Incentives in Computer Science

One-sided matching
TTCA
Kidney exchange

Participation

- Please do it!!!!!!!
- Use the chat feature to either write a question or in the chat box, type “hand” and I will call on you soon thereafter or just shout out!

- Also, I’d love it if you kept your video on so I can see you….

Today and especially Monday

- Covers some of the major results that resulted in the awarding of the 2012 Nobel Prize in economics to Lloyd Shapley and Al Roth

- *"The Prize concerns a central economic problem: how to match different agents as well as possible. For example, students have to be matched with schools, and donors of human organs with patients in need of a transplant. How can such matching be accomplished as efficiently as possible? What methods are beneficial to what groups? The prize rewards two scholars who answered these questions on a journey from abstract theory on stable allocations to practical design of market institutions."*

A basic definition

MECHANISM

An algorithm whose inputs come from agents with a strategic interest in the output. Each agent's input is their own private information.

Takes as input the reported preferences/data for a set of agents and produces as output an outcome, decision or action.

Examples:
auctions
voting
school choice

TODAY: MECHANISMS WITHOUT MONEY

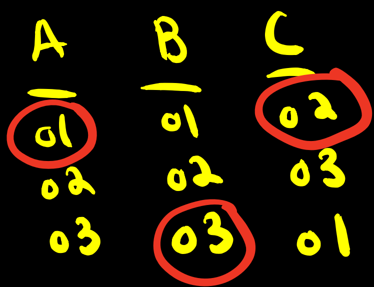
One-sided matching problems

Office Allocation

- n people, n offices; each person has private preference order over all offices.
- Mechanism for allocating offices to people?

Algorithm 1

- People report preferences to algorithm.
- Algorithm visit students in alphabetical order and matches them to their first choice if it’s available.
- Then, for all unmatched students, the algorithm visits them in alphabetical order and matches them to their second choice if available.
- And so on until everyone matched.



Pareto Optimality

- An outcome is Pareto optimal if you cannot make anyone better off without also making someone else worse off.

Lemma: Algorithm 1 is Pareto optimal

- People report preferences to algorithm.
- Algorithm visit students in alphabetical order and matches them to their first choice if it's available.
- For all unmatched students, the algorithm visits them in alphabetical order and matches them to their second choice if available.
- And so on until everyone matched.

→ M

Proof

M' any other matching

S_j : set of people who get their j^{th} choice

i : lowest index s.t. some person $p \in S_i$ is strictly happier in M'

p is matched to office allocated to, say p', in M, in round S_ℓ $1 \le \ell \le i-1$ or visited earlier in round i

p' is worse off.

∎

Is it truthful?

- That is, is it in each agents to report their preferences truthfully?

Not truthful.

A	B	C
o1	o1	o1
o2	o2	o2
o3	o3	o3

C
o2
o1
o3

Truthful mechanisms

- A mechanism is truthful or strategyproof or dominant strategy incentive-compatible (DSIC) if honesty is always the best policy.
- That is, no matter what other agents do, lying about your preferences cannot make you better off.

Algorithm 2: Serial dictatorship

- Pick an arbitrary ordering of the students. alphabetical.
- Visit the students in this order and let them pick their favorite available office that has not yet been picked.

A	B	C
o1	o1	o2
o2	o2	o3
o3	o3	o1

- Pareto optimal?
- Truthful?

Lemma: Serial Dictatorship is Pareto optimal

Pf Let M' be any other allocation.

Consider first person who gets different alloc in M than in M'

- Pick an arbitrary ordering of the students.
- Visit the students in this order and let them pick their favorite available office that has not yet been picked.

→ M

Lemma: Serial Dictatorship is truthful

Pick person p
Fix reports of everyone else.
p has no incentive to lie

- Pick an arbitrary ordering of the students.
- Visit the students in this order and let them pick their favorite available office that has not yet been picked.

Why should we care about truthfulness?

- difficult to reason about outcome.
- easier on agents.

Office allocation

- n people (agents), *each starts with an office*
- Each person has a total order over all the offices.
- How should we reallocate them to get to a better allocation?

Top Trading Cycle Algorithm (TTCA)

while agents remain (initially all)

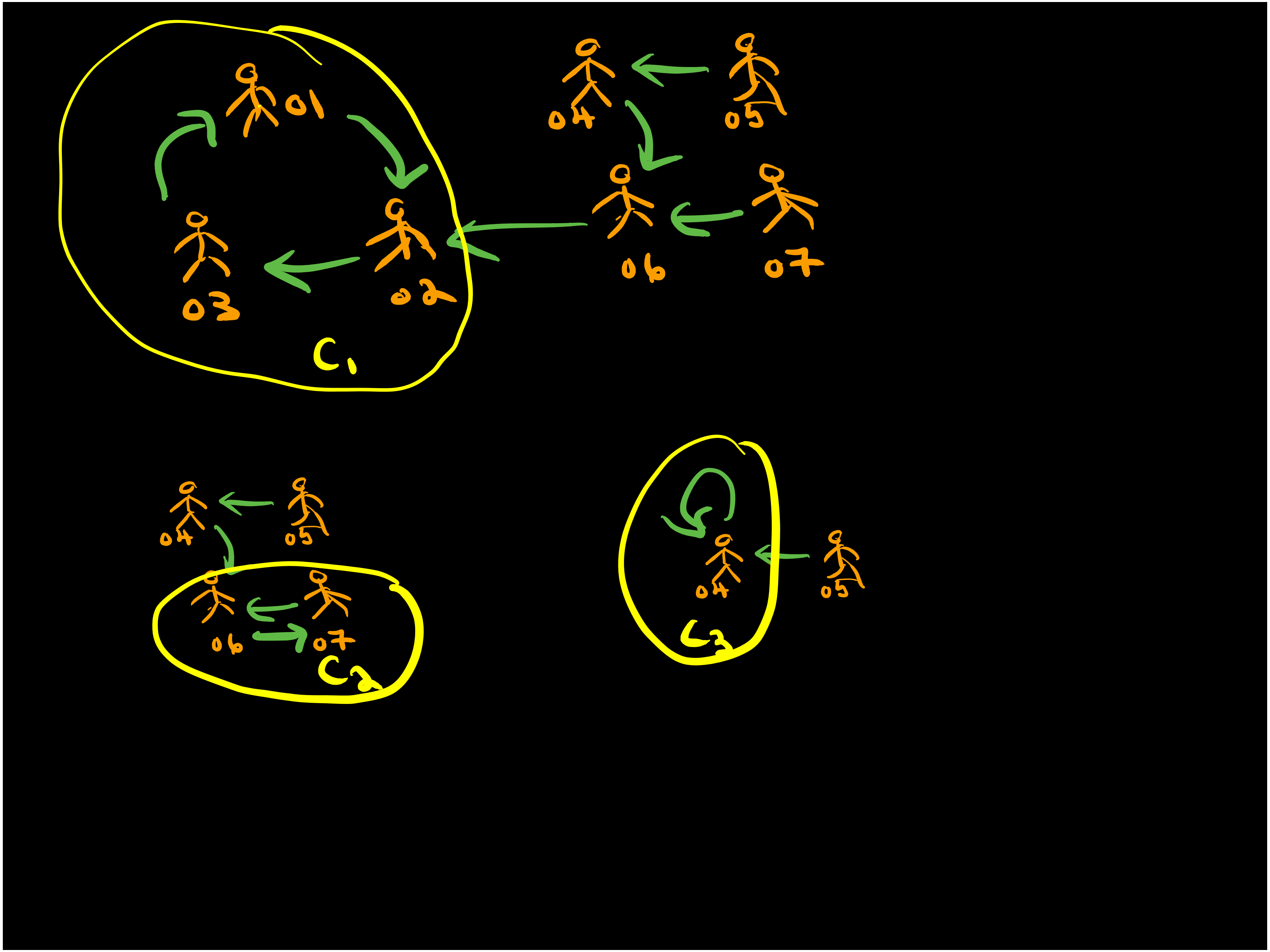
each remaining agent to point to their favorite office

<u>Claim:</u> $\exists$ always cycle in resulting directed graph.

reallocate according to that cycle

remove all those agents

repeat till no agents remain


01
02
03
04
05
06
07
C1
04
05
06
07
C2
04
05
C3

Theorem: TTCA is a truthful mechanism

Pf Fix reports of everyone but i

Suppose that if i truthful

$C_1, C_2, \ldots, C_k$

and i is allocated in cycle C_j

Claim: all the people in $C_1, \ldots, C_{j-1}$ prefer their allocation to any office in $C_j, \ldots, C_k$

This means that can not be any cycle that contains i & any agent in $C_1 \ldots C_{j-1}$

i can only get someone in $C_j \ldots C_k$ & getting his favorite by reporting truthfully

Theorem: The allocation produced by TTCA is stable

all do at least as well & at least one strictly better

- The allocation is stable if no subset of agents could have done better by not participating, but rather just reallocating amongst themselves.

Proof: by →←

- Suppose there is a subset A of agents that prefer to go off & reallocate among themselves
- Let $A' \subseteq A$ be agents in A that get a different alloc from what they would have gotten
- Let C_j be first cycle in TTCA containing $a \in A'$
- $C_1, \ldots C_{j-1}$ get the exact same allocation. so a has to be doing strictly worse ∎

Pareto Optimality

- An outcome is Pareto optimal if in any other outcome at least one agent is worse off.

- Is the outcome produced by TTCA Pareto optimal?

Kidney Exchange

Next set of slides created by Jason Hartline and Nicole Immorlica

Kidney failure

Dehydration

Diabetes

Sepsis

Without a transplant,
they will die.

Hypovolemia

High blood pressure

Rhabdomyolysis

Kidney supply

1. Cadavers

Kidney supply

Kidney Transplant Recipient Duane with wife and Donor, Anne.

2. Live donors

In 2008,

10,526 patients

received cadaver kidneys.

4,857 patients

received live donor kidneys.

Kidney demand

There are currently

93,000 people

waiting for a kidney transplant in the US.

http://optn.transplant.hrsa.gov

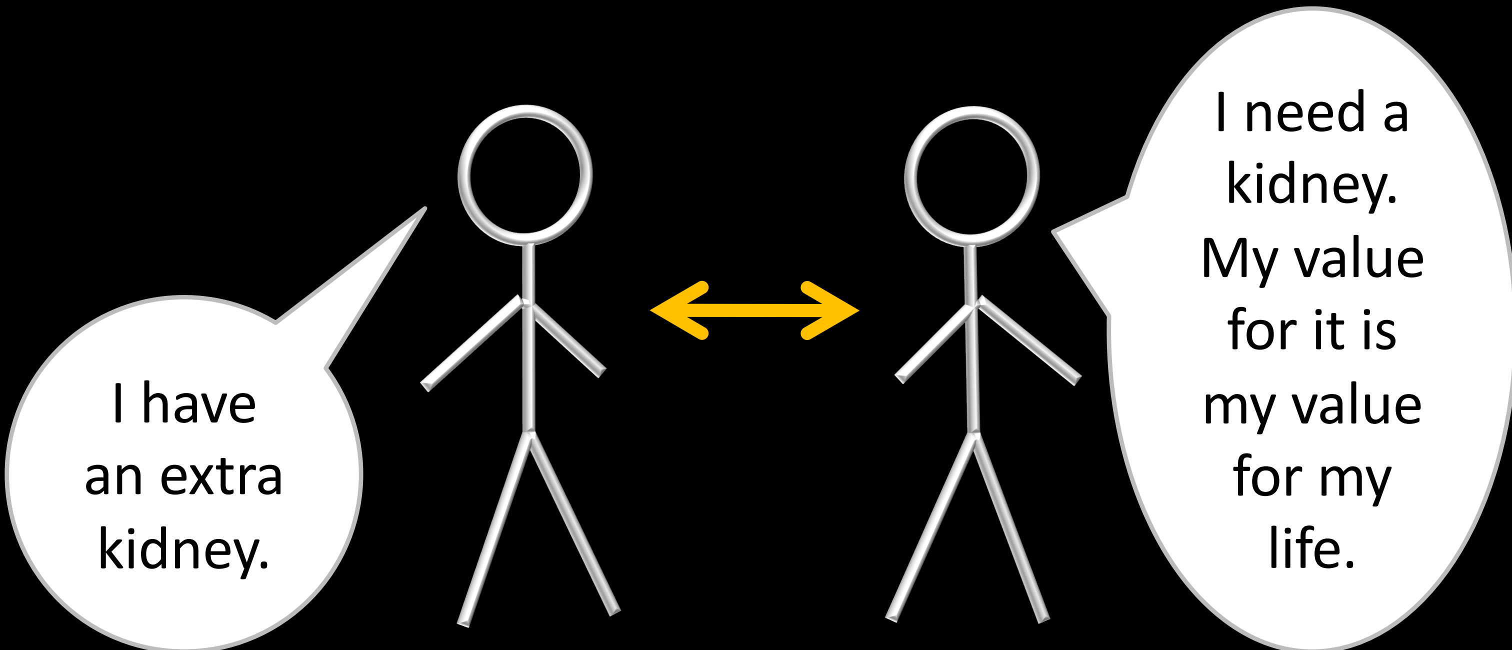
In 2014,

Over 8,000 patients died

waiting or became too sick for a transplant.

Making supply meet demand

The economic approach 101: Buying kidneys.



Repugnance

Often $x + \$$ is repugnant, even when x alone is not.

Interest on loans

Prostitution

Organ donation

"We didn't have time to pick up a bottle of wine, but this is what we would have spent."

Legality

Section 301 of the National Organ Transplant Act, “Prohibition of organ purchases” imposes criminal penalties on any person who

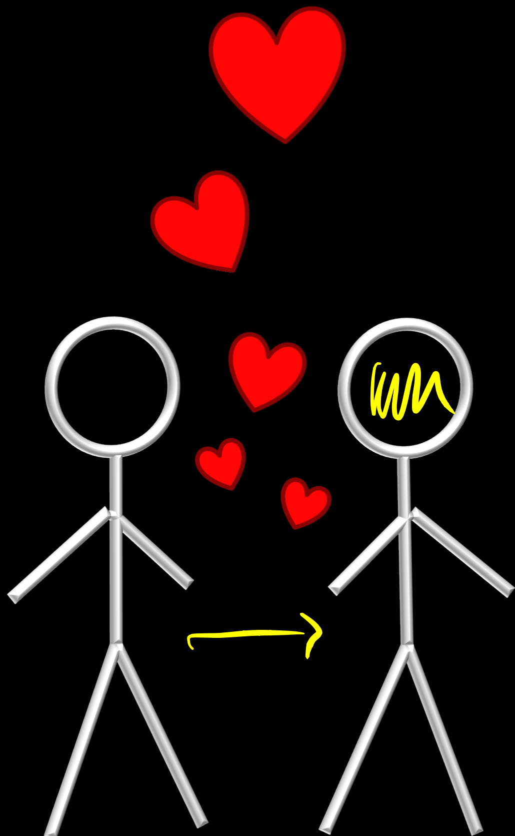
“knowingly acquire[s], receive[s], or otherwise transfer[s] any human organ for valuable consideration for use in human transplantation”

Making supply meet demand

Take two:

Kidney exchange.

Compatibility



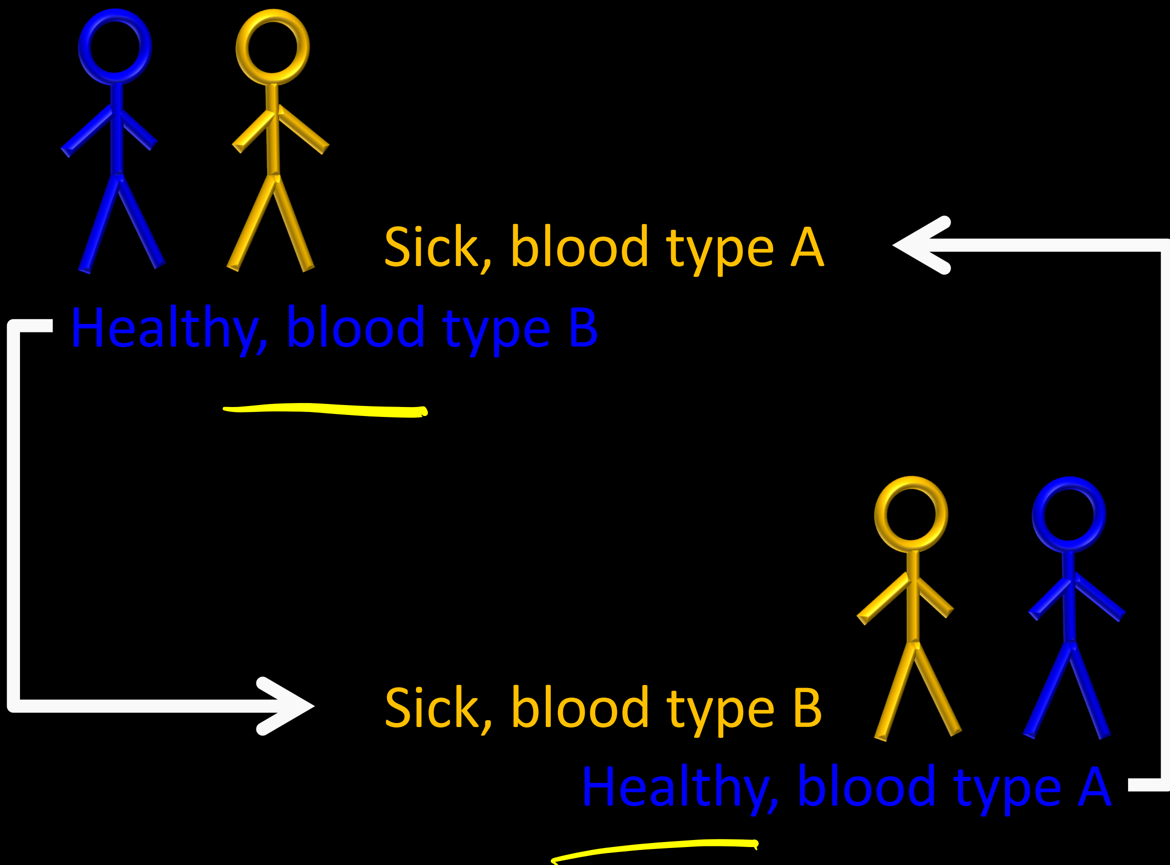
?

Blood
"O", "A", "B", "AB"

?

Tissue
(crosshatch test)

Kidney exchange



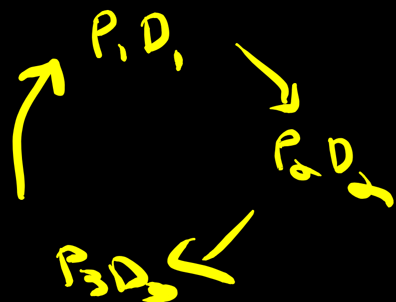
Have bunch of patient-donor pairs $P_1 - D_1, P_2 - D_2, \ldots, P_n - D_n$

Idea #1 Use TTCA

agent w/ office $\longleftrightarrow$ patient w/ donor

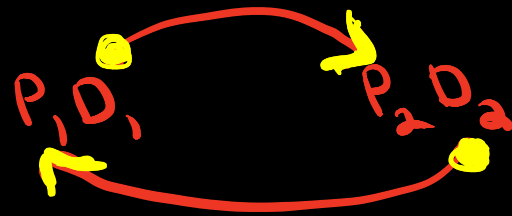
total ordering on offices $\longleftrightarrow$ each P_i having total order over donor kidney $\downarrow$ order of prob of transplant success

To run TTCA



can extend TTCA to deal patients w/o donor, donors w/o patient

Issue #1



4 surgeries
issue w/ doing sequentially.
$D_1 \to P_1$ first
D_2 can now renege.
can't legally coerce D_2 to follow thru.

$\Rightarrow$ always done simultaneously

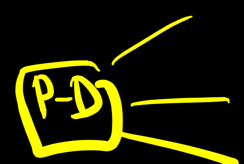
$\Rightarrow$ don't want to do long cycles

Issue #2 model is overkill.

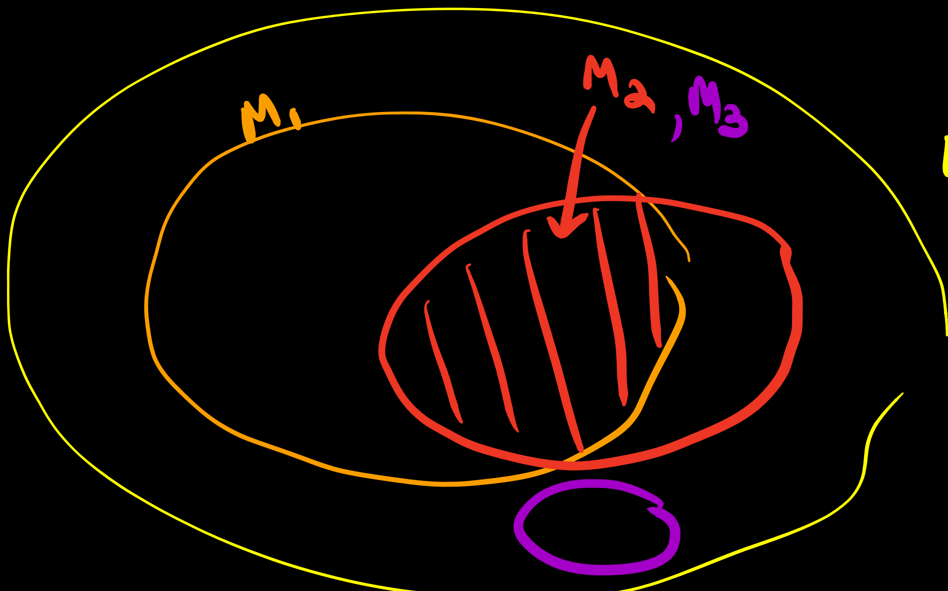
- Q: is a transplant highly likely to work or not.
- input to problem: $G = (V, E)$
  - V: set of PD pairs
  - E: PD — P'D' if both likely to work
- objective: max cardinality matching

Input reported to National Kidney exchange patients/doctors.

want to be sure incentivized to report all edges



Essential requirement: alg has to ensure that no patient can switch from matched to unmatched when they report additional edges



Matchings

Fix order
an order on patients

[redacted]