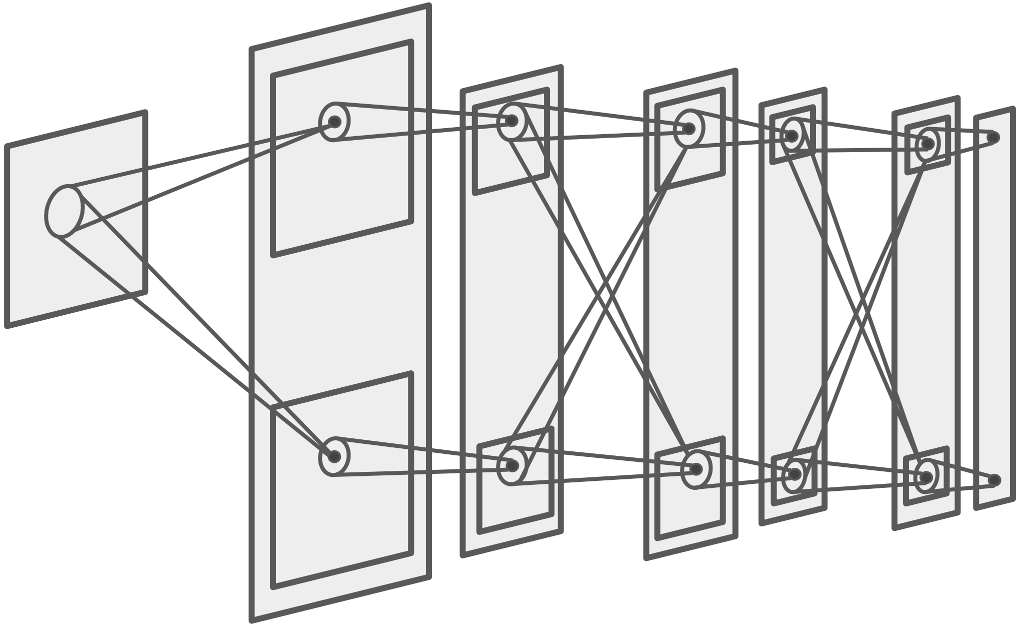
Complex cells: Response to light orientation and movement

Hypercomplex cells: response to movement with an end point



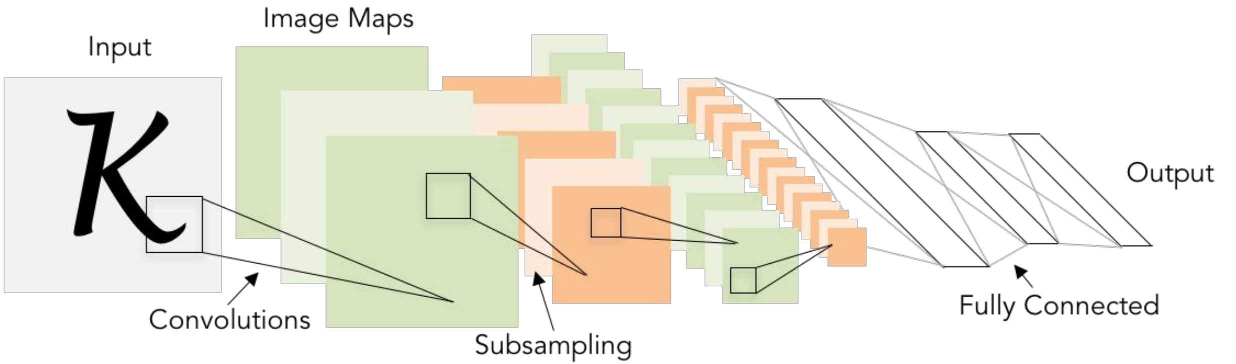
A bit of history:

Neocognitron
[Fukushima 1980]



"sandwich" architecture (SCSCSC…)
simple cells: modifiable parameters
complex cells: perform pooling

A bit of history:
Gradient-based learning applied to document recognition
[LeCun, Bottou, Bengio, Haffner 1998]

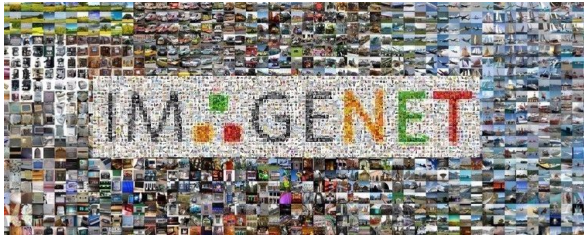


LeNet-5

A bit of history:

ImageNet Classification with Deep Convolutional Neural Networks

[Krizhevsky, Sutskever, Hinton, 2012]



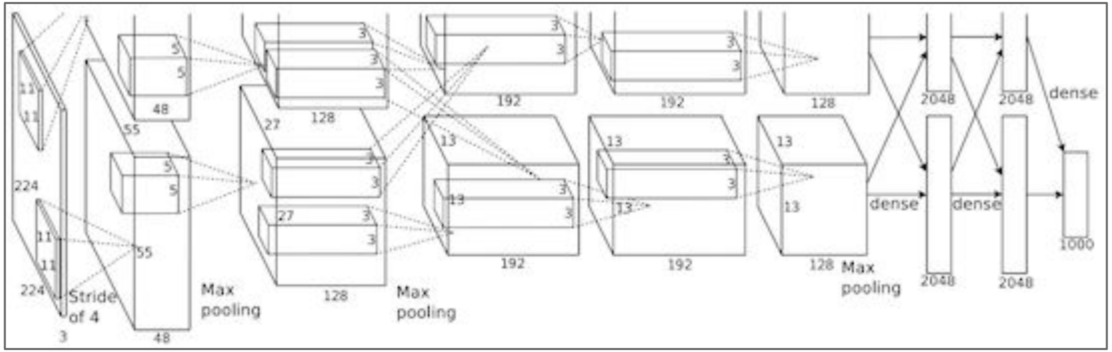
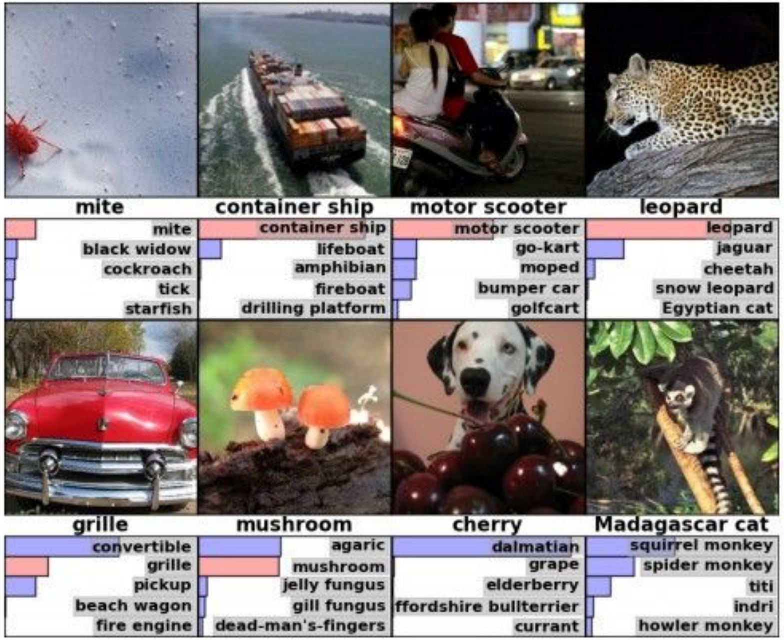


Figure copyright Alex Krizhevsky, Ilya Sutskever, and Geoffrey Hinton, 2012. Reproduced with permission.

“AlexNet”

Fast-forward to today: ConvNets are everywhere

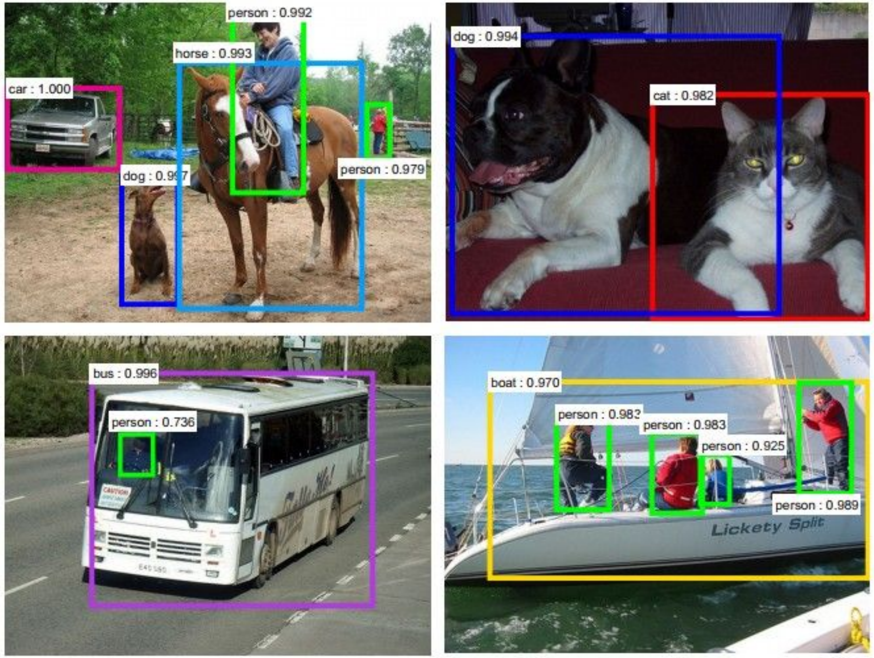
Classification



Retrieval

Fast-forward to today: ConvNets are everywhere

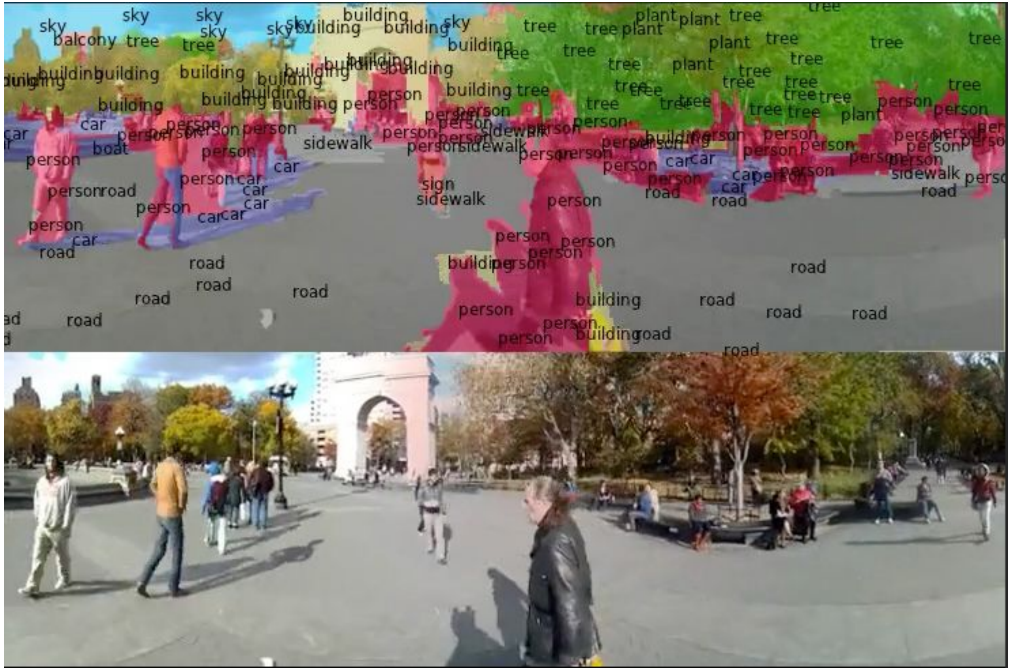
Detection



Figures copyright Shaoqing Ren, Kaiming He, Ross Girschick, Jian Sun, 2015. Reproduced with permission.

[Faster R-CNN: Ren, He, Girshick, Sun 2015]

Segmentation



Figures copyright Clement Farabet, 2012.
Reproduced with permission.

[Farabet et al., 2012]

Fast-forward to today: ConvNets are everywhere

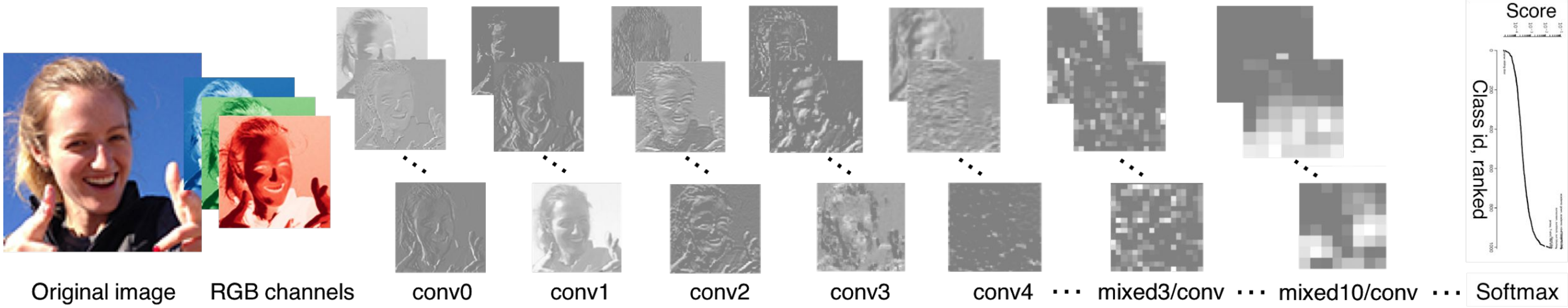
Photo by Lane McIntosh. Copyright CS231n 2017.

self-driving cars

NVIDIA Tesla line

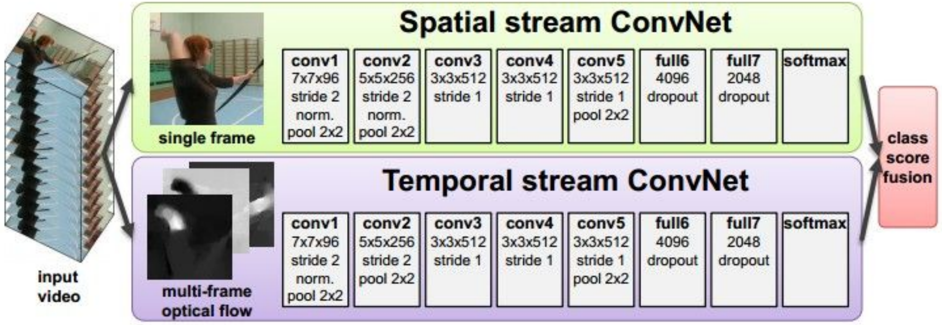
Note that for embedded systems a typical setup would involve NVIDIA Tegras, with integrated GPU and ARM-based CPU cores.

Fast-forward to today: ConvNets are everywhere



[Taigman et al. 2014]

Activations of [inception-v3 architecture] [Szegedy et al. 2015] to image of Emma McIntosh, used with permission. Figure and architecture not from Taigman et al. 2014.



[Simonyan et al. 2014]

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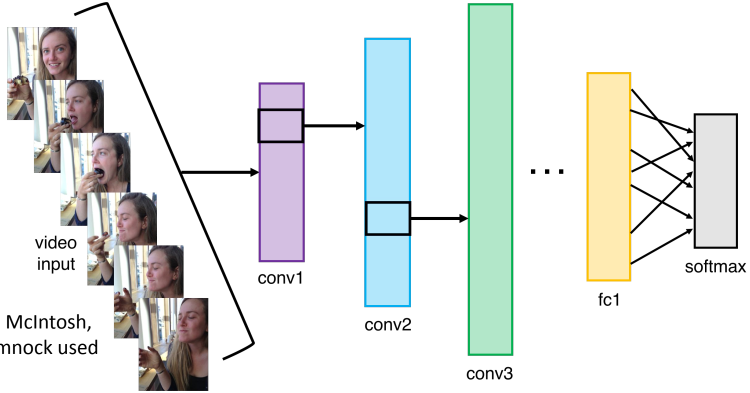
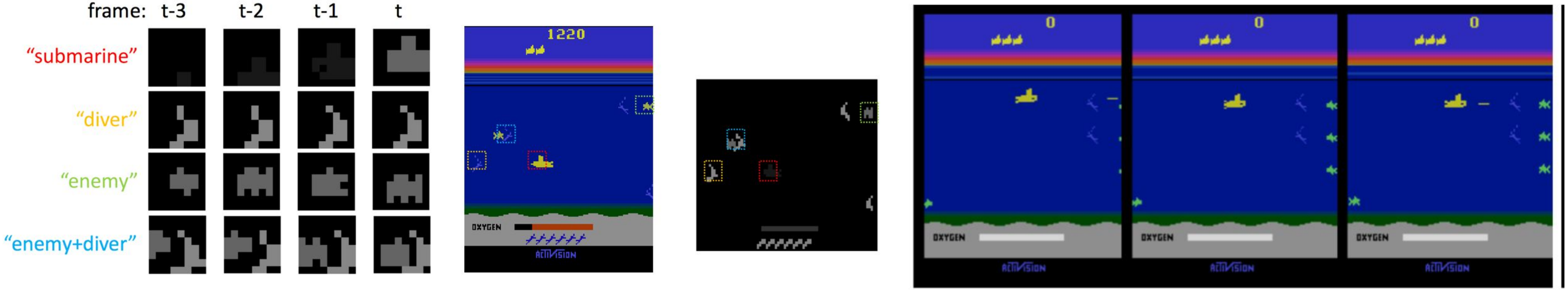


Illustration by Lane McIntosh, photos of Katie Cumnock used with permission.

Fast-forward to today: ConvNets are everywhere

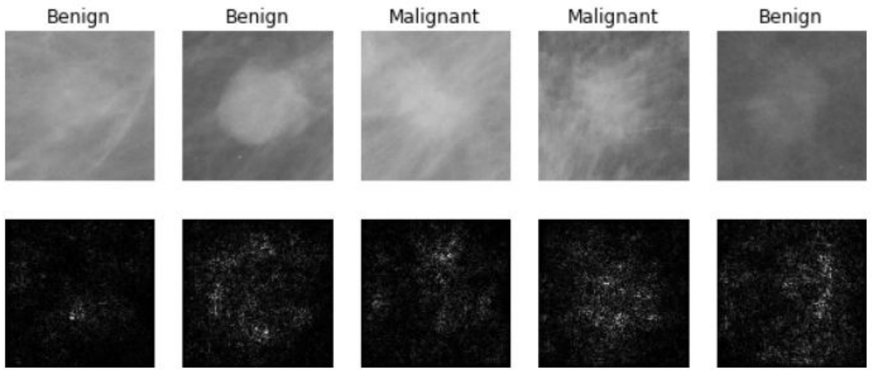
Images are examples of pose estimation, not actually from Toshev & Szegedy 2014. Copyright Lane McIntosh.

[Toshev, Szegedy 2014]



[Guo et al. 2014]

Fast-forward to today: ConvNets are everywhere



[Levy et al. 2016]

[Dieleman et al. 2014]

[Sermanet et al. 2011]
[Ciresan et al.]

This image by Christin Khan is in the public domain and originally came from the U.S. NOAA.

Whale recognition, Kaggle Challenge

Photo and figure by Lane McIntosh; not actual example from Mnih and Hinton, 2010 paper.

Mnih and Hinton, 2010

Image Captioning

[Vinyals et al., 2015]
[Karpathy and Fei-Fei, 2015]

No errors

A white teddy bear sitting in the grass

Minor errors

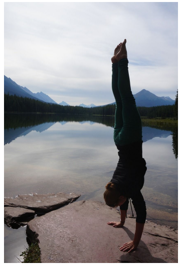
A man in a baseball uniform throwing a ball

Somewhat related

A woman is holding a cat in her hand

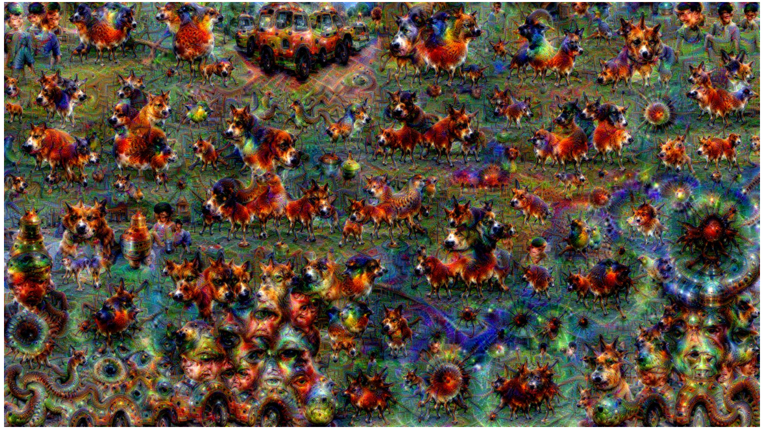
A man riding a wave on top of a surfboard

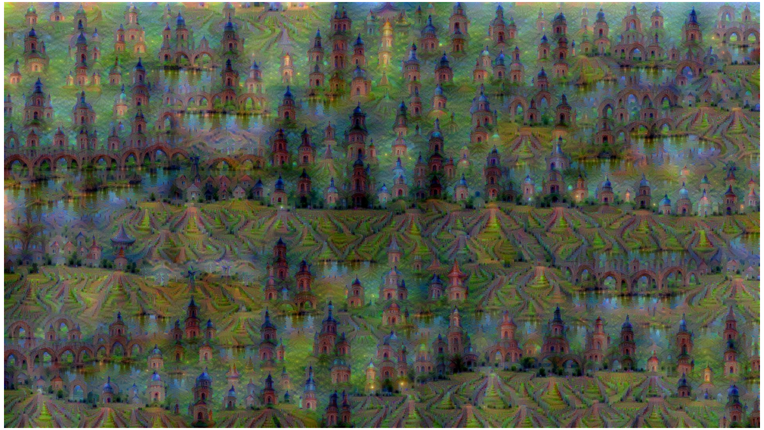
A cat sitting on a suitcase on the floor



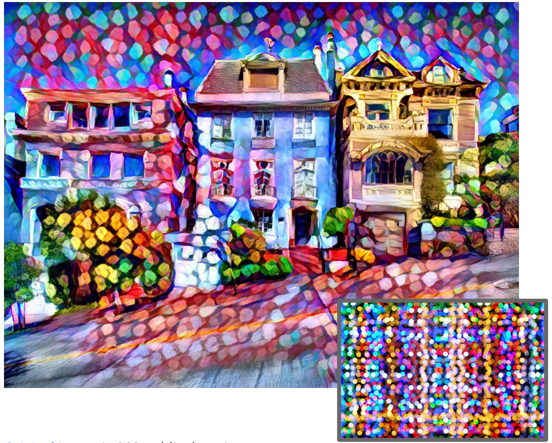
A woman standing on a beach holding a surfboard

Captions generated by Justin Johnson using Neuraltalk2





Figures copyright Justin Johnson, 2015. Reproduced with permission. Generated using the Inceptionism approach from a blog post by Google Research.



Original image is CC0 public domain
Starry Night and Tree Roots by Van Gogh are in the public domain
Bokeh image is in the public domain
Stylized images copyright Justin Johnson, 2017;
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Gatys et al, “Image Style Transfer using Convolutional Neural Networks”, CVPR 2016
Gatys et al, “Controlling Perceptual Factors in Neural Style Transfer”, CVPR 2017